

Definiții, notații

Piramide regulate

Realizarea desenelor

Formule de calcul

Aplicații

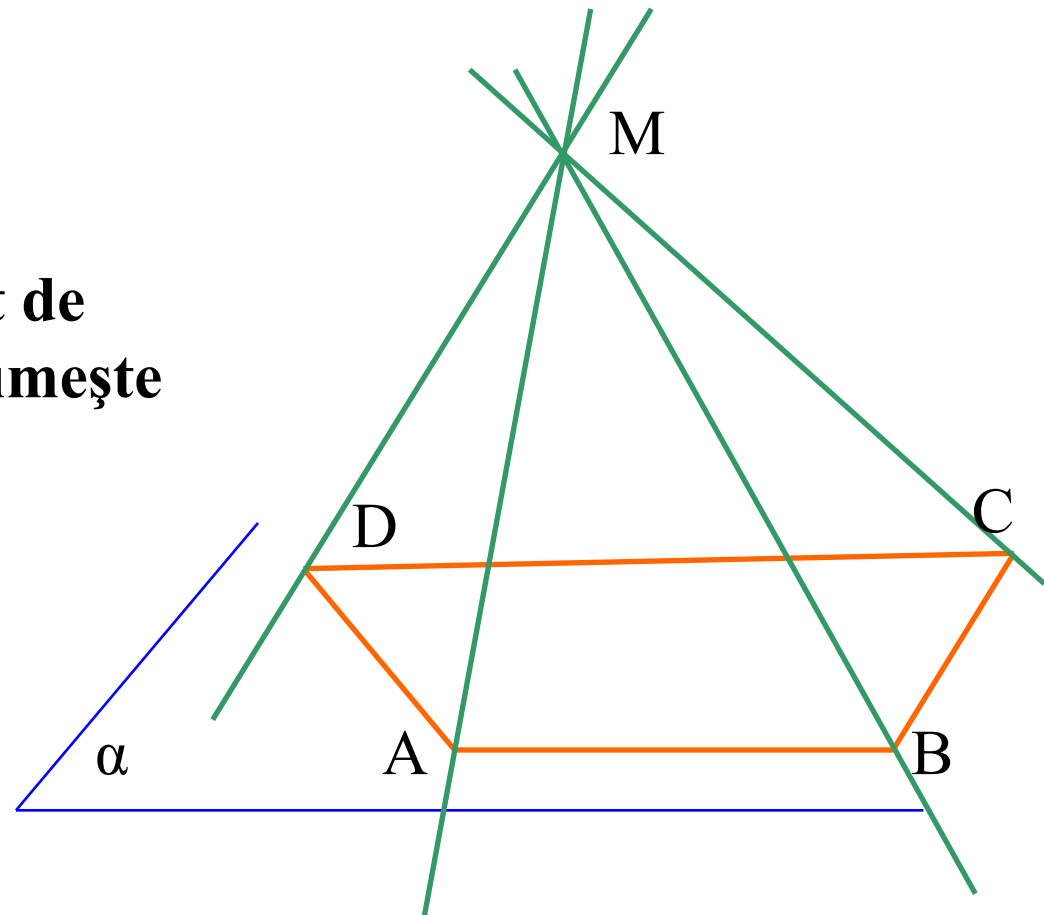
PIRAMIDA



Fie $ABCD$ un poligon în planul α , M un punct în afara planului.

Dacă prin punctul M se consideră o dreaptă d ce se deplasează pe laturile poligonului, aceasta descrie o suprafață de piramidă.

Corpul geometric delimitat de punctul M și planul α se numește *piramida* $MABCD$.



Elementele piramidei

Bază

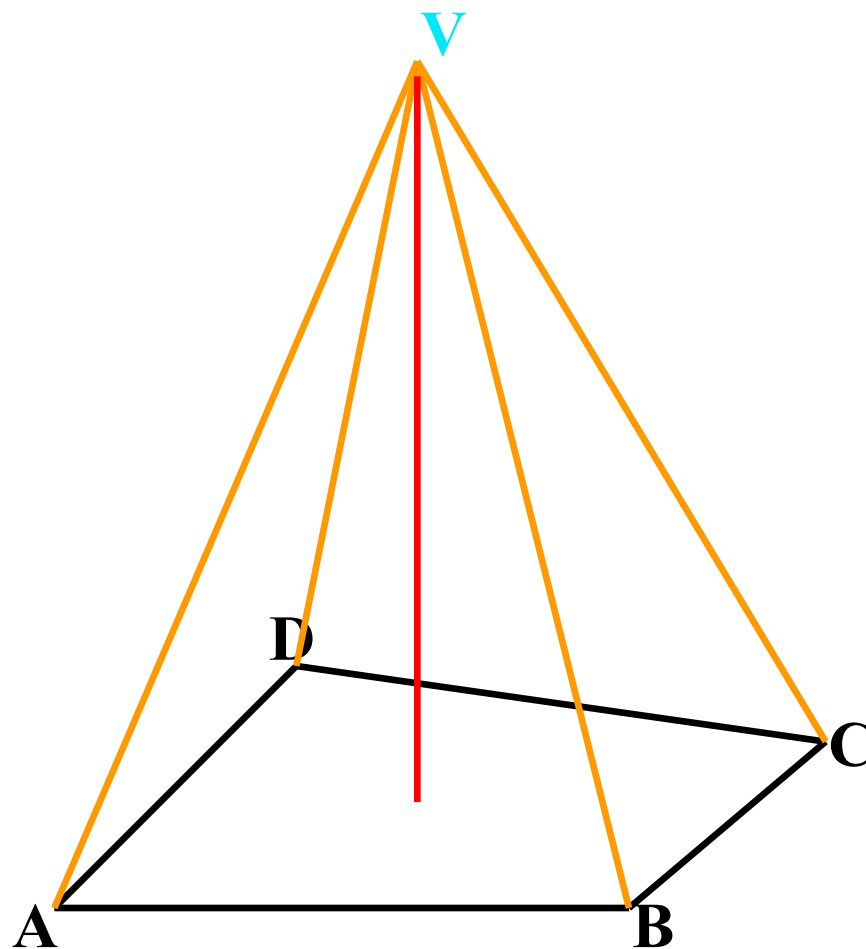
Muchii ale bazei

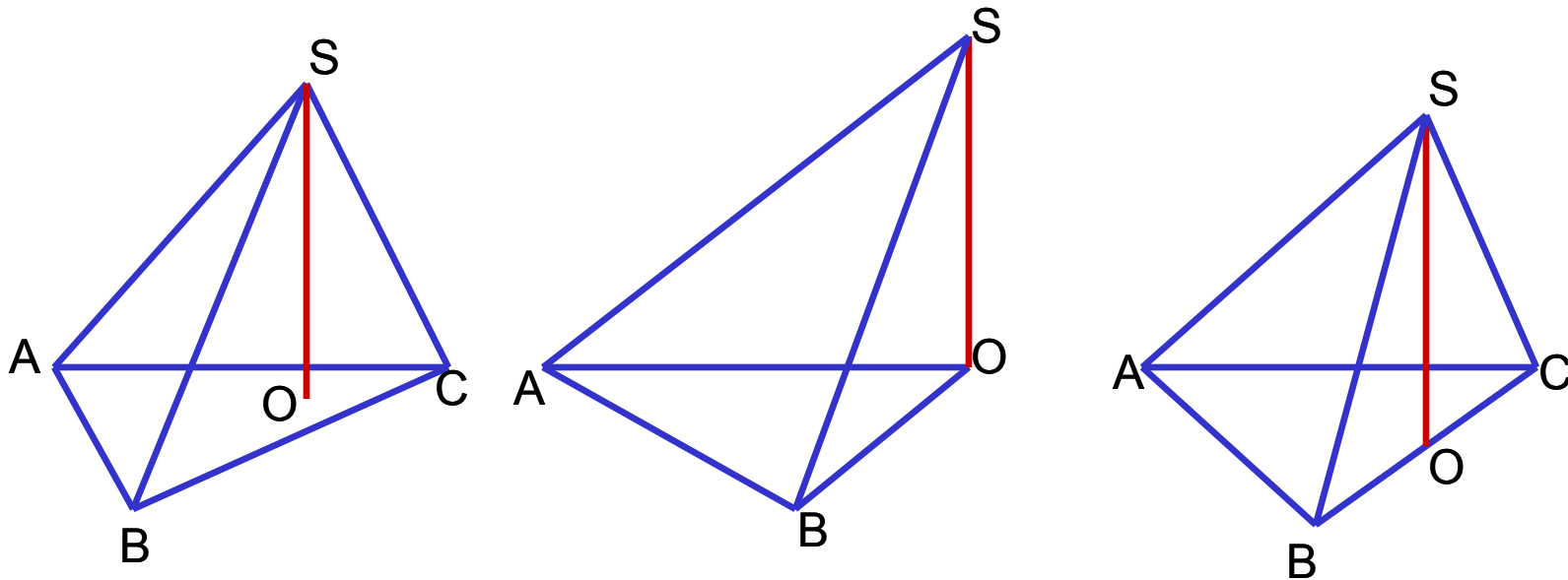
Fețe laterale

Muchii laterale

Vârf

Înălțime: distanța vârfului
piramidei de la planul bazei



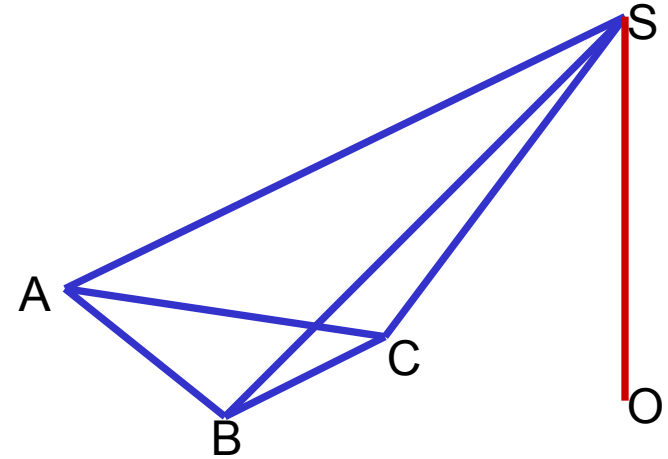
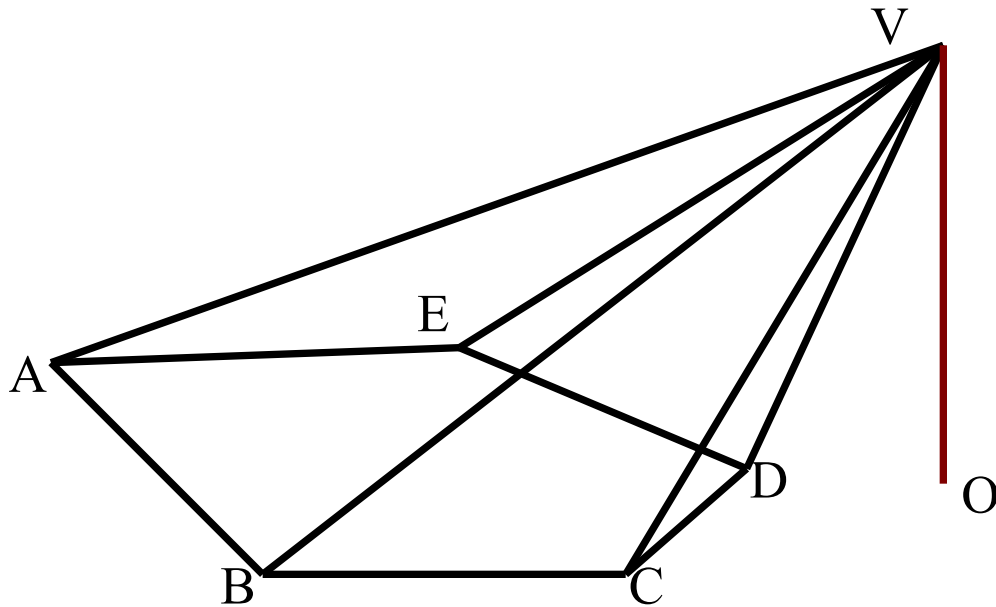


Dacă piciorul perpendicularei duse din vârful piramidei pe planul bazei este în domeniul interior al acestuia atunci piramida este o *piramidă dreaptă*.





În caz contrar *piramidă oblică*.





Piramida regulată

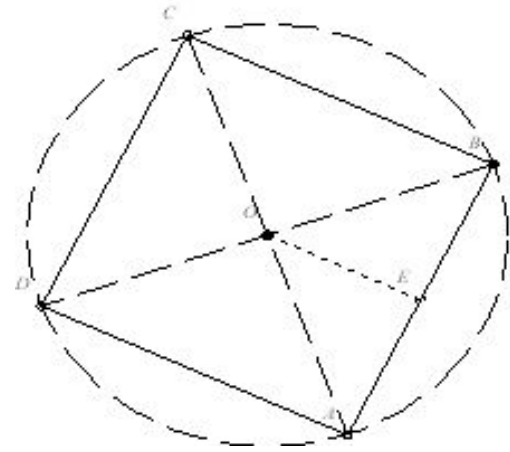
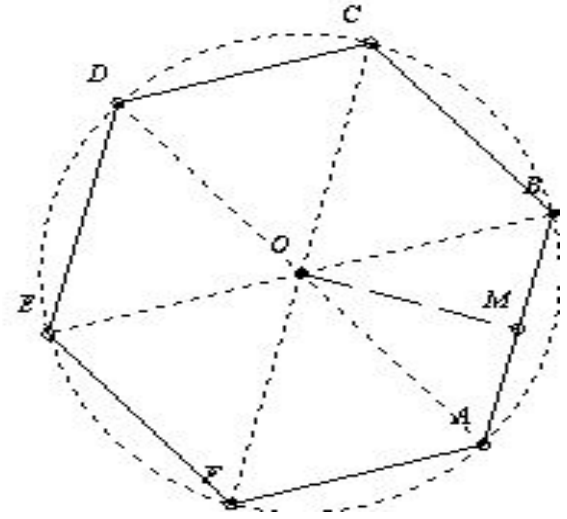
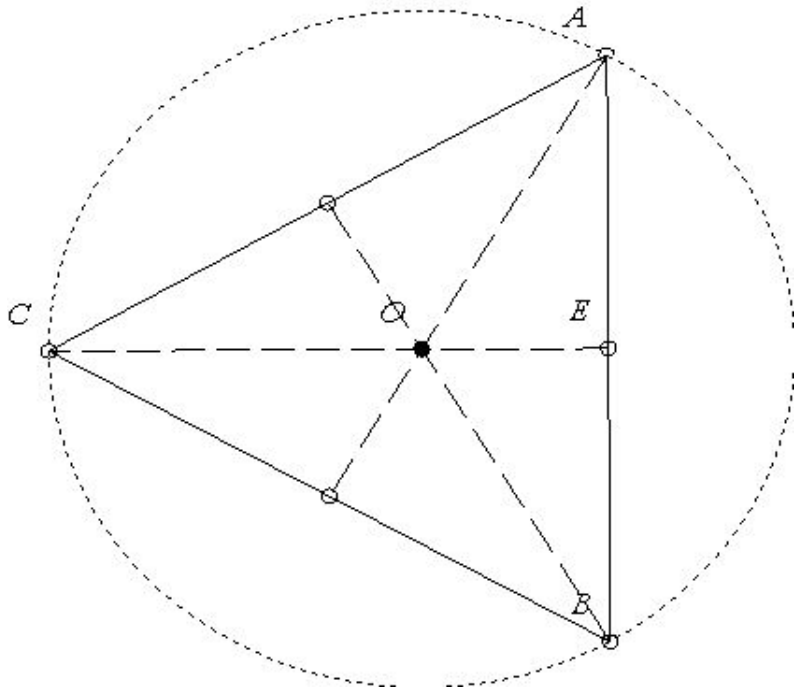
O piramidă se numește regulată, dacă:

- Are ca bază un poligon regulat (Δ echilateral, pătrat, hexagon regulat, etc)
- Picioarul perpendicularei duse din vârful piramidei coincide cu centrul poligonului
- (centrul poligonului = centrul cercului circumscris).

