PIRAMIDA

Definiţii, notaţii

Piramide regulate

Realizarea desenelor

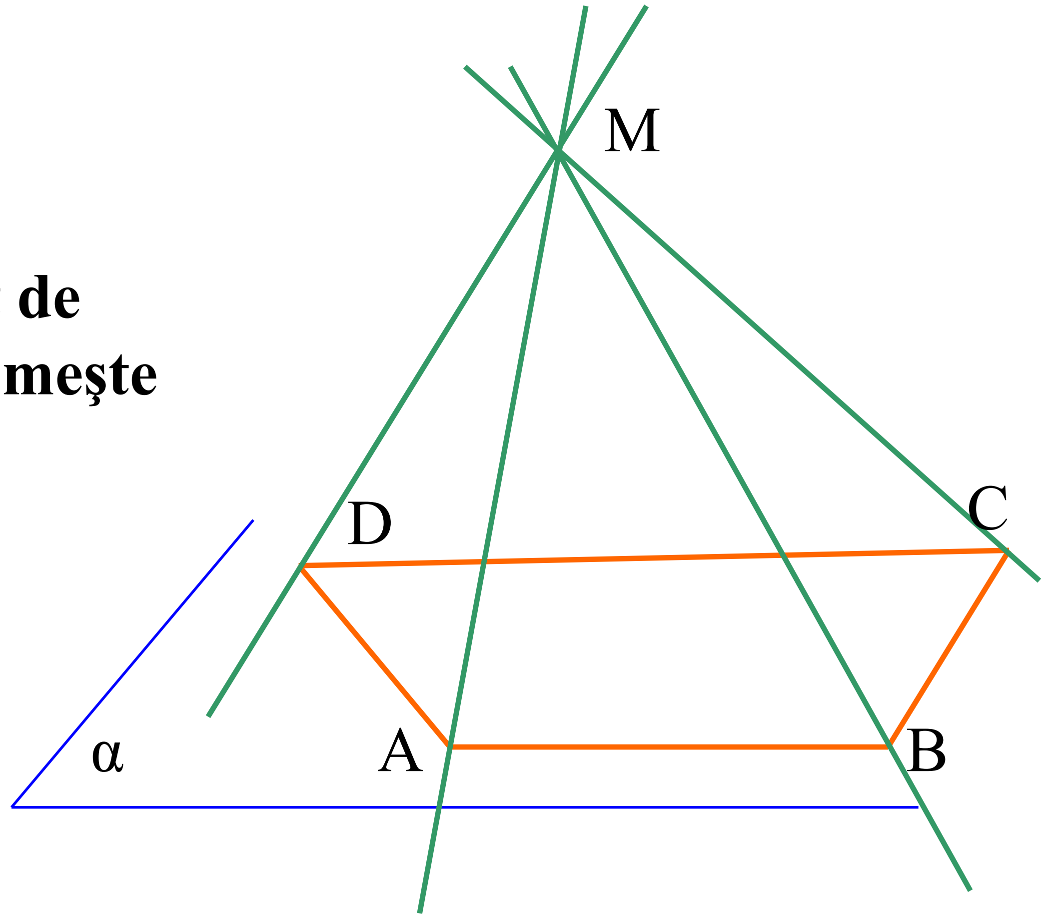
Formule de calcul

Aplicaţii

Fie ABCD un poligon în planul α, M un punct în afara planului.

Dacă prin punctul M se consideră o dreaptă d ce se deplasează pe laturile poligonului, aceasta descrie o suprafaţă de piramidă.

Corpul geometric delimitat de punctul M şi planul α se numeşte *piramida* MABCD.



Elementele piramidei

Bază

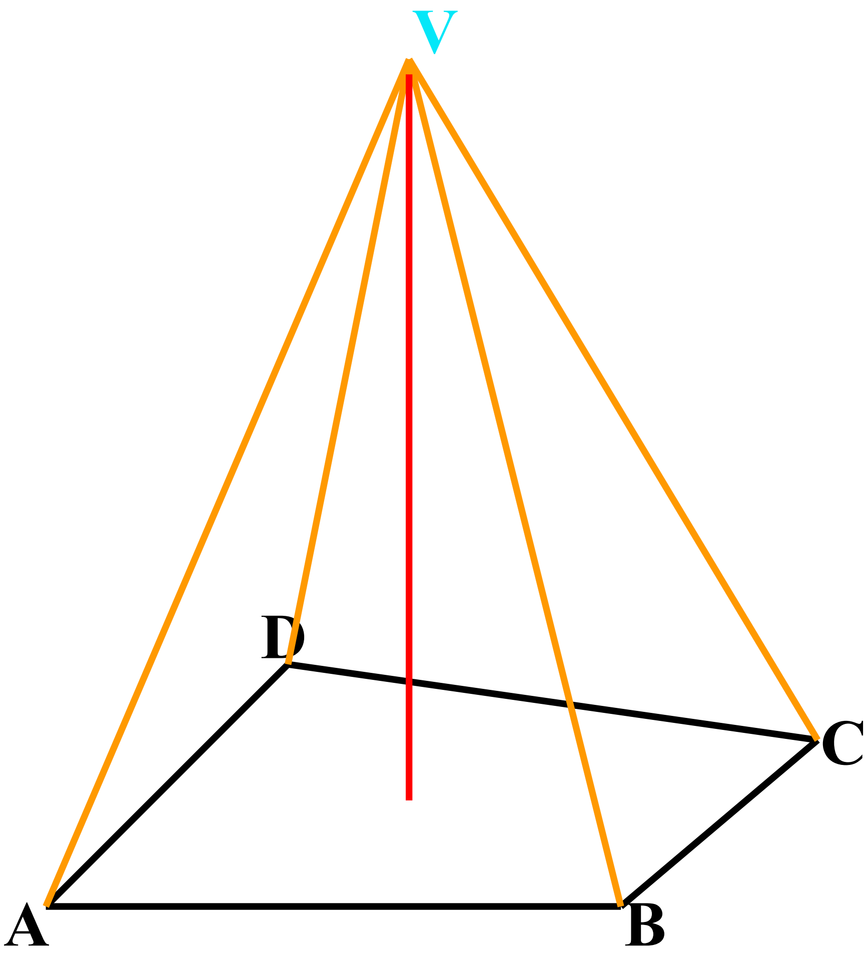
Muchii ale bazei

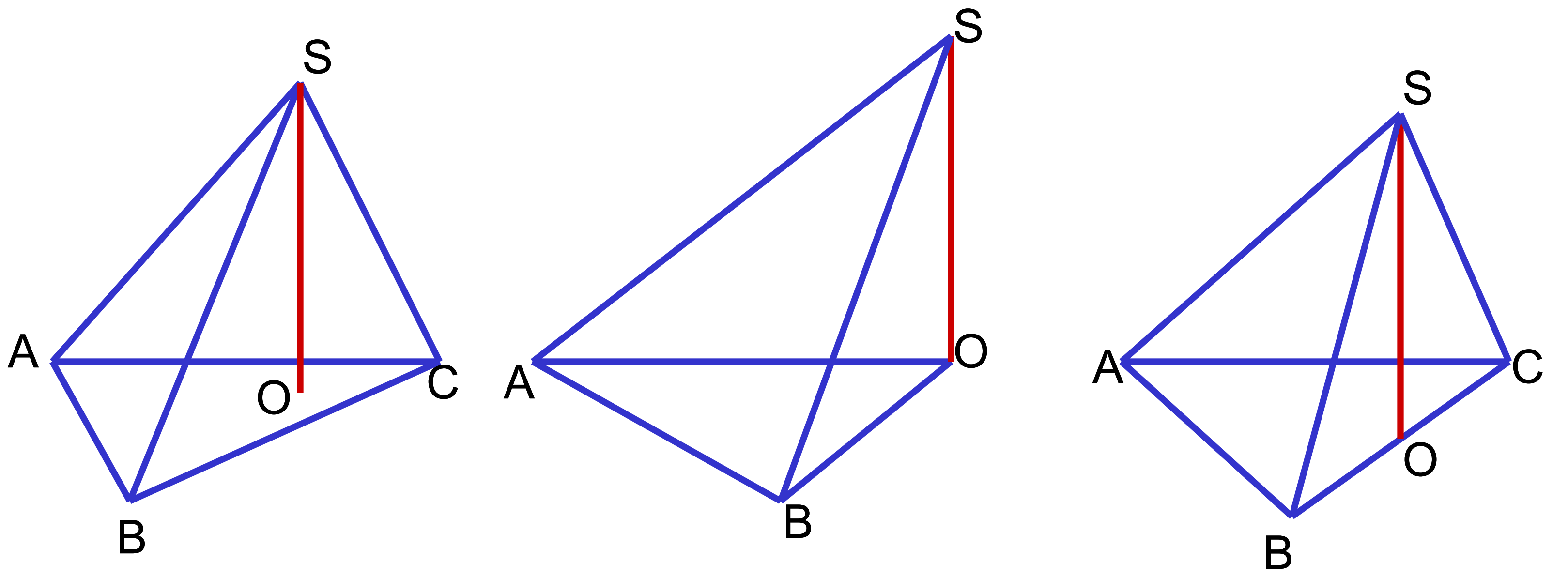
Feţe laterale

Muchii laterale

Vârf

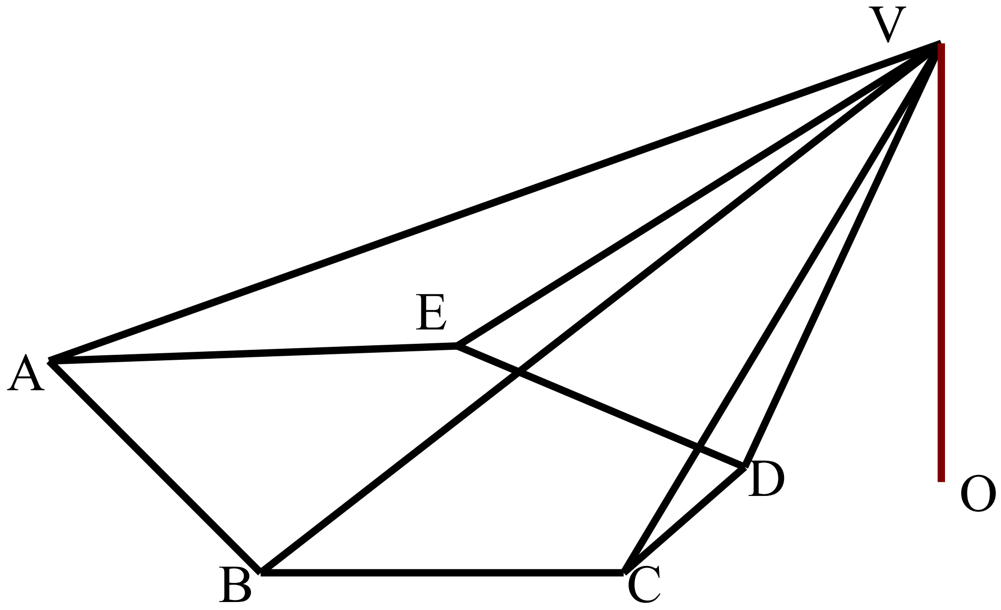
Înălţime: distanţa vârfului piramidei de la planul bazei

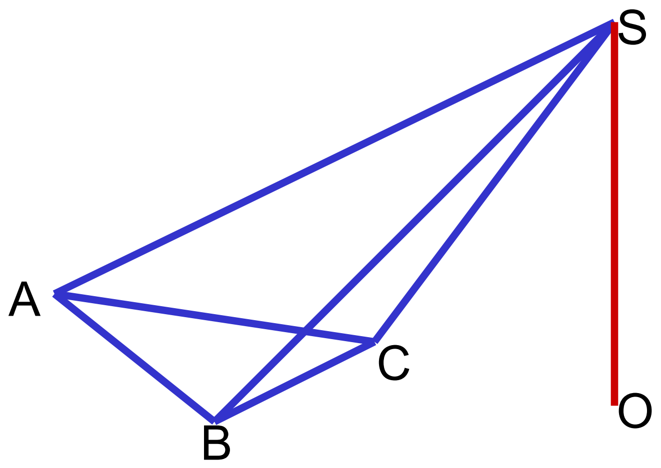




Dacă piciorul perpendicularei duse din vârful piramidei pe planul bazei este în domeniul interior al acestuia atunci piramida este o *piramidă dreaptă*.

În caz contrar *piramidă oblică.*





Piramida regulată

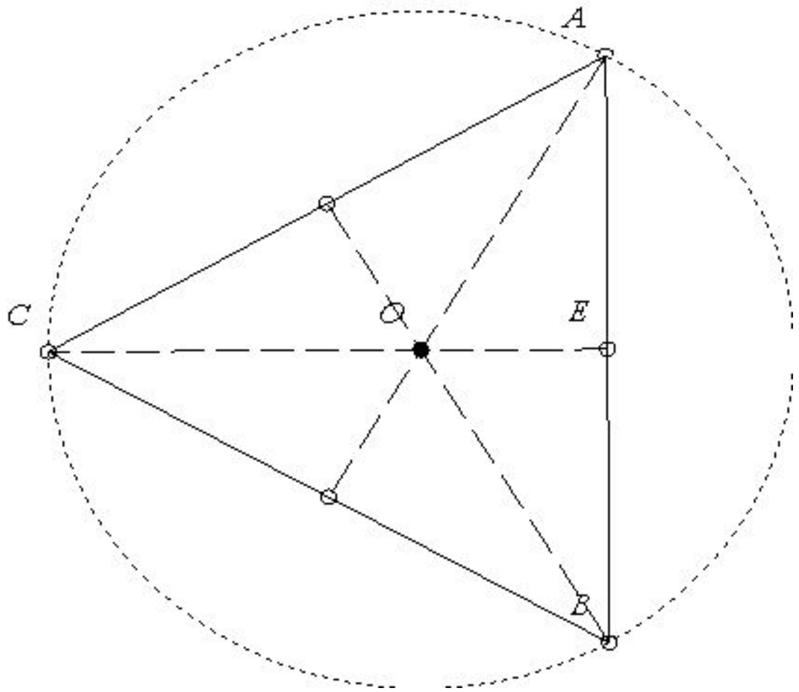
O piramidă se numeşte regulată, dacă:

- **Are ca bază un poligon regulat (Δ echilateral, pătrat, hexagon regulat, etc)**
- **Piciorul perpendicularei duse din vârful piramidei coincide cu centrul poligonului**
- **(centrul poligonului = centrul cercului circumscris).**

Toate muchiile unei piramide regulate sunt congruente, deci toate feţele sunt triunghiuri isoscele congruente.

Înălţimea unei feţe laterale se numeşte apotema piramidei.

