

# Разбор задач CryptoCTF 2020

<http://10.10.10.19:8080/s/eQK4y9924FINXgR>

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# Модульная арифметика

**Простое число** - натуральное число, имеющее ровно два различных натуральных делителя — единицу и самого себя.

**Взятие  $x$  по модулю** ( $x \% m$ ) – вычисление остатка от деления  $x$  на  $m$

**а и b сравнимы** по модулю  $m$ , если их остатки при делении на  $m$  равны ( $a \equiv b \% m$ ). **Класс вычетов** – множество всех чисел, сравнимых с  $a$  по модулю  $m$ .

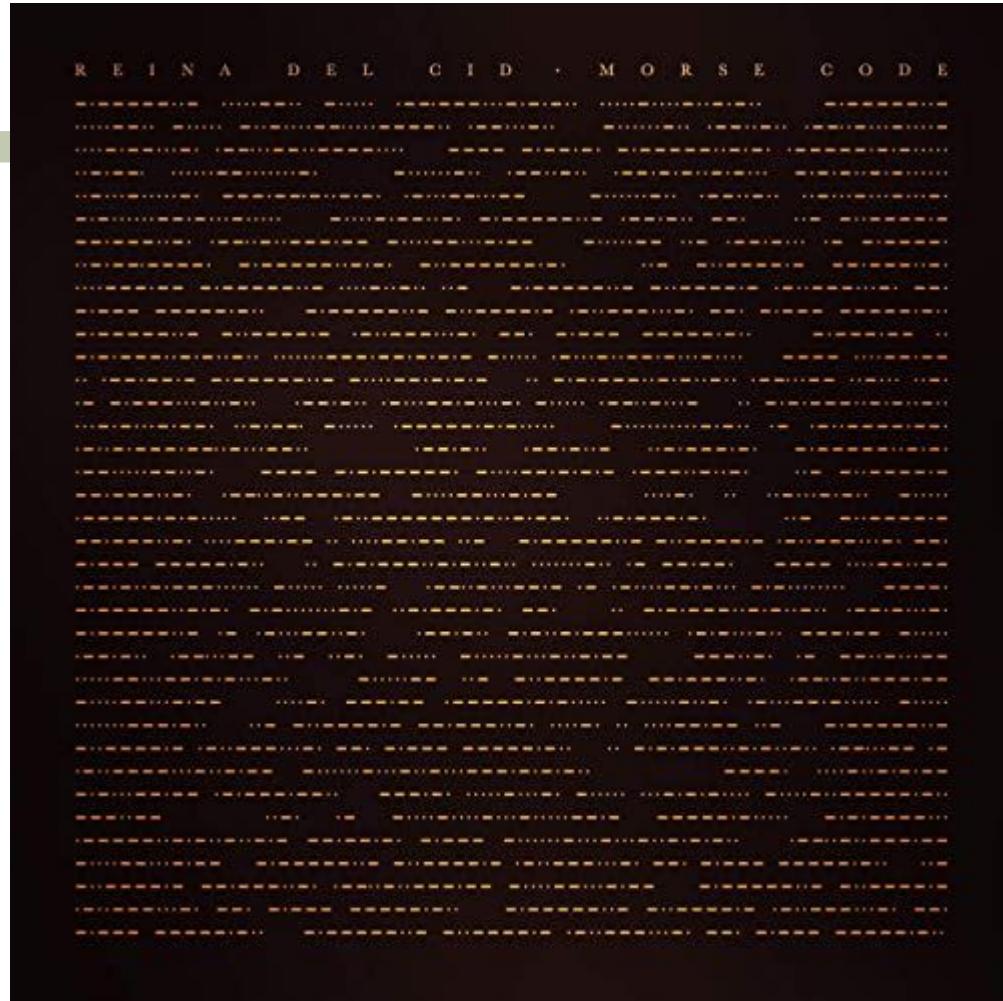
Отношение сравнимости является отношением эквивалентности (симметричным, транзитивным, рефлексивное)

Допустимые операции со сравнениями:  $a + m * x \equiv b \% m$ ;  $a - b \equiv 0 \% m$ ;  $a + x \equiv b + x \% m$ ;  
 $a * x \equiv b * x \% m$ ;  $a^x \equiv b^x \% m$ ;  $a / k \equiv b / k \% m$  если НОД( $k, m$ )=1

**Обратное число по модулю** – такое  $i$ , что  $a * i \equiv 1 \% m$ . Существует для всех  $a$ , взаимнопростых с  $m$

**Теорема Эйлера:**  $a^{\phi(n)} \equiv 1 \% m$ ,  $\phi(m)$  - функция Эйлера (количество натуральных чисел, не превышающих  $n$  и взаимно простых с ним)

# Encoding





# Abbot

```
import string
import random
from fractions import Fraction as frac
from secret import flag

def me(msg):
    if len(msg) == 1:
        return ord(msg)
    msg = msg[::-1]
    reducer = len(msg) - 1
    resultNum, resultDen = frac(ord(msg[0]), reducer).denominator, frac(ord(msg[0]), reducer)
    reducer *= -1
    for i in range(1, len(msg)-1):
        result = ord(msg[i]) + frac(resultNum, resultDen)
        resultDen, resultNum = result.denominator, result.numerator
        resultDen, resultNum = resultNum, reducer * resultDen
        reducer *= 1
    result = ord(msg[-1]) + frac(resultNum, resultDen)
    resultDen, resultNum = result.denominator, result.numerator
    return (resultNum, resultDen)

def you(msg):
    if len(msg) == 1:
        return ord(msg)
    msg = msg[::-1]
    reducer = (-1) ** len(msg)
    result = frac(ord(msg[0]), reducer)
    resultNum, resultDen = result.denominator, result.numerator
    reducer *= -1
    for i in range(1, len(msg)-1):
        result = ord(msg[i]) + frac(resultNum, resultDen)
        resultDen, resultNum = result.denominator, result.numerator
        resultDen, resultNum = resultNum, reducer * resultDen
        reducer *= -1

    result = ord(msg[-1]) + frac(resultNum, resultDen)
    resultDen, resultNum = result.denominator, result.numerator
    return (resultNum, resultDen)
```

```
def us(msg):
    if len(msg) == 1:
        return ord(msg)
    msg = msg[::-1]
    reducer = (-1) ** int(frac(len(msg), len(msg)**2))
    result = frac(ord(msg[0]), reducer)
    resultNum, resultDen = result.denominator, result.numerator
    reducer *= -1
    reducer = int(reducer)
    for i in range(1, len(msg)-1):
        result = ord(msg[i]) + frac(resultNum, resultDen)
        resultDen, resultNum = result.denominator, result.numerator
        resultDen, resultNum = resultNum, reducer * resultDen
        reducer *= -1
        reducer = int(reducer)
    result = ord(msg[-1]) + frac(resultNum, resultDen)
    resultDen, resultNum = result.denominator, result.numerator
    return (resultNum, resultDen)

dict_encrypt = {
    1: me,
    2: you,
    3: us,
    4: you,
    5: me
}
cipher = [[] for _ in range(5)]
S = list(range(1,6))
random.shuffle(S)
print("enc = [")
for i in range(4):
    cipher[i] = dict_encrypt[S[i]](flag[int(i * len(flag)) // 5] : int(i * len(flag)) // 5)
    print(cipher[i])
    print(", ")
i += 1
cipher[i] = dict_encrypt[S[i]](flag[int(i * len(flag)) // 5] : int(i * len(flag)) // 5 + len(flag))
print(cipher[i])
print( " ]")

enc = [(4874974328610108385835995981839358584964018454799387862L, 727446086721304042164046
```



# Abbot

```
def us(msg):
    if len(msg) == 1:
        return ord(msg)
    msg = msg[::-1]
    reducer = (-1) ** int(frac(len(msg), len(msg)**2))
    result = frac(ord(msg[0]), reducer)
    resultNum, resultDen = result.denominator, result.numerator
    reducer *= -1
    reducer = int(reducer)
    for i in range(1, len(msg)-1):
        result = _ord(msg[i]) + frac(resultNum, resultDen)
        resultDen, resultNum = result.denominator, result.numerator
        resultDen, resultNum = resultNum, reducer * resultDen
        reducer *= -1
        reducer = int(reducer)
    result = ord(msg[-1]) + frac(resultNum, resultDen)

    resultDen, resultNum = result.denominator, result.numerator
    return (resultNum, resultDen)
```



```
def us_(num, den):
    text = ""
    resultDen, resultNum = den, num
    r = int(resultNum / resultDen)
    text += chr(r)
    result = frac(resultNum, resultDen) - r
    resultDen, resultNum = result.numerator, result.denominator
    while resultNum != 0 and resultDen != 0:
        r = int(resultNum / resultDen)

        result = (frac(resultNum, resultDen) - r)
        resultDen, resultNum = result.numerator, result.denominator

        if result == 1:
            text_ += chr(r - 1)
        else:
            text_ += chr(r)

    return text_
```

```
for ct in enc:
    print(us_(ct[0], ct[1]))
    print(me_(ct[0], ct[1]))
    print(you_(ct[0], ct[1]))

#CCTF{This_is_n0t_Arthur_Who_
#_ASIS_Crypto_CTF_with_
#_very_m0d3rn_arthur_Enc0d1ng!
#_l0ves_Short_st0ries_This_IS_
#_!_D0_y0u_Enj0y_IT_as_w3ll??}

# CCTF{This_is_n0t_Arthur_Who_l0ves_Short_st0ries_This_IS__ASIS__Crypto_CTF__with_very_m0d3rn_arthur_Enc0d1ng!!_D0_y0u_Enj0y_IT_as_w3ll??}
```