Toate muchiile unei piramide regulate sunt congruente, deci toate fețele sunt triunghiuri isoscele congruente.

Înălțimea unei fețe laterale se numește **apotema** piramidei.







Realizarea desenelor

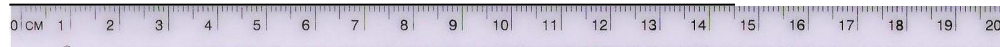
Piramidă triunghiulară regulată

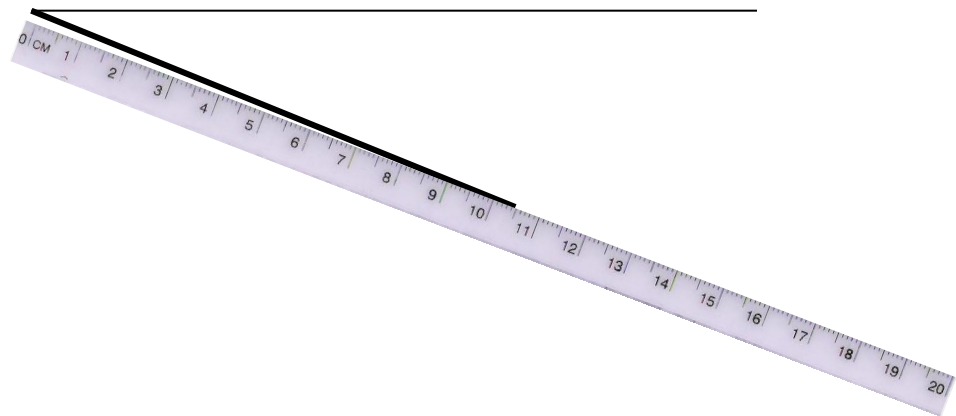
Piramidă patrulateră regulată

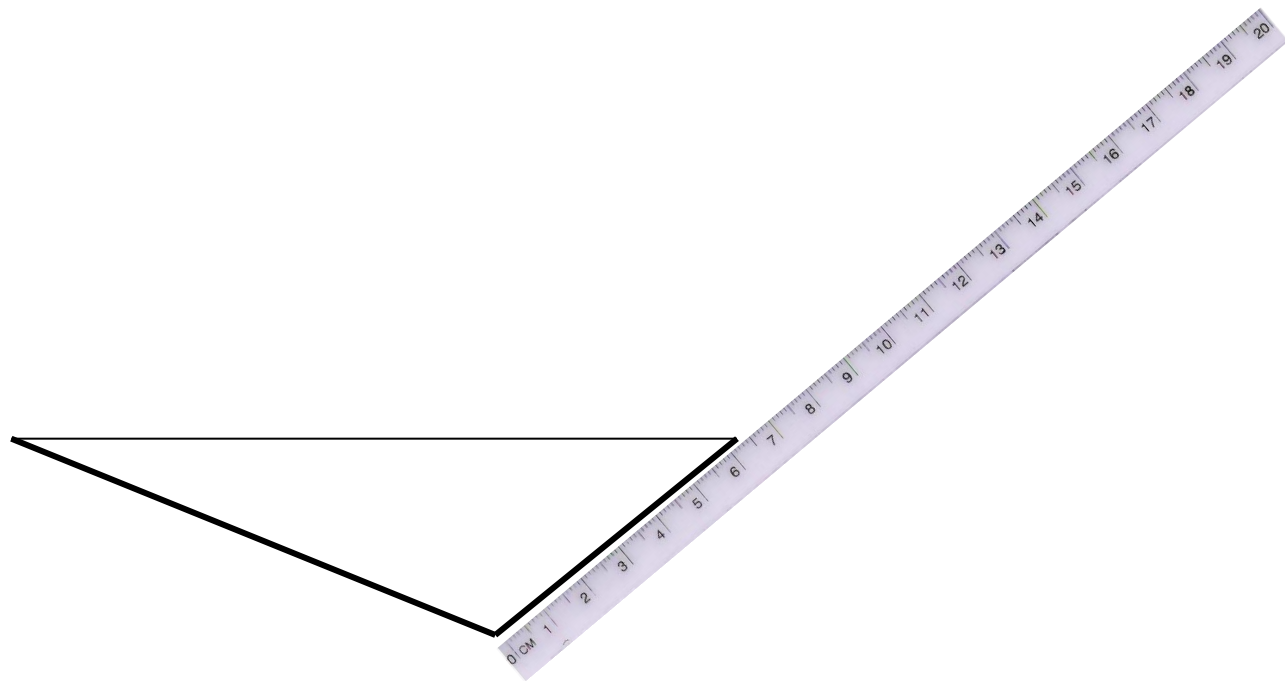
Piramidă hexagonală regulată

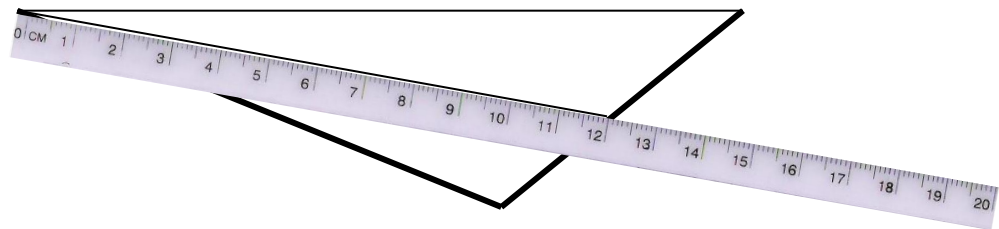