Triunghiuri de lucru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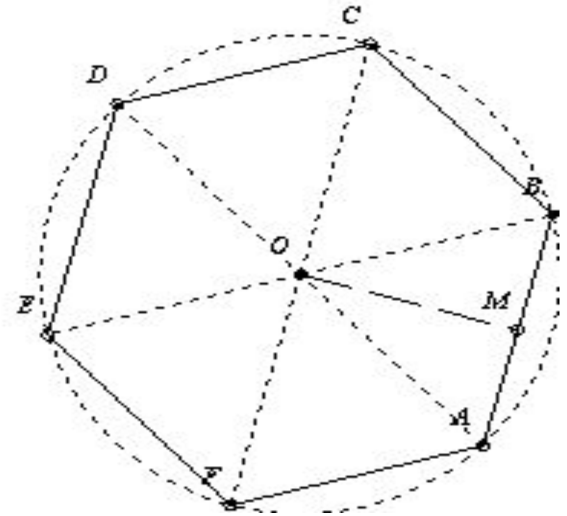
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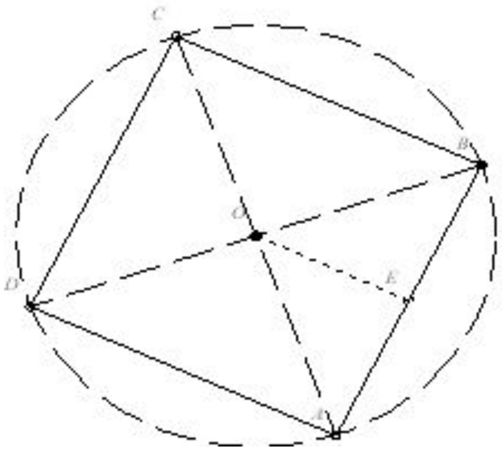
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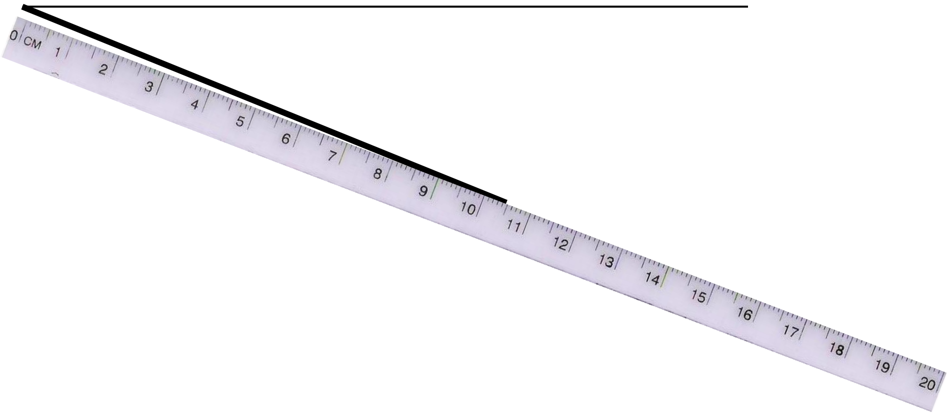
Realizarea desenelor

Piramidă triunghiulară regulată

Piramidă patrulateră regulată

Piramidă hexagonală regulată

Piramidă triunghiulară regulată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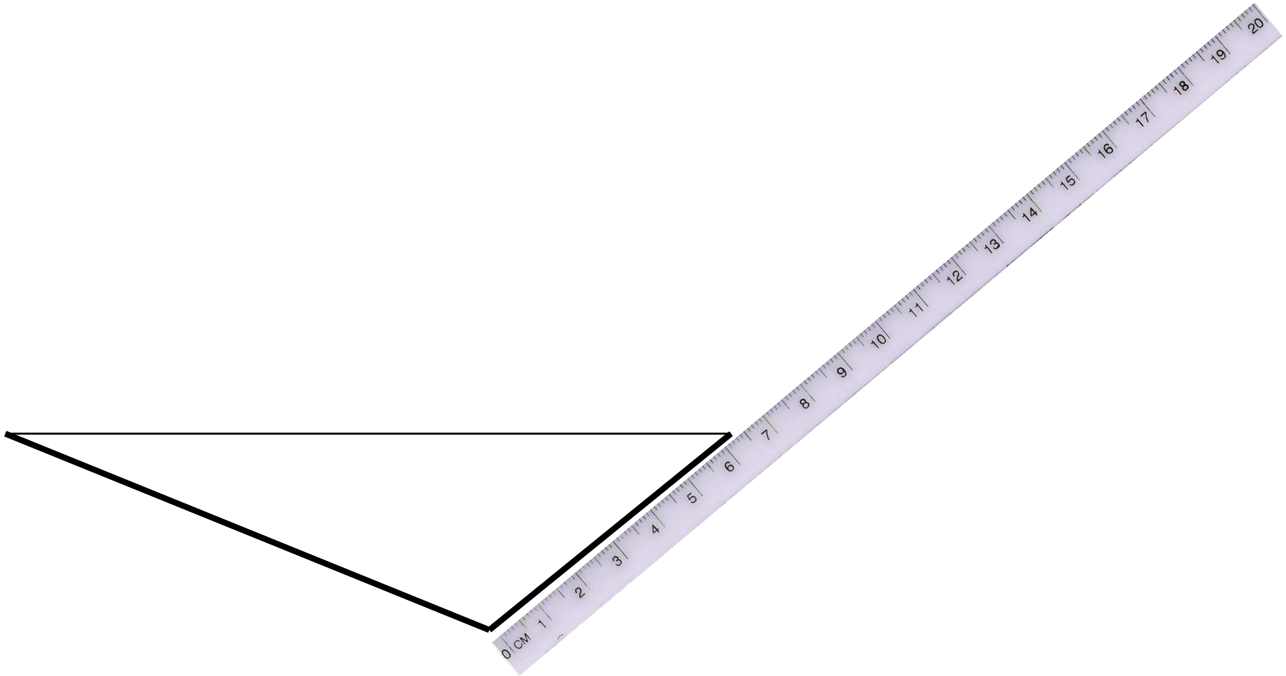
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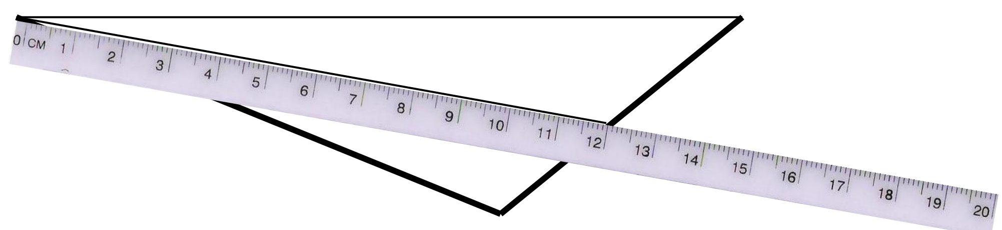
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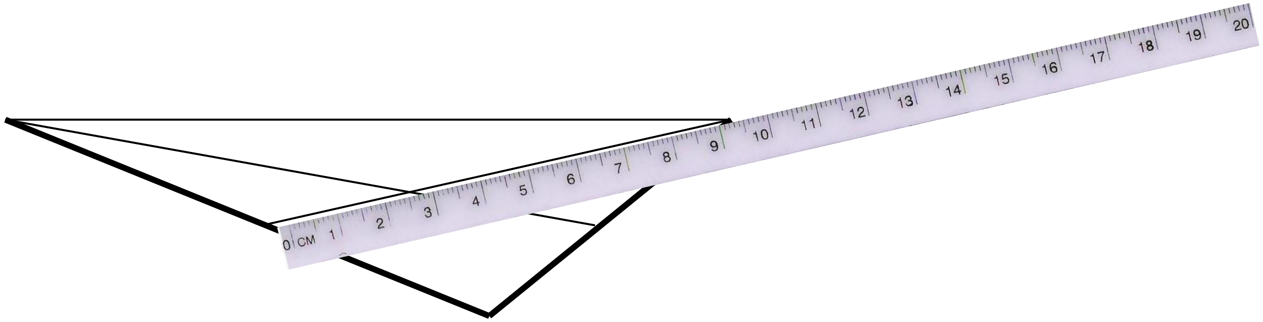
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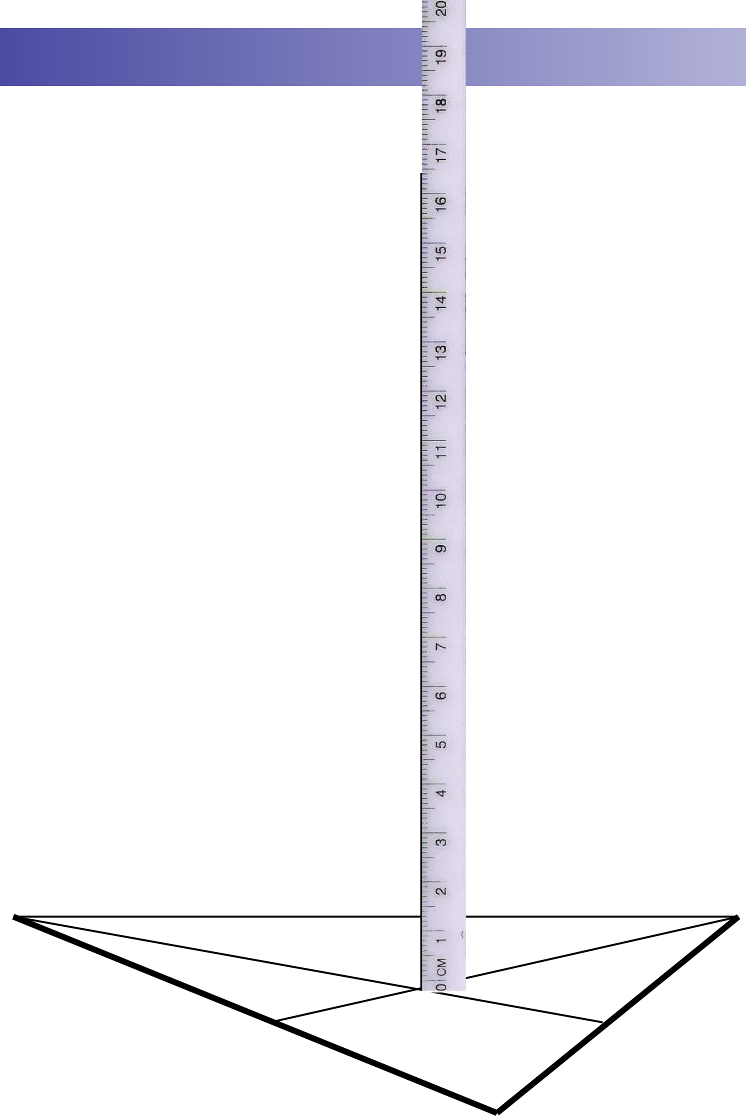

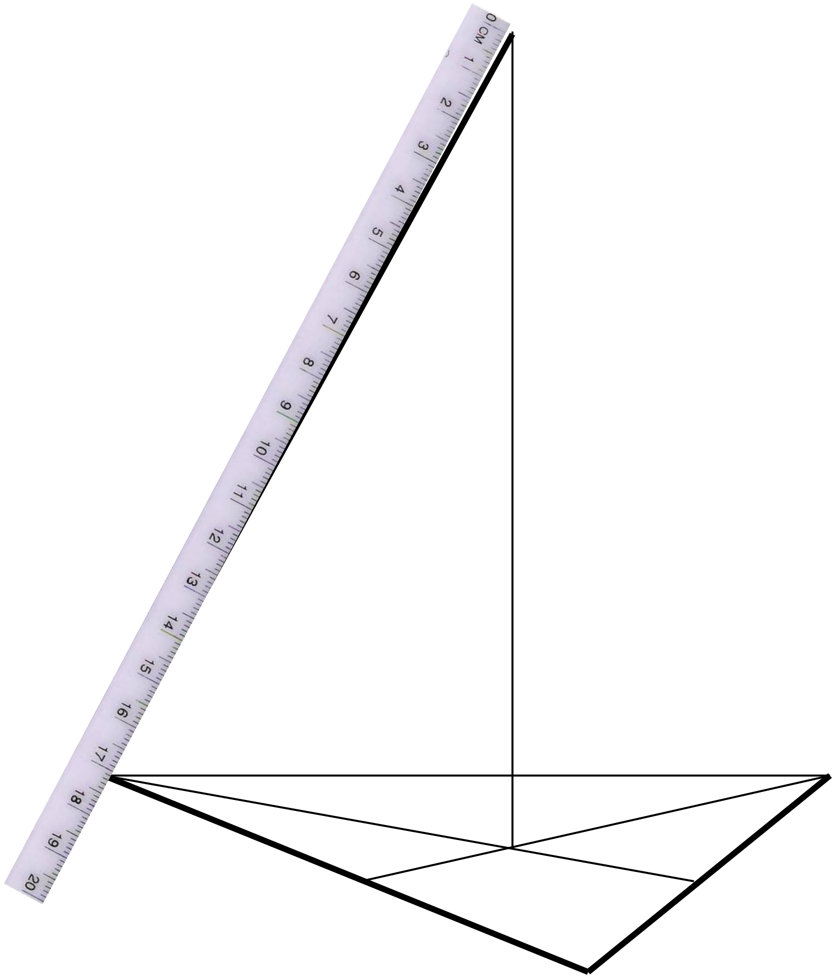
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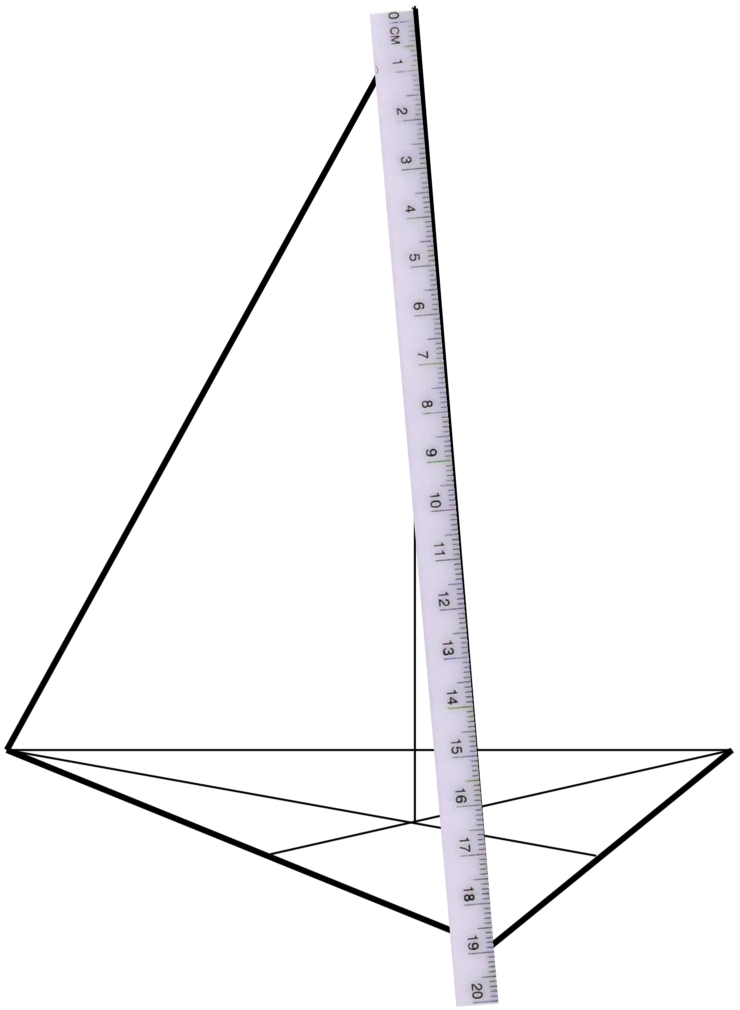

