Relations

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- Relations, properties of relations
- Equivalence relations
- Partial orderings

Exercise 1

- For the following relation on the set

$$\{x : x \in \mathbb{Z} \; and \; 1 \leq x \leq 12\}$$

list the ordered pairs belonging to the relation

$$R = \{(x, y) : xy = 9\}.$$

Exercise 2

Let R be a relation on $\{1, 2, 3, 4\}$ given by uRv if and only if $u + 2v$ is odd. Represent R in each of the following ways:

a) as a set of ordered pairs;

b) in graphical form.

Exercise 3

Determine which of the following relations on the set of people is reflexive, symmetric, antisymmetric or transitive:

a) has the same parents as;
b) is a brother of;
c) is older or younger than;
d) is at least as clever as.

Exercise 4

A relation R on the set $\mathbb{R}$ is given by

$aRb \overset{def}{\iff} a - b$ is integer.

Prove that R is an equivalence relation and determine the equivalence classes of $0, \frac{1}{2}, \sqrt{2}$.

Exercise 5

For each of the following equivalence relations R on the set A describe the blocks into which the relation partitions the set A:

a) A is the set of the books in a library; R is given by xRy if and only if the colour of x's cover is the same as colour of y's cover.

b) $A = \mathbb{Z}$; R is given by xRy if and only if $x - y$ is even.

Exercise 5

For each of the following equivalence relations R on the set A describe the blocks into which the relation partitions the set A:

c) A is a set of all people; R is given by xRy if and only if x has the same sex as y.

d) $A = \mathbb{R}^2$; R is given by: $(a, b)R(c, d)$ if and only if $a^2 + b^2 = c^2 + d^2$.

Exercise 6

A relation R on $\mathbb{Z}$ is given by xRy if and only if $x^2 - y^2$ is divisible by 3. Show that R is an equivalence relation and determine the corresponding partition of $\mathbb{Z}$ into distinct equivalence classes.