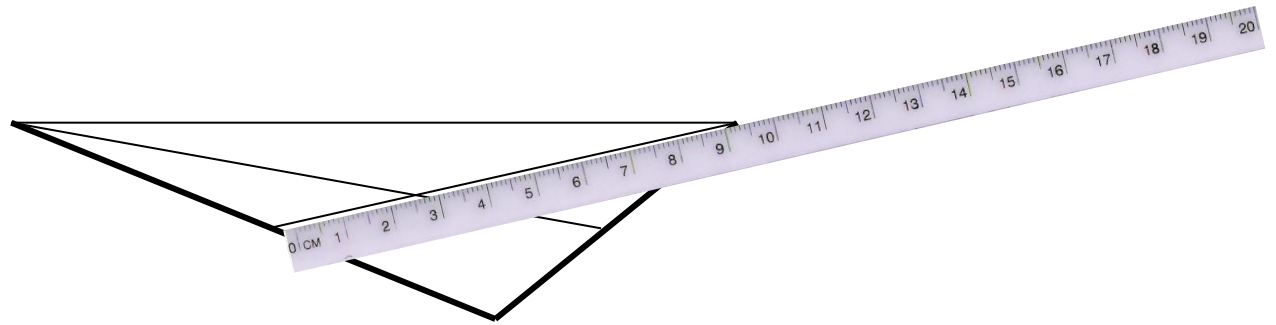
Piramidă triunghiulară regulată

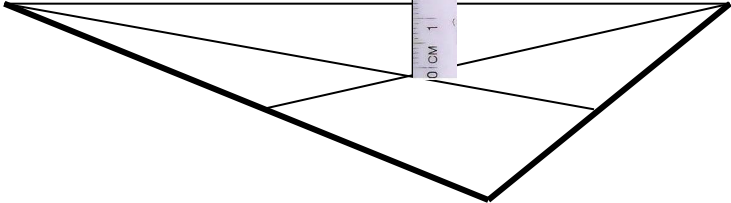


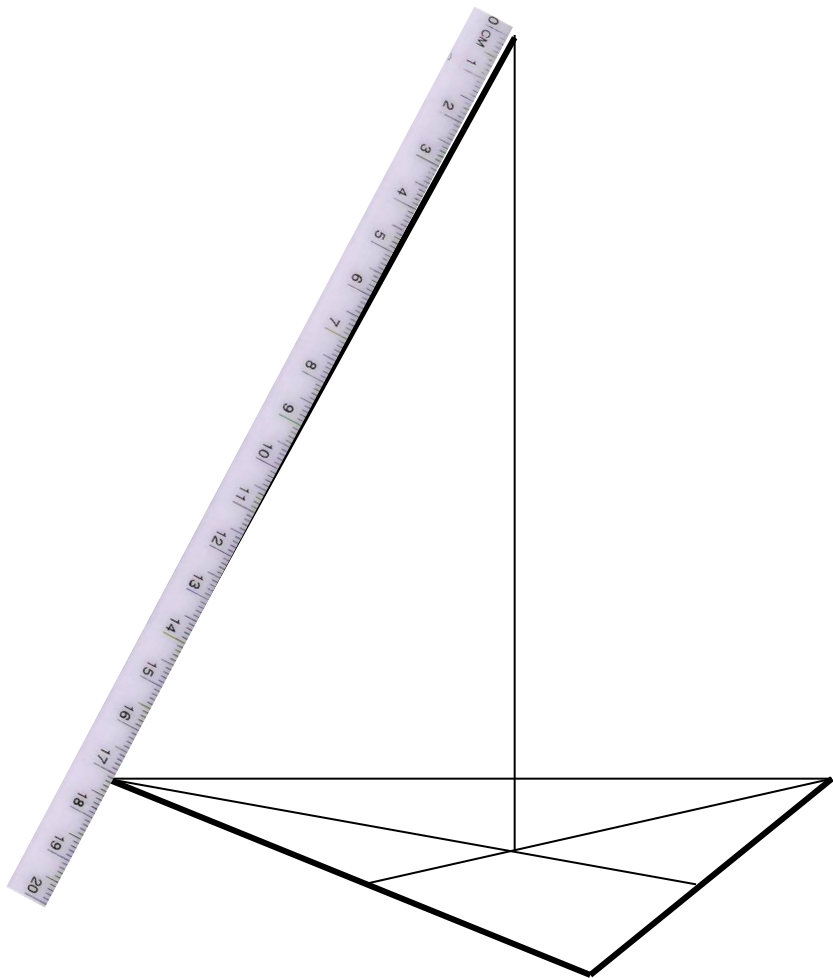


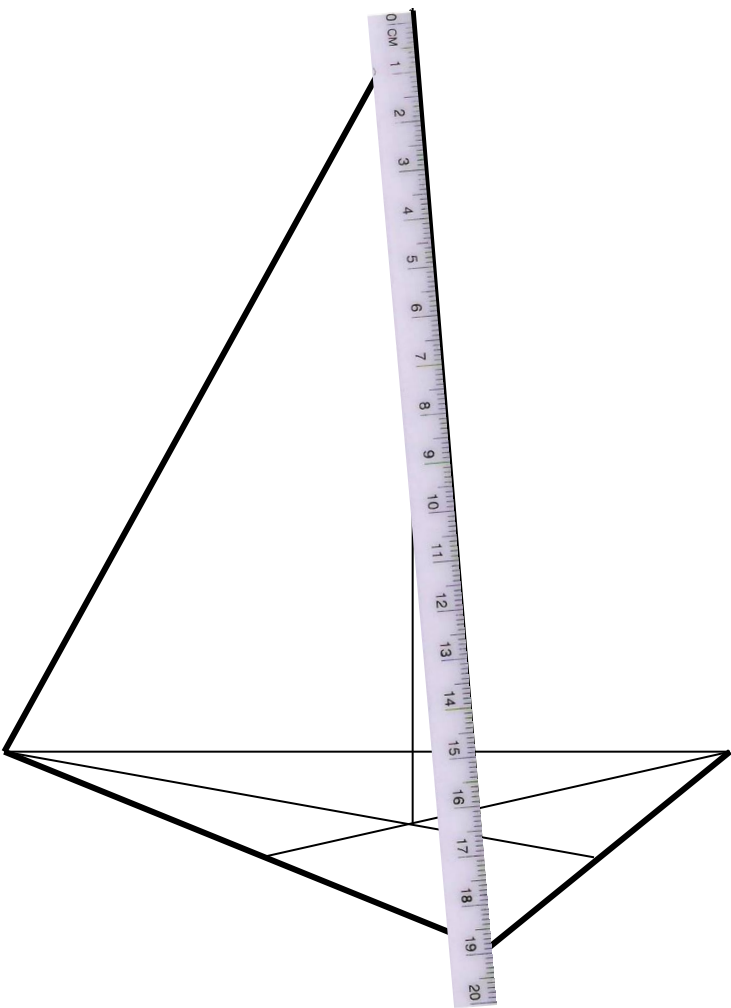


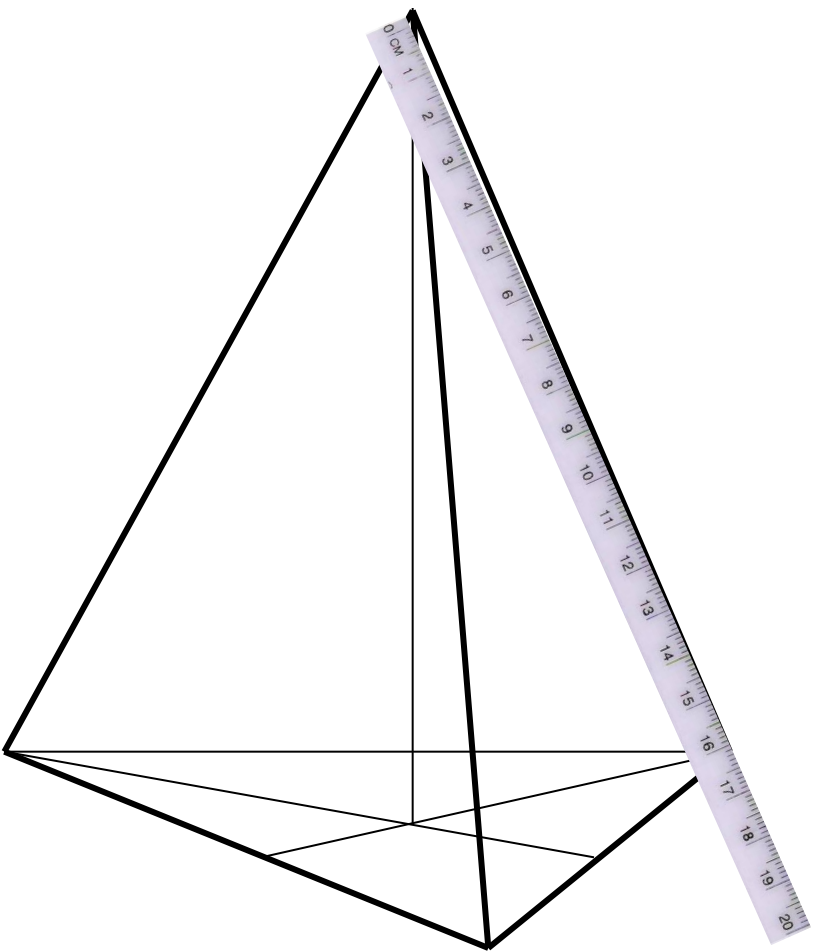


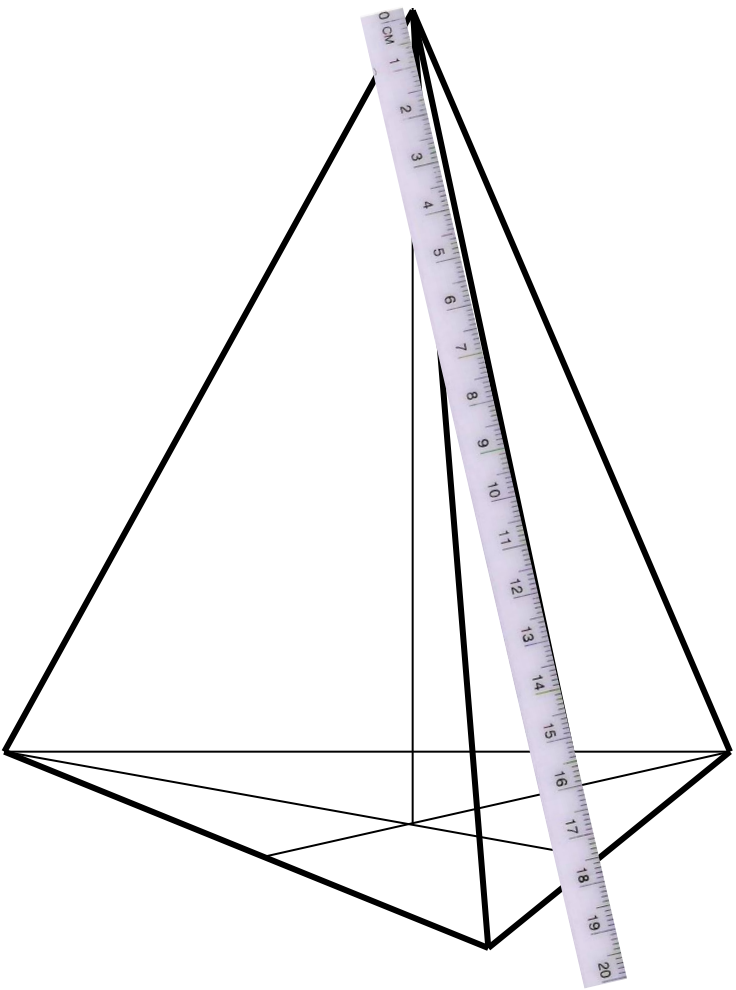


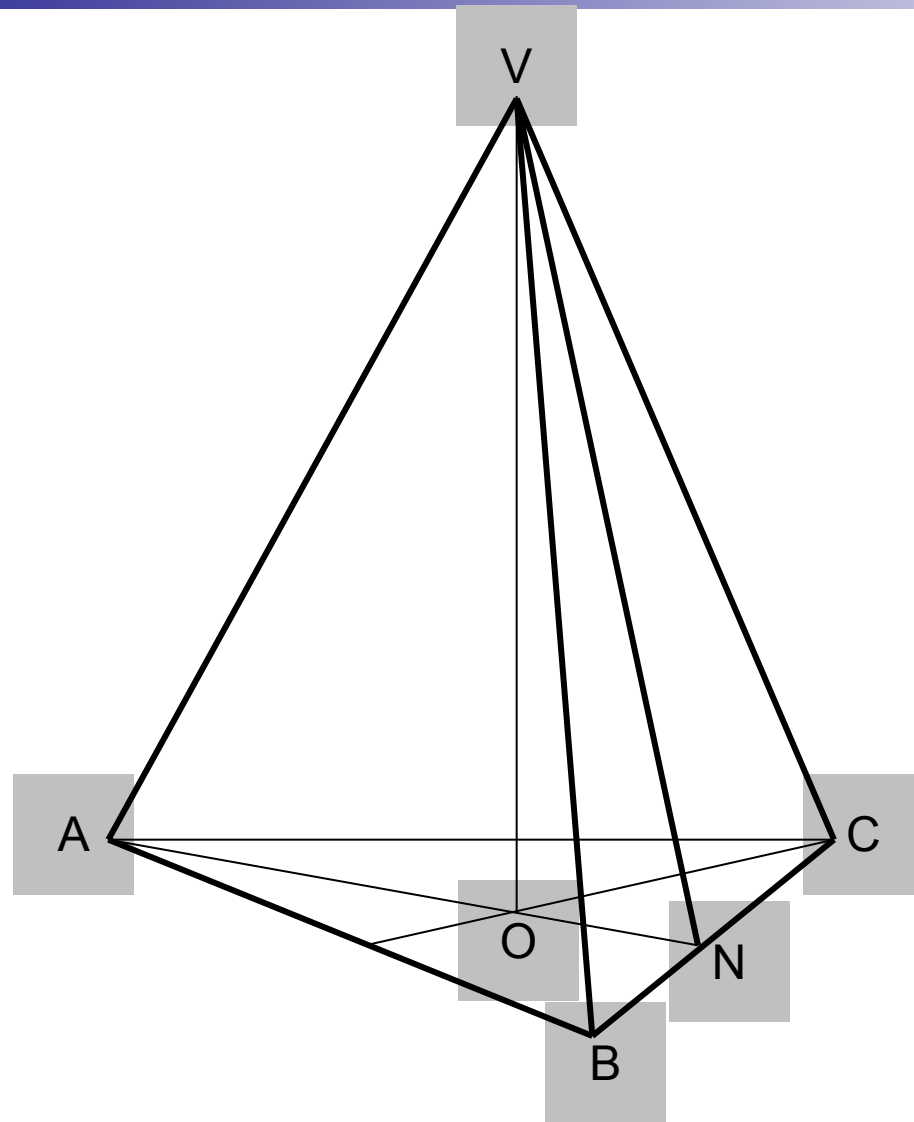




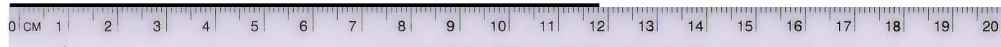


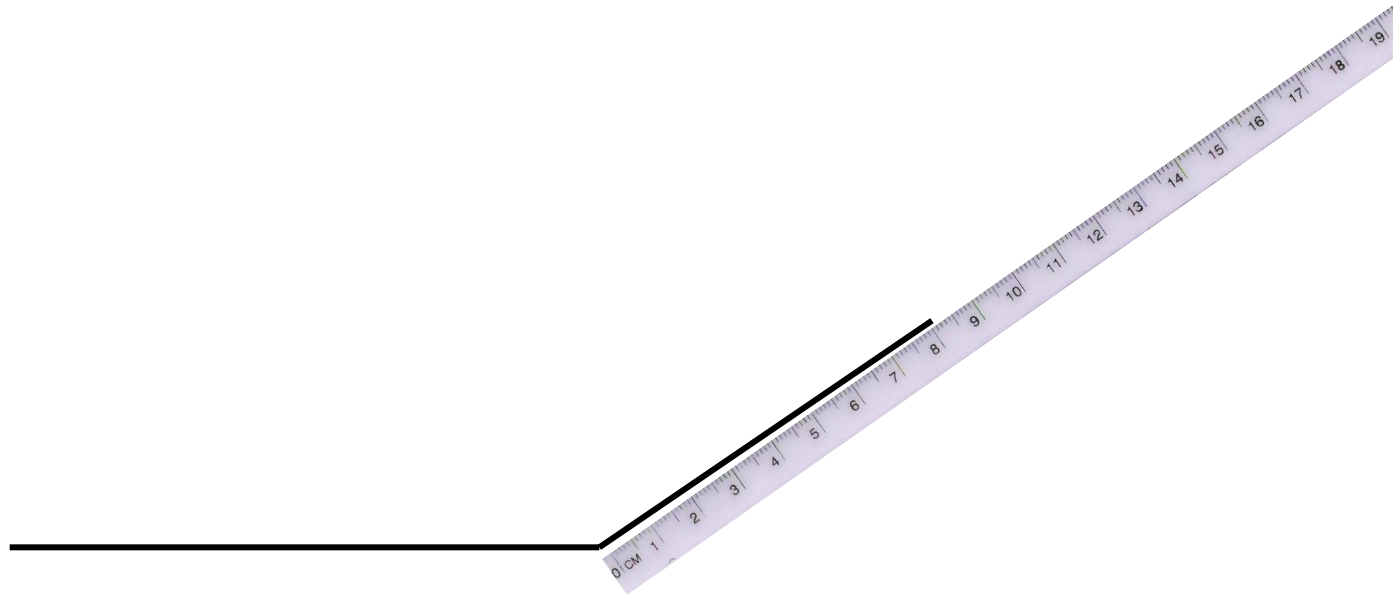


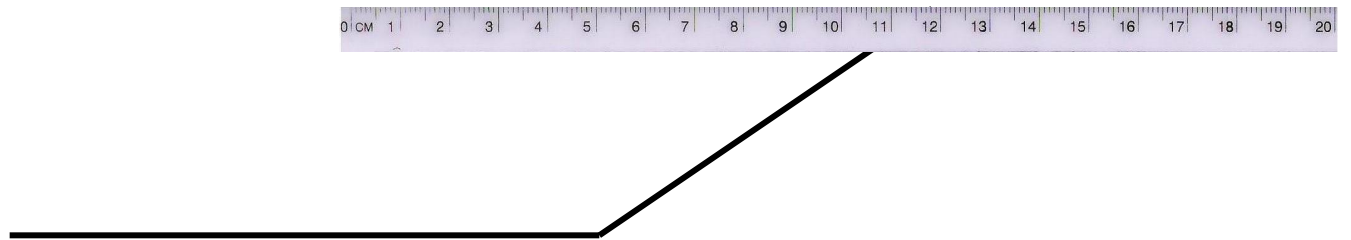