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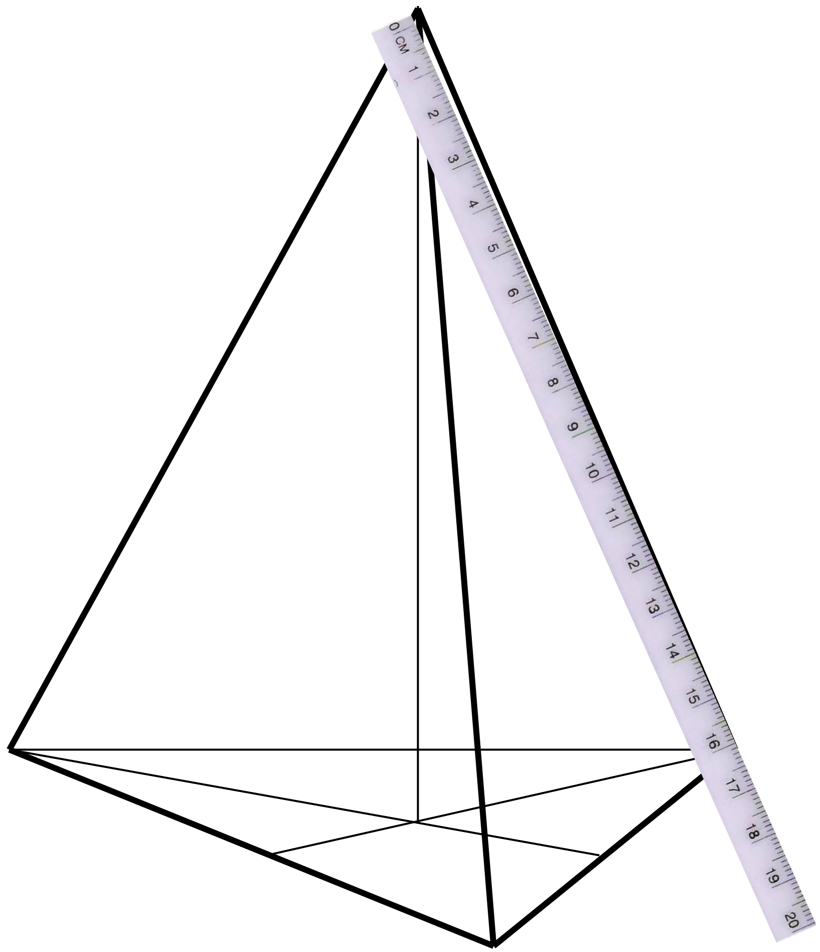
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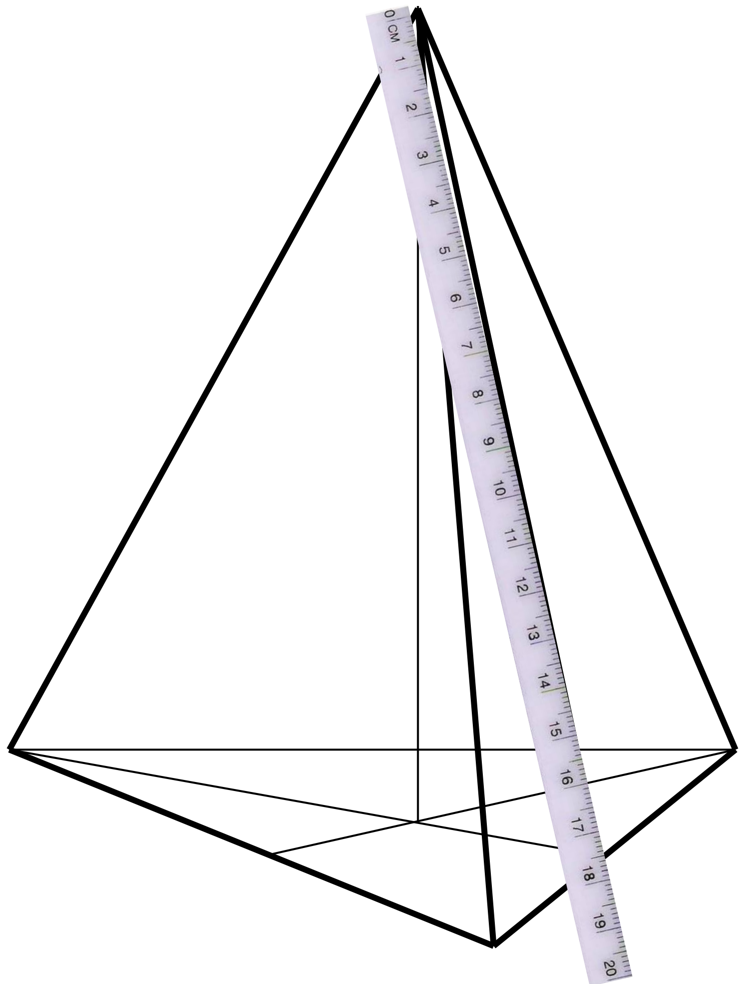
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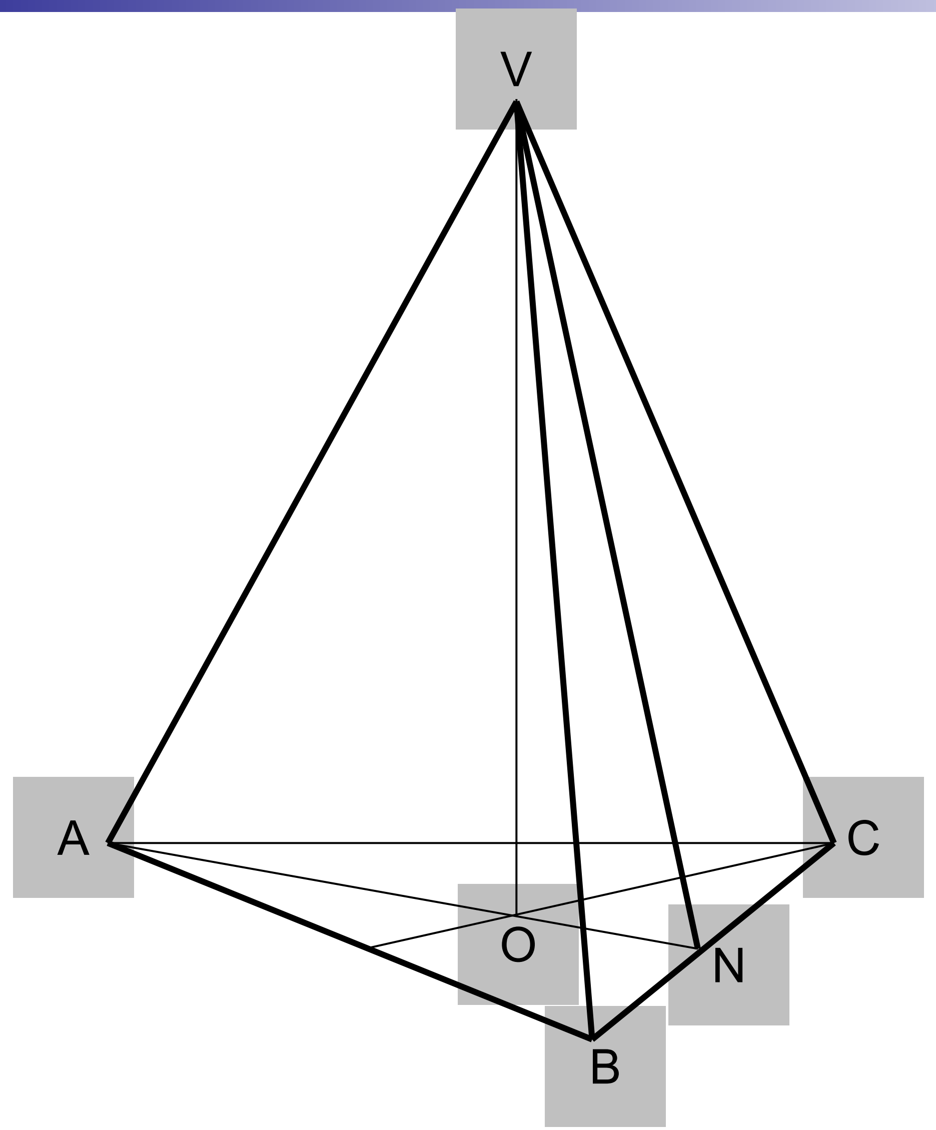
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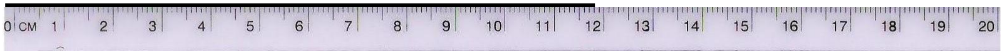
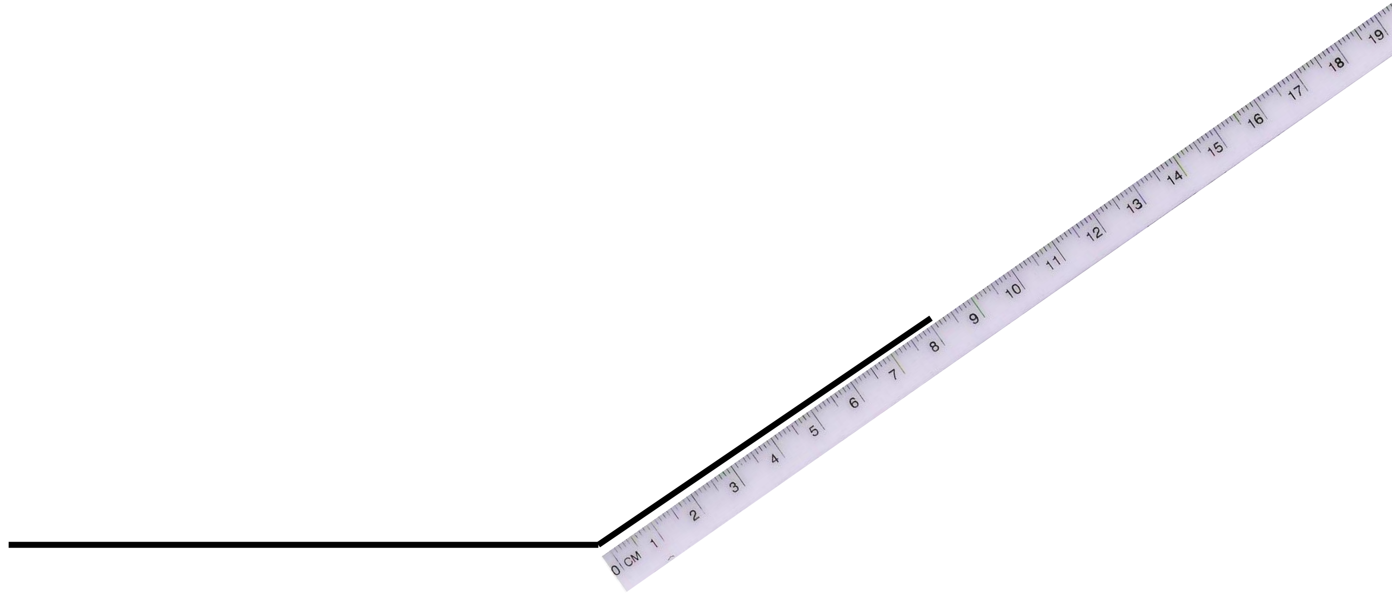
Piramidă patrulateră regulată

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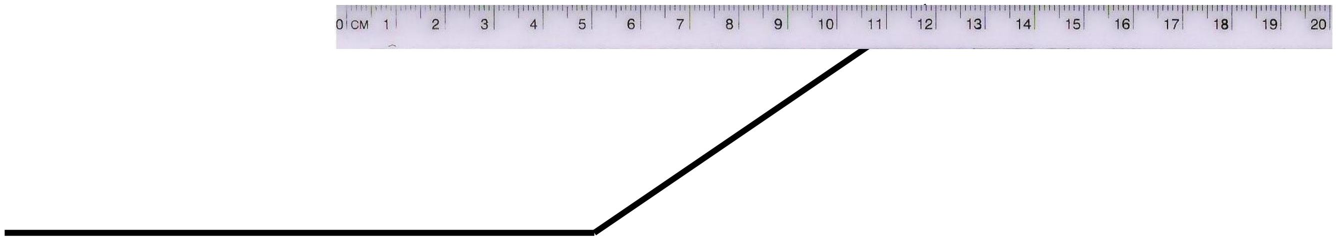
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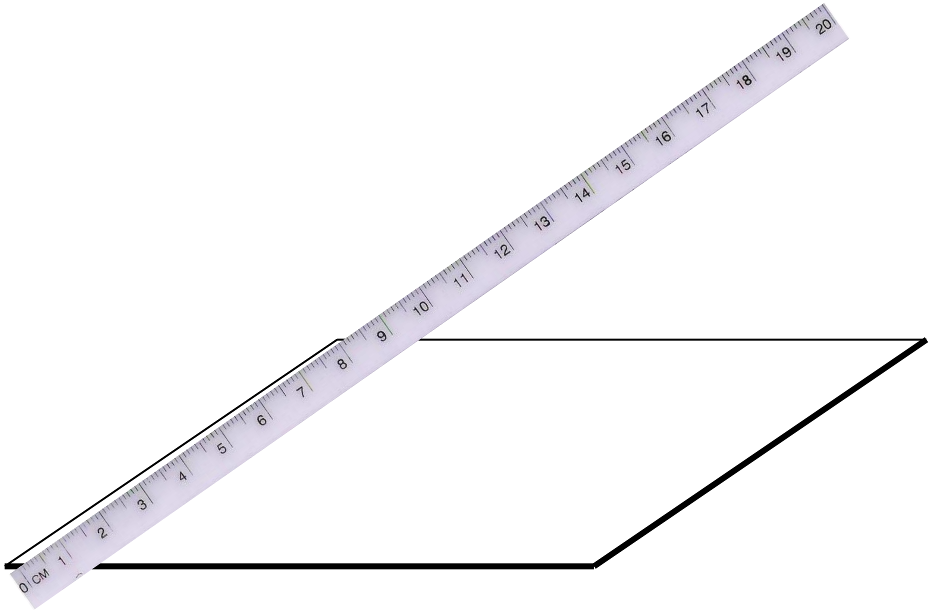
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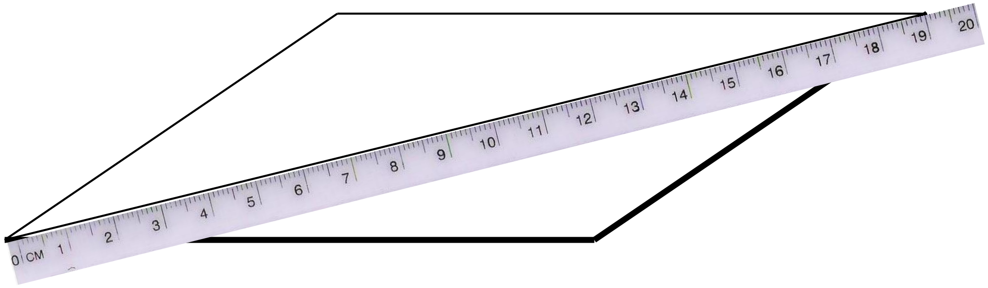

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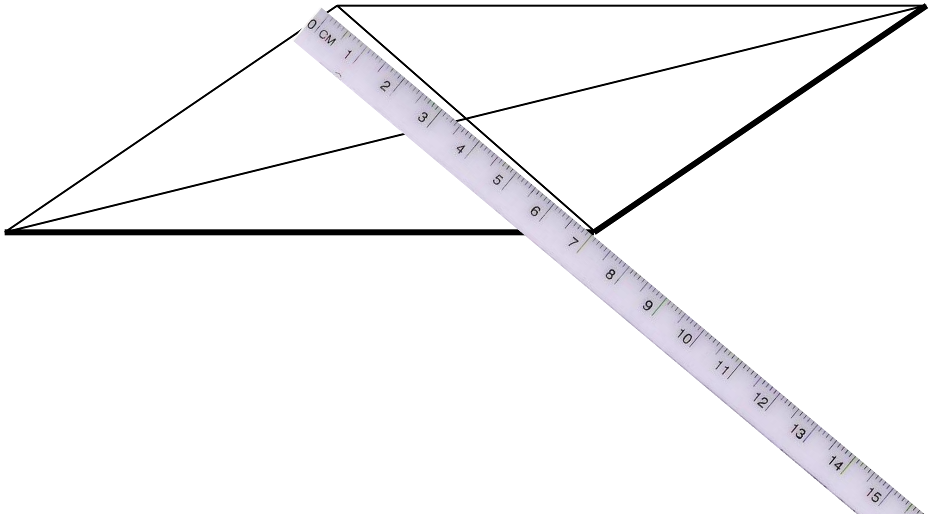
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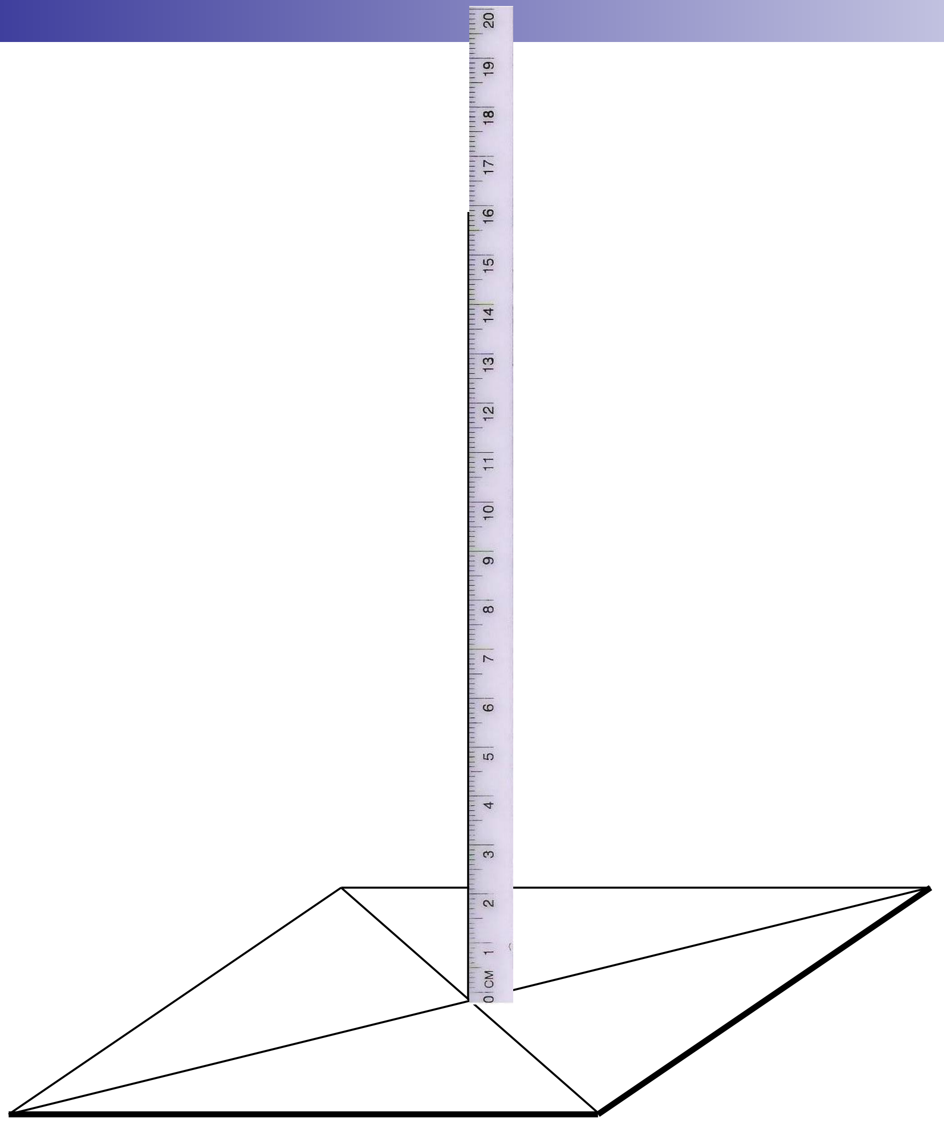
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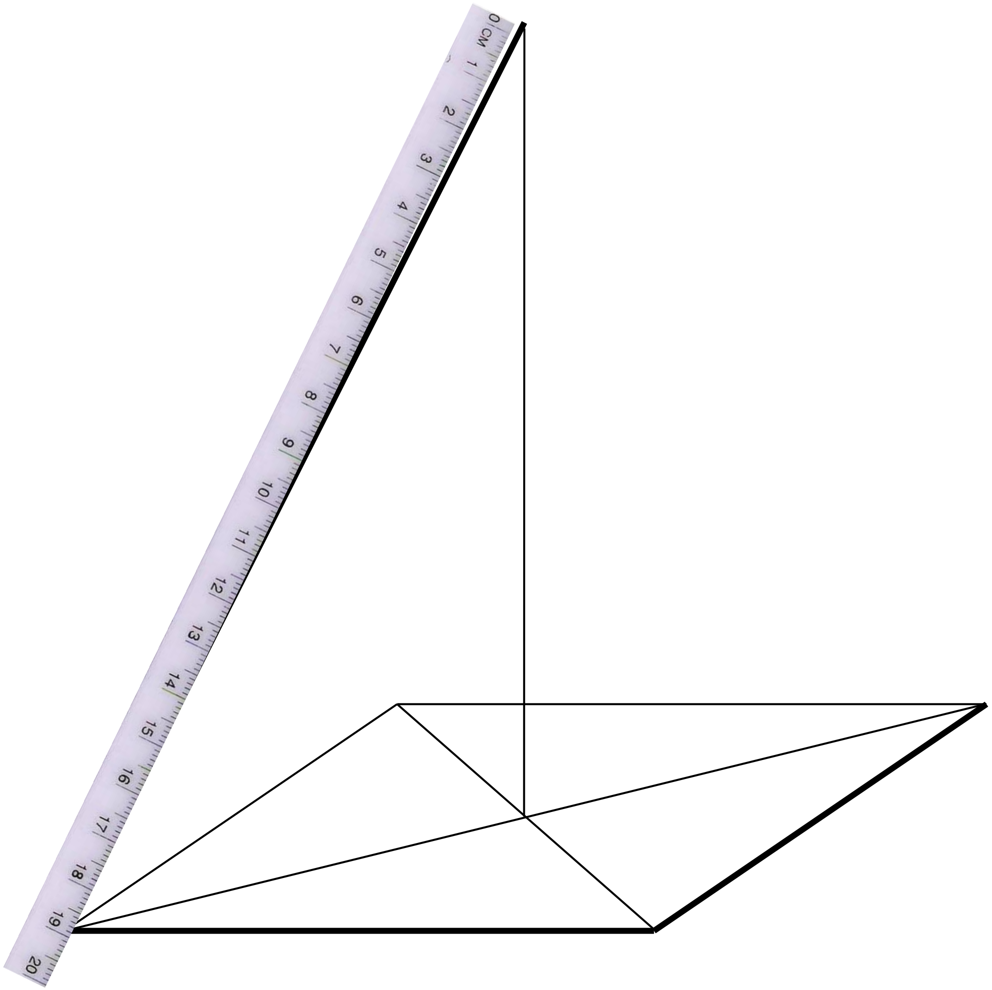