# Amsterdam



```
55
56     def comb(n, k):
57         if k > n:
58             return 0
59         k = min(k, n - k)
60         u = reduce(operator.mul, range(n, n - k, -1), 1)
61         d = reduce(operator.mul, range(1, k + 1), 1)
62         return u // d
63
64     def encrypt(msg, n, k):
65         msg = bytes_to_long(msg.encode('utf-8'))
66         print("msg", msg)
67         if msg >= comb(n, k):
68             return -1
69         m = ['1'] + ['0' for i in range(n - 1)]
70         for i in range(1, n + 1):
71             if msg >= comb(n - i, k):
72                 m[i-1] = '1'
73                 msg -= comb(n - i, k)
74                 print("sub", comb(n - i, k), n, i, k)
75                 k -= 1
76         m = int(''.join(m), 2)
77
78         print("encryption_res_1", bin(m))
79         i, z = 0, [0 for i in range(n - 1)]
80         c = 0
81         while (m > 0):
82             if m % 4 == 1:
83                 c += 3 ** i
84                 m -= 1
85             elif m % 4 == 3:
86                 c += 2 * 3 ** i
87                 m += 1
88                 m //= 2
89                 i += 1
90         print("enc_res_2", c)
91         return c
92
93     #enc = encrypt(flag, n, k)
94     #print('enc =', enc)
95
96     enc = 555033281787628016227499985599737847960923581713343829357167769965088680
97
```

# Amsterdam



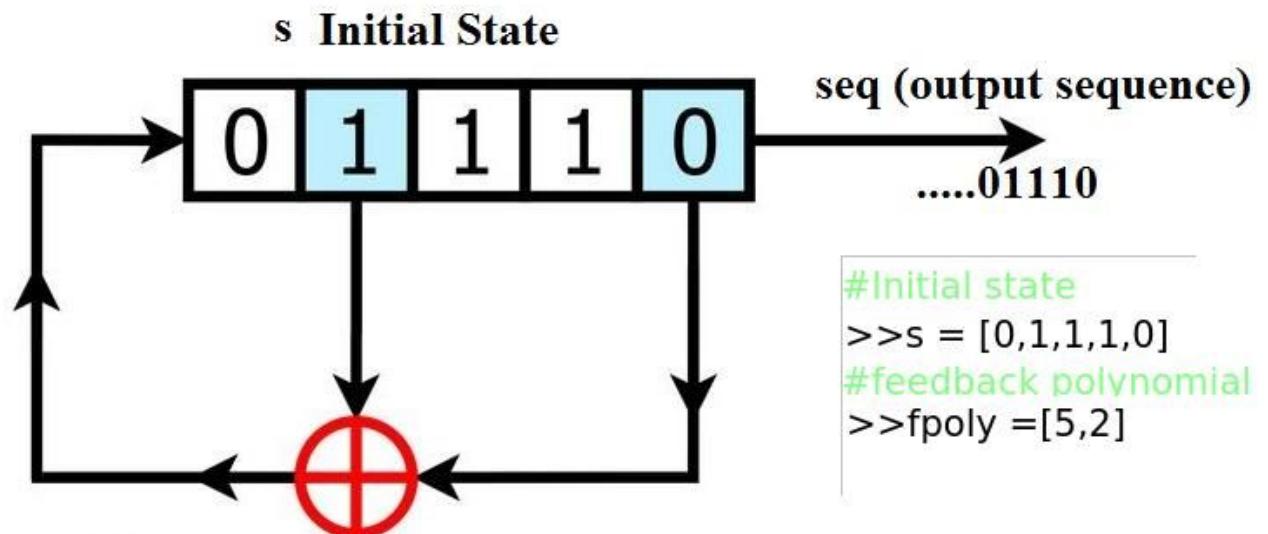
```
55
56     def comb(n, k):
57         if k > n:
58             return 0
59         k = min(k, n - k)
60         u = reduce(operator.mul, range(n, n - k, -1), 1)
61         d = reduce(operator.mul, range(1, k + 1), 1)
62         return u // d
63
64     def encrypt(msg, n, k):
65         msg = bytes_to_long(msg.encode('utf-8'))
66         print("msg", msg)
67         if msg >= comb(n, k):
68             return -1
69         m = ['1'] + ['0' for i in range(n - 1)]
70         for i in range(1, n + 1):
71             if msg >= comb(n - i, k):
72                 m[i-1] = '1'
73                 msg -= comb(n - i, k)
74                 print("sub", comb(n - i, k), n, i, k)
75                 k -= 1
76         m = int(''.join(m), 2)
77
78         print("encryption_res_1", bin(m))
79         i, z = 0, [0 for i in range(n - 1)]
80         c = 0
81         while (m > 0):
82             if m % 4 == 1:
83                 c += 3 ** i
84                 m -= 1
85             elif m % 4 == 3:
86                 c += 2 * 3 ** i
87                 m += 1
88             m //= 2
89             i += 1
90         print("enc_res_2", c)
91         return c
92
93     #enc = encrypt(flag, n, k)
94     #print('enc =', enc)
95
96     enc = 555033281787628016227499985599737847960923581713343829357167769965088680
97
```

```
128
129     def dec_(enc):
130         arr = []
131         i = 0
132         while enc > 0:
133             m = enc % 3
134             arr.append(m)
135             enc //= 3
136             i += 1
137         c = 0
138
139         for m in arr[::-1]:
140             c *= 2
141             if m == 1:
142                 c += 1
143             elif m == 2:
144                 c -= 1
145         return c
146
147     def decrypt(msg, n, k):
148         pt = 0
149
150         for i in range(2, n + 1):
151             if msg[i-1] == '1':
152                 pt += comb(n - i, k)
153                 k -= 1
154
155         return long_to_bytes(pt)
156
157
158     msg_ = dec_(enc)
159     print(msg_)
160     msg_ = bin(msg_)[2:]
161     n = len(msg_)
162
163     for k in range(n):
164         pt = decrypt(msg_, n, k)
165         if b'CCTF{' in pt:
166             print(k, pt)
167             break
168
169     # b'...: CCTF{With_Re3p3ct_for_Sch4lkwijk_dec3nt_Encoding!} ...'
```

# Shift registers



LFSR





# Heaven

```
three_ravens.py  flow.py  magic.py  magic1.py  magic_.py  hamara.py  con...
1  from bitstring import BitArray
2  from heaven import seventh_seal, oh, no, new_testament
3
4  def matthew_effect(shire, rohan):
5      gandalf = ''
6      for every, hobbit in enumerate(shire):
7          gandalf += oh if ord(hobbit) ^ ord(rohan[every]) == 0 else no
8      return gandalf
9
10 def born_to_die(isengard):
11     luke = 0
12     for book in new_testament:
13         luke ^= ord(isengard[book])
14     lizzy_grant = oh + isengard[:-1] if luke == 0 else no + isengard[:-1]
15     return lizzy_grant
16
17
18 david = len(seventh_seal)
19 elf = seventh seal
20 lord = BitArray(bytes=bytes(open('flag.jpg', 'rb').read())).bin
21 bilbo = len(lord)
22 matthew = 0
23 princess_leia = ''
24 destiny = bilbo // david
25 apocalypse = bilbo % david
26 for i in range(32):
27     elf = born_to_die(elf)
28 while matthew < destiny:
29     princess_leia += matthew_effect(elf, lord[matthew * david_: (matthew + 1) * david])
30     elf = born_to_die(elf)
31     matthew += 1
32 princess_leia += matthew_effect(elf[:apocalypse], lord[matthew * david_:])
33 res = open('flag.enc', 'wb')
34 res.write(bytes(int(princess_leia[i: i + 8]), 2) for i in range(0, bilbo, 8)))
35
```



