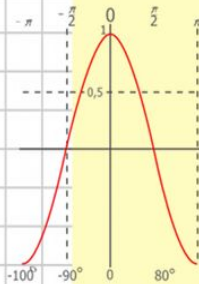
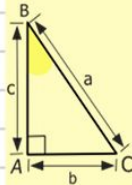
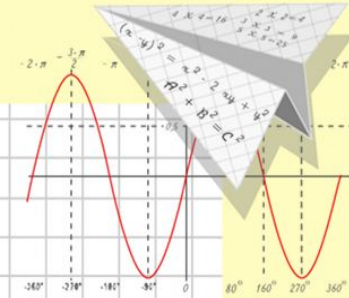
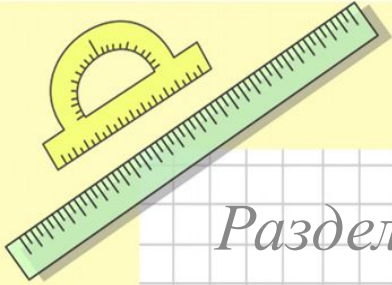


Математик

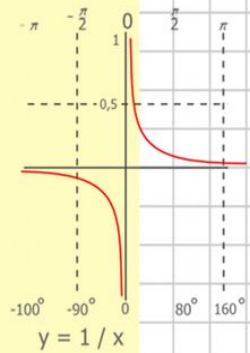
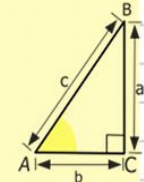
Раздел 6. Метод **а** координат в пространстве

Занятие 63. Координаты точек и векторов



$y = \cos x$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

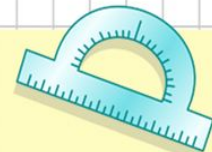


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

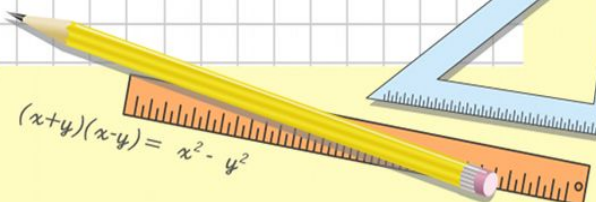
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$



$\sin 90^\circ = 1$

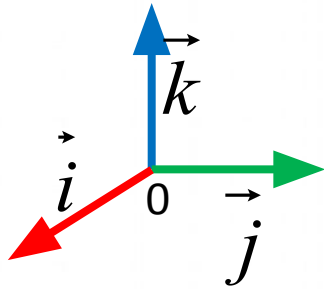


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$



$$(x+y)(x-y) = x^2 - y^2$$

Координатные векторы



$$\vec{i} \perp \vec{j} \perp \vec{k}$$

$$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$$

$\vec{i}, \vec{j}, \vec{k}$ - координатные векторы

Теорема. Любой вектор пространства можно разложить по трем некомпланарным векторам, причем коэффициенты разложения определяются единственным образом:

$$\vec{p} = x\vec{a} + y\vec{b} + z\vec{c}$$

где $\vec{a}, \vec{b}, \vec{c}$ - тройка некомпланарных
 $x, y, z \in \mathbb{R}$ - коэффициенты разложения

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

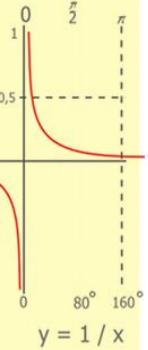
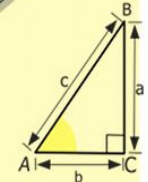
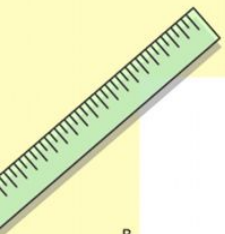
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