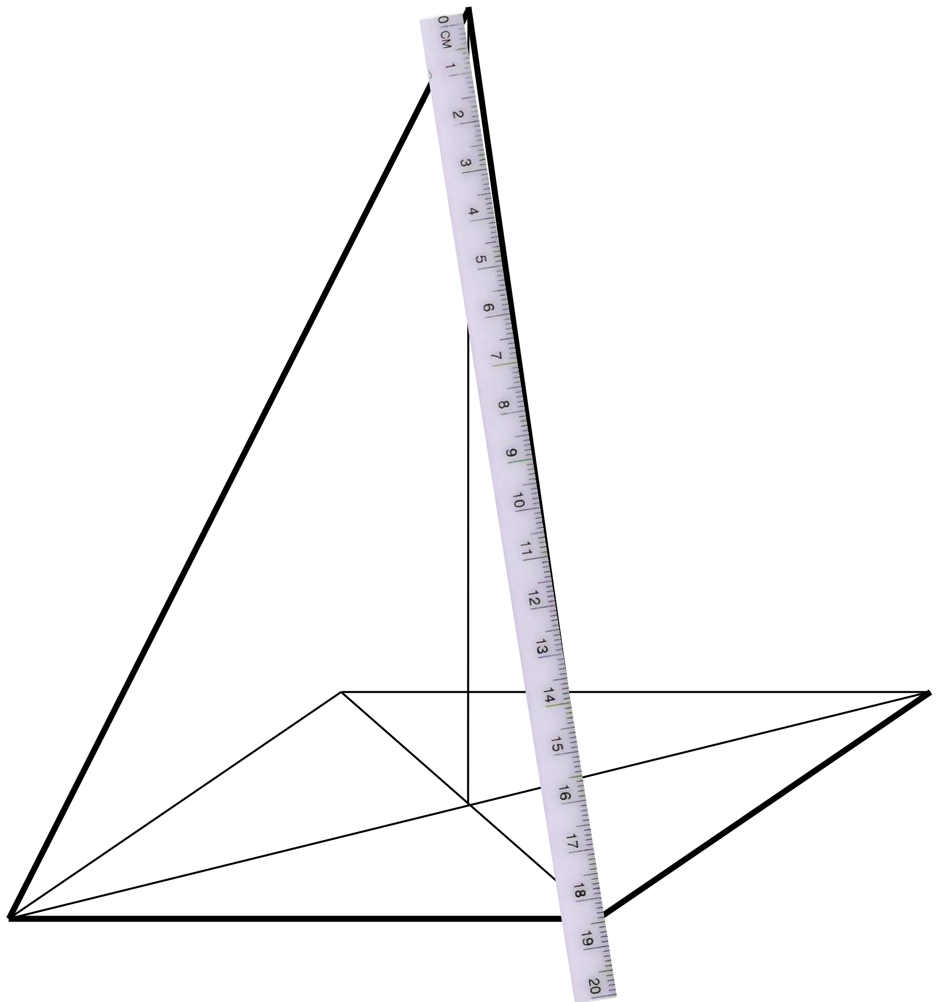
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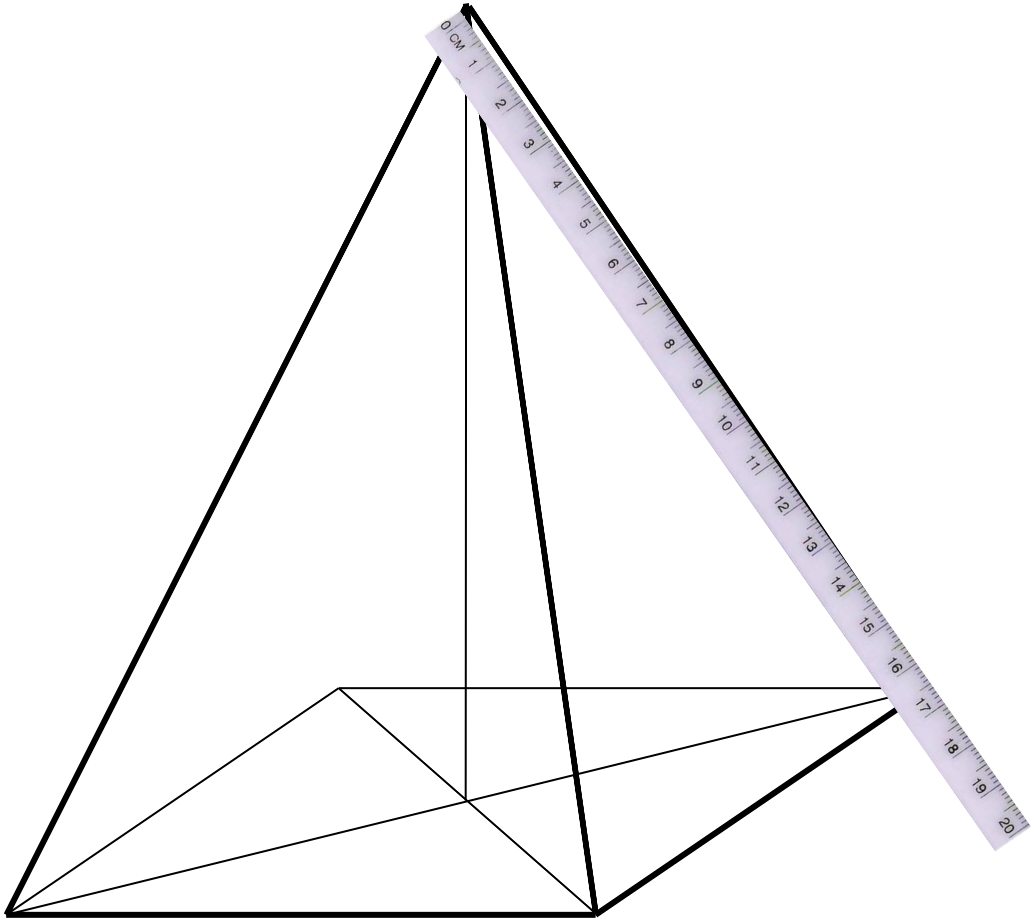