# Heaven

```
three_ravens.py flow.py magic.py magic1.py magic_.py hamara.py cor...  
1  from bitstring import BitArray  
2  from heaven import seventh_seal, oh, no, new_testament  
3  
4  def matthew_effect(shire, rohan):  
5      gandalf = ''  
6      for every, hobbit in enumerate(shire):  
7          gandalf += oh if ord(hobbit) ^ ord(rohan[every]) == 0 else no  
8      return gandalf  
9  
10 def born_to_die(isengard):  
11     luke = 0  
12     for book in new_testament:  
13         luke ^= ord(isengard[book])  
14     lizzy_grant = oh + isengard[:-1] if luke == 0 else no + isengard[:-1]  
15     return lizzy_grant  
16  
17     |  
18     david = len(seventh_seal)  
19     elf = seventh seal  
20     lord = BitArray(bytes=bytes(open('flag.jpg', 'rb').read())).bin  
21     bilbo = len(lord)  
22     matthew = 0  
23     princess_leia = ''  
24     destiny = bilbo // david  
25     apocalypse = bilbo % david  
26     for i in range(32):  
27         elf = born_to_die(elf)  
28     while matthew < destiny:  
29         princess_leia += matthew_effect(elf, lord[matthew * david_: (matthew + 1) * david_])  
30         elf = born_to_die(elf)  
31         matthew += 1  
32     princess_leia += matthew_effect(elf[:apocalypse], lord[matthew * david_:])  
33     res = open('flag.enc', 'wb')  
34     res.write(bytes(int(princess_leia[i_: i + 8]), 2) for i in range(0, bilbo, 8)))  
35
```

```
three_ravens.py flow.py magic.py magic1.py magic_.py hamara.py cor...  
1  from bitstring import BitArray  
2  #from heaven import seventh seal, oh, no, new testament  
3  
4  oh = '0'  
5  no = '1'  
6  
7  def xor(shire, rohan):  
8      gandalf = ''  
9      for every, hobbit in enumerate(shire):  
10         gandalf += oh if ord(hobbit) ^ ord(rohan[every]) == 0 else no  
11     return gandalf  
12  
13  def shift(isengard):  
14      luke = 0  
15      for book in new_testament:  
16          luke ^= ord(isengard[book])  
17      lizzy_grant = oh + isengard[:-1] if luke == 0 else no + isengard[:-1]  
18      return lizzy_grant  
19  
20      key = ""  
21      key_len = len(key)  
22  
23      msg = BitArray(bytes=bytes(open('flag.jpg', 'rb').read())).bin  
24      msg_len = len(msg)  
25      ctr = 0  
26      ct = ''  
27      for i in range(32):  
28          key = shift(key)  
29      while ctr < msg_len // key_len:  
30          ct += xor(key, msg[ctr * key_len_: (ctr + 1) * key_len])  
31          key = shift(key)  
32          ctr += 1  
33  
34          ct += xor(key[:msg_len % key_len], msg[ctr * key_len_:])  
35          print(bytes(int(ct[i_: i + 8]), 2) for i in range(0, msg_len, 8)))
```



# Heaven

1)

| Offset   | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | - | 8  | 9  | A  | B  | C  | D  | E  | F  | ASCII            |
|----------|----|----|----|----|----|----|----|----|---|----|----|----|----|----|----|----|----|------------------|
| 00000000 | FF | D8 | FE | E0 | 00 | 10 | 4A | 46 |   | 49 | 46 | 00 | 01 | 01 | 01 | 00 | 48 | яшия..JFIF.....H |
| 00000010 | 00 | 48 | 00 | 00 | FF | DB | 00 | 43 |   | 00 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | .H..яи.C.....    |
| 00000020 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 |   | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | .....            |
| 00000030 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 |   | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | .....            |
| 00000040 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 |   | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | .....            |
| 00000050 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 |   | 01 | FF | DB | 00 | 43 | 01 | 01 | 01 | .....яи.C...     |
| 00000060 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 |   | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | .....            |
| 00000070 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 |   | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | .....            |
| 00000080 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 |   | 01 | 01 | 01 | 01 | 01 | 01 | 01 | 01 | .....            |

```
f = open("flag.enc", "rb")
ct = f.read()
f.close()

k = "FFD8FFE0"
print(xor_(tobit(ct[:4].hex()), tobit(k)))

stream = "11000111001010110000110001110010"

keys = [
    "1100011100101011000",
    "0110001110010"
]
```

```
three_ravens.py  flow.py  magic.py  magic.py  magic_.py  main.py
1  from bitstring import BitArray
2  #from heaven import seventh_seal, oh, no, new_testament
3
4  oh = '0'
5  no = '1'
6
7  def xor(shire, rohan):
8      gandalf = ''
9      for every, hobbit in enumerate(shire):
10          gandalf += oh if ord(hobbit) ^ ord(rohan[every]) == 0 else no
11
12  return gandalf
13
14  def shift(isengard):
15      luke = 0
16      for book in new_testament:
17          luke ^= ord(isengard[book])
18      lizzy_grant = oh + isengard[:-1] if luke == 0 else no + isengard[:-1]
19
20  return lizzy_grant
21
22  key = ""
23  key_len = len(key)
24
25  msg = BitArray(bytes=bytes(open('flag.jpg', 'rb').read())).bin
26  msg_len = len(msg)
27  ctr = 0
28  ct = ''
29
30  for i in range(32):
31      key = shift(key)
32  while ctr < msg_len // key_len:
33      ct += xor(key, msg[ctr * key_len : (ctr + 1) * key_len])
34      key = shift(key)
35      ctr += 1
36
37  ct += xor(key[:msg_len % key_len], msg[ctr * key_len:])
38  print(bytes(int(ct[i:i + 8]), 2) for i in range(0, msg_len, 8)))
```



# Heaven

2)

## JFIF APP0 marker segment

| Field         | Size (bytes) | Description   |
|---------------|--------------|---|
| APP0 marker   | 2            | FF E0   |
| Length        | 2            | Length of segment excluding APP0 marker   |
| Identifier    | 5            | 4A 46 49 46 00 = "JFIF" in ASCII, terminated by a null byte   |
| JFIF version  | 2            | First byte for major version, second byte for minor version ( 01 02 for 1.02)   |
| Density units | 1            | Units for the following pixel density fields <ul style="list-style-type: none"><li>• 00 : No units; width:height pixel aspect ratio = Ydensity:Xdensity</li><li>• 01 : Pixels per inch (2.54 cm)</li><li>• 02 : Pixels per centimeter</li></ul> |

[https://asecuritysite.com/information/jpeg?file=new\\_logo4.jpg](https://asecuritysite.com/information/jpeg?file=new_logo4.jpg)

```
key = "1100011100101011000"
key_len = len(key)

ct = BitArray(bytes=open('flag.enc', 'rb').read()).bin
msg_len = len(ct)
ctr = 0
pt = ''
add = "000101" + " " * 100000

print(ct)
stream = ""

while ctr < msg_len // key_len:
    pt += xor_(key, ct[ctr * key_len: (ctr + 1) * key_len])
    stream += key
    key = shift_(key, add[0])
    add = add[1:]

    ctr += 1

pt += xor_(key[:msg_len % key_len], ct[ctr * key_len:])

print(stream)
print(pt)
print(tobyte(pt))

k = "FFD8FFE000104a4649460001"

#11110000000011111001000100001111010011010011011001000011001111
#1100011100101011000 110001110010101100 11000111001010110 11000111
#111111111011000111 11111110000000000000 00010000010010100 01100100
#f f d 8 f e 0 0 1 0 4 a 6 4

#11110000000011111001000100001111010011010011011001000011001111
#1100011100101011000 110001110010101100 110001110010101100 11000111
#1111111110111111100000000000000000000000000000000000000000000000000000
#f f d 8 f f e 0 0 0 1 0 4 a 4 6 4
```

What is Huffman Coding?