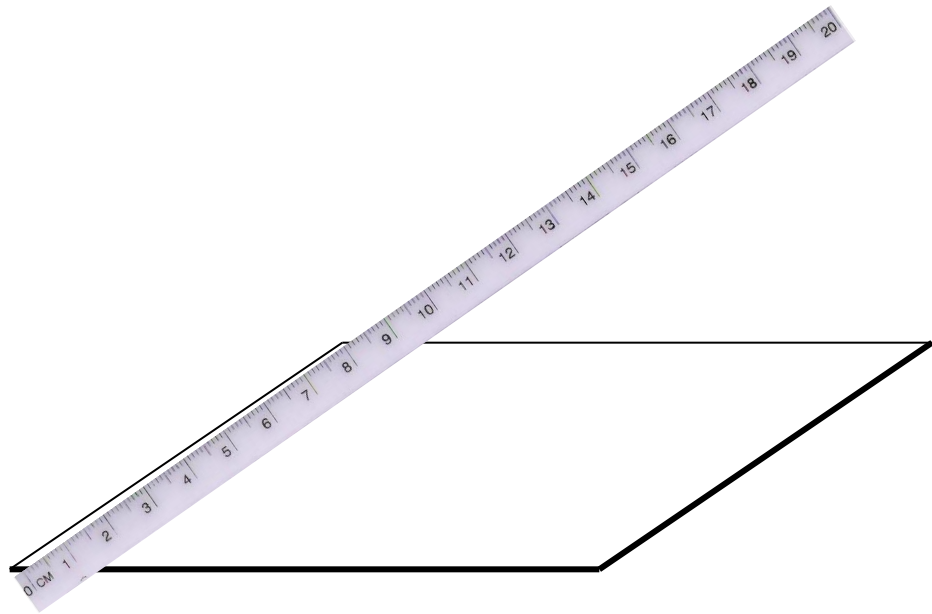


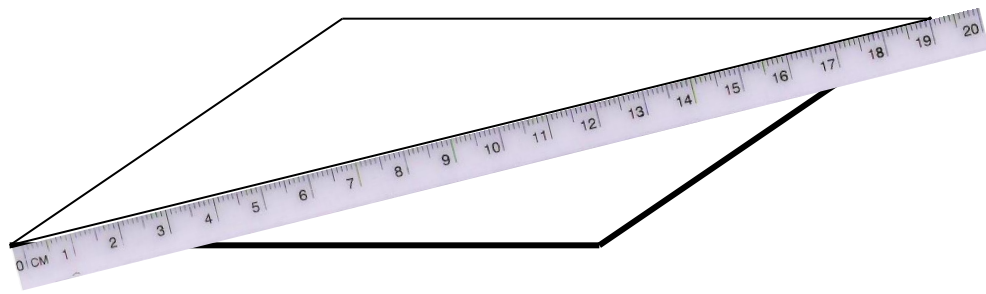
Piramidă patrulateră regulată

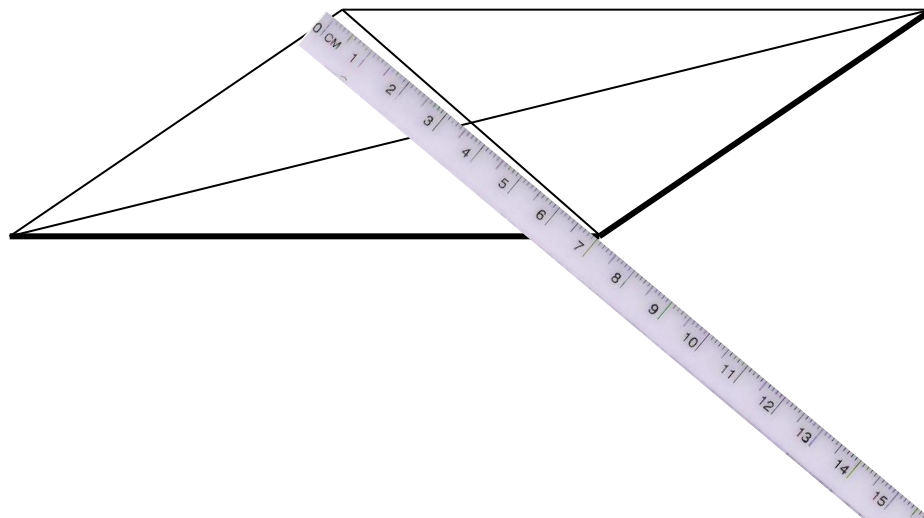


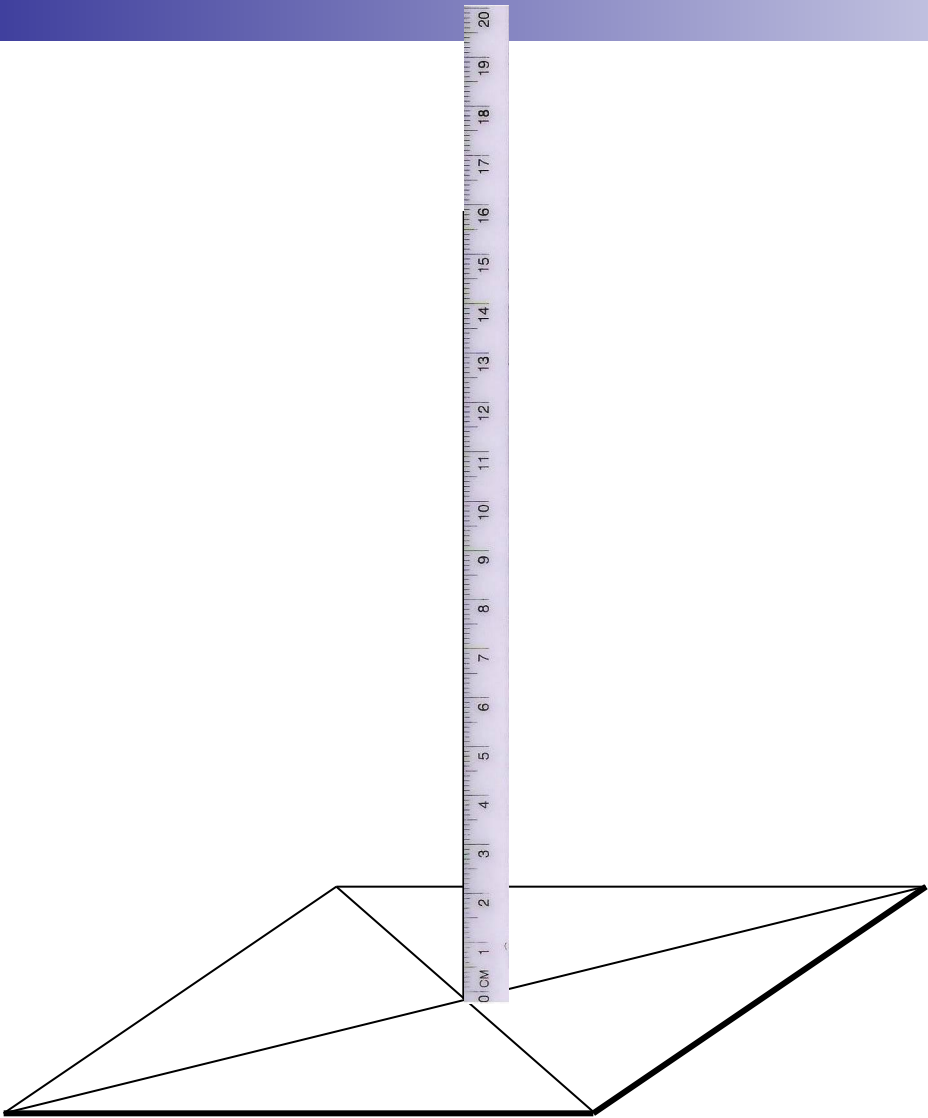


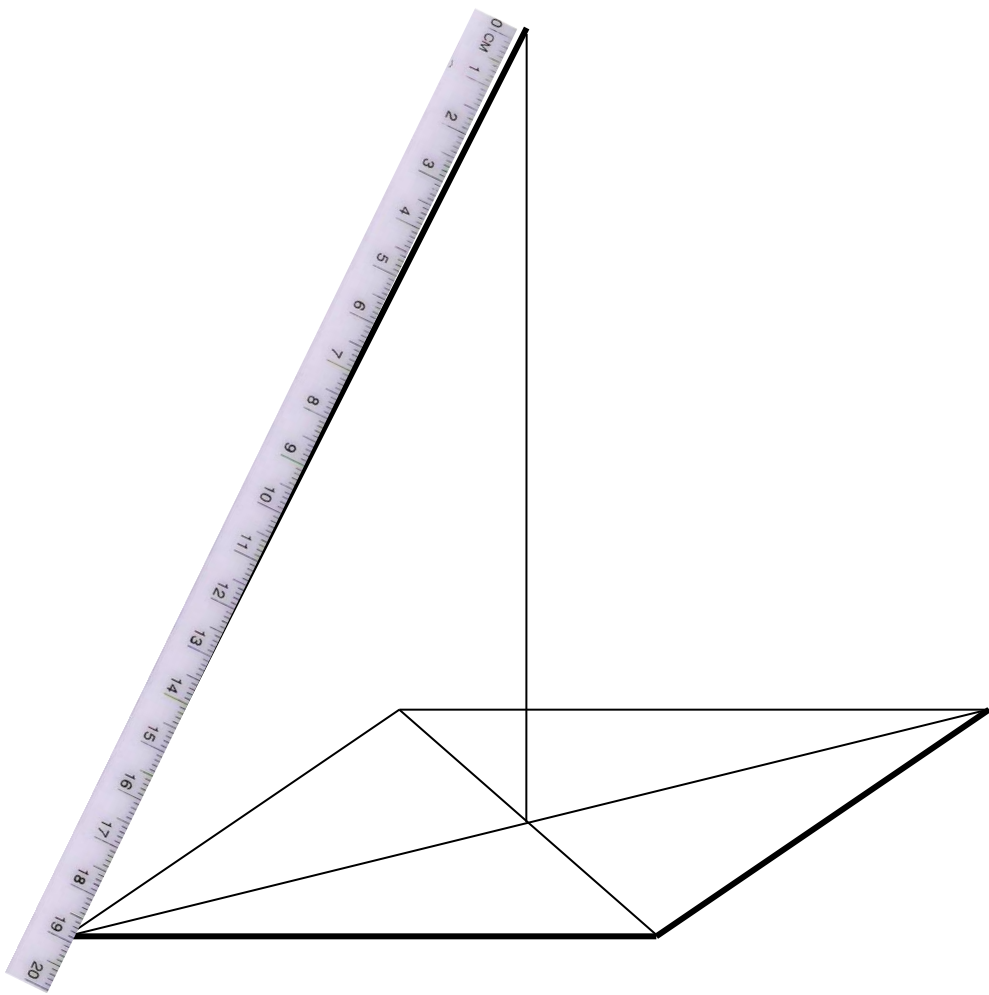


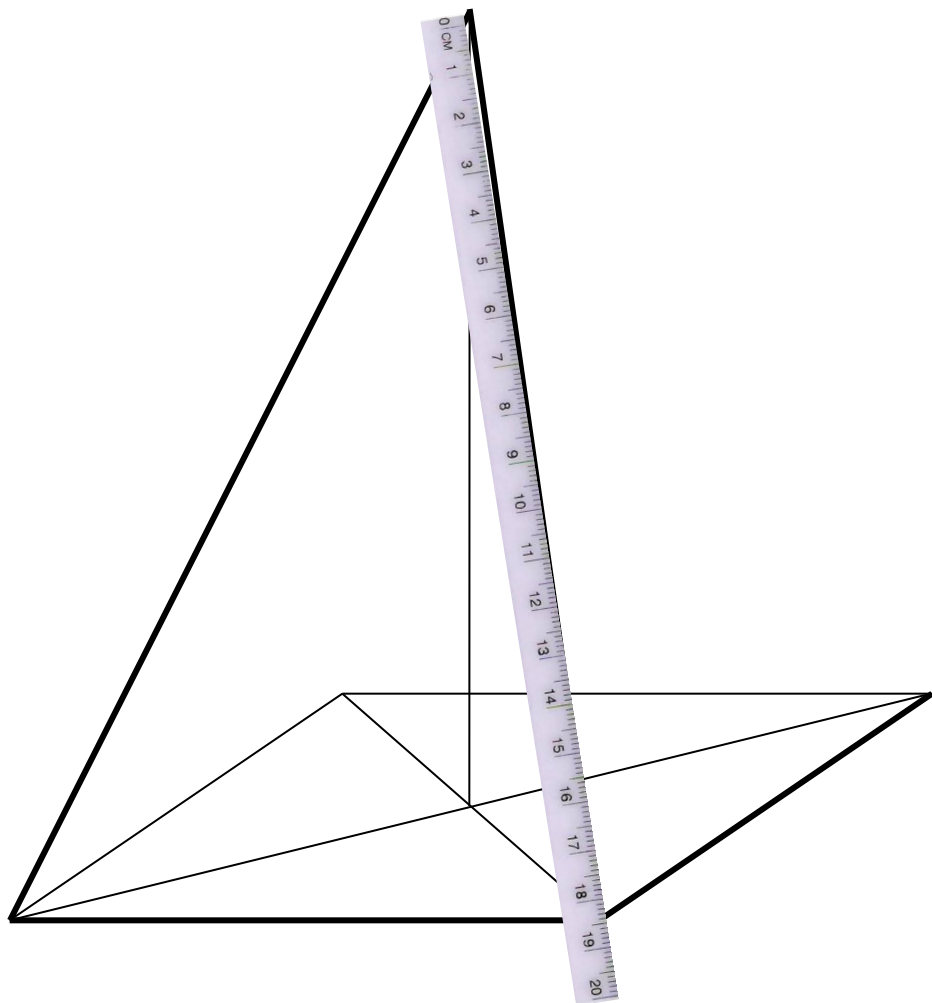


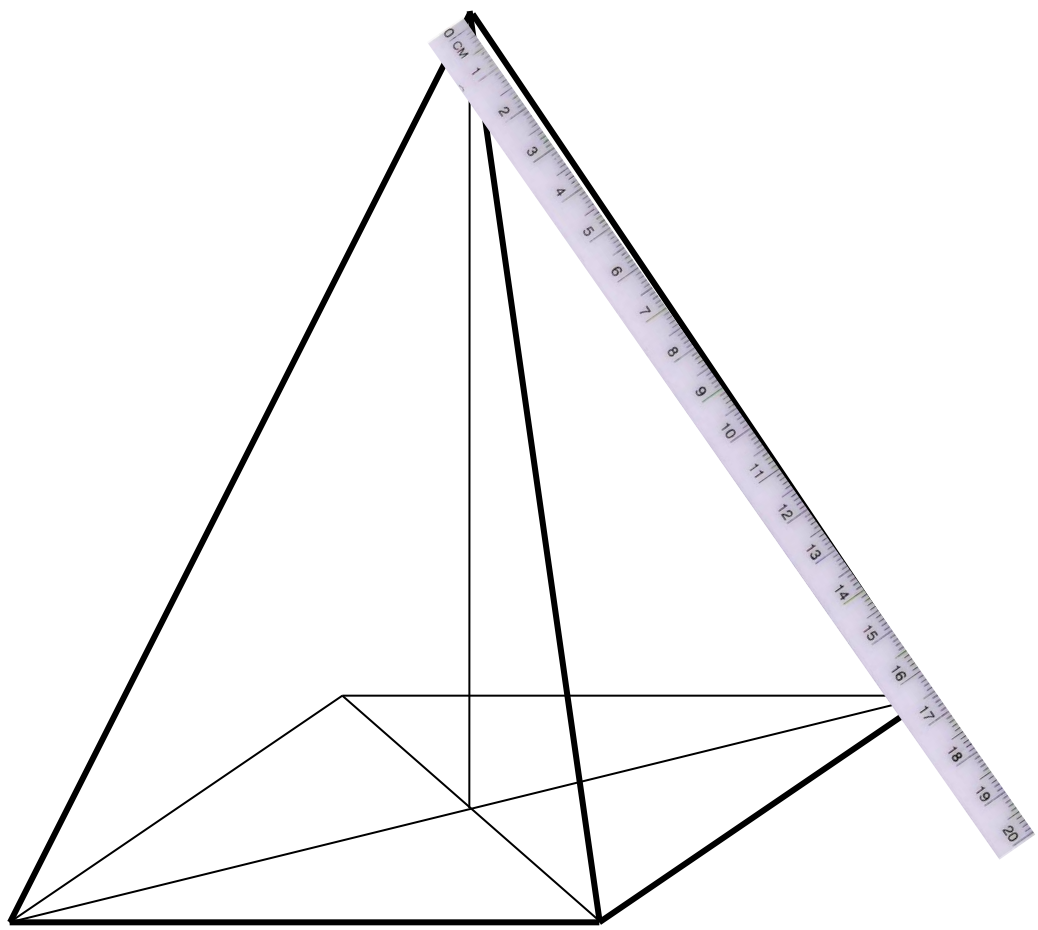


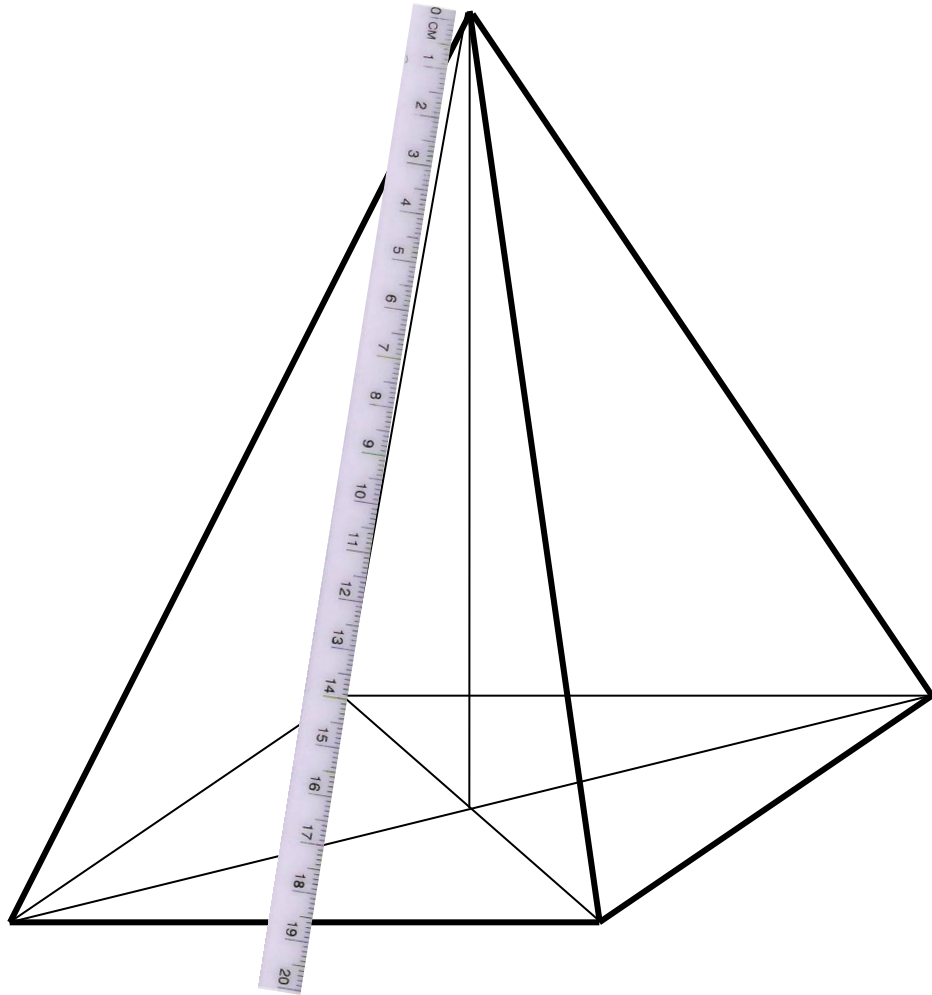


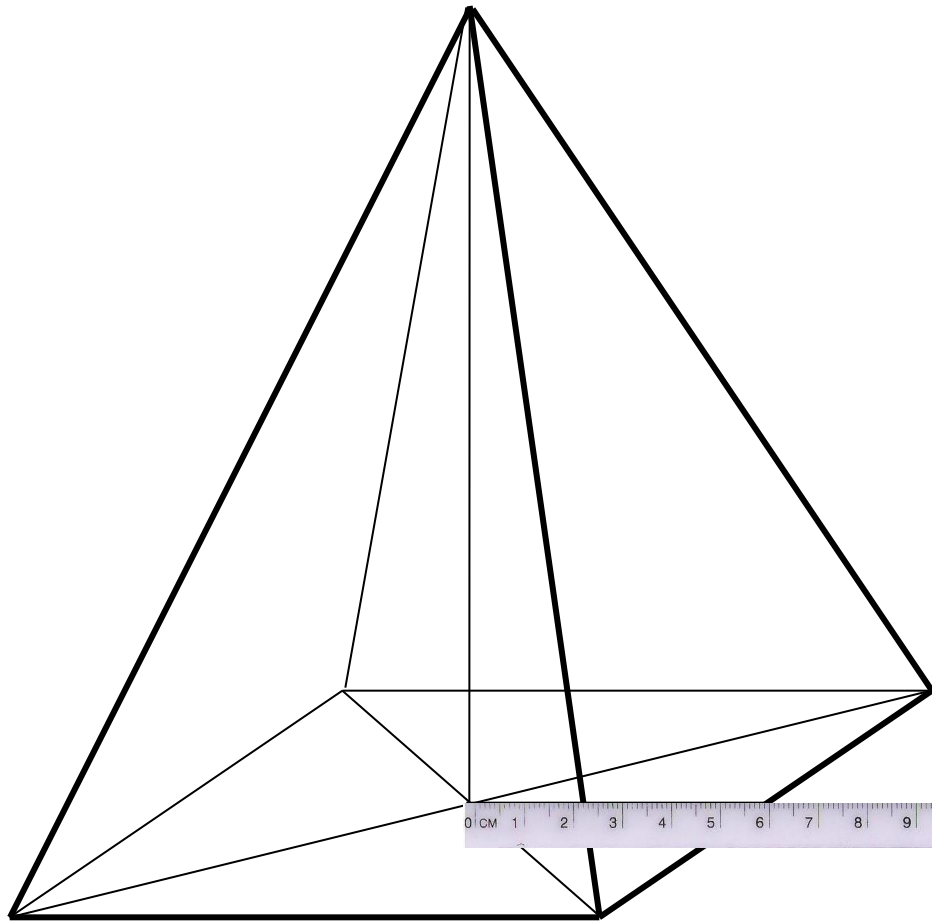


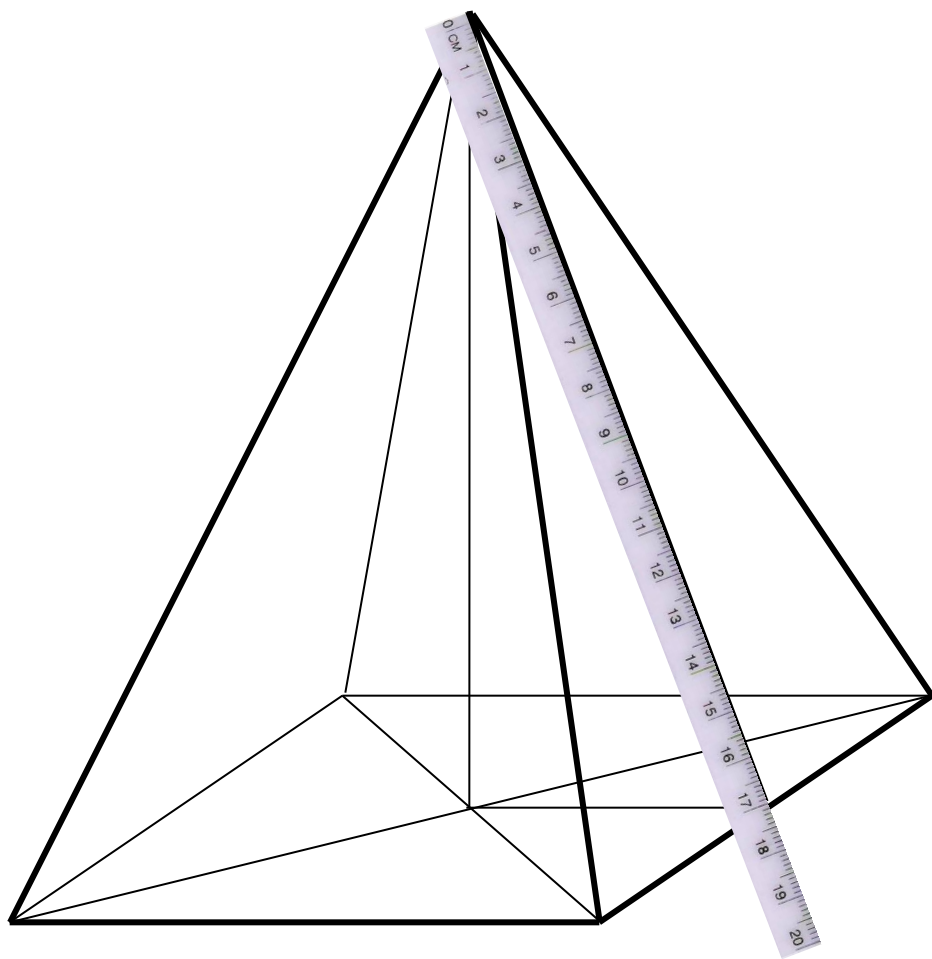