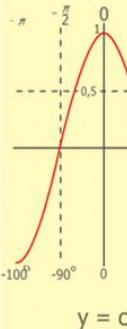
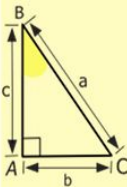
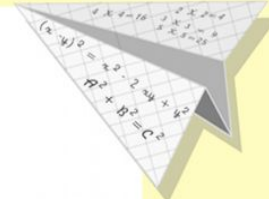
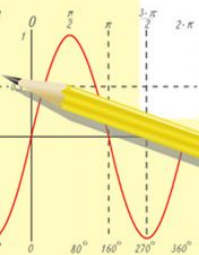
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

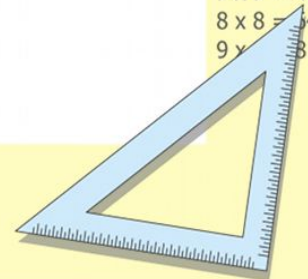
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} \frac{1}{2} 5 00 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105 0 00 \end{array}$$



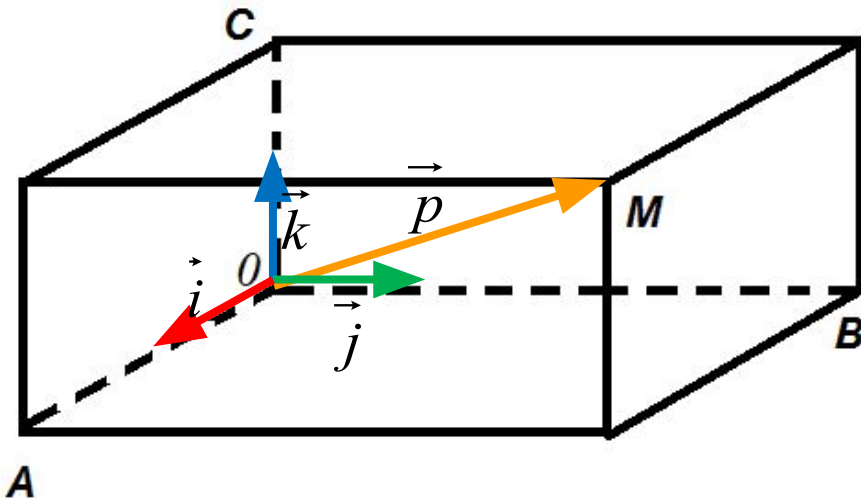
$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Координаты вектора

В качестве тройки некопланарных векторов, по которым производится разложение произвольного вектора, удобно выбрать координатные векторы

$$\vec{i}, \vec{j}, \vec{k}$$



$$\vec{OM} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{OM} = \vec{OA} + \vec{OB} + \vec{OC}$$

$$\vec{OA} = 2\vec{i}$$

$$\vec{OB} = 4\vec{j}$$

$$\vec{OC} = 2\vec{k}$$

$$\vec{OM} = 2\vec{i} + 4\vec{j} + 2\vec{k}$$

т.о. $x = 2, y = 4, z = 2$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

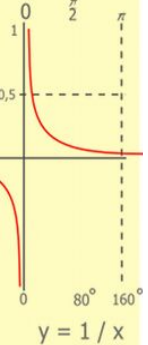
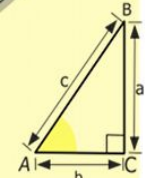
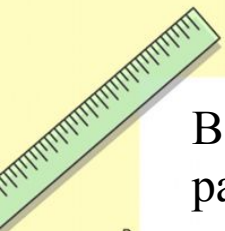
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

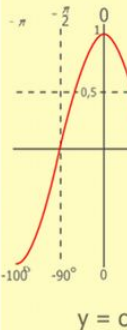
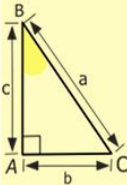
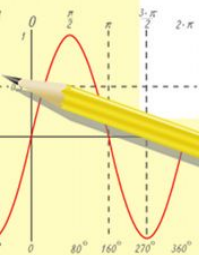
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