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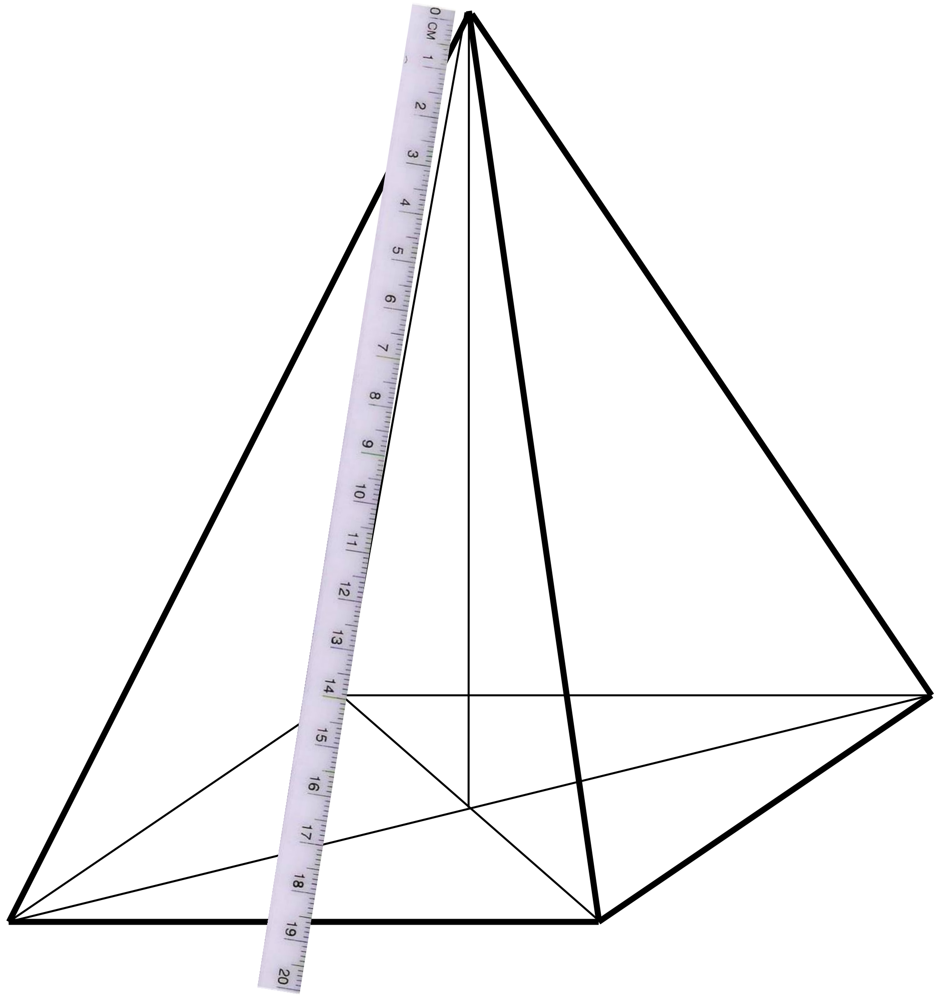
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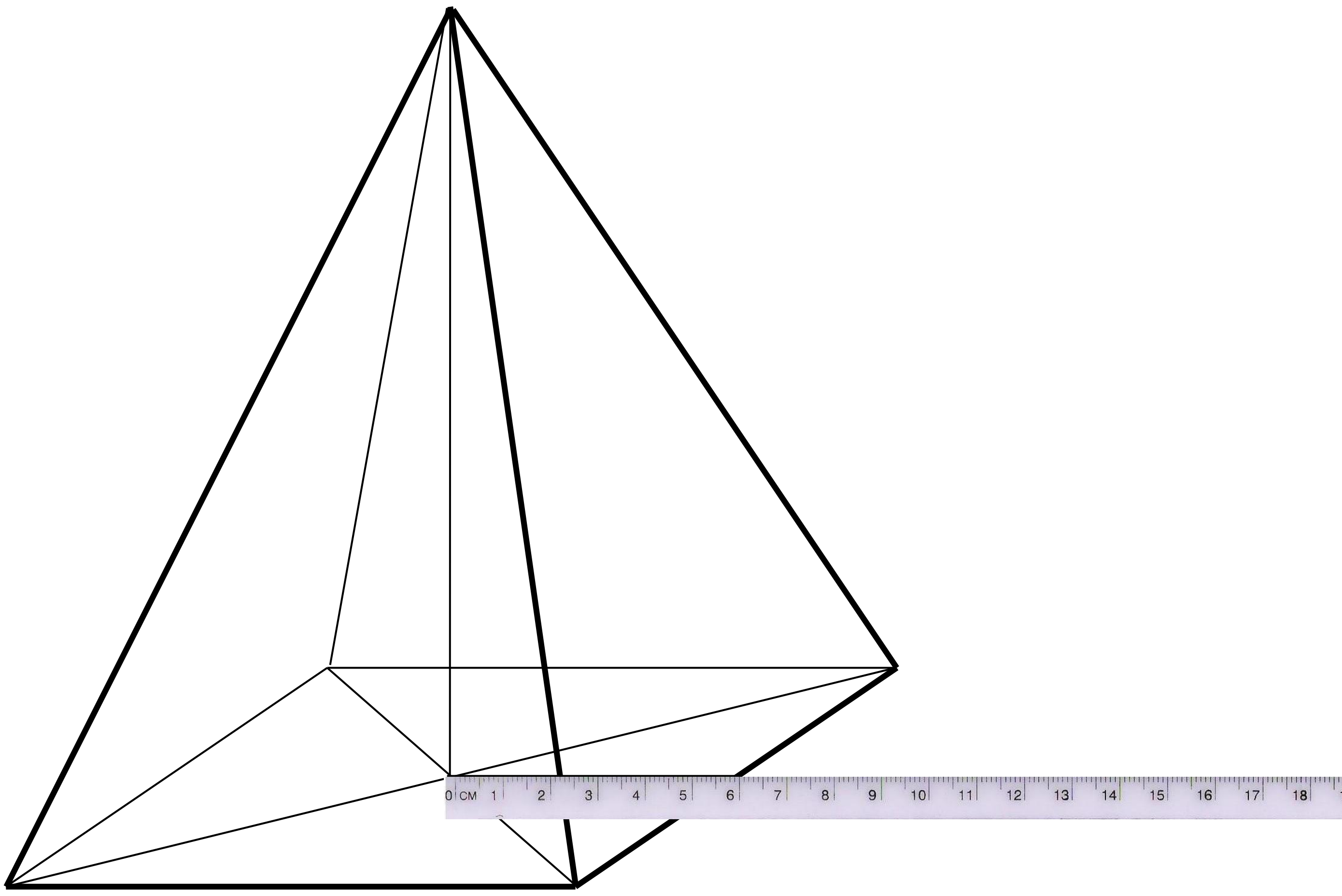
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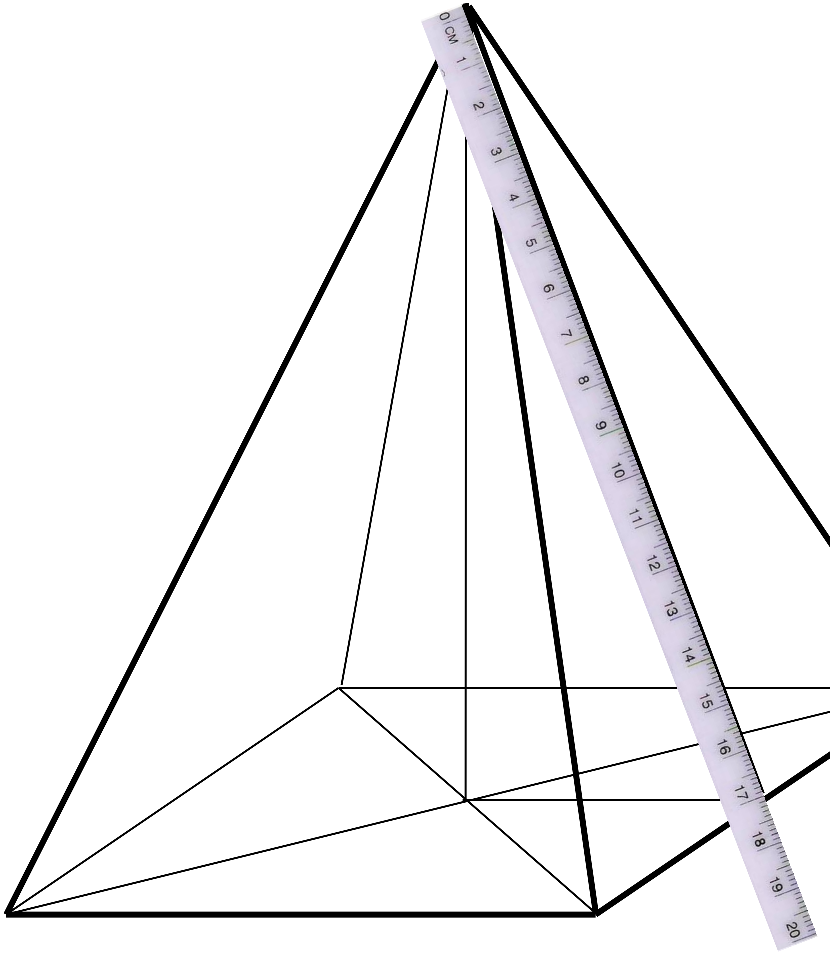
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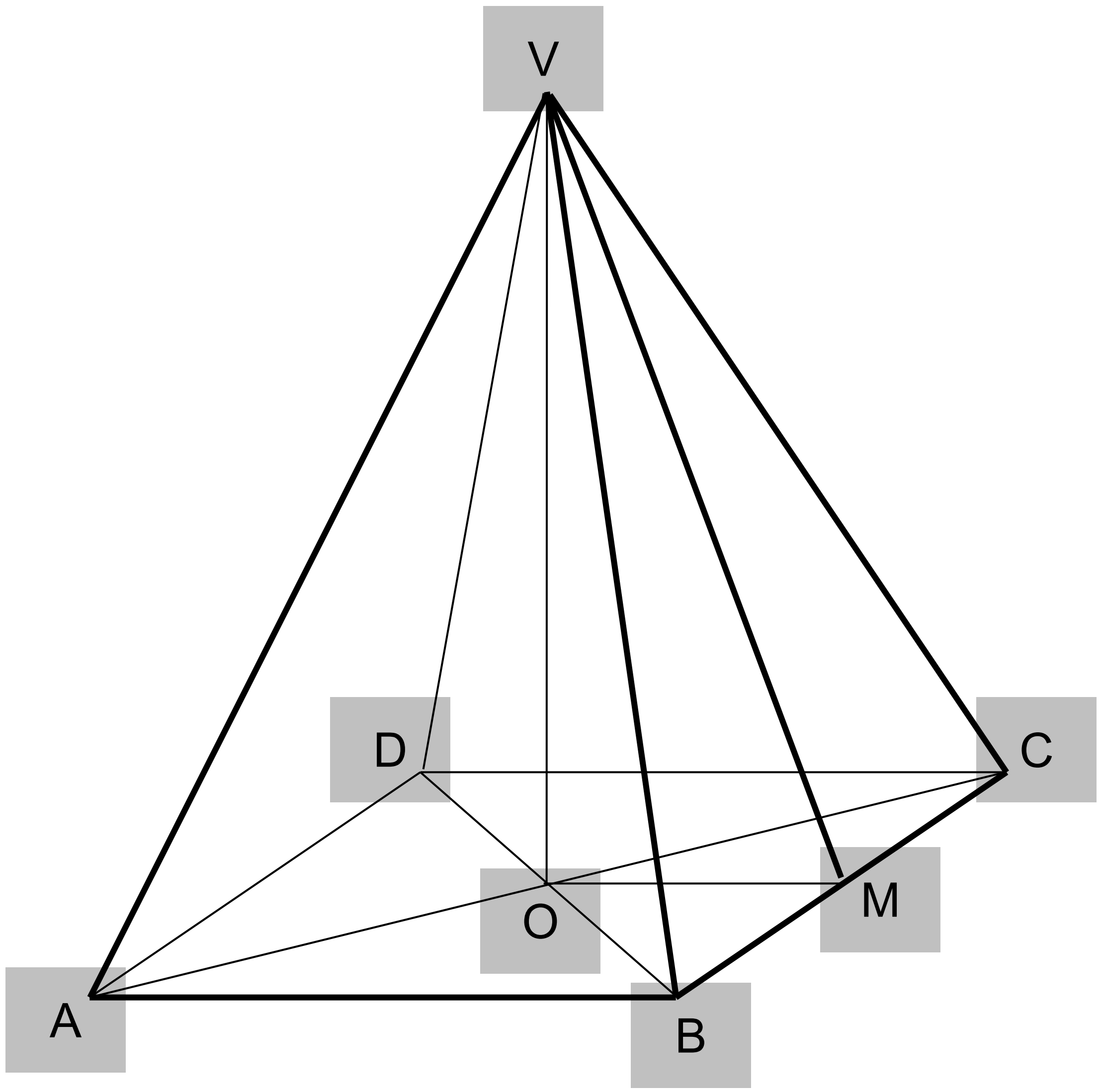
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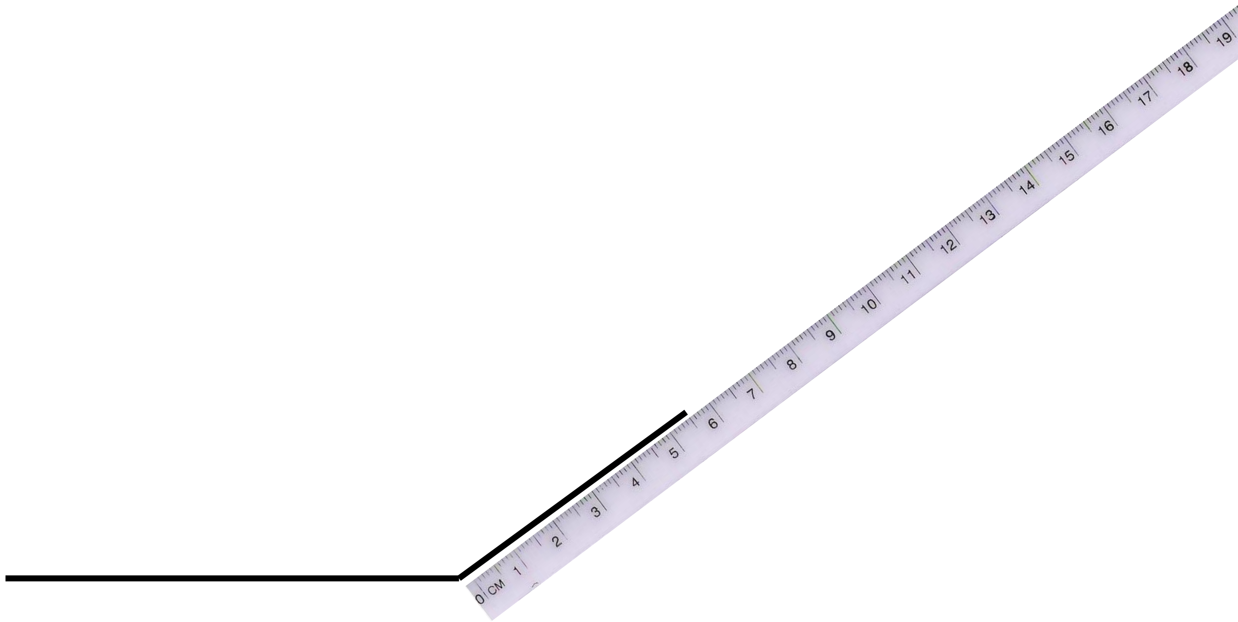
Piramidă hexagonală regulată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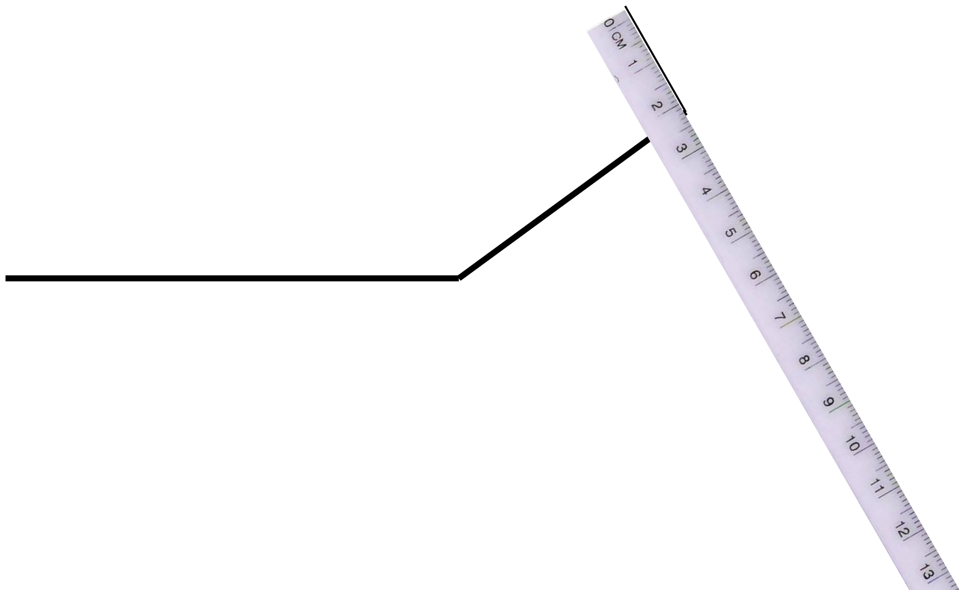
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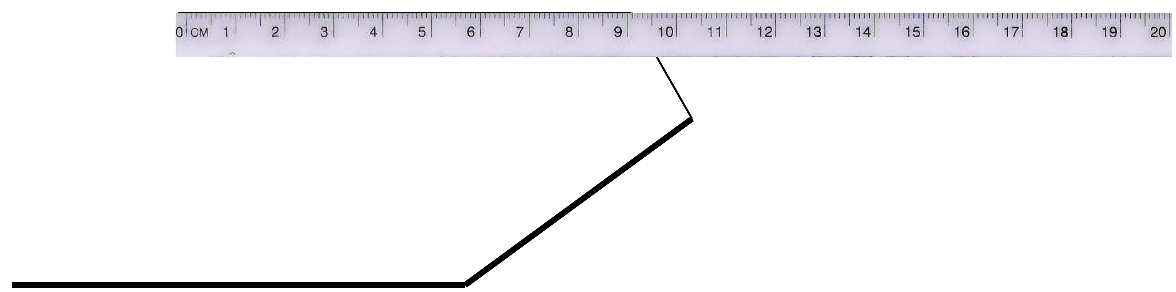
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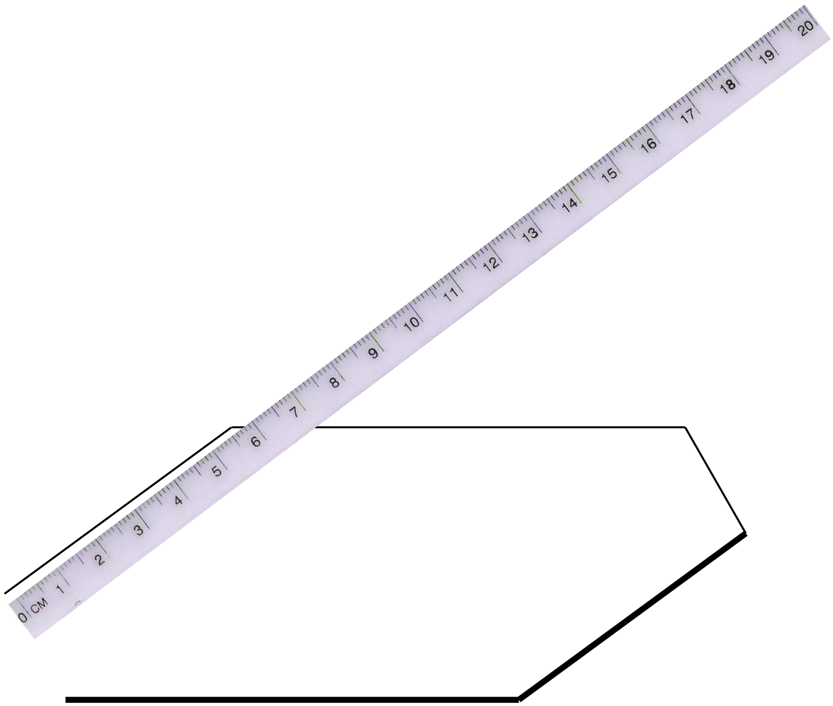
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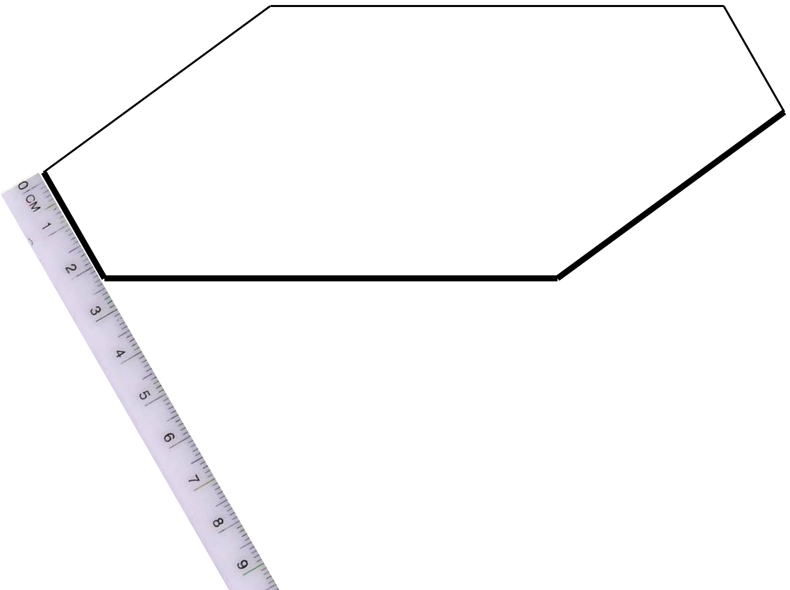
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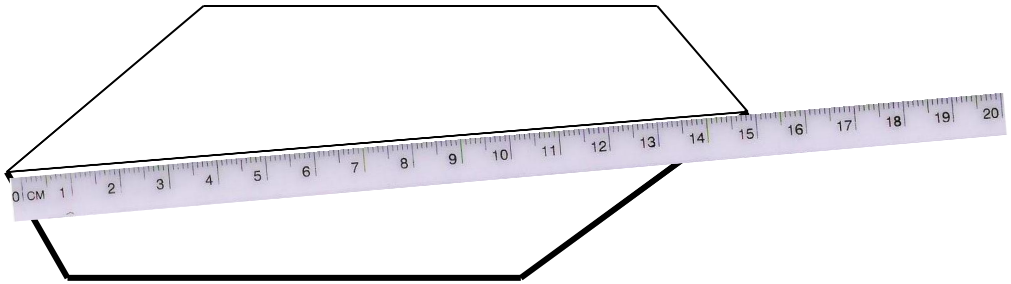
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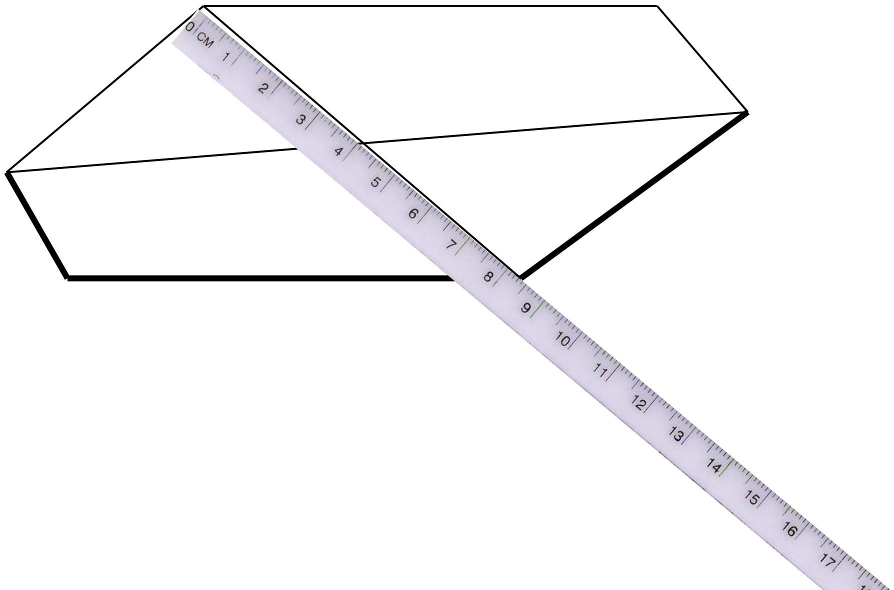