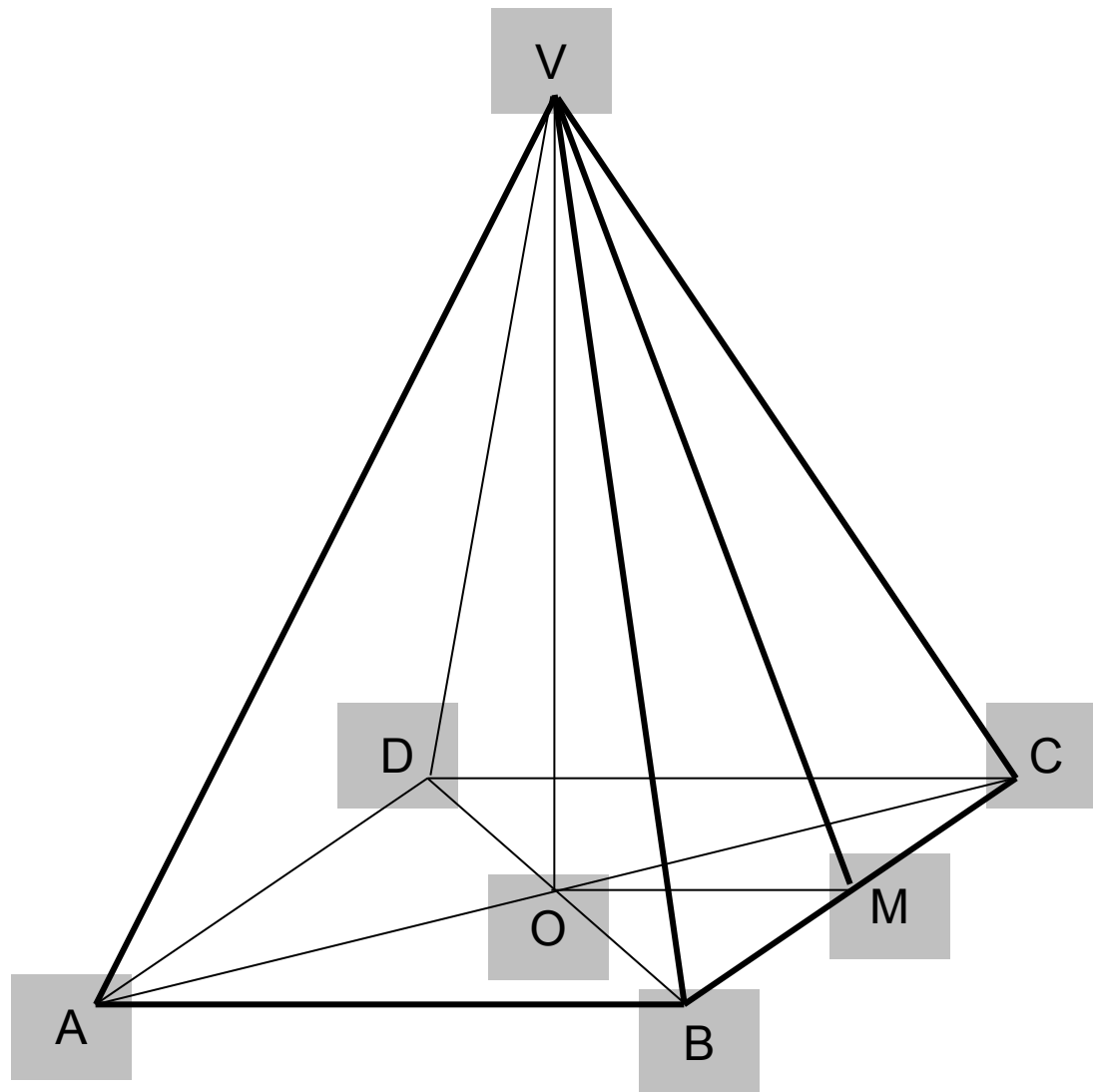






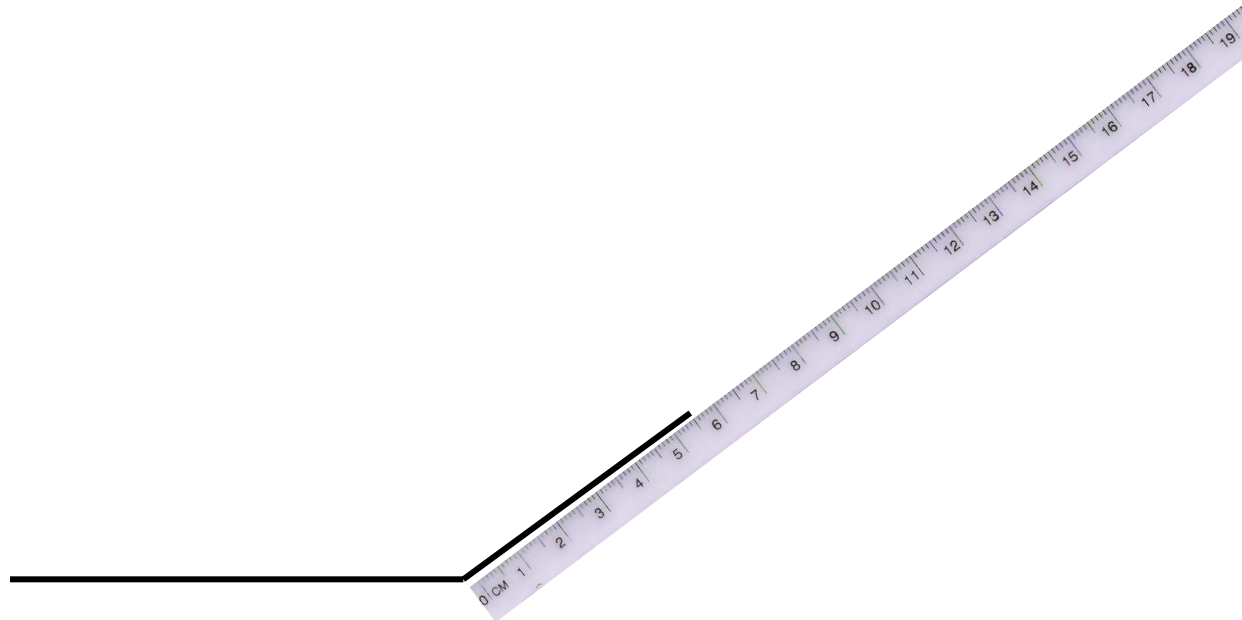


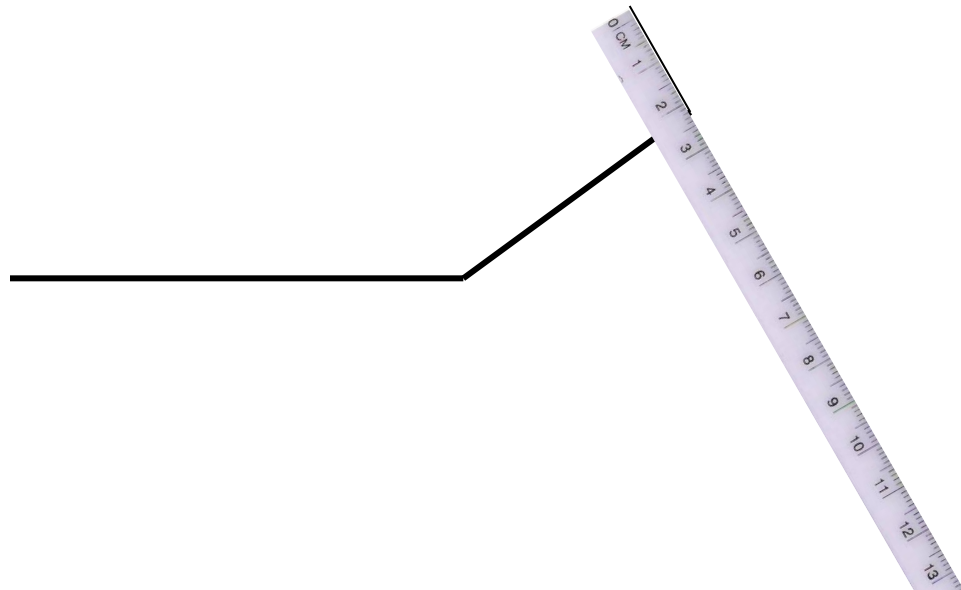


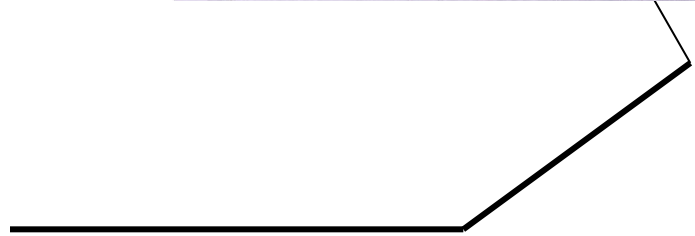


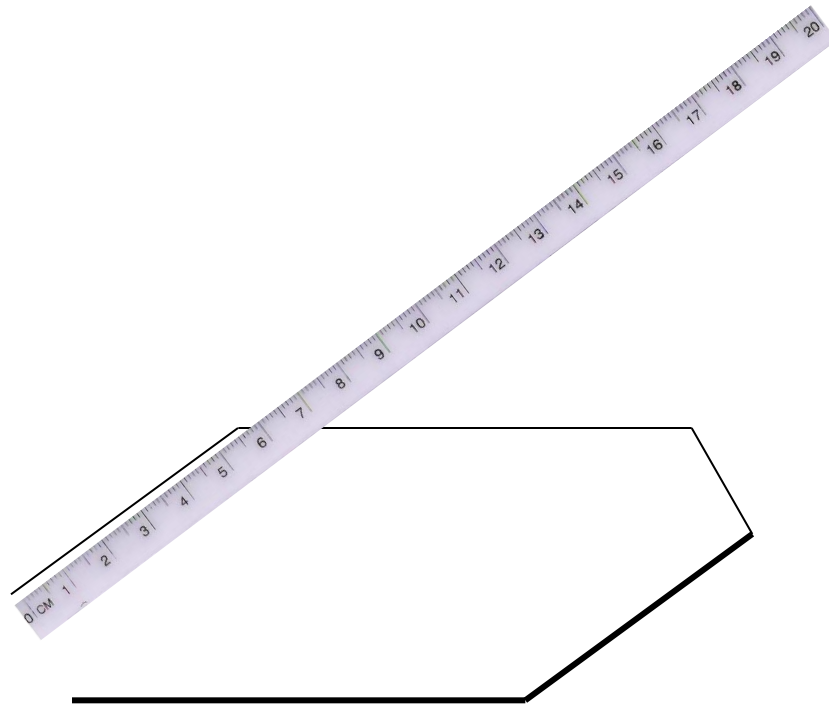
Piramidă hexagonală regulată

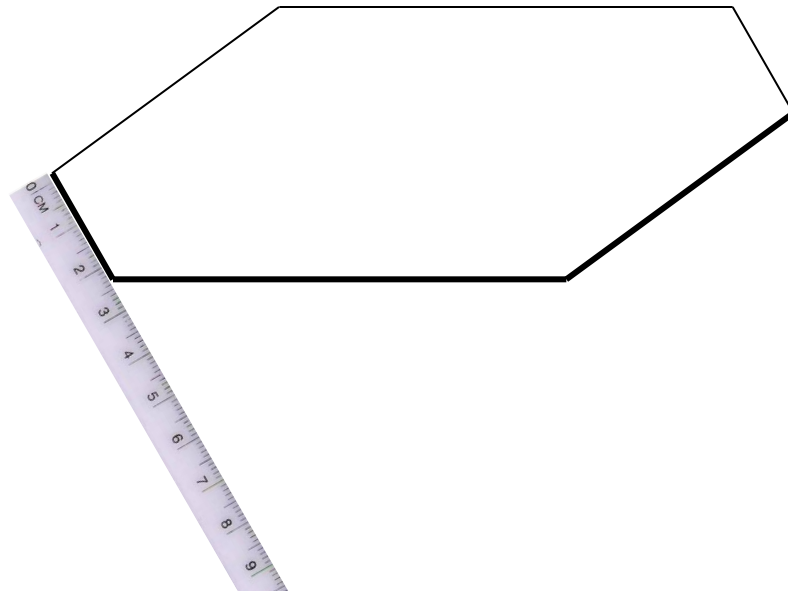


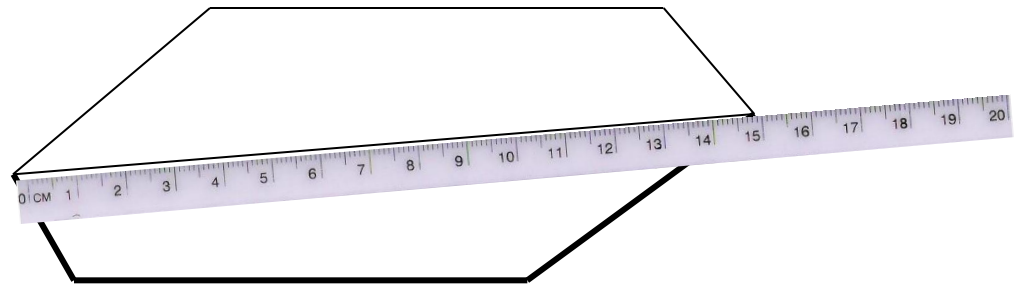


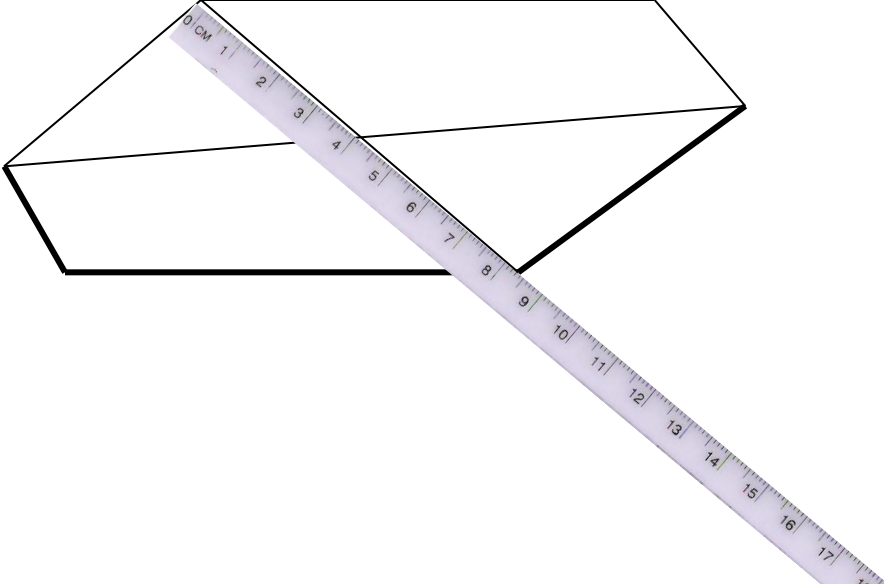


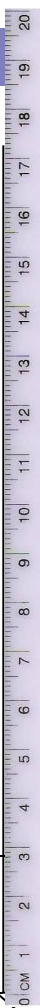
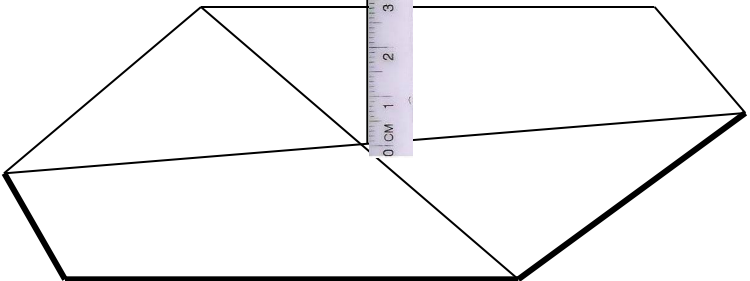