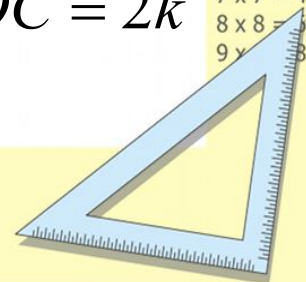


$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$y = \cos$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Координаты вектора

Координаты вектора равны коэффициентам разложения этого вектора по координатным векторам, т.е.

$$\vec{p} = x\vec{i} + y\vec{j} + z\vec{k}$$



$$\vec{p} = (x; y; z)$$

Например:

$$\vec{a} = 3\vec{i} + \vec{j} + 2\vec{k} \blacktriangleright \vec{a} = (3; 1; 2)$$

$$\vec{a} = (-6; 3; 1) \blacktriangleright \vec{a} = -6\vec{i} + 3\vec{j} + \vec{k}$$

$$\vec{b} = \vec{i} + 4\vec{j} - 3\vec{k} \blacktriangleright \vec{b} = (1; 4; -3)$$

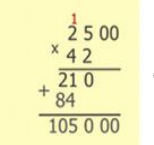
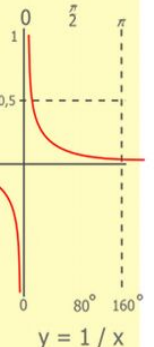
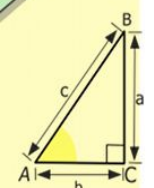
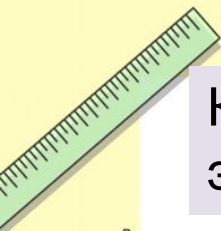
$$\vec{b} = (2; -5; 0) \blacktriangleright \vec{b} = 2\vec{i} - 5\vec{j}$$

$$\vec{c} = 5\vec{i} - \vec{k} \blacktriangleright \vec{c} = (5; 0; -1)$$

$$\vec{c} = (0; -1; 3) \blacktriangleright \vec{c} = -\vec{j} + 3\vec{k}$$

$$\vec{d} = 7\vec{j} \blacktriangleright \vec{d} = (0; 7; 0)$$

$$\vec{d} = (1; -2; -5) \blacktriangleright \text{?????}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

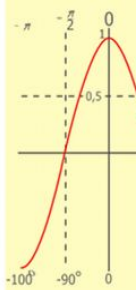
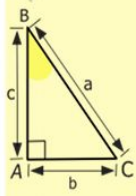
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

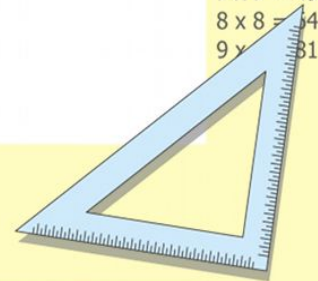
$$\begin{cases} y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

2 x 2 =	4
3 x 3 =	9
4 x 4 =	16
5 x 5 =	25
6 x 6 =	36
7 x 7 =	49
8 x 8 =	64
9 x 9 =	81



Действия с векторами в координатах

Пусть $\vec{a} = (x_1; y_1; z_1)$ $\vec{b} = (x_2; y_2; z_2)$ $\lambda \in \mathbb{R}$

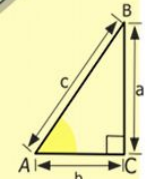
даны:
тогда:

0. $\vec{a} = \vec{b} \Leftrightarrow x_1 = x_2; y_1 = y_2; z_1 = z_2$

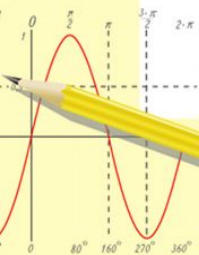
1. $\vec{a} + \vec{b} = (x_1 + x_2; y_1 + y_2; z_1 + z_2)$

