OTUS

ОНЛАЙН-ОБРАЗОВАНИЕ

Точечные и интервальные оценки

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Меня хорошо слышно && видно?

Напишите в чат, если есть проблемы!

Ставьте + если все хорошо

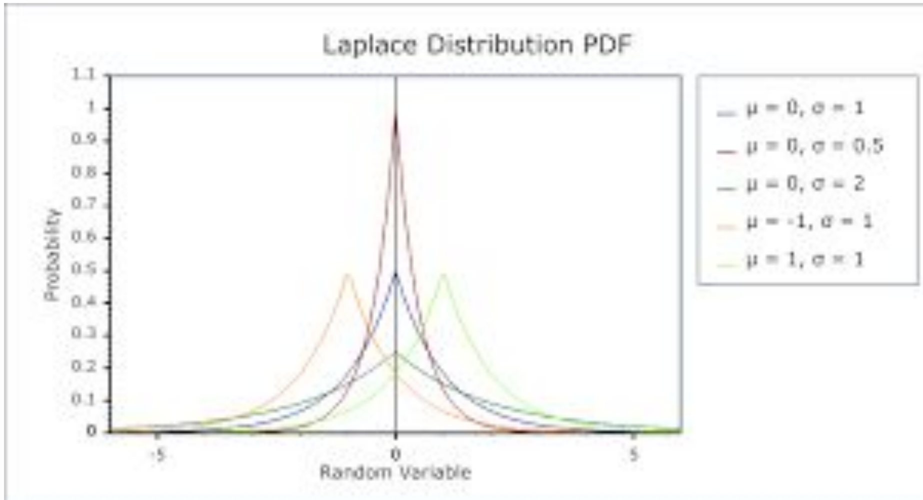
Правила вебинара

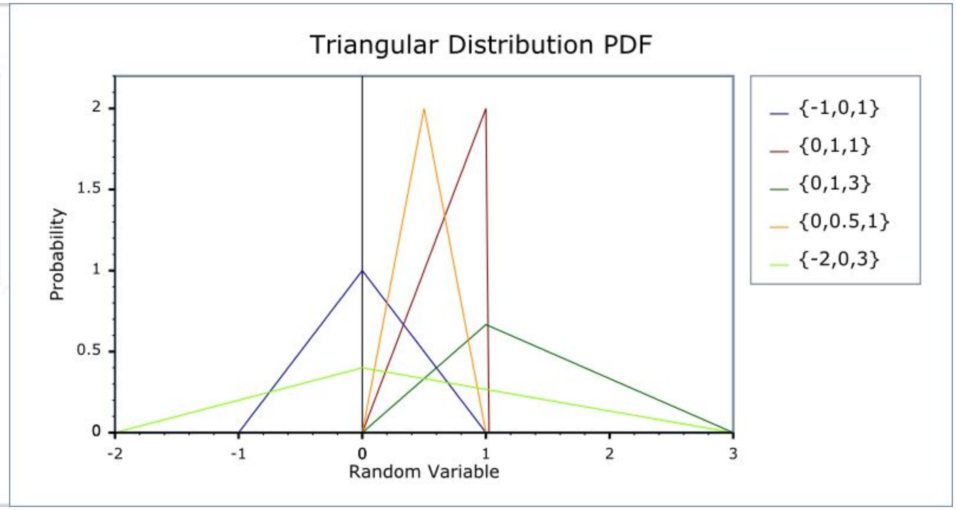
Активно участвуем

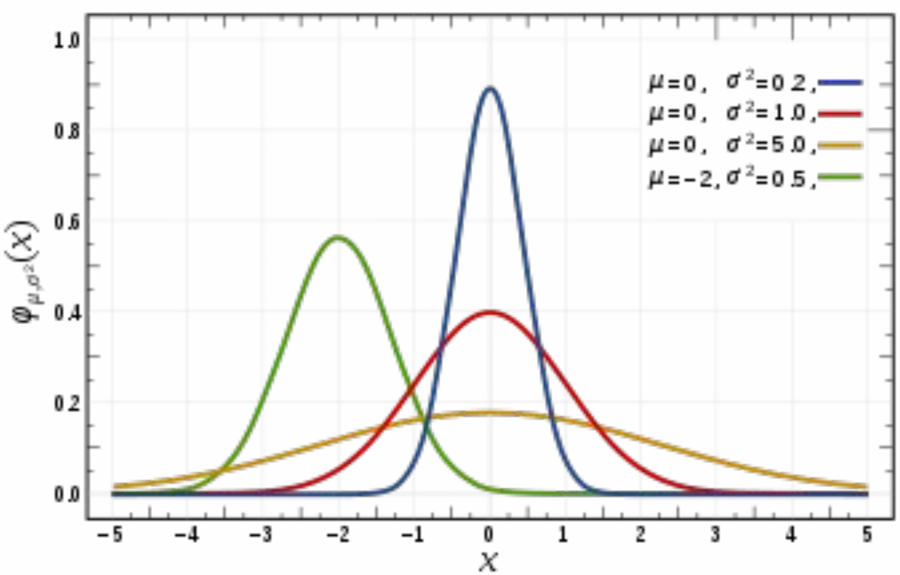
Задаем вопросы в чат

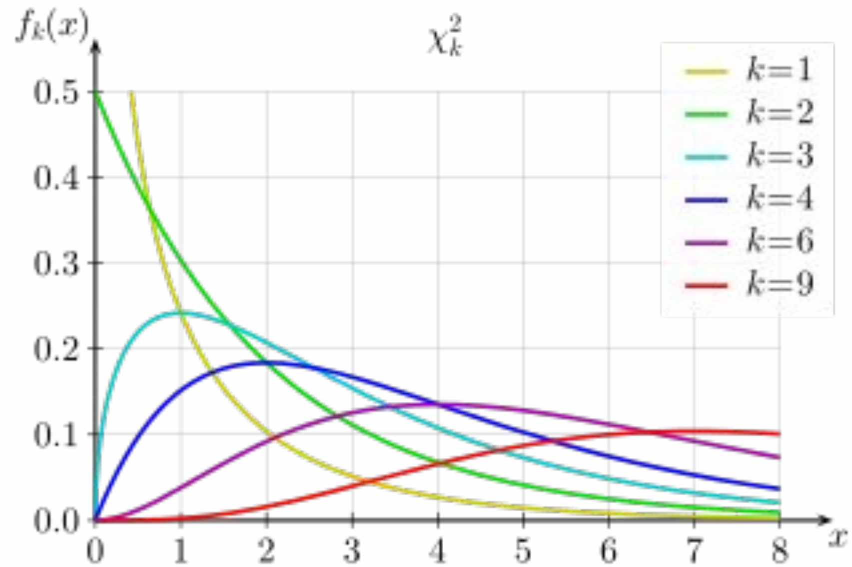
Вопросы вижу в чате, могу ответить не сразу

Point Estimators









Point Estimators

Статистикой называется произвольная функция $\theta^* = \theta^*(X_1, \ldots, X_n)$ от элементов выборки.

Статистика $\theta^* = \theta^*(X_1, \ldots, X_n)$ называется несмещенной оценкой параметра θ, если для любого $\theta \in \Theta$ выполнено равенство

$$E_\theta \theta^* = \theta.$$

Статистика $\theta^* = \theta^*(X_1, \ldots, X_n)$ называется состоятельной оценкой параметра θ, если для любого $\theta \in \Theta$ имеет место сходимость

$$\theta^* \xrightarrow{p} \theta \text{ при } n \to \infty.$$

Point Estimators

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Let $\hat{\theta}$ be an estimator of θ, which means that $\hat{\theta}$ can be expressed as a function applied to a sample: $\hat{\theta} = f(X_1, \ldots, X_n)$.

(a) $\hat{\theta}$ is called unbiased if $\mathrm{E}\,\hat{\theta} = \theta$ for all possible values of the unknown parameters.

(b) If $\hat{\theta}_1$ and $\hat{\theta}_2$ are two unbiased estimators we say that $\hat{\theta}_1$ is more efficient than $\hat{\theta}_2$ if $\mathrm{Var}(\hat{\theta}_1) \leq \mathrm{Var}(\hat{\theta}_2)$ for all possible values of the unknown parameters (and the inequality is strict for some values of the parameters).

(c) $\hat{\theta}$ is called consistent if $\hat{\theta} \to \theta$ as the sample size $n \to \infty$. We will understand this convergence in the following sense: $\mathrm{E}((\hat{\theta} - \theta)^2) \to 0$ as $n \to \infty$. If $\hat{\theta}$ is unbiased, this means $\mathrm{Var}(\hat{\theta}) \to 0$.