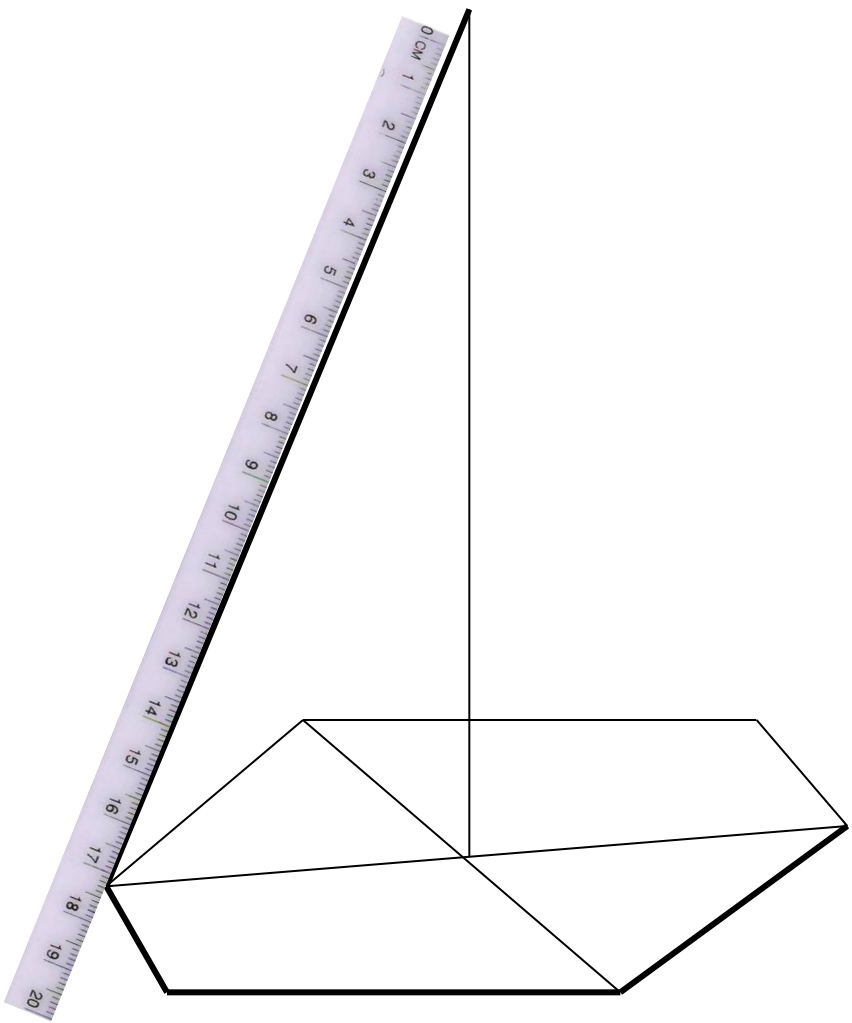


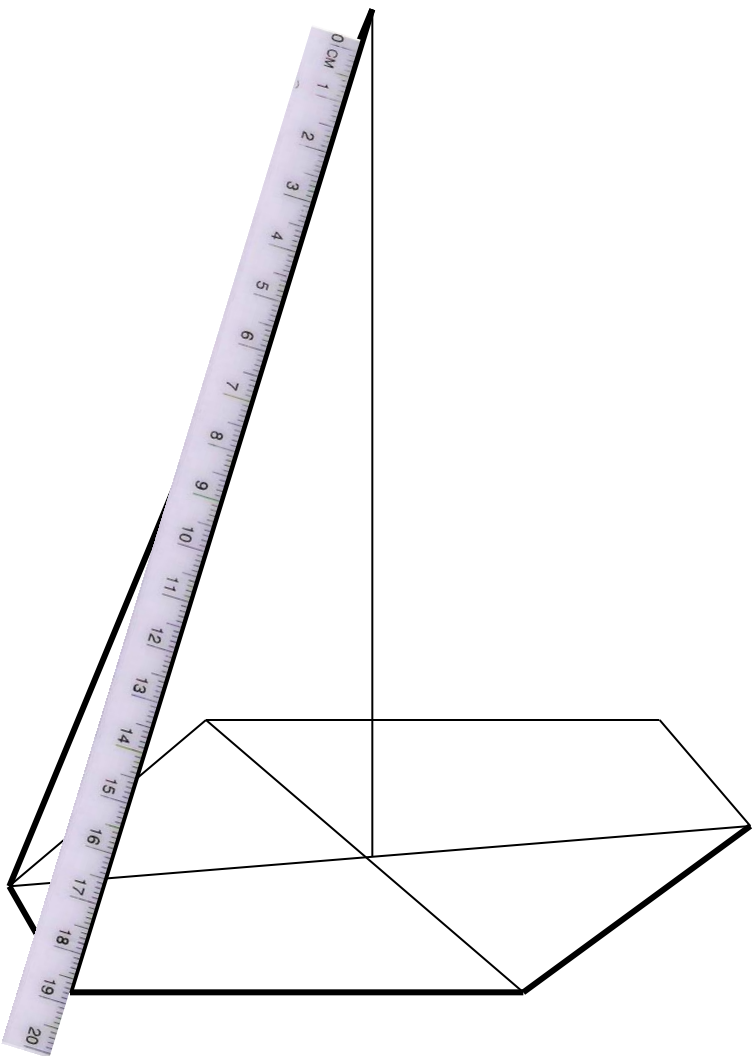


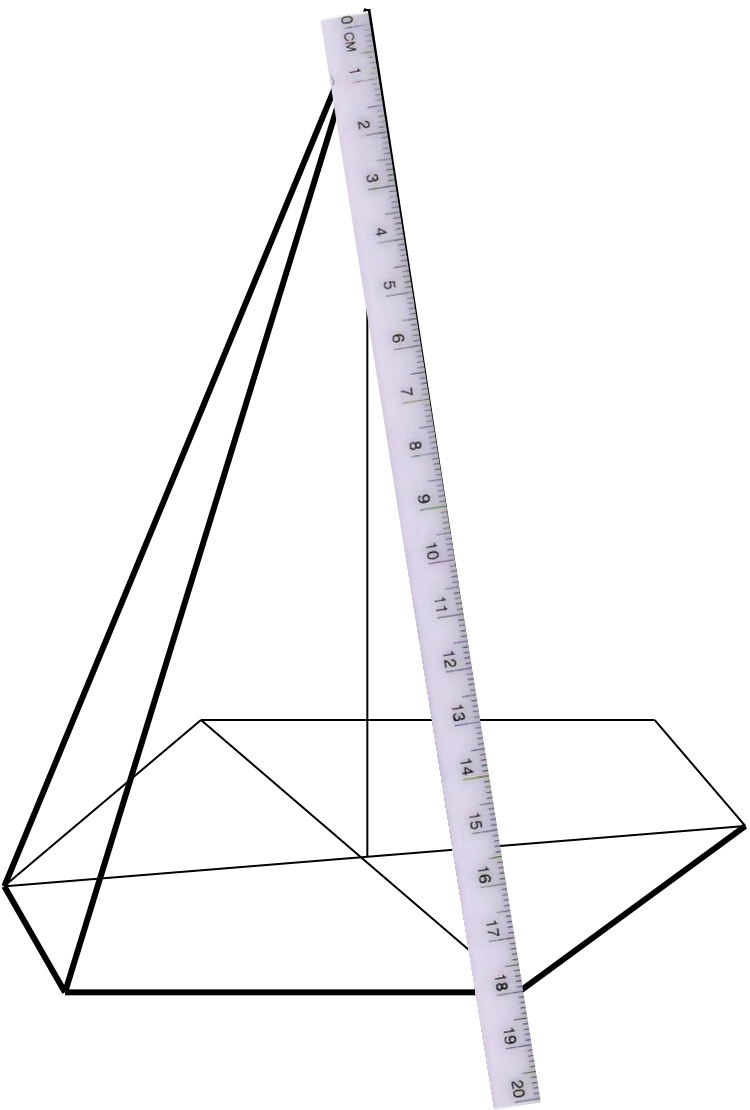


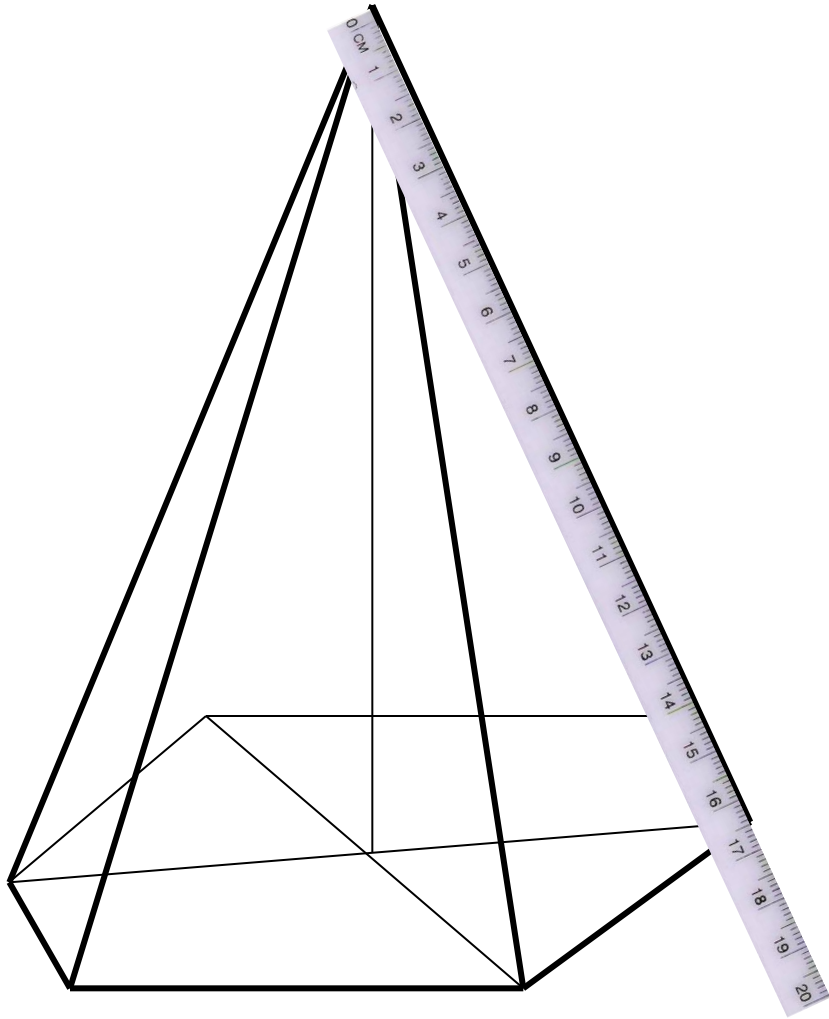


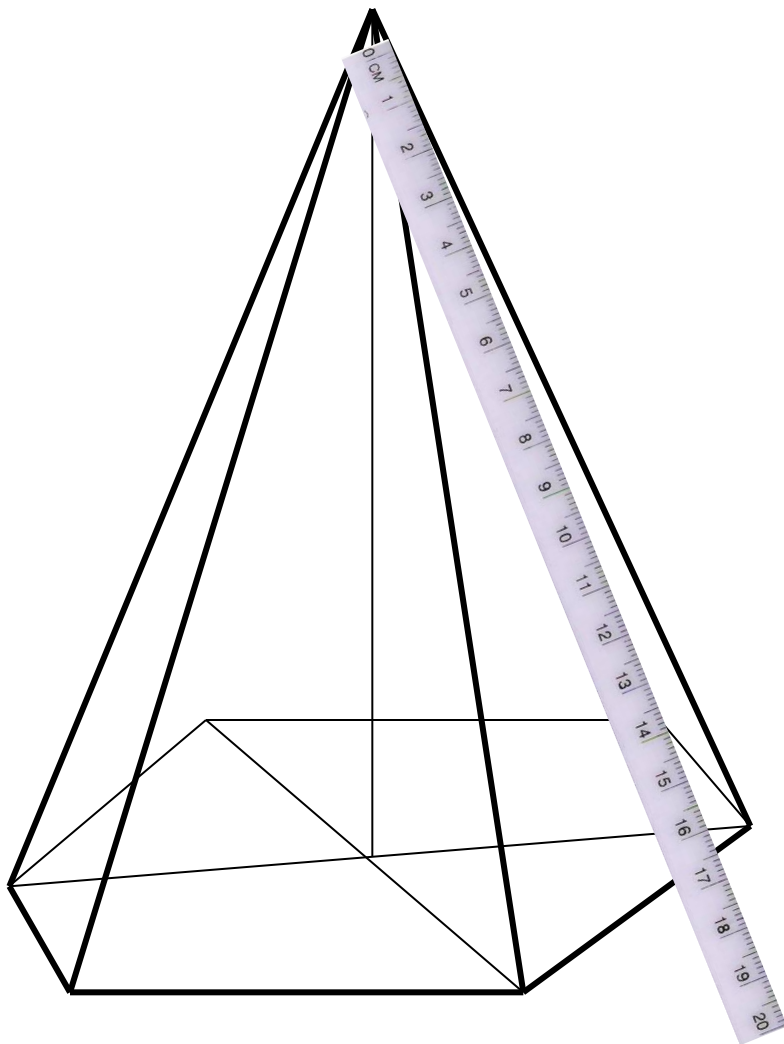


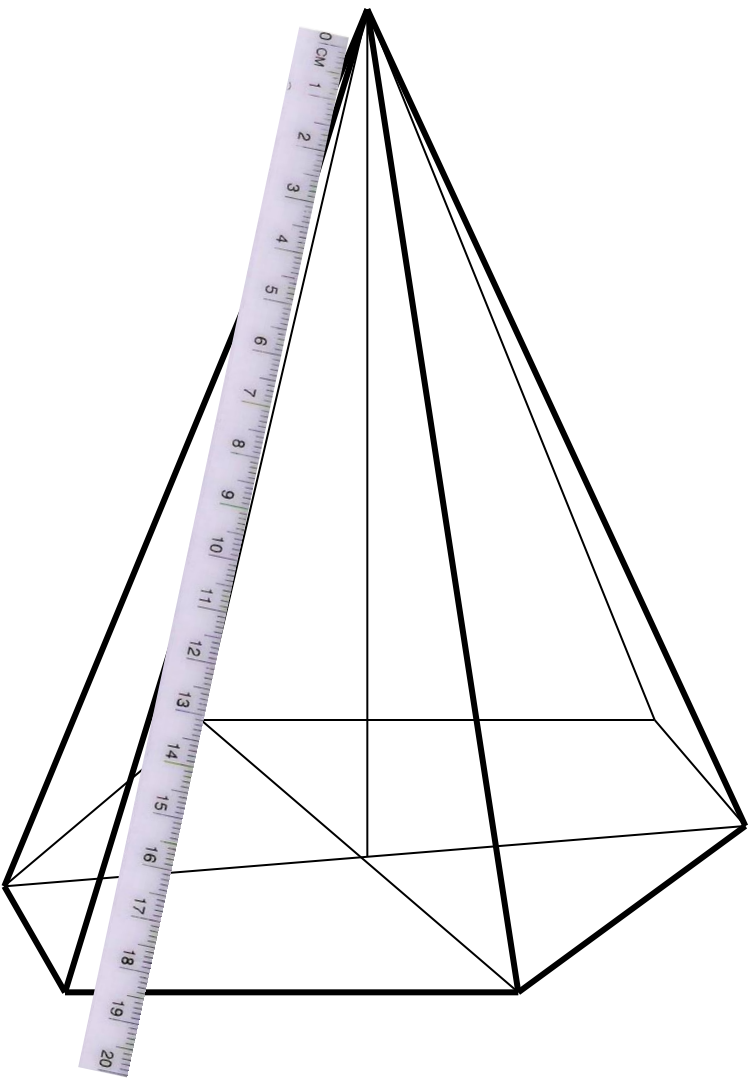


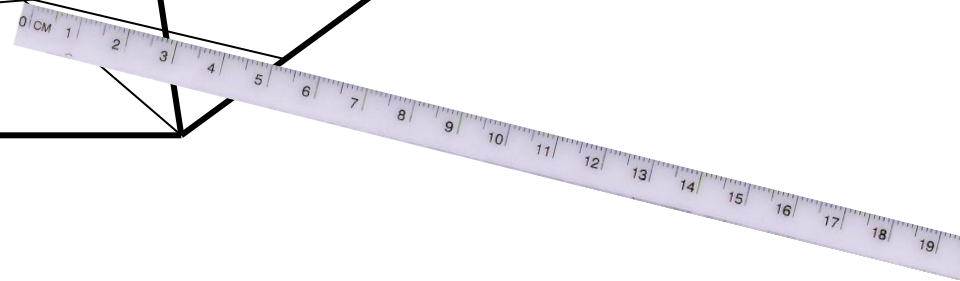
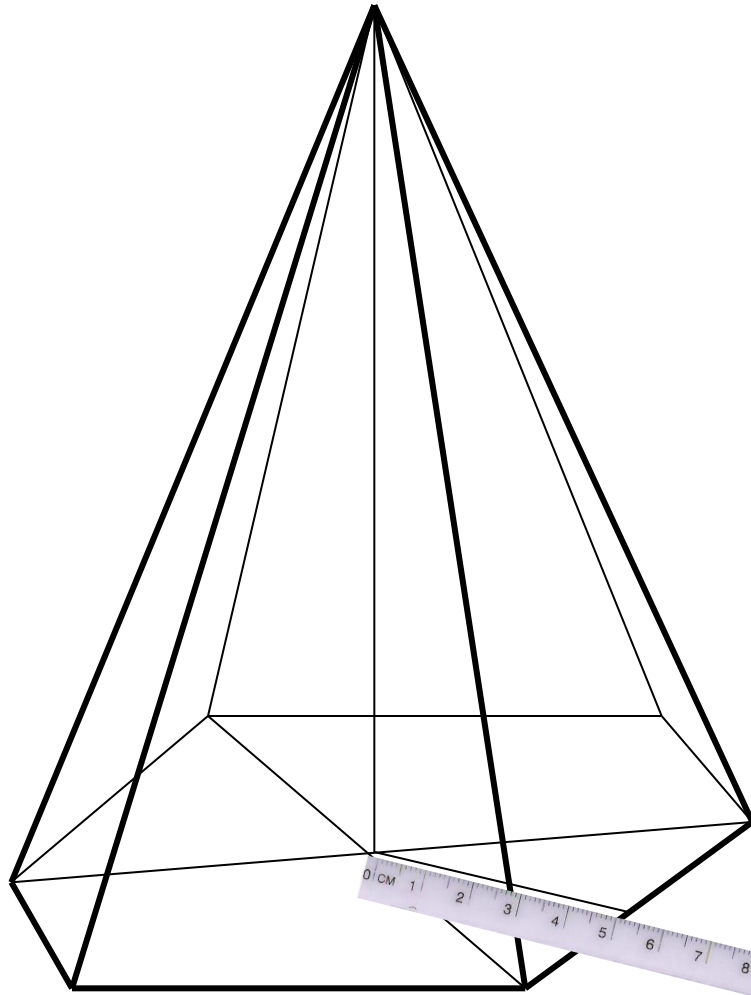


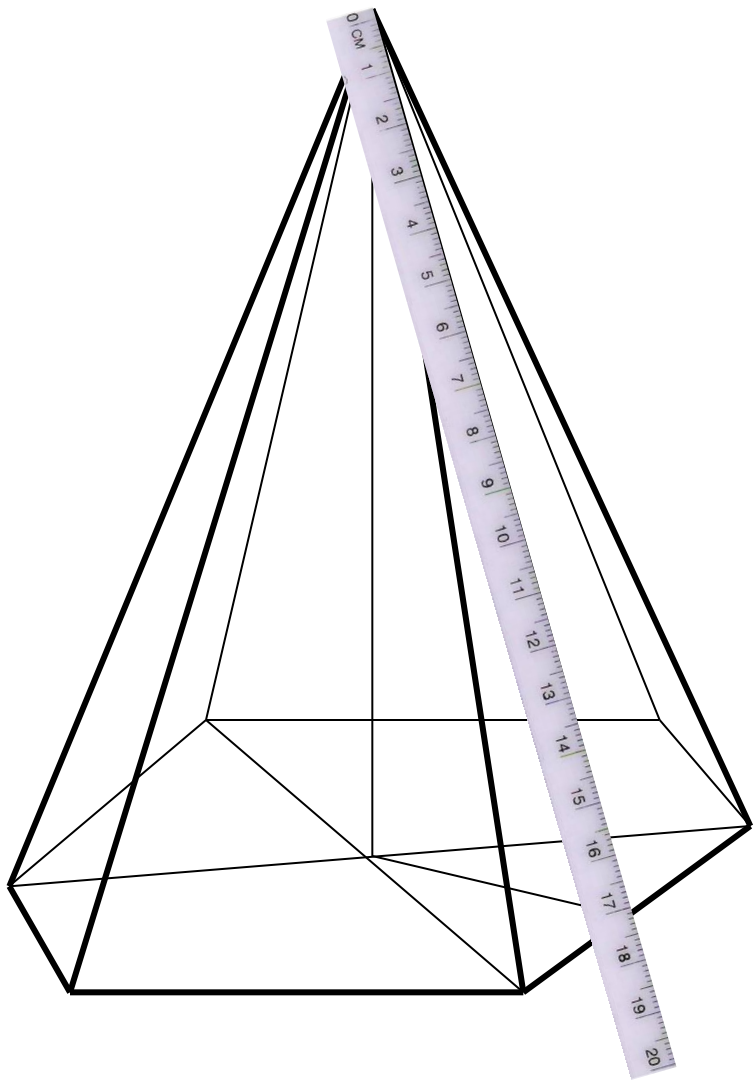


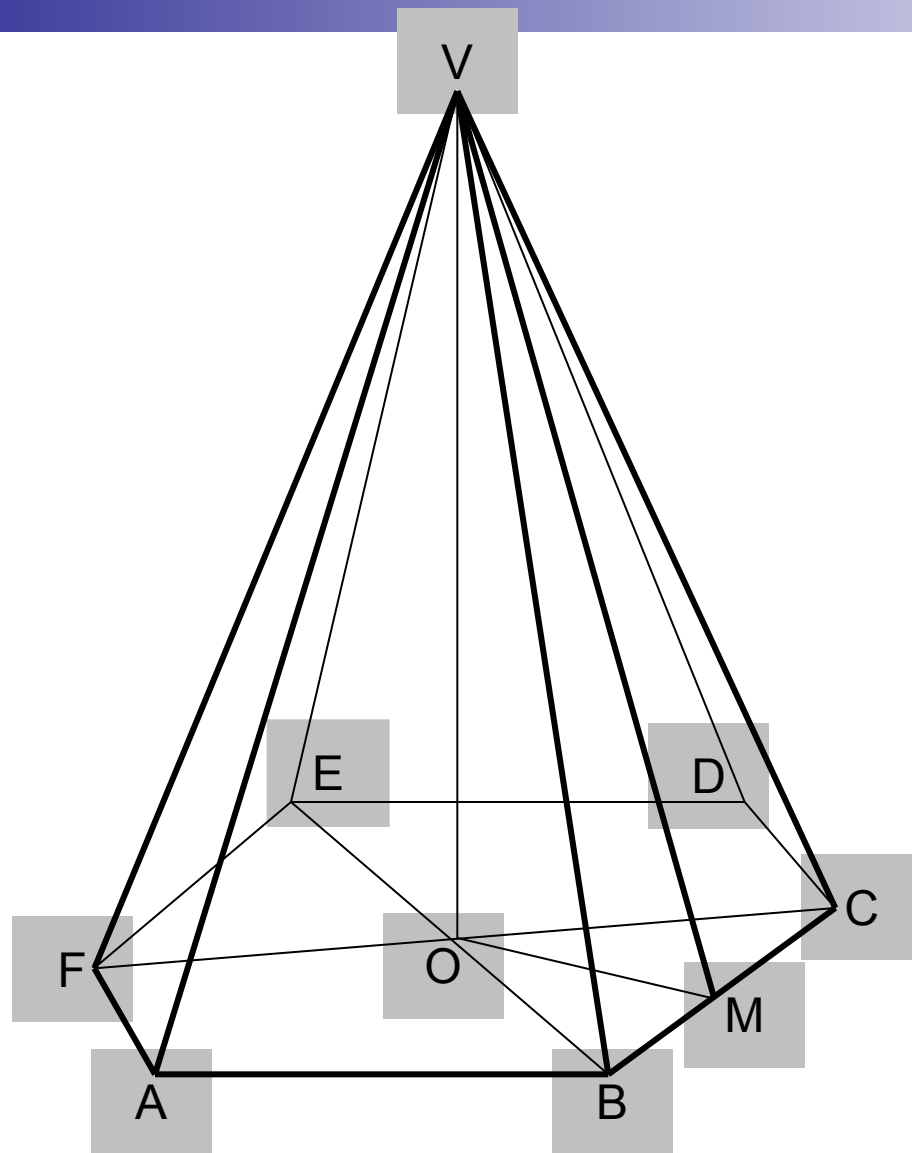






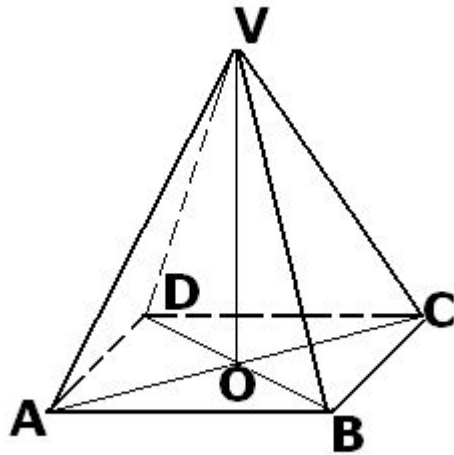






Elementele piramidelor regulate

Piramidă patrulateră regulată