2. $\vec{a} - \vec{b} = (x_1 - x_2; y_1 - y_2; z_1 - z_2)$

3. $\lambda \vec{a} = (\lambda x_1; \lambda y_1; \lambda z_1)$



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

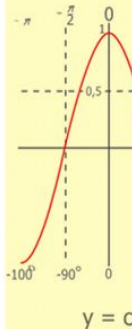
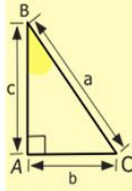
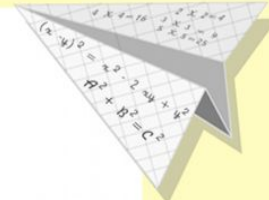
$$\sin 90^\circ = 1$$



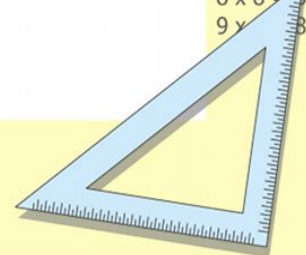
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



Действия с векторами в

координатах

Пусть $\vec{a} = (3; -1; 5)$ $\vec{b} = (2; 1; -3)$ $\vec{c} = (-4; 0; -1)$

даны
Найти координаты

$$\vec{p} = 3\vec{a} + 4\vec{b} - 5\vec{c}$$

вектора
Решение:

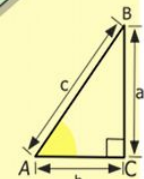
$$\vec{p} = 3\vec{a} + 4\vec{b} - 5\vec{c} = 3 \cdot \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} + 4 \cdot \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} - 5 \cdot \begin{pmatrix} -4 \\ 0 \\ -1 \end{pmatrix} =$$

$$= \begin{pmatrix} 3 \cdot 3 \\ -1 \cdot 3 \\ 5 \cdot 3 \end{pmatrix} + \begin{pmatrix} 2 \cdot 4 \\ 1 \cdot 4 \\ -3 \cdot 4 \end{pmatrix} - \begin{pmatrix} -4 \cdot 5 \\ 0 \cdot 5 \\ -1 \cdot 5 \end{pmatrix} = \begin{pmatrix} 9 \\ -3 \\ 15 \end{pmatrix} + \begin{pmatrix} 8 \\ 4 \\ -12 \end{pmatrix} - \begin{pmatrix} -20 \\ 0 \\ -5 \end{pmatrix} =$$

$$= \begin{pmatrix} 9 + 8 - (-20) \\ -3 + 4 - 0 \\ 15 + (-12) - (-5) \end{pmatrix} = \begin{pmatrix} 37 \\ 1 \\ 8 \end{pmatrix}$$

Ответ:

$$\vec{p} = (37; 1; 8)$$



$$\begin{array}{r} \frac{1}{2} \ 5 \ 00 \\ \times \ 4 \ 2 \\ \hline 21 \ 0 \\ + \ 84 \\ \hline 105 \ 0 \ 00 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

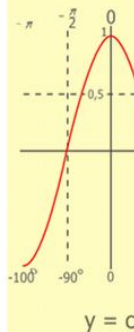
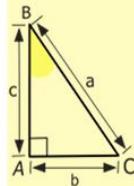
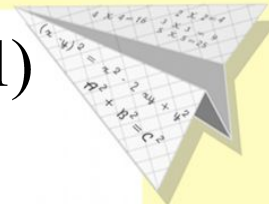


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

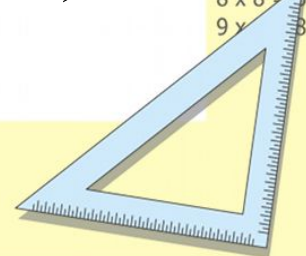
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