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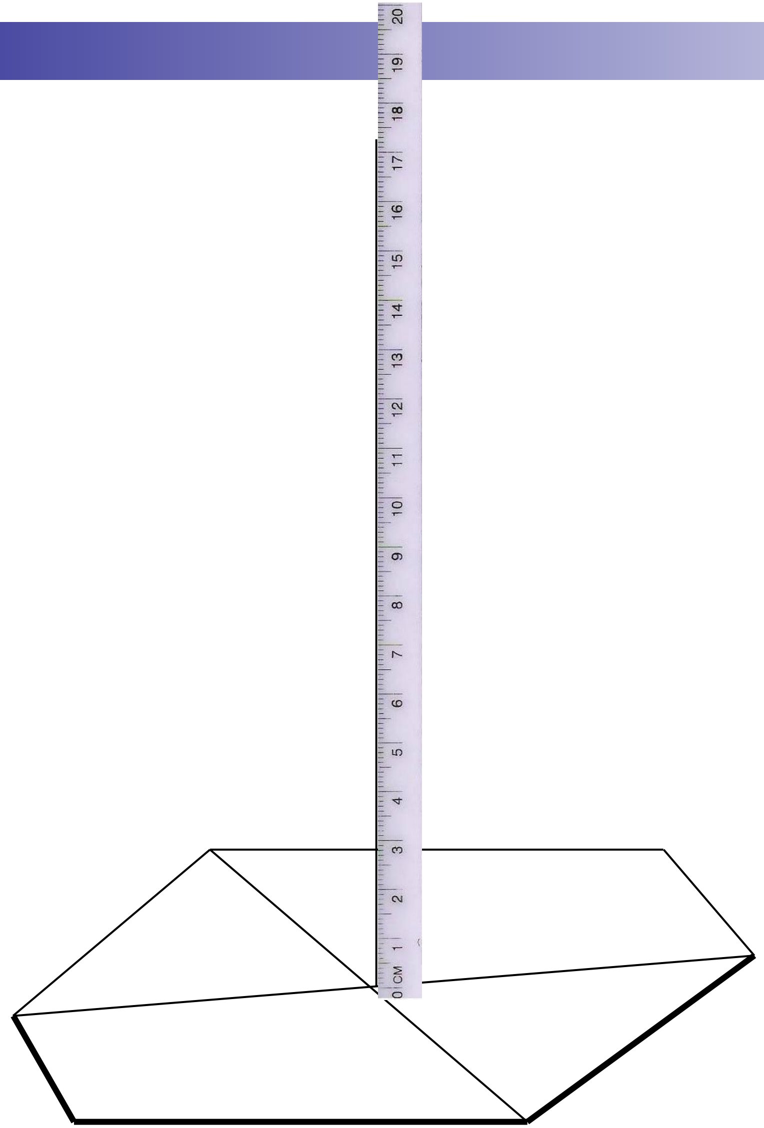
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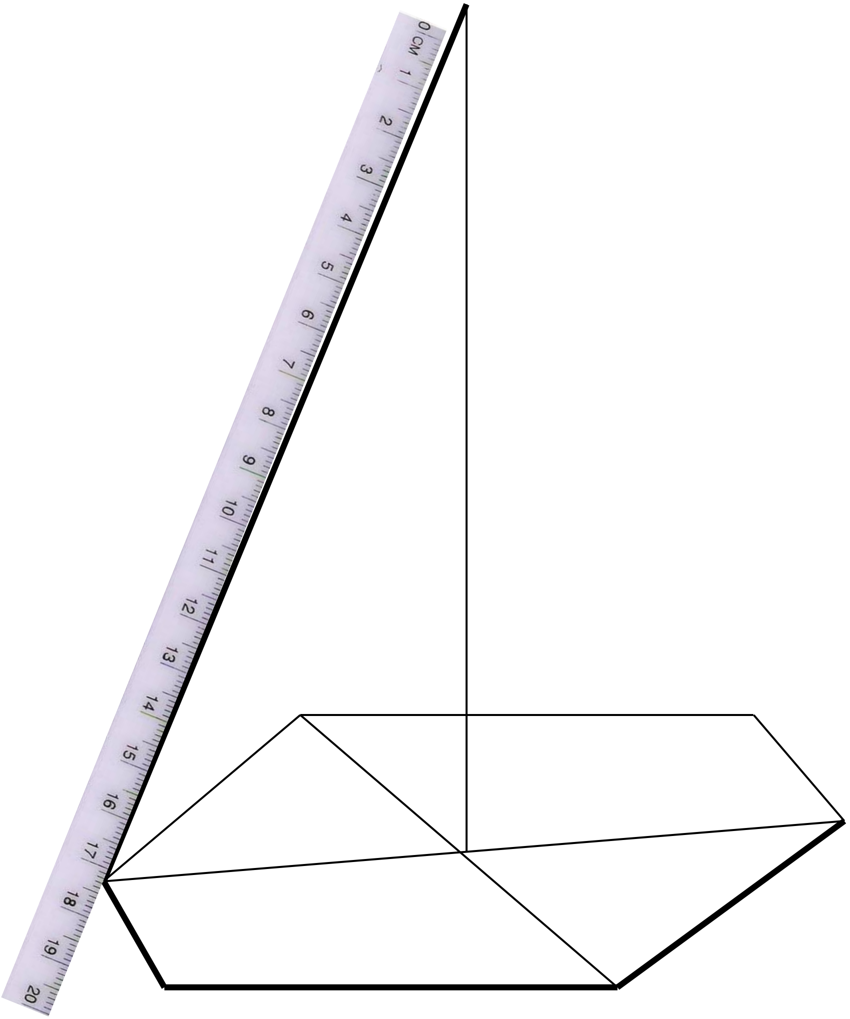

