

# Relations

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- Relations, properties of relations
- Equivalence relations
- Partial orderings

## Exercise 1

For the following relation on the set

$$\{x: x \in \mathbb{Z} \text{ and } 1 \leq x \leq 12\}$$

list the ordered pairs belonging to the relation

$$R = \{(x, y): xy = 9\}.$$

## Exercise 2

Let  $R$  be a relation on  $\{1, 2, 3, 4\}$  given by  $uRv$  if and only if  $u + 2v$  is odd. Represent  $R$  in each of the following ways:

- a) as a set of ordered pairs;
- b) in graphical form.

## Exercise 3

Determine which of the following relations on the set of people is reflexive, symmetric, antisymmetric or transitive:

- a) has the same parents as;
- b) is a brother of;
- c) is older or younger than;
- d) is at least as clever as.

## Exercise 4

A relation  $R$  on the set  $\mathbb{R}$  is given by

$$aRb \stackrel{\text{def}}{\iff} a - b \text{ is integer.}$$

Prove that  $R$  is an equivalence relation and determine the equivalence classes of  $0, \frac{1}{2}, \sqrt{2}$ .

## Exercise 5

For each of the following equivalence relations  $R$  on the set  $A$  describe the blocks into which the relation partitions the set  $A$ :

- a)  $A$  is the set of the books in a library;  $R$  is given by  $xRy$  if and only if the colour of  $x$ 's cover is the same as colour of  $y$ 's cover.
- b)  $A = \mathbb{Z}$ ;  $R$  is given by  $xRy$  if and only if  $x - y$  is even.

## Exercise 5

For each of the following equivalence relations  $R$  on the set  $A$  describe the blocks into which the relation partitions the set  $A$ :

- c)  $A$  is a set of all people;  $R$  is given by  $xRy$  if and only if  $x$  has the same sex as  $y$ .
- d)  $A = \mathbb{R}^2$ ;  $R$  is given by:  $(a, b)R(c, d)$  if and only if  $a^2 + b^2 = c^2 + d^2$ .

## Exercise 6

A relation  $R$  on  $\mathbb{Z}$  is given by  $xRy$  if and only if  $x^2 - y^2$  is divisible by 3. Show that  $R$  is an equivalence relation and determine the corresponding partition of  $\mathbb{Z}$  into distinct equivalence classes.