## Additional scanning signature analysis

Possible: [JPEG files, sig: FFD8FF] Count: 1  
Possible: [JPEG files, sig: 4A464946] Count: 1

## File hex characters

```
[00000000] FF D8 FF E0 00 10 4A 46 49 46 00 01 01 01 00 48 .....JFIF.....H
[00000016] 00 48 00 00 FF DB 00 43 00 01 01 01 01 01 01 01 .H.....C.....
[00000032] 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 01 .....
[00000048] 01 01 01 01 01 02 01 01 01 01 01 02 02 02 02 02 .....
[00000064] 02 02 02 02 02 02 02 02 02 02 02 02 02 02 02 02 .....
[00000080] 02 02 02 02 02 02 02 02 02 02 02 02 02 02 02 02 .....
[00000096] 01 01 01 01 02 02 02 02 02 02 02 02 02 02 02 02 .....
[00000112] 02 02 02 02 02 02 02 02 02 02 02 02 02 02 02 02 .....
```



# Heaven

2)

```
100
101     key = "1100011100101011000"
102     key_len = len(key)
103
104     ct = BitArray(bytes=open('flag.enc', 'rb').read()).bin
105     msg_len = len(ct)
106     ctr = 0
107     pt = ''
108     add = "0001011001011001011101010001" + " " * 100000
109     "#0,0,0,1,1,0,1,0,1,0,0,1,1,0,0,0,1,1,0,0,0,1,0,1,1,0,0,1,0,1,1,1,0,1,0,1,0,0,0,1"
110
111     print(ct[400:])
112     stream = ""
113
114     while ctr < msg_len // key_len:
115         pt += xor_(key, ct[ctr * key_len: (ctr + 1) * key_len])
116         stream += key
117         key = shift_(key, add[0])
118         add = add[1:]
119
120         ctr += 1
121
122     pt += xor_(key[:msg_len % key_len], ct[ctr * key_len:])
123
124     print(stream[400:])
125     print(pt[400:])
126     print(tobyte(pt)[400:])
127
128     k = "FFD8FFE000104a4649460001"
129
```

CCTF{0Ne\_k3y\_t0\_rU1e\_7hem\_A11\_
4Nd\_7o\_d3crYp7\_th3\_fl4g!}



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## An Online Calculator of Berlekamp-Massey Algorithm

Berlekamp-Massey algorithm is an algorithm that will find the shortest linear feedback shift register (LFSR) for a given binary output sequence. Here we present a web-based implementation to compute the shortest LFSR and linear span of a given binary sequence. If you have any questions or suggestions, please do not hesitate to contact [Bo Zhu](#).

Please enter the binary sequence (separated by commas):

```
0,0,0,1,1,0,1,0,1,0,0,1,1,1,0,0,0,1,0,1,1,0,0,1,0,1,1,1,0,1,0,1,1,1,0,1,1,1,0,1,1,1,
```

LFSR:   $x^{19} + x^5 + x^2 + x^1 + 1$  Linear Span:  19 Time Used:  0.00 sec

Please note the output polynomial is using the form that its degree is always equal to the linear span. For example,  $x^3 + x + 1$  means the LFSR has degree 3.

**new\_testament = [18, 17, 16, 13]**

If there is no output, please check the input sequence and make sure it is a valid binary sequence. You can download the source code of this calculator and run it on local computers.

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MAKING THE FUTURE  
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# Knapsack



|                       |                           |                    |                    |                    |
|-----------------------|---------------------------|--------------------|--------------------|--------------------|
| <b>открытый текст</b> | <b>1 1 1 1 1 0</b>        | <b>0 0 1 1 0 0</b> | <b>0 0 0 0 0 0</b> | <b>0 0 0 0 0 1</b> |
| вещи в рюкзаке        | 3 4 6 7 10 11             | 3 4 6 7 10 11      | 3 4 6 7 10 11      | 3 4 6 7 10 11      |
| шифротекст            | $3 + 4 + 6 + 7 + 10 = 30$ | $6 + 7 = 13$       | 0                  | 11                 |

# Namura



```
def encrypt(pubkey, msg):  
    C = 0  
    for i in range(n):  
        C += pubkey[i] * int(msg[i])  
    return C  
  
flag = flag.lstrip('CCTF{').rstrip('}')  
bflag = bin(bytes_to_long(flag.encode('utf-8')))[2:]  
n = len(bflag)  
u = n - 30  
  
pubkey = keygen((n+1) // 3, n, u)  
  
print('pubkey =', pubkey)  
enc = encrypt(pubkey, bflag)  
print('enc =', enc)  
  
pubkey = [63673042463428268415078750502463684687819253083430137304541  
enc = 154657917005376465967753276253676484467260782425419406781078357]
```

# Namura



```
def encrypt(pubkey, msg):  
    C = 0  
    for i in range(n):  
        C += pubkey[i] * int(msg[i])  
    return C  
  
flag = flag.lstrip('CCTF{').rstrip('}')  
bflag = bin(bytes_to_long(flag.encode('utf-8')))[2:]  
n = len(bflag)  
u = n - 30  
  
pubkey = keygen((n+1) // 3, n, u)  
  
print('pubkey =', pubkey)  
enc = encrypt(pubkey, bflag)  
print('enc =', enc)  
  
pubkey = [63673042463428268415078750502463684687819253083430137304541  
enc = 154657917005376465967753276253676484467260782425419406781078357]
```

Knapsack Public-Key Cryptosystem Using  
Chinese Remainder Theorem

Yasuyuki MURAKAMI\*  
yasuyuki@isc.osakac.ac.jp

Takeshi NASAKO †  
nasako@m.ieice.org

$$B = \begin{pmatrix} 1 & 0 & \dots & 0 & -\lambda a_1 \\ 0 & 1 & \dots & 0 & -\lambda a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & -\lambda a_n \\ -1/2 & -1/2 & \dots & -1/2 & \lambda C \end{pmatrix},$$

# Namura

```
def encrypt(pubkey, msg):  
    C = 0  
    for i in range(n):  
        C += pubkey[i] * int(msg[i])  
    return C  
  
flag = flag.lstrip('CCTF{').rstrip('}')  
bflag = bin(bytes_to_long(flag.encode('utf-8')))[2:]  
n = len(bflag)  
u = n - 30  
  
pubkey = keygen((n+1) // 3, n, u)  
  
print('pubkey =', pubkey)  
enc = encrypt(pubkey, bflag)  
print('enc =', enc)  
  
pubkey = [63673042463428268415078750502463684687819253083430137304541  
enc = 154657917005376465967753276253676484467260782425419406781078357]
```

```
def LLL_for_knapsack_problem(vector, w):  
    K = 100000000  
    def check_01(line):  
        for l in line:  
            if l not in [0, 1]:  
                return False  
        return True  
  
    def check_0_1(line):  
        for l in line:  
            if l not in [0, -1]:  
                return False  
        return True  
  
    for i in range(len(vector)):  
        vector[i] *= K  
  
    l = len(vector)  
    A = matrix.identity(l)  
    B = zero_matrix(ZZ, l, 1)  
    C = matrix([vector])  
    D = [[-w * K]]  
    M = block_matrix(SR, [[A, B], [C, D]], subdivide=False)  
    M = Matrix(ZZ, l + 1, l + 1, M)  
    MR = M.T.LLL()  
    R = []  
    for line in MR:  
        if line[-1] == 0 and (check_01(line) or check_0_1(line)):  
            R.append(line[:-1])  
    return(R)  
pubkey = [636730424634282684150787505024636846878192530834301373045417941, 4434437  
enc = 154657917005376465967753276253676484467260782425419406781078357515  
  
print(LLL_for_knapsack_problem(pubkey, enc))  
for i in range(10000):  
    #print(shuffle(pubkey))  
    l = LLL_for_knapsack_problem(pubkey, enc)  
    if len(l) > 0:  
        print(l)  
        print(pubkey)  
        break
```



# Matrices

$$b_A = \begin{bmatrix} 4 & 1 & 11 & 10 \\ 5 & 5 & 9 & 5 \\ 3 & 9 & 0 & 10 \\ 1 & 3 & 3 & 2 \\ 12 & 7 & 3 & 4 \\ 6 & 5 & 11 & 4 \\ 3 & 3 & 5 & 0 \end{bmatrix} \times \begin{bmatrix} 6 \\ 9 \\ 11 \\ 11 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} \pmod{13} = \begin{bmatrix} 4 \\ 7 \\ 2 \\ 11 \\ 5 \\ 12 \\ 8 \end{bmatrix}$$



```

import random
#from secret import p, flag

def transpose(x):...

def multiply(A, B): ...

def sum_matrix(A, B): ...

def keygen(p):
    d = random.randint(1, 2**64)
    if p % 4 == 1:
        Q = []
        for i in range(p):
            q = []
            for j in range(p):
                if i == j:
                    q.append(0)
                elif pow((i-j), int_((p-1) // 2), p) == 1:
                    q.append(1)
                else:
                    q.append(-1)
            Q.append(q)
        H = []
        r = []
        r.append(0)
        r.extend([1 for i in range(p)])
        H.append(r)
        for i in range(1, p + 1):
            r = []
            for j in range(p + 1):
                r.append(1) if j == 0 else r.append(Q[i-1][j-1])
            H.append(r)

        H2 = [[0 for j in range(2*(p+1))] for i in range(2*(p+1))]
        for i in range(0, p+1):
            for j in range(0, p+1):
                if H[i][j] == 0:
                    H2[i*2][j*2] = 1
                    H2[i*2][j*2+1] = -1
                    H2[i*2+1][j*2] = -1
                    H2[i*2+1][j*2+1] = -1
                elif H[i][j] == 1:
                    H2[i*2][j*2] = 1
                    H2[i*2][j*2+1] = 1
                    H2[i*2+1][j*2] = 1
                    H2[i*2+1][j*2+1] = -1
                else:
                    H2[i*2][j*2] = -1
                    H2[i*2][j*2+1] = -1
                    H2[i*2+1][j*2] = -1
                    H2[i*2+1][j*2+1] = +1
        ID = [[(-1)**d if i == j else 0 for i in range(len(H2))] for j in range(len(H2))]
        H2 = multiply(ID, H2)
        return(H2, d)
    else:
        Q = []
        for i in range(p):
            q = []
            for j in range(p):
                if i == j:
                    q.append(0)
                elif pow(-(i-j), int_((p-1) // 2), p) == 1:
                    q.append(1)
                else:
                    q.append(-1)
            Q.append(q)
        H1 = []
        H1.append([1 for i in range(p+1)])
        for i in range(1, p + 1):
            r = []
            for j in range(p + 1):
                if j == 0:
                    r.append(-1)
                elif i == j:
                    r.append(1 + Q[i-1][j-1])
                else:
                    r.append(Q[i-1][j-1])
            H1.append(r)
        ID = [[(-1)**d if i == j else 0 for i in range(len(H1))] for j in range(len(H1))]
        H1 = multiply(ID, H1)
        return(H1, d)