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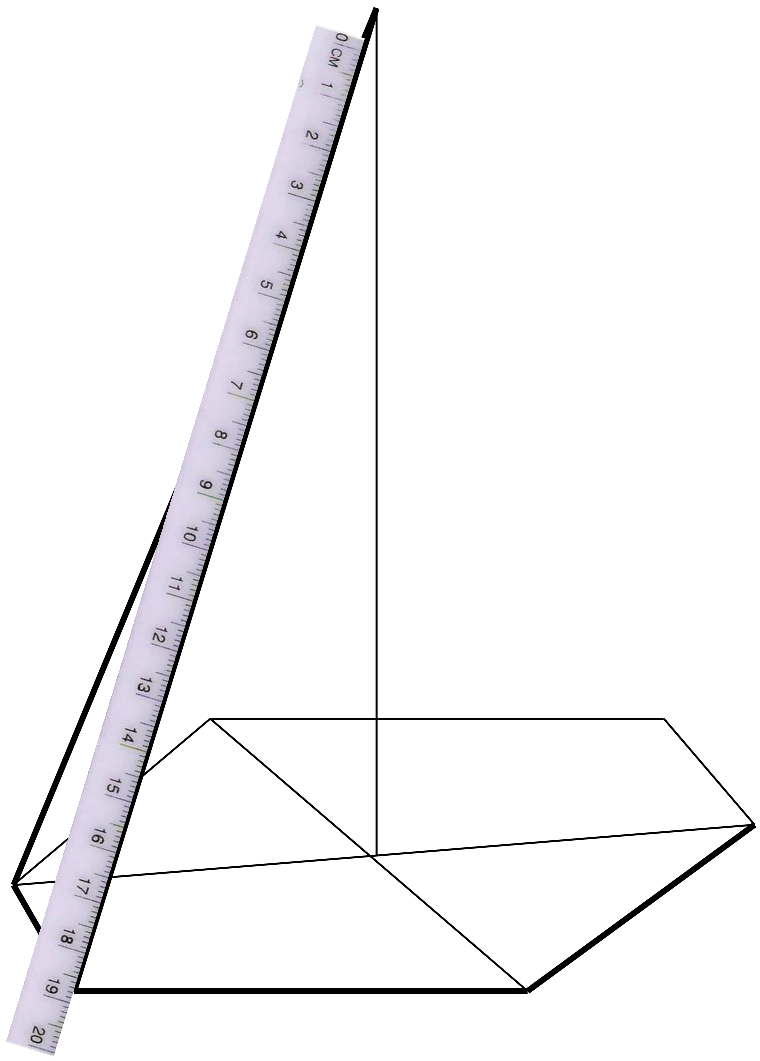
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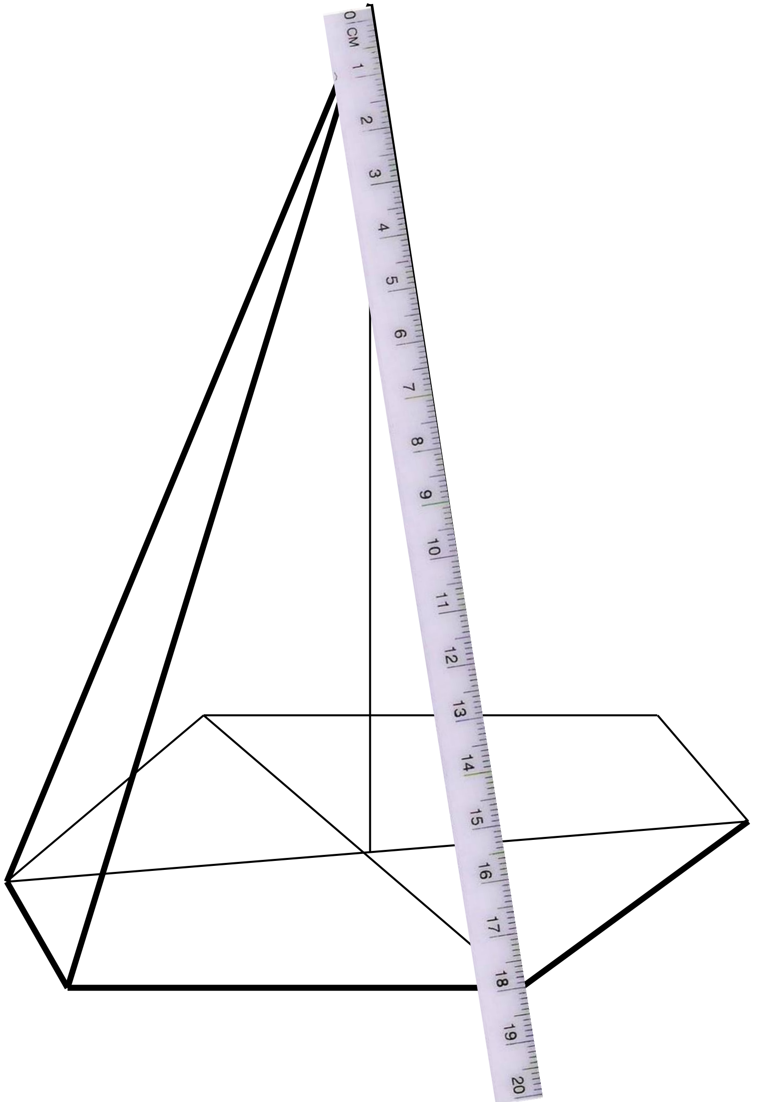
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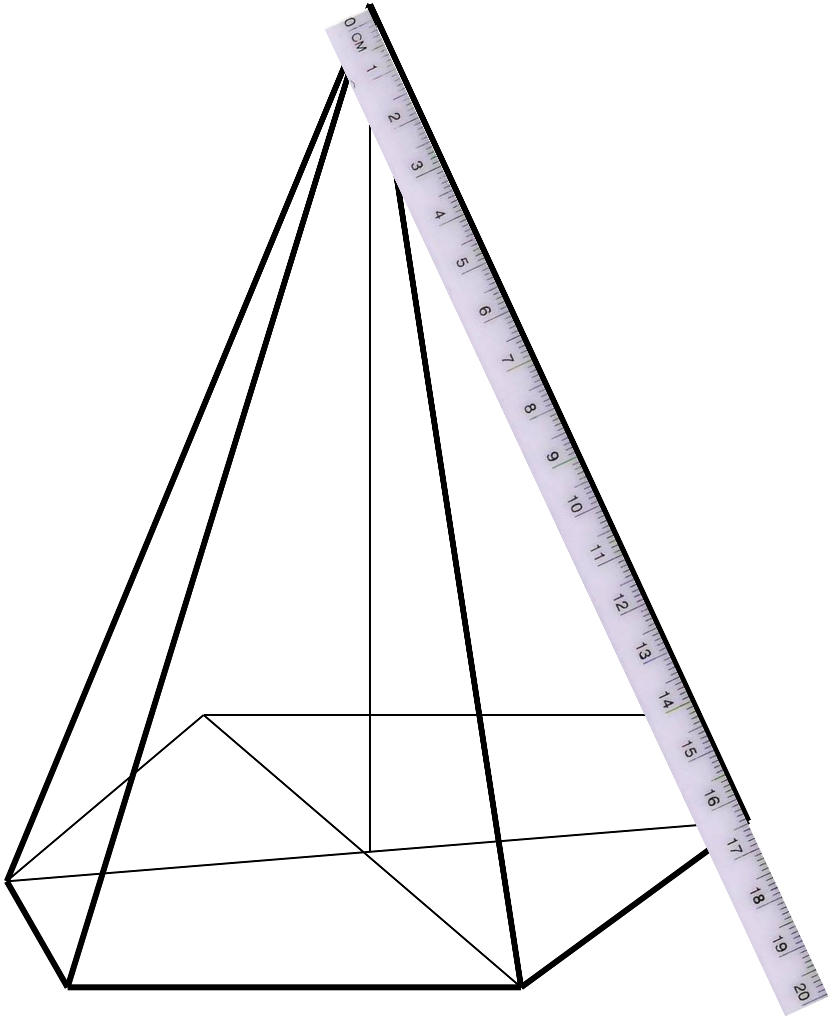
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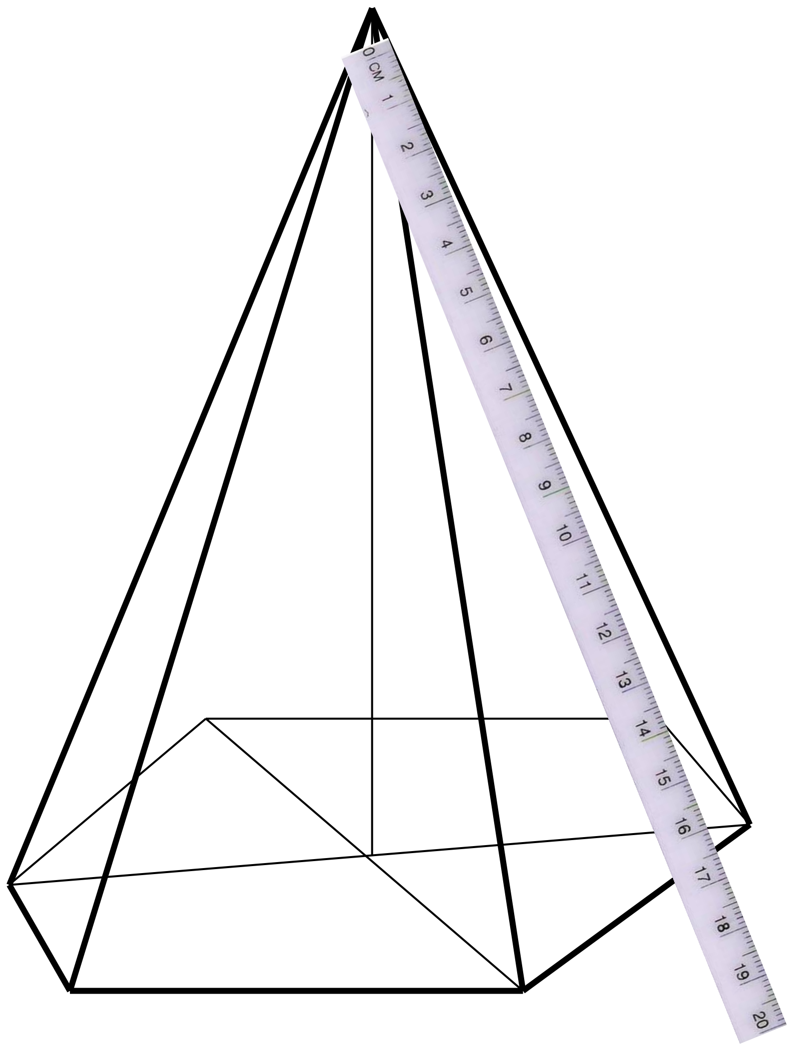
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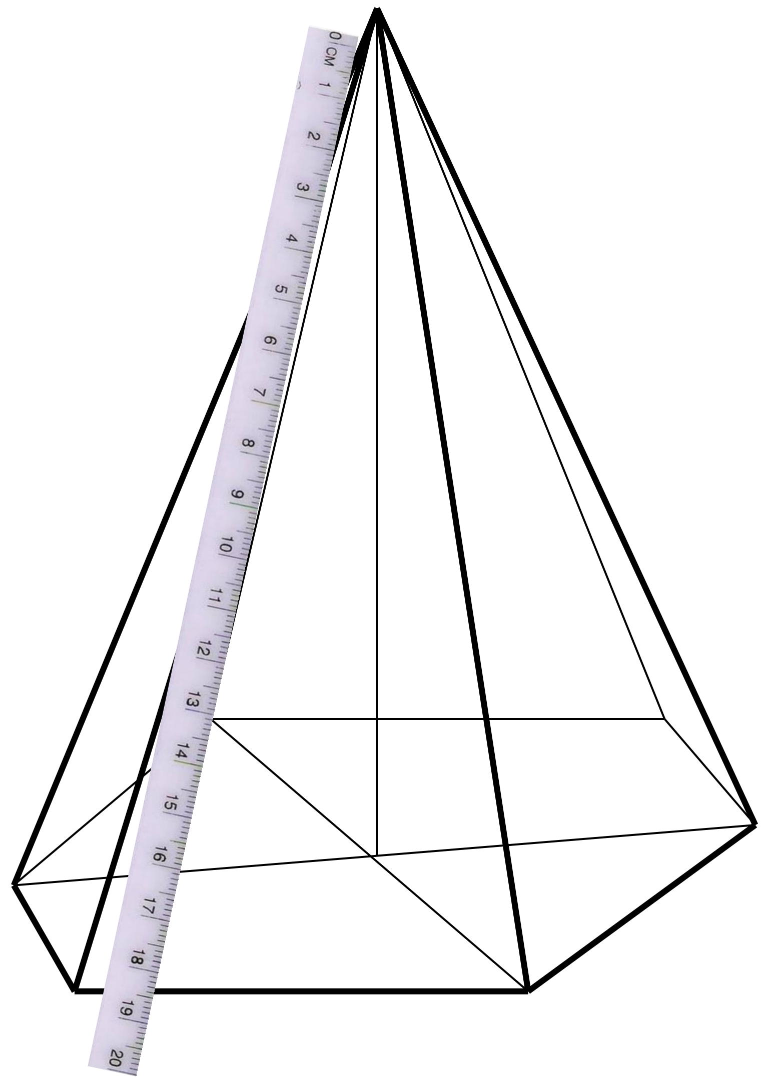
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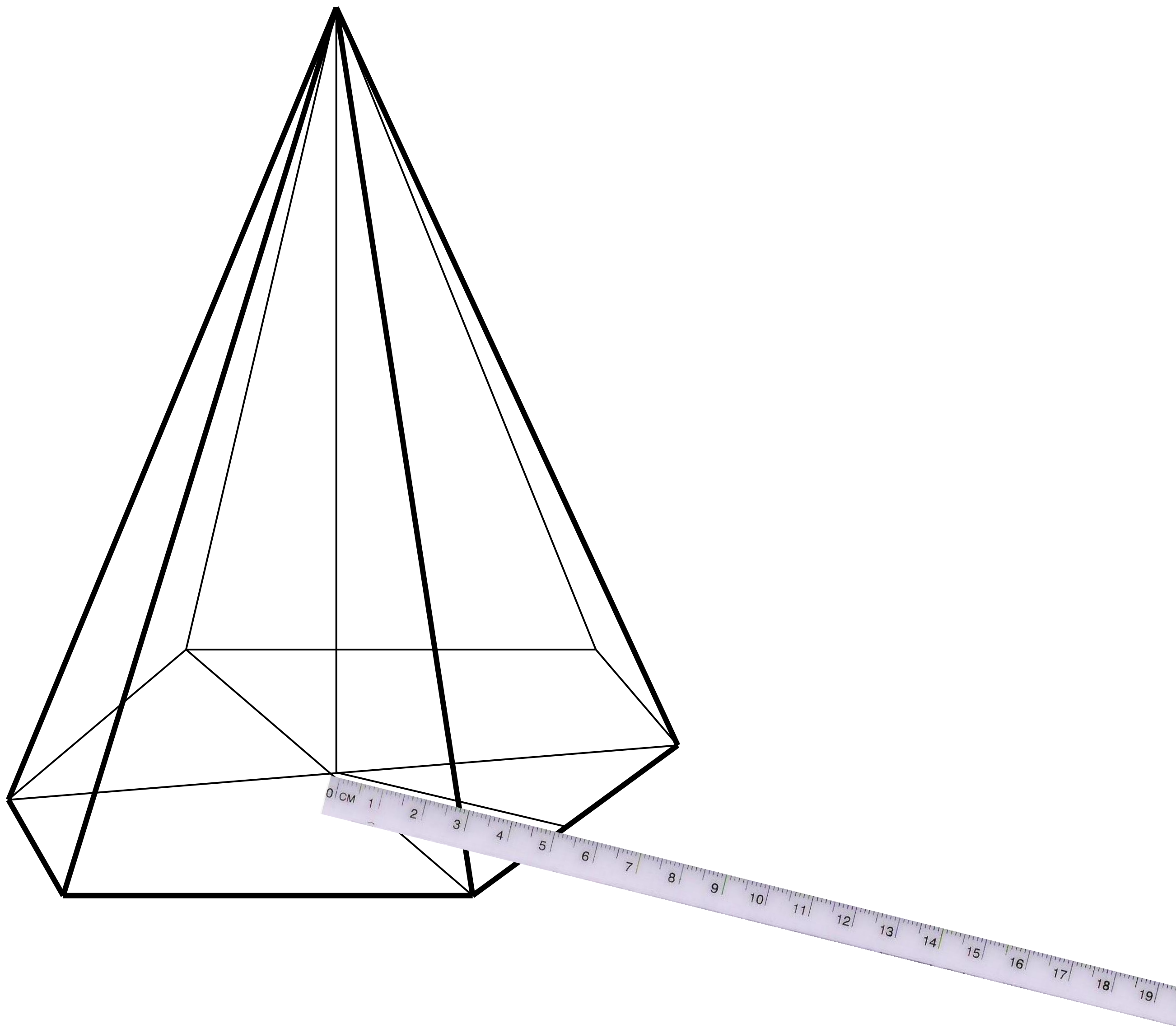
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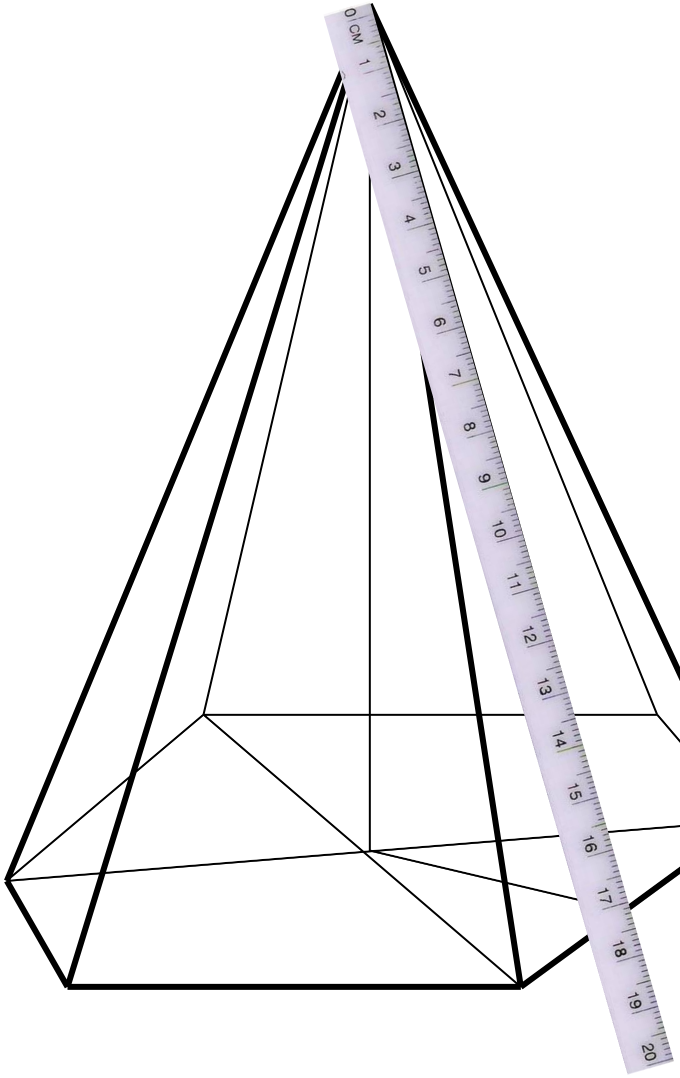
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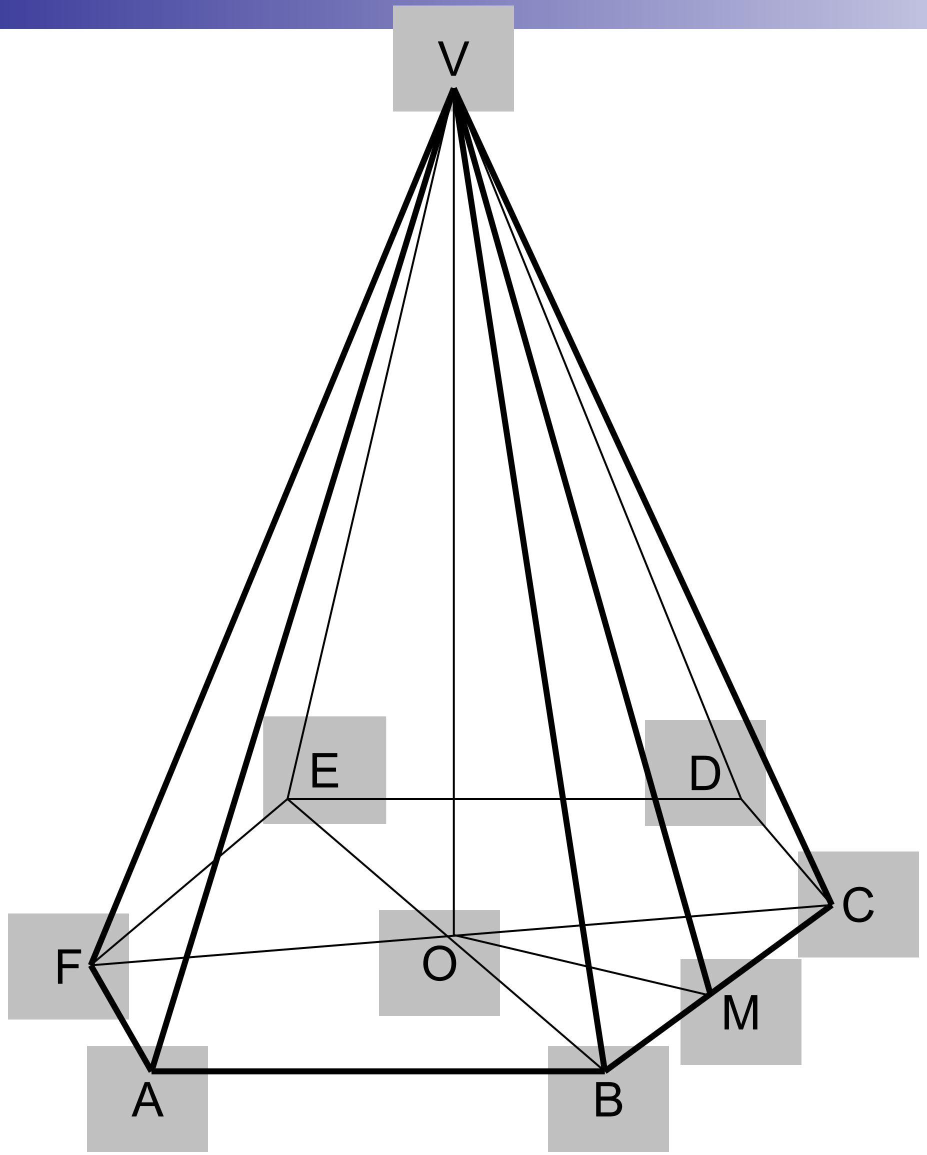
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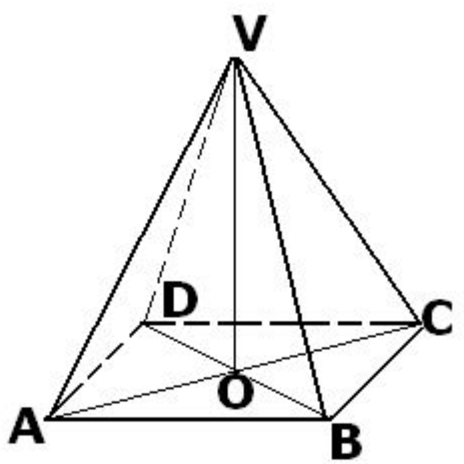
Elementele piramidelor regulate

Piramidă patrulateră regulată

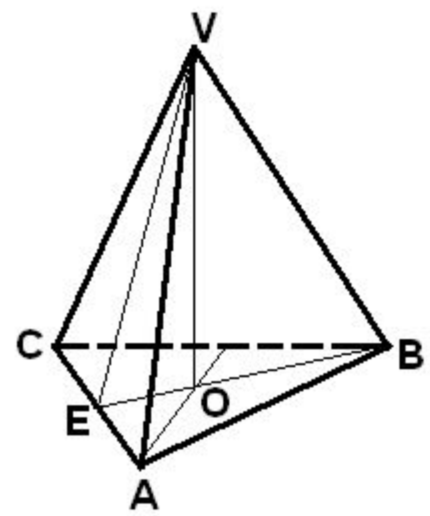


ABCD pătrat

VO înălţime ({O}=AC ∩ BD)

VBC, VCD, VDA, VAB feţe laterale (Triunghiuri isoscele congruente)

Piramidă triunghiulară regulată

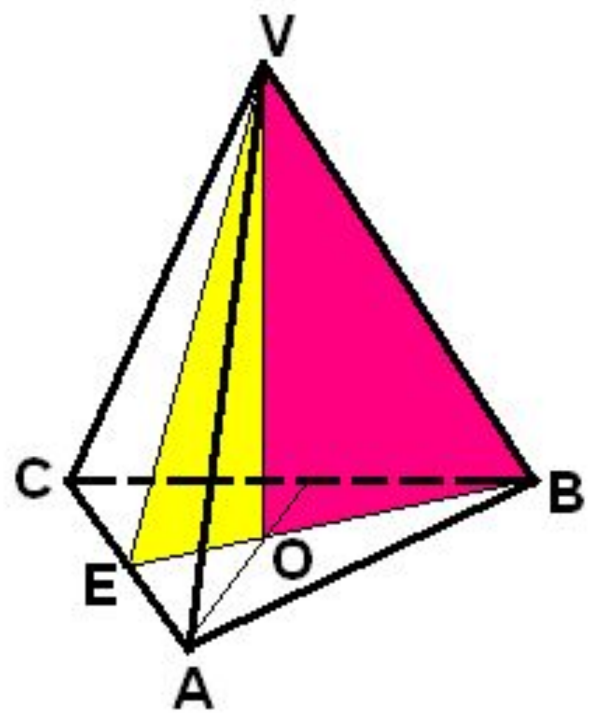


ABC triunghi echilateral

VO înălţime; O –centru de greutate

VBC, VCA, VAB feţe laterale (Triunghiuri isoscele congruente)