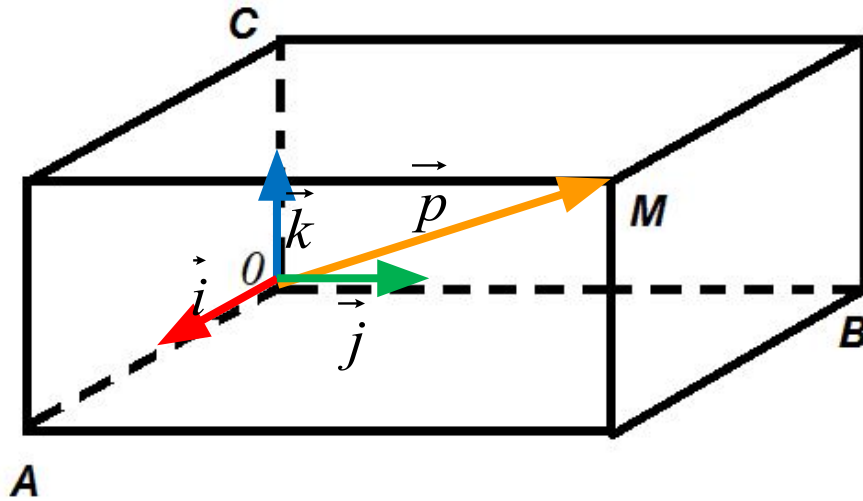


$$y = \cos$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



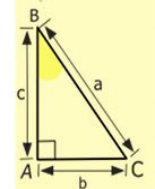
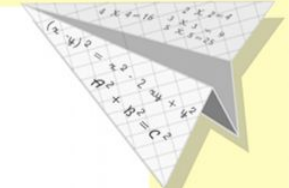
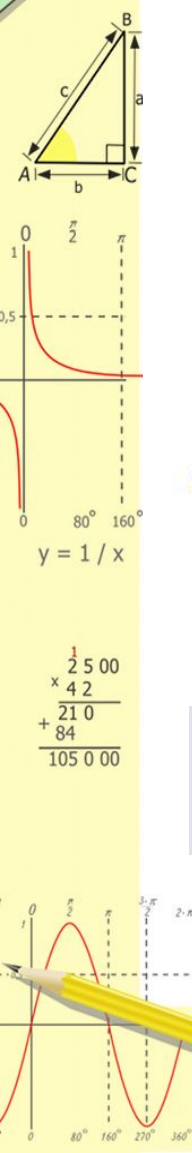
Связь между координатами точек и векторов



$$M = (2; 4; 2)$$

$$\vec{OM} = 2\vec{i} + 4\vec{j} + 2\vec{k} \quad \Rightarrow \quad \vec{OM} = (2; 4; 2)$$

Координаты точки совпадают с координатами вектора, который начинается в начале координат, а заканчивается в данной точке.



2 x 2 = 4
3 x 3 = 9
4 x 4 = 16
5 x 5 = 25
6 x 6 = 36
7 x 7 = 49
8 x 8 = 64
9 x 9 = 81

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

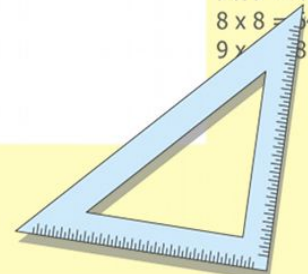
$$\sin 90^\circ = 1$$



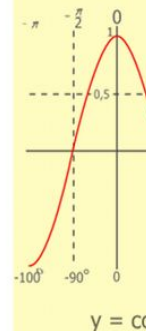
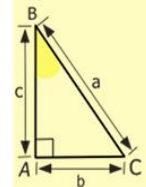
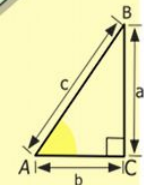
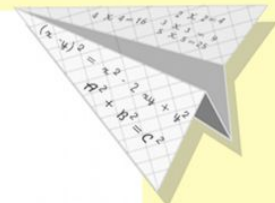
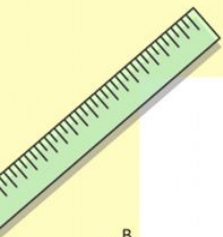
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

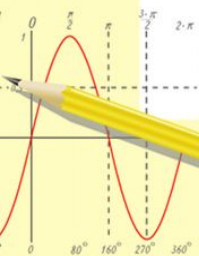


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Юрьевич
преподаватель математики
ГБПОУ ЗКНО
Москва, 2021г.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