```

m – ИСКОМЫЙ  
вектор

K, d –  
закрытый ключ  
K – матрица, d –  
натуральное  
число

$$c = m * K + \\ + [d, d \dots d]$$



```

def encrypt(msg, key):
    matrix = key[0]
    d = key[1]
    m = [[ord(char) for char in msg_]]
    de = [[-d for i in range(len(msg))]]
    C = multiply(m, matrix)
    cipher = sum_matrix(C, de)
    return cipher

key = keygen(p)
flag = flag + (len(key[0][0]) - len(flag)) * flag[-1]
cipher = encrypt(flag, key)
print('cipher =', cipher)
## 1 x 76
cipher = [-3459749918754130611, -3459749918754138177, -3459749918754137803, -3459749918754138385,
           -3459749918754138025, -3459749918754138097, -3459749918754138073, -3459749918754138245,
           -3459749918754138183, -3459749918754138445, -3459749918754137991, -3459749918754138597,

```

# Mad hat

```

import random
#from secret import p, flag

def transpose(x):...

def multiply(A, B): ...

def sum_matrix(A, B): ...

def keygen(p):
    d = random.randint(1, 2**64)
    if p % 4 == 1:
        Q = []
        for i in range(p):
            q = []
            for j in range(p):
                if i == j:
                    q.append(0)
                elif pow((i-j), int_((p-1) // 2), p) == 1:
                    q.append(1)
                else:
                    q.append(-1)
            Q.append(q)
        H = []
        r = []
        r.append(0)
        r.extend([1 for i in range(p)])
        H.append(r)
        for i in range(1, p + 1):
            r = []
            for j in range(p + 1):
                r.append(1 if j == 0 else r.append(Q[i-1][j-1]))
            H.append(r)

        H2 = [[0 for j in range(2*(p+1))] for i in range(2*(p+1))]
        for i in range(0, p+1):
            for j in range(0, p+1):
                if H[i][j] == 0:
                    H2[i*2][j*2] = 1
                    H2[i*2][j*2+1] = -1
                    H2[i*2+1][j*2] = -1
                    H2[i*2+1][j*2+1] = -1
                elif H[i][j] == 1:
                    H2[i*2][j*2] = 1
                    H2[i*2][j*2+1] = 1
                    H2[i*2+1][j*2] = 1
                    H2[i*2+1][j*2+1] = -1
                else:
                    H2[i*2][j*2] = -1
                    H2[i*2][j*2+1] = -1
                    H2[i*2+1][j*2] = -1
                    H2[i*2+1][j*2+1] = +1
        ID = [[(-1)**d if i == j else 0 for i in range(len(H2))] for j in range(len(H2))]
        H2 = multiply(ID, H2)
        return(H2, d)
    else:

```

1) Какие передаваемые или генерируемые в самой функции параметры определяют ключевую матрицу?

# Mad hat

```

        return(H2, d)

    else:
        Q = []
        for i in range(p):
            q = []
            for j in range(p):
                if i == j:
                    q.append(0)
                elif pow_((i-j), int_((p-1) // 2), p) == 1:
                    q.append(1)
                else:
                    q.append(-1)
            Q.append(q)
        H1 = []
        H1.append([1 for i in range(p+1)])
        for i in range(1, p + 1):
            r = []
            for j in range(p + 1):
                if j == 0:
                    r.append(-1)
                elif i == j:
                    r.append(1 + Q[i-1][j-1])
                else:
                    r.append(Q[i-1][j-1])
            H1.append(r)
        ID = [[(-1)**d if i == j else 0 for i in range(len(H1))] for j in range(len(H1))]
        H1 = multiply(ID, H1)
        return(H1, d)

def encrypt(msg, key):
    matrix = key[0]
    d = key[1]
    m = [[ord(char) for char in msg_]]
    de = [[-d for i in range(len(msg))]]
    C = multiply(m, matrix)
    cipher = sum_matrix(C, de)
    return cipher

key = keygen(p)
flag = flag + (len(key[0][0]) - len(flag)) * flag[-1]
cipher = encrypt(flag, key)
print('cipher =', cipher)
## 1 x 76
cipher = [-3459749918754130611, -3459749918754138177, -3459749918754137803, -3459749918754138385,
           -3459749918754138025, -3459749918754138097, -3459749918754138073, -3459749918754138245,
           -3459749918754138183, -3459749918754138445, -3459749918754137991, -3459749918754138597,

```



```

import random
#from secret import p, flag

def transpose(x):...
def multiply(A, B):...
def sum_matrix(A, B):...

def keygen(p):
    d = random.randint(1, 2**64)
    if p % 4 == 1:
        Q = []
        for i in range(p):
            q = []
            for j in range(p):
                if i == j:
                    q.append(0)
                elif pow((i-j), int_((p-1) // 2), p) == 1:
                    q.append(1)
                else:
                    q.append(-1)
            Q.append(q)
        H = []
        r = []
        r.append(0)
        r.extend([1 for i in range(p)])
        H.append(r)
        for i in range(1, p + 1):
            r = []
            for j in range(p + 1):
                r.append(1 if j == 0 else r.append(Q[i-1][j-1]))
            H.append(r)

        H2 = [[0 for j in range(2*(p+1))] for i in range(2*(p+1))]
        for i in range(0, p+1):
            for j in range(0, p+1):
                if H[i][j] == 0:
                    H2[i*2][j*2] = 1
                    H2[i*2][j*2+1] = -1
                    H2[i*2+1][j*2] = -1
                    H2[i*2+1][j*2+1] = -1
                elif H[i][j] == 1:
                    H2[i*2][j*2] = 1
                    H2[i*2][j*2+1] = 1
                    H2[i*2+1][j*2] = 1
                    H2[i*2+1][j*2+1] = -1
                else:
                    H2[i*2][j*2] = -1
                    H2[i*2][j*2+1] = -1
                    H2[i*2+1][j*2] = -1
                    H2[i*2+1][j*2+1] = +1
        ID = [[(-1)**d if i == j else 0 for i in range(len(H2))] for j in range(len(H2))]
        H2 = multiply(ID, H2)
        return(H2, d)
    else:

```

2) как именно они влияют и что нужно для их нахождения

Для  $d$  имеет значение только четность  
р можно определить по размерности шифртекста:  
 $p_1 = 37$   
 $p_2 = 75$



```

        return(H2, d)

else:
    Q = []
    for i in range(p):
        q = []
        for j in range(p):
            if i == j:
                q.append(0)
            elif pow((i-j), int_((p-1) // 2), p) == 1:
                q.append(1)
            else:
                q.append(-1)
        Q.append(q)
    H1 = []
    H1.append([1 for i in range(p+1)])
    for i in range(1, p + 1):
        r = []
        for j in range(p + 1):
            if j == 0:
                r.append(-1)
            elif i == j:
                r.append(1 + Q[i-1][j-1])
            else:
                r.append(Q[i-1][j-1])
        H1.append(r)
    H = H1
    ID = [[(-1)**d if i == j else 0 for i in range(len(H1))] for j in range(len(H1))]
    H1 = multiply(ID, H1)
    return(H1, d)

def encrypt(msg, key):
    matrix = key[0]
    d = key[1]
    m = [[ord(char) for char in msg]]
    de = [[-d for i in range(len(msg))]]
    C = multiply(m, matrix)
    cipher = sum_matrix(C, de)
    return cipher

key = keygen(p)
flag = flag + (len(key[0][0]) - len(flag)) * flag[-1]
cipher = encrypt(flag, key)
print('cipher =', cipher)
## 1 x 76
cipher = [-3459749918754130611, -3459749918754138177, -3459749918754137803, -3459749918754138385,
           -3459749918754138025, -3459749918754138097, -3459749918754138073, -3459749918754138245,
           -3459749918754138183, -3459749918754138445, -3459749918754137991, -3459749918754138597,