Triunghiuri de lucru în piramida triunghiulară regulată



$$Fie\ AB = AC = BC = a\,(muchia\quad bazei)$$

$$VO = h$$

$$VE = Ap\ (\,apotema\quad piramidei\,)$$

$$OE = ap\ (apotema\quad bazei\,)$$

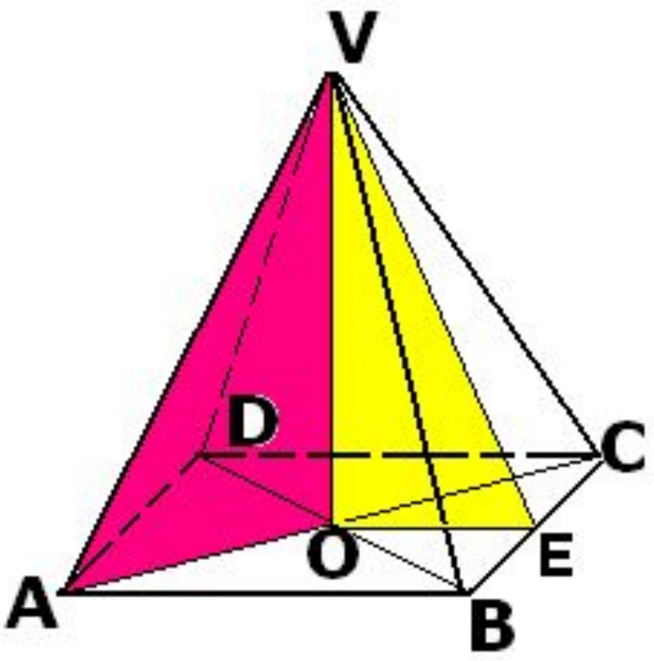
$$VB = l\,(muchie\quad lateral\breve{a})$$

$$\Delta VOE : OE = \frac{1}{3}\cdot\frac{a\sqrt{3}}{2};\ VO^2 + OE^2 = VE^2$$

$$\Delta VOB : OB = \frac{2}{3}\cdot\frac{a\sqrt{3}}{2};\ VO^2 + OB^2 = VB^2$$

$$\Delta VEA : EA = \frac{a}{2};\ VE^2 + EA^2 = VA^2$$

Triunghiuri de lucru în piramida patrulateră regulată



$Fie\ AB = BC = CD = DA = a\left(muchia\quad bazei\right)$

$VO = h\left(înaltimea\right)$

$OE = ap\ \left(apotema\, bazei\right)$

$VE = Ap\left(\ apotema\quad piramidei\ \right)$

$VA = l\ \left(muchia\quad laterala\right)$

$\Delta VOE : OE = \frac{1}{2}a;\ VO^2 + OE^2 = VE^2$

$\Delta VOA : AO = \frac{1}{2}a\sqrt{2};\ VO^2 + AO^2 = VA^2$

$\Delta VEC : EC = \frac{1}{2}a;\ VE^2 + EC^2 = VC^2$

Triunghiuri de lucru în piramida hexagonală regulată

$$Fie\ AB = BC = BD = \ldots = EF = a\,(muchia\ bazei)$$
$$VO = h\ (\hat{\imath}naltimea\ piramidei)$$
$$OM = ap\ (apotema\ \ bazei)$$
$$VM = Ap\ (apotema\ piramidei)$$
$$VA = VB = VC = \ldots = VE = VF = l\,(muchii\ laterale)$$
$$\Delta VOM : OM = \frac{a\sqrt{3}}{2};\ VO^2 + OM^2 = VM^2$$
$$\Delta VOE : OE = \frac{EB}{2} = a;\ VO^2 + OE^2 = VE^2$$
$$\Delta VMA : AM = \frac{a}{2};\ VM^2 + MA^2 = VA^2$$

Formule

A_l= arie laterală=suma ariilor feţelor laterale

A_t= arie totală = A_l + aria bazei

$A_t = A_l + A_{bază}$

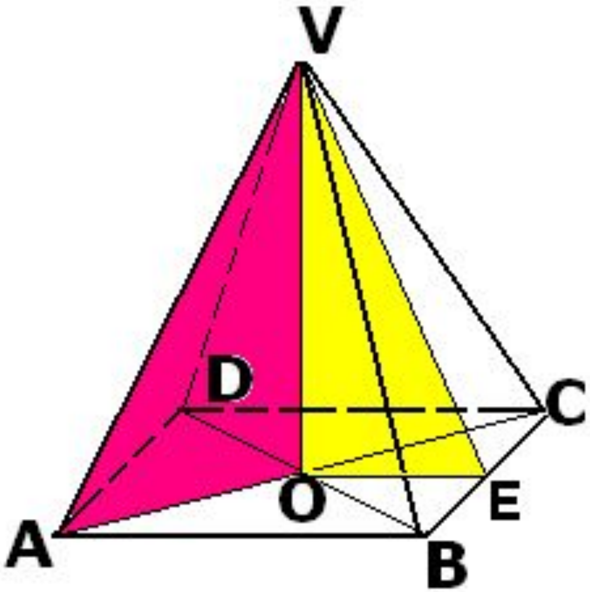
$$V = volum = \frac{1}{3} \cdot A_{bazei} \cdot \hat{i}naltime$$

Formule de calcul pentru piramide regulate

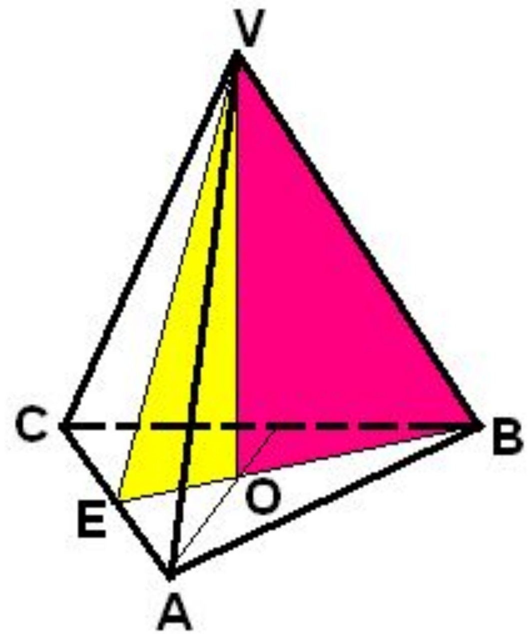
$$A_l = \frac{P_{bazei} \cdot Apotemă}{2}$$

$$V = \frac{1}{3} \cdot A_{baza} \cdot h \qquad A_t = A_l + A_{bază}$$

pentru orice tip de piramidă



$$A_l = 4 \cdot A_{VBC} = 4 \cdot \frac{BC \cdot VE}{2} = \frac{(4BC) \cdot Ap}{2} = \frac{P_{baza} \cdot Ap}{2}$$



$$A_l = 3 \cdot A_{VAC} = 3 \cdot \frac{AC \cdot VE}{2} = \frac{(3AC) \cdot Ap}{2} = \frac{P_{baza} \cdot Ap}{2}$$

Piramidă triunghiulară regulată cu muchia bazei “*a*”

$$P_{baza} = 3a$$

$$A_{baza} = \frac{a^2\sqrt{3}}{4}$$

Piramidă patrulateră regulată cu muchia bazei “*a*”

$$P_{baza} = 4a$$

$$A_{baza} = a^2$$

Piramidă hexagonală regulată cu muchia bazei “*a*”

$$P_{baza} = 6a$$

$$A_{baza} = 6 \cdot \frac{a^2\sqrt{3}}{4}$$

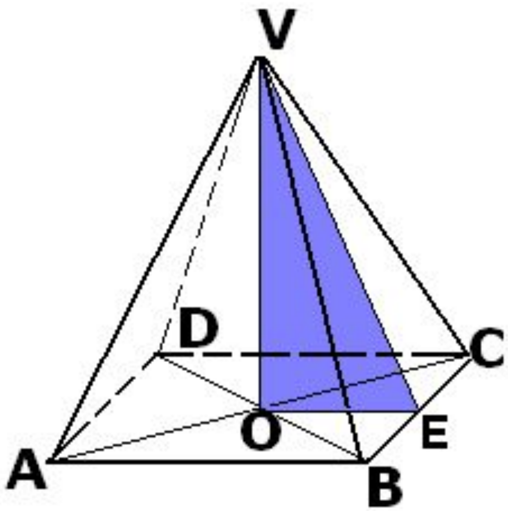
Aplicaţii

1. Fie VABCD o piramidă patrulateră regulată. Dacă se cunoaşte muchia bazei AB=4 cm, înălţimea VO=12 cm, să se calculeze aria şi volumul.

R 1

2. Dacă muchia laterală a unei piramide patrulatere regulate este de 10 cm, iar muchia bazei de 12 cm, să se calculeze volumul şi aria laterală.

R2



$$Avem\ V=\frac{1}{3}A_b\cdot h,\ unde\ A_b=a^2,\ A_b=4^2=16(cm^2).$$

$$Deci\ V=\frac{1}{3}\cdot 16\cdot 12=64(cm^3).$$

$$\Delta VOE: VO^2+OE^2=VE^2$$

$$12^2+2^2=VE^2$$

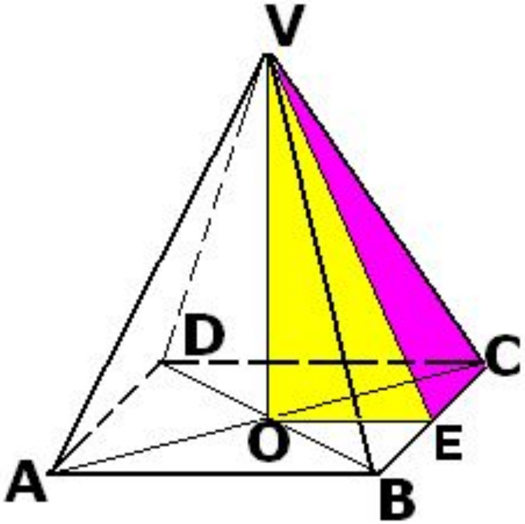
$$144+4=VE^2$$

$$VE=\sqrt{148}=2\sqrt{37}\Rightarrow Ap_p=2\sqrt{37}cm.$$

$$A=A_l+A_b\ ;\ A_l=\frac{1}{2}P_b\cdot Ap_p$$

$$A_l=\frac{1}{2}\cdot 16\cdot 2\sqrt{37}=16\sqrt{37}(cm^2)$$

$$A=16\sqrt{37}+16=16\left(1+\sqrt{37}\right)cm^2$$



$$AB = BC = CD = DA = 12cm \Rightarrow EC = 6cm$$

$$\Delta VEC : VC = 10cm,\ EC = 6cm \Rightarrow VE = 8cm.(numere\ \ pitagorice)$$

$$\Delta VOE : OE = \frac{AB}{2} = 6cm \Rightarrow VO^2 = VE^2 - OE^2$$

$$VO^2 = 64 - 36 \Rightarrow VO = \sqrt{28} = 2\sqrt{7}cm$$

$$V = \frac{1}{3} \cdot A_b h = \frac{1}{3} \cdot 144 \cdot 2\sqrt{7} = 72\sqrt{7}(cm^3)$$

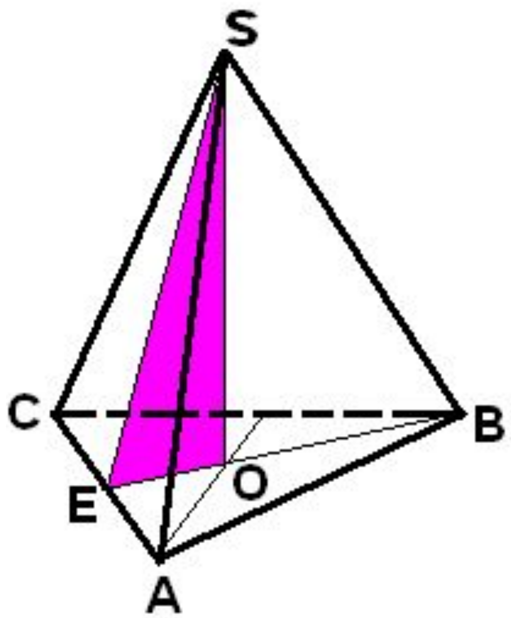
$$A_l = \frac{1}{2} \cdot P_b \cdot Ap = \frac{1}{2} \cdot 48 \cdot 8 = 192(cm^2)$$

1. Fie SABC o piramidă triunghiulară regulată cu muchia bazei AB=6 cm şi înălţimea SO=2 cm. Să se calculeze aria şi volumul piramidei!

R

2. Fie VABC o piramidă triunghiulară regulată. Dacă se cunoaşte muchia VB=10 cm, înălţimea VO=8 cm să se calculeze aria şi volumul piramidei!

R



$$V=\frac{1}{3}A_b h \Rightarrow V=\frac{1}{3}\cdot\frac{6^2\sqrt{3}}{4}\cdot 2=6\sqrt{3}\left(cm^3\right)$$

$$A=A_l+A_b\ ;\quad A_l=\frac{1}{2}P_b Ap;\quad P_b=3\cdot 6=18(cm)$$

$$\Delta SOE : OE=\frac{1}{3}\cdot\frac{6\sqrt{3}}{2}=\sqrt{3}$$

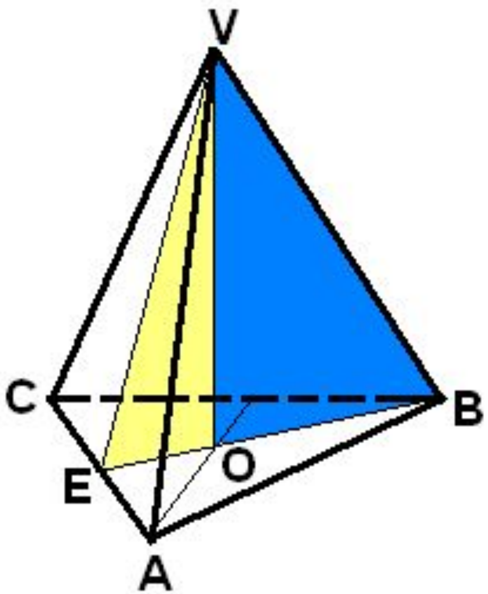
$$SO^2+OE^2=SE^2$$

$$2^2+\left(\sqrt{3}\right)^2=SE^2$$

$$SE=\sqrt{7}cm=Ap$$

$$A_l=\frac{1}{2}\cdot 18\cdot\sqrt{7}=9\sqrt{7}(cm)$$

$$A=9\sqrt{7}+9\sqrt{3}=9\left(\sqrt{7}+\sqrt{3}\right)cm^2$$



$$\Delta VOB: VO^2 + OB^2 = VB^2 \Rightarrow OB^2 = 100 - 64 \Rightarrow OB = \sqrt{36} = 6(cm)$$

$$dar \quad OB = \frac{2}{3} \cdot \frac{AB\sqrt{3}}{2} \Rightarrow AB = \frac{18}{\sqrt{3}} = 6\sqrt{3}(cm)$$

$$V = \frac{1}{3} \cdot A_{ABC} \cdot VO \Rightarrow V = \frac{1}{3} \cdot \frac{\left(6\sqrt{3}\right)^2 \sqrt{3}}{4} \cdot 8 = 72\sqrt{3}(cm^3)$$

$$\Delta VEC: \left.\begin{matrix} VC = 10cm \\ EC = \dfrac{6\sqrt{3}}{2}cm \end{matrix}\right\} \Rightarrow VE^2 = VC^2 - EC^2 \Rightarrow VE^2 = 10^2 - \left(3\sqrt{3}\right)^2 \Rightarrow VE = \sqrt{73}cm = Ap$$

$$A = A_l + A_b \Rightarrow A = \frac{1}{2} \cdot P_b \cdot h + A_b \Rightarrow A = \frac{1}{2} \cdot 18\sqrt{3} \cdot \sqrt{73} + 27\sqrt{3} = 9\sqrt{3}\left(\sqrt{73} + 3\right)(cm^2)$$