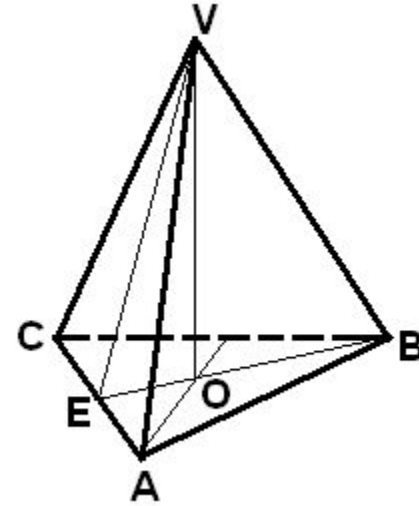


ABCD pătrat

VO înălțime ($\{O\} = AC \cap BD$)

**VBC, VCD, VDA, VAB fețe laterale
(Triunghiuri isoscele congruente)**

Piramidă triunghiulară regulată



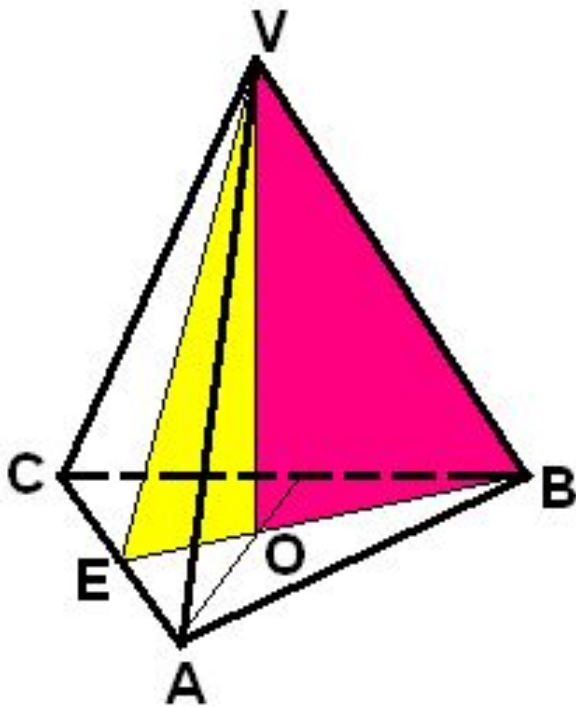
ABC triunghi echilateral

VO înălțime; O –centru de greutate

**VBC, VCA, VAB fețe laterale
(Triunghiuri isoscele congruente)**



Triunghiuri de lucru în piramida triunghiulară regulată



Fie $AB = AC = BC = a$ (muchia bazei)

$VO = h$

$VE = Ap$ (apotema piramidei)

$OE = ap$ (apotema bazei)