```

# Mad hat



```
import numpy as np

p1 = 37
p2 = 75

matrices = [np.matrix(keygen(p1, 1)[0]), np.matrix(keygen(p2, 1)[0]),
            np.matrix(keygen(p1, 2)[0]), np.matrix(keygen(p2, 2)[0])]
inv_matrices = [np.linalg.inv(a) for a in matrices]
d0 = -3459749918754130000

for m in inv_matrices:
    for i in range(10000):
        cipher_ = sum_matrix([cipher], [[-d0 + i] * 76])
        cipher_ = np.matrix(cipher_).dot(m)
        flag = ""
        b = True
        for c in range(76):
            c_ = round(cipher_[0, c])
            if c_ < 0 or c_ >= 256:
                b = False
                break
            flag += chr(c_)
        if b and "CTF{" in flag:
            print(flag)

# CTF{TH13_i3_Hadamard_rip_y0ung_&brilliant_Paley!}{}{}{}{}{}{}{}{}{}{}{}{}{}{}{}{}
```

```
136 import numpy as np
137 R = IntegerModRing(257)
138
139 p1 = 37
140 p2 = 75
141
142 matrices = [Matrix(R, keygen(p1, 1)[0]), Matrix(R, keygen(p2, 1)[0]),
143               Matrix(R, keygen(p1, 2)[0]), Matrix(R, keygen(p2, 2)[0])]
144 inv_matrices = [a.inverse() for a in matrices]
145 d0 = -3459749918754130000
146
147
148 for m in inv_matrices:
149     for i in range(257):
150         cipher_ = Matrix(R, [cipher]) + Matrix(R, [[-d0 + i] * 76])
151         cipher_ = cipher_ * m
152         flag = ""
153         b = True
154         for c in range(76):
155             c_ = cipher_[0, c]
156             if c_ < 0 or c_ >= 256:
157                 b = False
158                 break
159             flag += chr(c_)
160     if b and "CTF{" in flag:
161         print(flag)
162         #break
163
```

# Algebraic

```
sage: R.<x> = ZZ['x']; A.<Dx> = OreAlgebra(R)  
  
sage: L = (5*x^2+3*x-7)*Dx^2 + (3*x^2+8*x-1)*Dx + (9*x^2-3*x+8)  
  
sage: L.parent()
```

$\mathbf{Z}[x]\langle Dx \rangle$

```
sage: B = OreAlgebra(QQ['x'], 'Dx')  
  
sage: L = B(L)  
  
sage: L.parent()
```

$\mathbf{Q}[x]\langle Dx \rangle$





# Gambling

```
nc 05.cr.yp.toc.tf 33371
```

```
+++++
+ Hi, there is a strong relation between philosophy and the gambling! +
+ Gamble as an ancient philosopher and find the flag :) +
+++++
| Options:
| [C]ipher flag!
| [E]ncryption function!
| [T]ry the encryption
| [Q]uit
```

```
def encrypt(m, p, a, b):
    assert m < p and isPrime(p)
    return (m ** 3 + a * m + b) % p
```

p, a, b – не известны

$$f(x) \equiv x^3 + ax + b \% p$$

$$f(flag) = c$$

Мы можем получить f(x) для любого x



# Gambling

```
nc 05.cr.yp.toc.tf 33371
```

```
+ Hi, there is a strong relation between philosophy and the gambling! +
+ Gamble as an ancient philosopher and find the flag :) +
+-----+
| Options:
| [C]ipher flag!
| [E]ncryption function!
| [T]ry the encryption
| [Q]uit
```

```
def encrypt(m, p, a, b):
    assert m < p and isPrime(p)
    return (m ** 3 + a * m + b) % p
```

$p, a, b$  – не известны

$f(x) \equiv x^3 + ax + b \% p$

$f(flag) = c$

Мы можем получить  $f(x)$  для любого  $x$

$$f(0) \equiv b \% p \quad - \text{получаем } b$$
$$f(1) \equiv 1 + a + b \% p \quad - \text{получаем } a$$

$$c_1 = x_1^3 + ax_1 + b = f(x_1) + k_1 p$$

$$c_2 = x_2^3 + ax_2 + b = f(x_2) + k_2 p$$

НОД ( $c_1 - f(x_1)$ ,  $c_2 - f(x_2)$ ) делится на  $p$

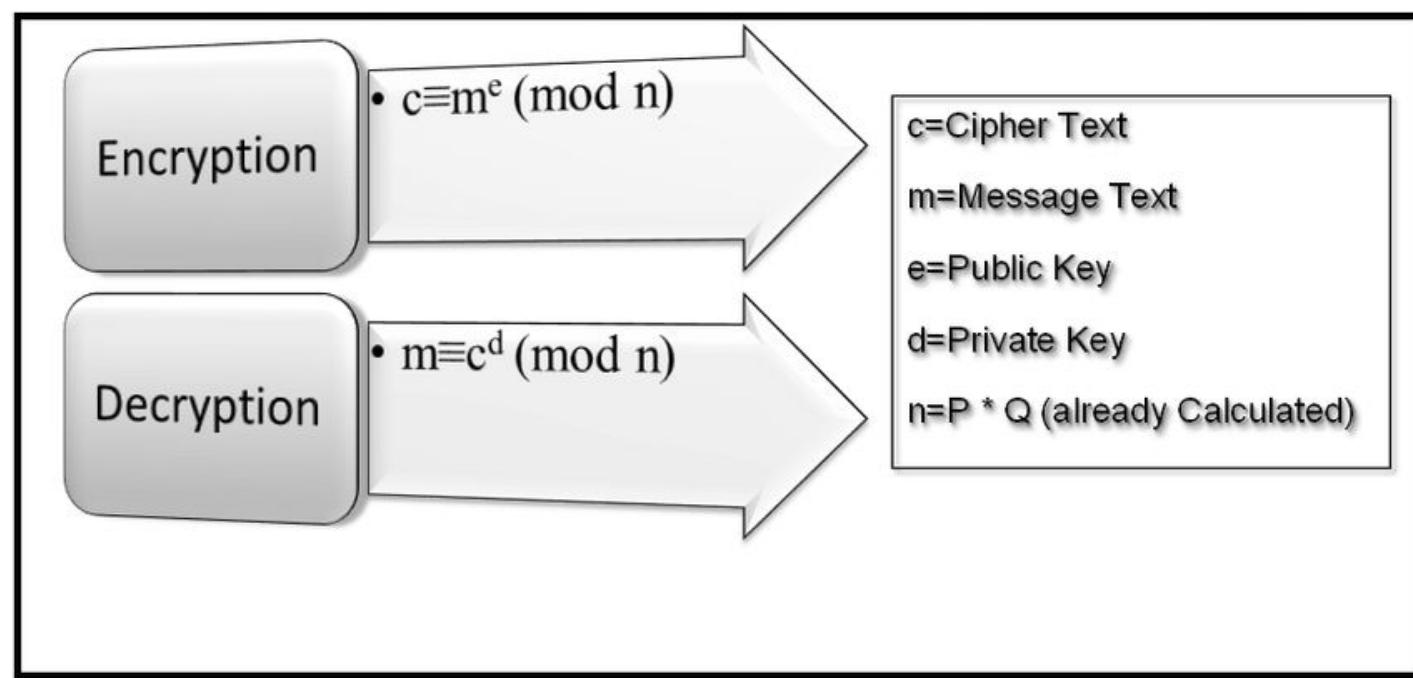
Решаем сравнение:

$$x^3 + ax + b - c \equiv 0 \% p$$

```
PR.<x> = PolynomialRing(GF(p))
f = x^3 + a * x + b - enc
rts = f.roots()
print(rts)

for root in rts:
    flag = root[0]
    print(long_to_bytes(flag))
```

# Algebraic, pow



# One line crypto

```
from Crypto.Util.number import *
from secret import m, n, x, y, flag

p, q = x**(m+1) - (x+1)**m, y**(n+1) - (y+1)**n
assert isPrime(p) and isPrime(q) and p < q < p << 3 and len(bin(p*q)[2:]) == 2048
enc = bytes_to_long(flag)
print(pow(enc, 0x10001, p*q))
```

$$p = x^{m+1} - (x+1)^m$$
$$q = y^{n+1} - (y+1)^n$$



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```

$$p = x^{m+1} - (x+1)^m$$
$$q = y^{n+1} - (y+1)^n$$

```
from Crypto.Util.number import *
from gmpy2 import invert

res = 1460847413295235232889708071732546430843832262331984742844793394320

def f(p, i):
    return pow(p, i + 1) - pow(p + 1, i)

d13 = pow(2, 1028)
d7 = pow(2, 1021)
primes = []
for i in range(1000):
    for j in range(-1000, 1000):
        fn = f(j, i)
        if fn < d13 and fn > d7 and isPrime(fn):
            print(fn, i, j)
            primes.append(fn)

for i in range(len(primes)):
    for j in range(i, len(primes)):
        n = primes[i]*primes[j]
        if len(bin(n)[2:]) == 2048:
            phi = (primes[i]-1)*(primes[j]-1)
            d = invert(0x10001, phi)
            m = long_to_bytes(pow(res, d, n))
            if b"CCTF{" in m:
                print(m)

# b'CCTF{0N3_1!nE_CrYp70_iN_2020}'
```



# Three raven

```
from Crypto.Util.number import *
from flag import flag

def keygen(nbit):
    while True:
        p, q, r = [getPrime(nbit) for _ in range(3)]
        if isPrime(p + q + r):
            pubkey = (p * q * r, p + q + r)
            privkey = (p, q, r)
            return pubkey, privkey

def encrypt(msg, pubkey):
    enc = pow(bytes_to_long(msg.encode('utf-8')), 0x10001, pubkey[0] * pubkey[1])
    return enc

nbit = 512
pubkey, _ = keygen(nbit)
print('pubkey =', pubkey)

enc = encrypt(flag, pubkey)
print('enc =', enc)
```



$p, q, r$  – простые  
(секретный ключ, не известны)  
 $s = p + q + r$  - простое  
 $n = p * q * r * s$   
 $s, n / s$  – открытый ключ, известны

$c = m^e \% n$   
 $m = ?$

# Three ravens

Короткое сообщение (< длины любого из множителей  $n$ )

Известен один из множителей  $n$

$$c \equiv m^e \% p * q * r * s$$

$$c = m^e = k * (p * q * r * s) + c$$

$$m^e \equiv c \% s$$

$$d \equiv e^{-1} \% s$$

$$c^d \equiv m^{ed} \equiv m \% s$$

```
def keygen(nbit):
    while True:
        p, q, r = [getPrime(nbit) for _ in range(3)]
        if isPrime(p + q + r):
            pubkey = (p * q * r, p + q + r)
            privkey = (p, q, r)
            return pubkey, privkey

def encrypt(msg, pubkey):
    enc = pow(bytes_to_long(msg.encode('utf-8')), 0x10001, pubkey[0] * pubkey[1])
    return enc

nbit = 512
pubkey, _ = keygen(nbit)
print('pubkey =', pubkey)

pubkey = (11180735511505417603835067658683342890958492172073834287759921283748260379
enc = 821805228222601189722970390776352121405425478527551188647686132806711749218379

def decrypt(mess, pubkey):
    d = invert(0x10001, pubkey[1] - 1)
    m = pow(mess, d, pubkey[1])
    return long_to_bytes(m)

print(decrypt(enc, pubkey))
# b'CCTF{tH3_r4V3n5_ThRe3_cR0w5}'
```





# Model

```
nc 04.cr.yp.toc.tf 8001
```

```
def genkey(nbit):
    while True:
        p, q = getPrime(nbit), getPrime(nbit)
        if gcd((p-1) // 2, (q-1) // 2) == 1:
            P, Q = (q-1) // 2, (p-1) // 2
            r = inverse(Q, P)
            e = 2 * r * Q - 1
            return(p, q, e)

def encrypt(msg, pubkey):
    e, n = pubkey
    return pow(bytes_to_long(msg), e, n)
```

p, q – простые

$$P = (q-1) // 2$$

$$Q = (p-1) // 2$$

$$r \equiv Q^{-1} \% P$$

$$e = 2 * r * Q - 1$$

$$c \equiv m^e \% p * q$$



# Model

nc 04.cr.yp.toc.tf 8001

```
def genkey(nbit):
    while True:
        p, q = getPrime(nbit), getPrime(nbit)
        if gcd((p-1) // 2, (q-1) // 2) == 1:
            P, Q = (q-1) // 2, (p-1) // 2
            r = inverse(Q, P)
            e = 2 * r * Q - 1
            return(p, q, e)

def encrypt(msg, pubkey):
    e, n = pubkey
    return pow(bytes_to_long(msg), e, n)
```

p, q – простые

$$P = (q-1) // 2$$

$$Q = (p-1) // 2$$

$$r \equiv Q^{-1} \% P$$

$$e = 2 * r * Q - 1$$

$$c \equiv m^e \% p * q$$

$$e = 2 * Q^{-1} * Q - 1 = 1 \% P$$

$$\begin{aligned} 1) \quad & e \equiv 1 \% (q-1) \\ & c \equiv m \% q \end{aligned}$$

$$1) \quad e \equiv 1 + P \% (q-1)$$

$$c = m^{1+(q-1)/2} \% q$$

$$c = m * 1^{1/2} \% q$$

$$c = \pm m \% q$$

$$q = \gcd(c \pm m, n)$$



# Butterfly effect

```
386
387     def hq_prng(x, p, g):
388         rng = 0
389         for _ in range(getRandomNBitInteger(14)):
390             x = pow(g, x, p)
391             for i in range(nbit):
392                 x = pow(g, x, p)
393                 if x < (p-1) // 2:
394                     rng += 2**i - 1
395                 elif x > (p-1) // 2:
396                     rng -= 2**i + 1
397                 else:
398                     rng ^= 2***(i + 1)
399             if rng <= 0:
400                 return -rng
401         return rng
402
403     def keygen(p, g):
404         r, s = hq_prng(getRandomNBitInteger(nbit), p, g), hq_prng(getRandomNBitInteger(nbit), p, g)
405         u, v = gmpy2.next_prime(r**2 + s**2), gmpy2.next_prime(2*r*s)
406         e, n = 0x10001, u * v
407         return e, n
408
409     def encrypt(msg, e, n):
410         return pow(bytes_to_long(msg.encode('utf-8')), e, n)
411
412 #encrypt(flag, e, n) = 117667582947026307482709850318214820165964980495414423711608614681075036546959
413
414 #(p, g, n) = (68396932999729141946282927360590169590631231980913314894620521363257317833167L, 1114840
415
```



# Butterfly effect

```
386
387     def hq_prng(x, p, g):
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389         for _ in range(getRandomNBitInteger(14)):
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394                     rng += 2**i - 1
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396                     rng -= 2**i + 1
397                 else:
398                     rng ^= 2***(i + 1)
399             if rng <= 0:
400                 return -rng
401             return rng
402
403     def keygen(p, g):
404         r, s = hq_prng(getRandomNBitInteger(nbit), p, g), hq_prng(getRandomNBitInteger(nbit), p, g)
405         u, v = gmpy2.next_prime(r**2 + s**2), gmpy2.next_prime(2*r*s)
406         e, n = 0x10001, u * v
407         return e, n
408
409     def encrypt(msg, e, n):
410         return pow(bytes_to_long(msg.encode('utf-8')), e, n)
411
412     #encrypt(flag, e, n) = 117667582947026307482709850318214820165964980495414423711608614681075036546959
413
414     #(p, g, n) = (68396932999729141946282927360590169590631231980913314894620521363257317833167L, 1114840
415
```



Type some Sage code below and press Evaluate.

```
1 Zp = Zmod(68396932999729141946282927360590169590631231980913314894620521363257317833167)
2 a = Zp(11148408907716636563689390048104567047599159688378384611325239859308138541650)
3 print(a.multiplicative_order())
4
```