$VB = l$ (muchie laterală)

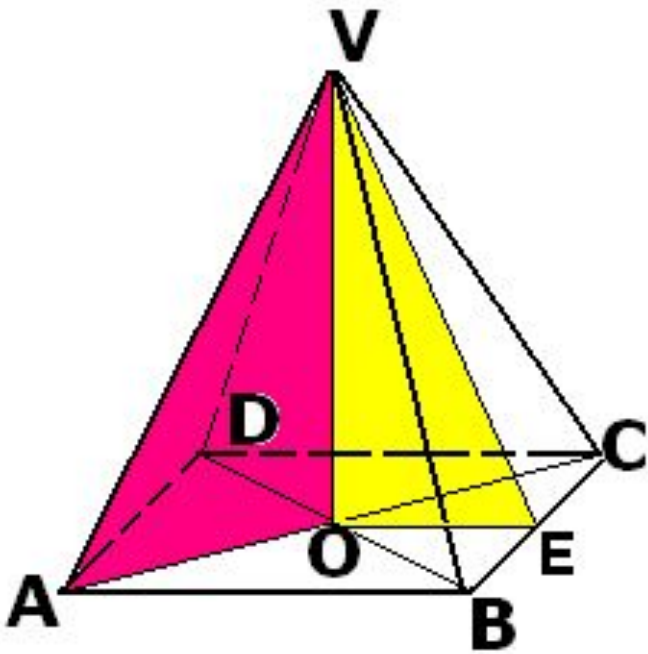
$$\Delta VOE : OE = \frac{1}{3} \cdot \frac{a\sqrt{3}}{2}; VO^2 + OE^2 = VE^2$$

$$\Delta VOB : OB = \frac{2}{3} \cdot \frac{a\sqrt{3}}{2}; VO^2 + OB^2 = VB^2$$

$$\Delta VEA : EA = \frac{a}{2}; VE^2 + EA^2 = VA^2$$



Triunghiuri de lucru în piramida patrulateră regulată



Fie $AB = BC = CD = DA = a$ (muchia bazei)

$VO = h$ (înălțimea)

$OE = ap$ (apotema bazei)

$VE = Ap$ (apotema piramidei)

$VA = l$ (muchia laterala)

$$\Delta VOE : OE = \frac{1}{2}a; VO^2 + OE^2 = VE^2$$

$$\Delta VOA : AO = \frac{1}{2}a\sqrt{2}; VO^2 + AO^2 = VA^2$$

$$\Delta VEC : EC = \frac{1}{2}a; VE^2 + EC^2 = VC^2$$



Triunghiuri de lucru în piramida hexagonală regulată

Fie $AB = BC = CD = \dots = EF = a$ (muchia bazei)

$VO = h$ (înălțimea piramidei)

$OM = ap$ (apotema bazei)

$VM = Ap$ (apotema piramidei)

$VA = VB = VC = \dots = VE = VF = l$ (muchii laterale)

$$\Delta VOM : OM = \frac{a\sqrt{3}}{2}; VO^2 + OM^2 = VM^2$$

$$\Delta VOE : OE = \frac{EB}{2} = \frac{a}{2}; VO^2 + OE^2 = VE^2$$

$$\Delta VMA : AM = \frac{a}{2}; VM^2 + MA^2 = VA^2$$





Formule

A_l = arie laterală = suma ariilor fețelor laterale

A_t = arie totală = A_l + aria bazei

$A_t = A_l + A_{\text{bază}}$

$V = \text{volum} = \frac{1}{3} \cdot A_{\text{bazei}} \cdot \text{înaltime}$





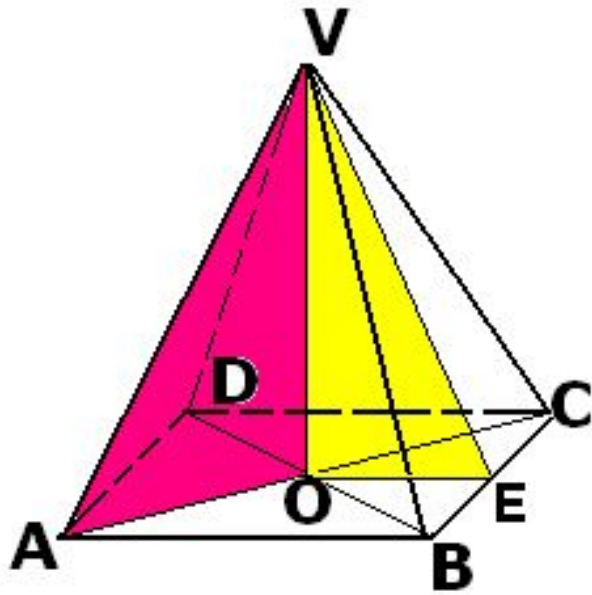
Formule de calcul pentru piramide regulate

$$A_l = \frac{P_{bazei} \cdot Apotemă}{2}$$

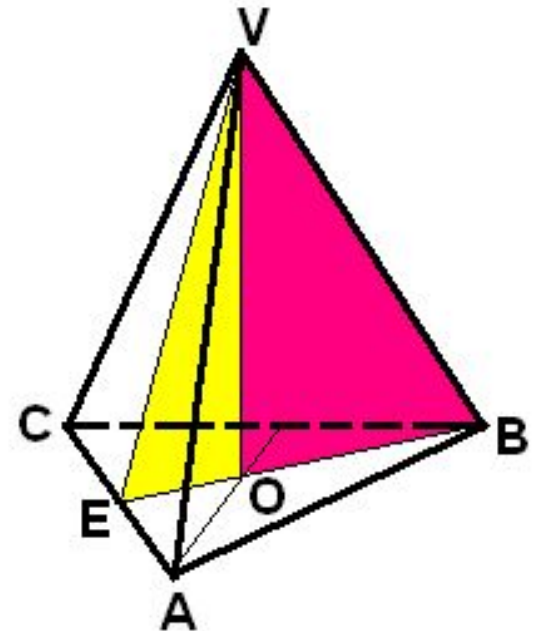
$$V = \frac{1}{3} \cdot A_{baza} \cdot h \quad A_t = A_l + A_{bază}$$

⏟
pentru orice tip de piramidă





$$A_l = 4 \cdot A_{VBC} = 4 \cdot \frac{BC \cdot VE}{2} = \frac{(4BC) \cdot Ap}{2} = \frac{P_{baza} \cdot Ap}{2}$$



$$A_l = 3 \cdot A_{VAC} = 3 \cdot \frac{AC \cdot VE}{2} = \frac{(3AC) \cdot Ap}{2} = \frac{P_{baza} \cdot Ap}{2}$$



Piramidă triunghiulară regulată cu muchia bazei “a”

$$P_{baza} = 3a$$

$$A_{baza} = \frac{a^2 \sqrt{3}}{4}$$

Piramidă patrulateră regulată cu muchia bazei “a”

$$P_{baza} = 4a$$

$$A_{baza} = a^2$$

Piramidă hexagonală regulată cu muchia bazei “a”

$$P_{baza} = 6a$$

$$A_{baza} = 6 \cdot \frac{a^2 \sqrt{3}}{4}$$





Aplicații



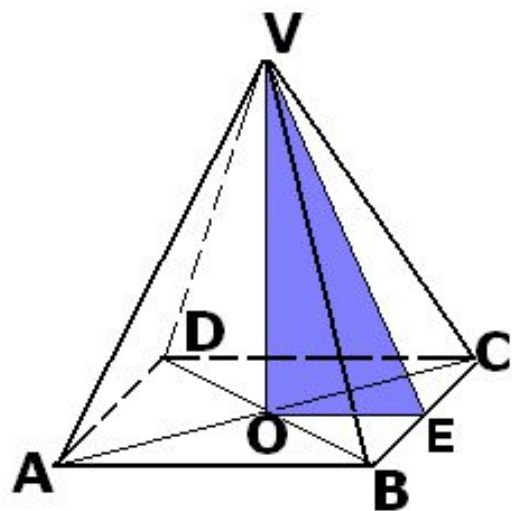
1. Fie $VABCD$ o piramidă patrulateră regulată. Dacă se cunoaște muchia bazei $AB=4$ cm, înălțimea $VO=12$ cm, să se calculeze aria și volumul.

R
1

2. Dacă muchia laterală a unei piramide patrulatere regulate este de 10 cm, iar muchia bazei de 12 cm, să se calculeze volumul și aria laterală.

R2





$$\text{Avem } V = \frac{1}{3} A_b \cdot h, \text{ unde } A_b = a^2, A_b = 4^2 = 16(\text{cm}^2).$$

$$\text{Deci } V = \frac{1}{3} \cdot 16 \cdot 12 = 64(\text{cm}^3).$$

$$\Delta VOE : VO^2 + OE^2 = VE^2$$

$$12^2 + 2^2 = VE^2$$

$$144 + 4 = VE^2$$

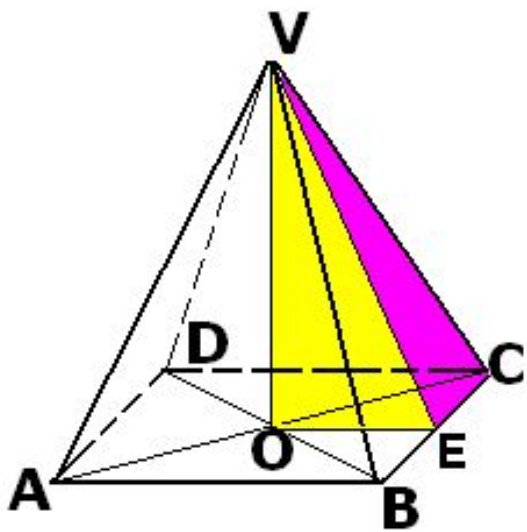
$$VE = \sqrt{148} = 2\sqrt{37} \Rightarrow Ap_p = 2\sqrt{37}\text{cm}.$$

$$A = A_l + A_b ; A_l = \frac{1}{2} P_b \cdot Ap_p$$

$$A_l = \frac{1}{2} \cdot 16 \cdot 2\sqrt{37} = 16\sqrt{37}(\text{cm}^2)$$

$$A = 16\sqrt{37} + 16 = 16(1 + \sqrt{37})\text{cm}^2$$





$$AB = BC = CD = DA = 12\text{cm} \Rightarrow EC = 6\text{cm}$$

$$\triangle VEC : VC = 10\text{cm}, EC = 6\text{cm} \Rightarrow VE = 8\text{cm. (numere pitagorice)}$$

$$\triangle VOE : OE = \frac{AB}{2} = 6\text{cm} \Rightarrow VO^2 = VE^2 - OE^2$$

$$VO^2 = 64 - 36 \Rightarrow VO = \sqrt{28} = 2\sqrt{7}\text{cm}$$

$$V = \frac{1}{3} \cdot A_b \cdot h = \frac{1}{3} \cdot 144 \cdot 2\sqrt{7} = 72\sqrt{7}(\text{cm}^3)$$

$$A_l = \frac{1}{2} \cdot P_b \cdot Ap = \frac{1}{2} \cdot 48 \cdot 8 = 192(\text{cm}^2)$$



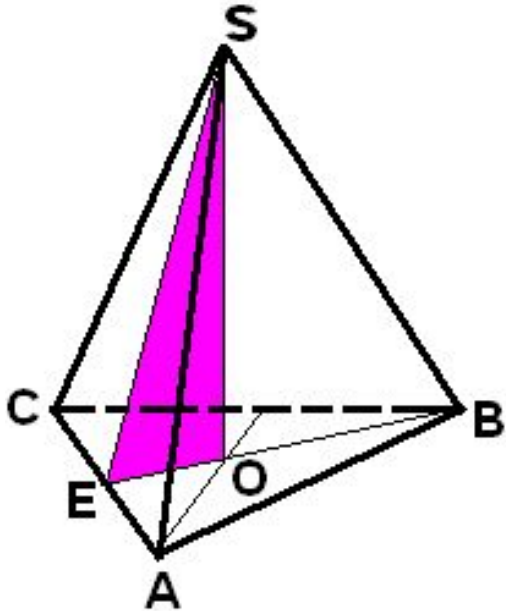
1. Fie $SABC$ o piramidă triunghiulară regulată cu muchia bazei $AB=6$ cm și înălțimea $SO=2$ cm. Să se calculeze aria și volumul piramidei!

R

2. Fie $VABC$ o piramidă triunghiulară regulată. Dacă se cunoaște muchia $VB=10$ cm, înălțimea $VO=8$ cm să se calculeze aria și volumul piramidei!

R





$$V = \frac{1}{3} A_b h \Rightarrow V = \frac{1}{3} \cdot \frac{6^2 \sqrt{3}}{4} \cdot 2 = 6\sqrt{3} (\text{cm}^3)$$

$$A = A_l + A_b; \quad A_l = \frac{1}{2} P_b A_p; \quad P_b = 3 \cdot 6 = 18 (\text{cm})$$

$$\Delta SOE : OE = \frac{1}{3} \cdot \frac{6\sqrt{3}}{2} = \sqrt{3}$$

$$SO^2 + OE^2 = SE^2$$

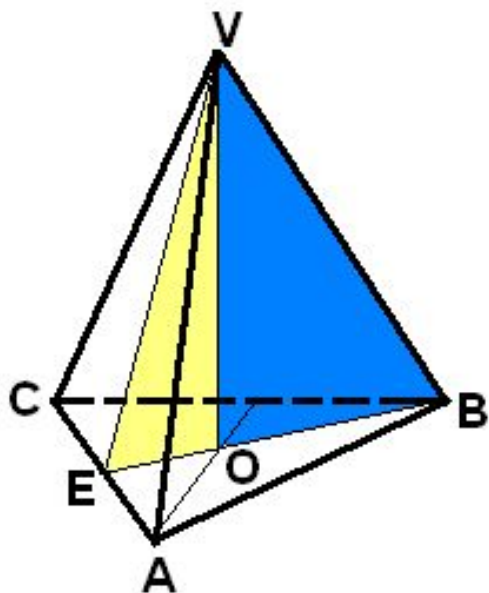
$$2^2 + (\sqrt{3})^2 = SE^2$$

$$SE = \sqrt{7} \text{ cm} = A_p$$

$$A_l = \frac{1}{2} \cdot 18 \cdot \sqrt{7} = 9\sqrt{7} (\text{cm})$$

$$A = 9\sqrt{7} + 9\sqrt{3} = 9(\sqrt{7} + \sqrt{3}) \text{ cm}^2$$





$$\Delta VOB: VO^2 + OB^2 = VB^2 \Rightarrow OB^2 = 100 - 64 \Rightarrow OB = \sqrt{36} = 6(\text{cm})$$

$$\text{dar } OB = \frac{2}{3} \cdot \frac{AB\sqrt{3}}{2} \Rightarrow AB = \frac{18}{\sqrt{3}} = 6\sqrt{3}(\text{cm})$$

$$V = \frac{1}{3} \cdot A_{ABC} \cdot VO \Rightarrow V = \frac{1}{3} \cdot \frac{(6\sqrt{3})^2 \sqrt{3}}{4} \cdot 8 = 72\sqrt{3}(\text{cm}^3)$$

$$\Delta VEC: \left. \begin{array}{l} VC = 10\text{cm} \\ EC = \frac{6\sqrt{3}}{2}\text{cm} \end{array} \right\} \Rightarrow VE^2 = VC^2 - EC^2 \Rightarrow VE^2 = 10^2 - (3\sqrt{3})^2 \Rightarrow VE = \sqrt{73}\text{cm} = A_p$$

$$A = A_l + A_b \Rightarrow A = \frac{1}{2} \cdot P_b \cdot h + A_b \Rightarrow A = \frac{1}{2} \cdot 18\sqrt{3} \cdot \sqrt{73} + 27\sqrt{3} = 9\sqrt{3}(\sqrt{73} + 3)(\text{cm}^2)$$