Evaluate

```
31337
```



# Butterfly effect

```
387     def hq_prng(x, p, g):
388         rng = 0
389         for _ in range(getRandomNBitInteger(14)):
390             x = pow(g, x, p)
391             for i in range(nbit):
392                 x = pow(g, x, p)
393                 if x < (p-1) // 2:
394                     rng += 2**i - 1
395                 elif x > (p-1) // 2:
396                     rng -= 2**i + 1
397                 else:
398                     rng ^= 2***(i + 1)
399                 if rng <= 0:
400                     return -rng
401             return rng
402
403     def keygen(p, g):
404         r, s = hq_prng(getRandomNBitInteger(nbit), p, g),
405         u, v = gmpy2.next_prime(r**2 + s**2), gmpy2.next_
406         e, n = 0x10001, u * v
407         return e, n
408
409     def encrypt(msg, e, n):
410         return pow(bytes_to_long(msg.encode('utf-8')), e,
411
412 # encrypt(flag, e, n) = 117667582947026307482709850318
413
414 #(p, g, n) = (683969329997291419462829273605901695906
415
```

```
46     import gmpy2
47     from Crypto.Util.number import bytes_to_long, getRandomNB
48
49     nbit = 256
50     def hq_prng(x, p, g):
51         rng = 0
52         global gs
53         for i in range(nbit):
54             x = gs[x % 31337]
55             if x < (p-1) // 2:
56                 rng += 2**i - 1
57             elif x > (p-1) // 2:
58                 rng -= 2**i + 1
59             else:
60                 rng ^= 2***(i + 1)
61             if rng <= 0:
62                 return -rng
63             return rng
64
65     def keygen(p, g):
66         r, s = hq_prng(getRandomNBitInteger(nbit), p, g), hq_
67         u, v = gmpy2.next_prime(r**2 + s**2), gmpy2.next_prime_
68         e, n = 0x10001, u * v
69         return e, n
70
71     def encrypt(msg, e, n):
72         return pow(bytes_to_long(msg.encode('utf-8')), e, n)
73
74 # encrypt(flag, e, n) = 117667582947026307482709850318214
75
76 #(p, g, n) = (68396932999729141946282927360590169590631231
77             11148408907716636563689390048104567047599159
78             17442100312381003338179023622183785651571732
79
80     print(gmpy2.is_prime(p))
81
82 #sage:
```

```
46     ord = 31337
47
48     gs = [0] * 31337
49     prns = [0] * 31337
50     curr = 1
51     for i in range(31337):
52         gs[i] = curr
53         curr = curr * g % p
54     print("done1")
55     for i in range(0, 31337):
56         prns[i] = hq_prng(i, p, g)
57     print("done2")
58     prns.sort()
59
60
61     for i in range(0, 31337):
62         print(i)
63         lef = i+1
64         rig = 31336
65         mid, best = 0, 0
66         while lef <= rig:
67             mid = (lef + rig) // 2
68             if (prns[i]*prns[i] + prns[mid] * prns[mid]) * 2 * prns[i] * prns[mid] >= n:
69                 best = mid
70                 rig = mid - 1
71             else:
72                 lef = mid + 1
73
74         if best == 0:
75             continue
76
77         for j in range(best-1, min(best+30, 31337)):
78             u = prns[i] * prns[i] + prns[j] * prns[j]
79             v = 2 * prns[i] * prns[j]
80
81             u = gmpy2.next_prime(u)
82             v = gmpy2.next_prime(v)
83             if u * v == n:
84                 print(u, v)
85                 exit()
86
```



# Decent RSA

-----BEGIN PUBLIC KEY-----

MIIBIjANBgkqhkiG9w0BAQEFAOCAQ8AMIIBCgKCAQEA/Ug8rlEPci1UXMsT  
+UDo  
y8DfxbTHX/3BK2oU+FPWiJf+EiUBM2x4ep04qZ1SO9Pmqj/WH9skMrF1J/LX  
uY3I  
fjvJCh0DXa9VUyX2dAJidja9Ior7GpFwwjYdKh+OETNV+2/CcX4RiPvj+8Apm  
edW  
gn4Fxaeivki+f/UwDa+ws1fTUzmI325v8yvcryHhbgeUWiF85EP6HFAavTsVPI  
xb  
LikVMAB1fuzDbqqJvW2u138w6b2FH3WrezYF6tbAyZej2HX46phwDm9C7MX  
YJ/sU  
oS+E8P7S1jMTCWjfMCOKU3SGrkWtXuTaoMZ2nZ+HVfJV8xJOjWez1OxQ  
5P3F1w  
GQIDAQAB  
-----END PUBLIC KEY-----

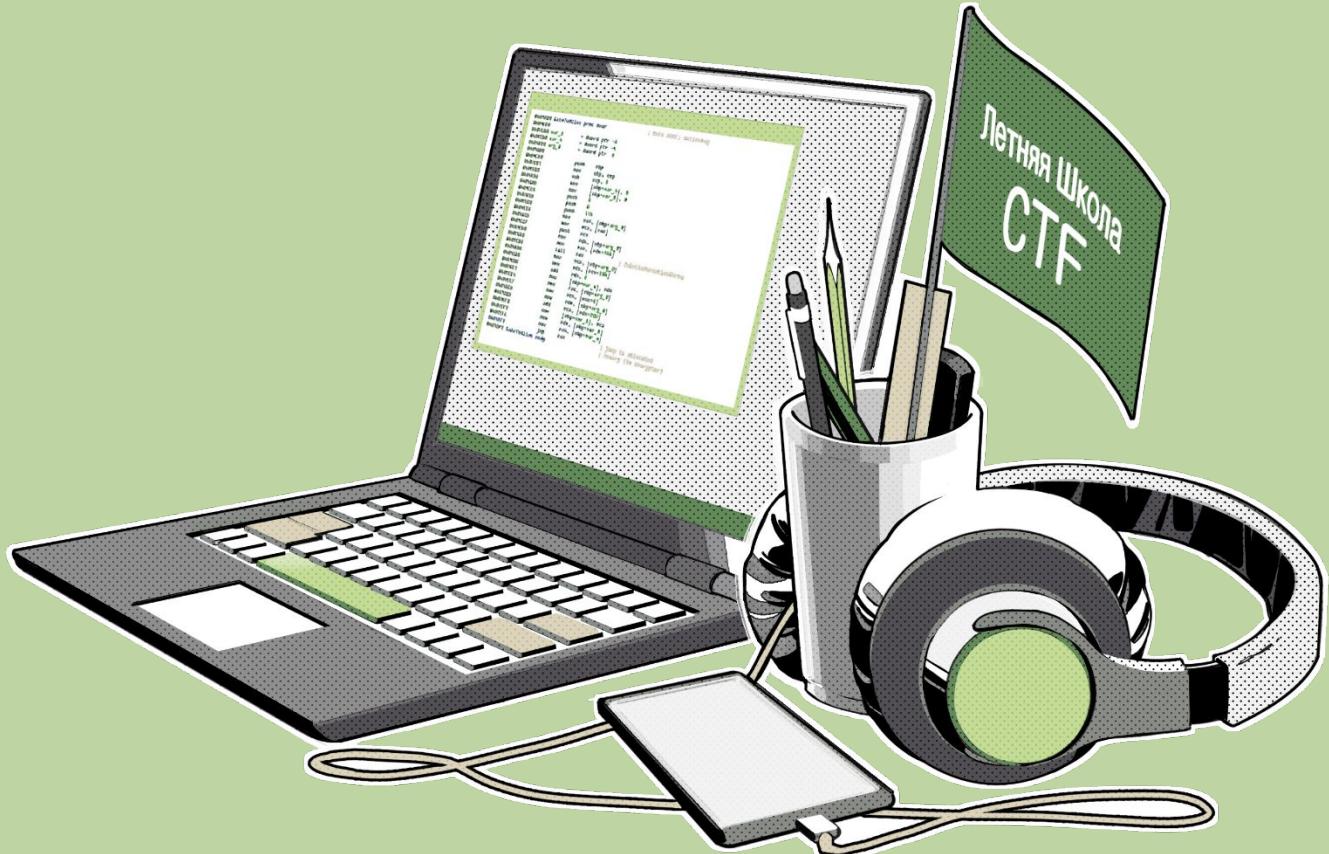


# Decent RSA

```
-----BEGIN PUBLIC KEY-----
MIIBIjANBgkqhkiG9w0BAQEFAOCAQ8AMIIBCgKCAQEA/Ug8rlEPci1UXMsT
+UDo
y8DfxbTHX/3BK2oU+FPWiJf+EiUBM2x4ep04qZ1SO9Pmj/WH9skMrF1J/LX
uY3I
fvJCh0DXa9VUyX2dAJidja9Ior7GpFwwjYdKh+OETNV+2/CcX4RiPvj+8Apm
edW
gn4Fxaeivki+f/UwDa+ws1fTUzmI325v8vcryHhbgeUWiF85EP6HFAavTsVPI
xb
LikVMAB1fuzDbqqJvW2u138w6b2FH3WrezYF6tbAyZej2HX46phwDm9C7MX
YJ/sU
oS+E8R7S1jMTCWjfWMCOKU3SGrkWtXuTaoMZ2nZ+HVfJV8xJOjWez1OxQ
5P3F1w
from Crypto.PublicKey import RSA
from Crypto.Util.number import long_to_bytes, bytes_to_long
GQIDAQAB
-----END PUBLIC KEY-----
flag = bytes_to_long(open("flag.enc", "rb").read())
key = RSA.import_key(open("mykey.pem").read())
n = Integer(key.n)

poly = sum(e * x^i for i,e in enumerate(Integer(key.n).digits(11)))
(p, _), (q, _) = poly.factor_list()
p, q = p(x=11), q(x=11)
assert p*q == n

d = inverse_mod(key.e, (p-1)*(q-1))
print(long_to_bytes(pow(flag, d, n)))
```



# ЛЕТНЯЯ ШКОЛА СТФ 2021



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