Point Estimators

$$X_1, \ldots, X_n \sim \textit{i.i.d.} \ (\mu, \sigma^2)$$

$\hat{\mu}_n \equiv \frac{1}{n} \sum_i X_i$, estimator of μ
$\hat{\sigma}_n^2 \equiv \frac{1}{n} \sum_i (X_i - \bar{X}_n)^2$, estimator of σ^2

Point Estimators

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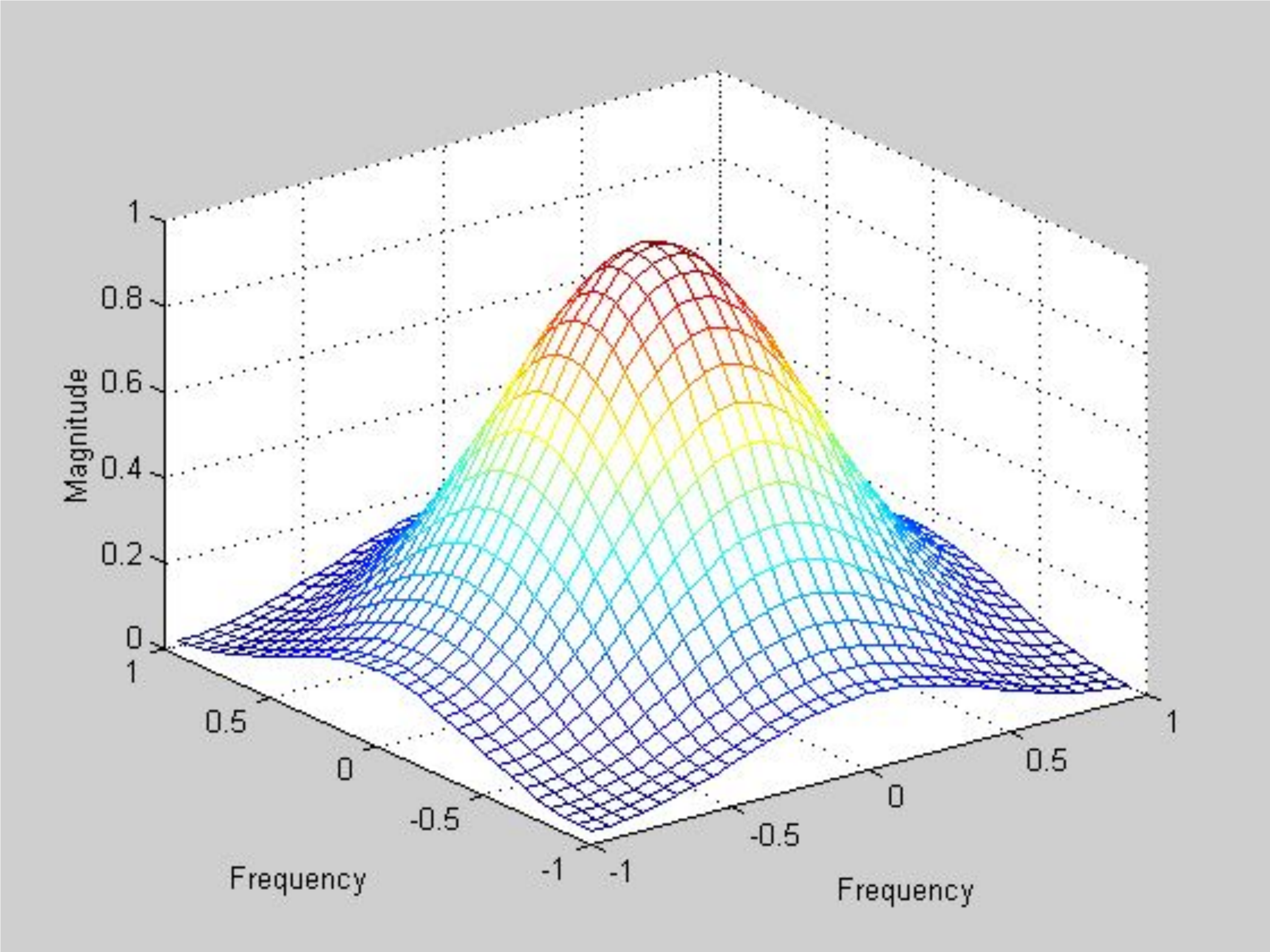
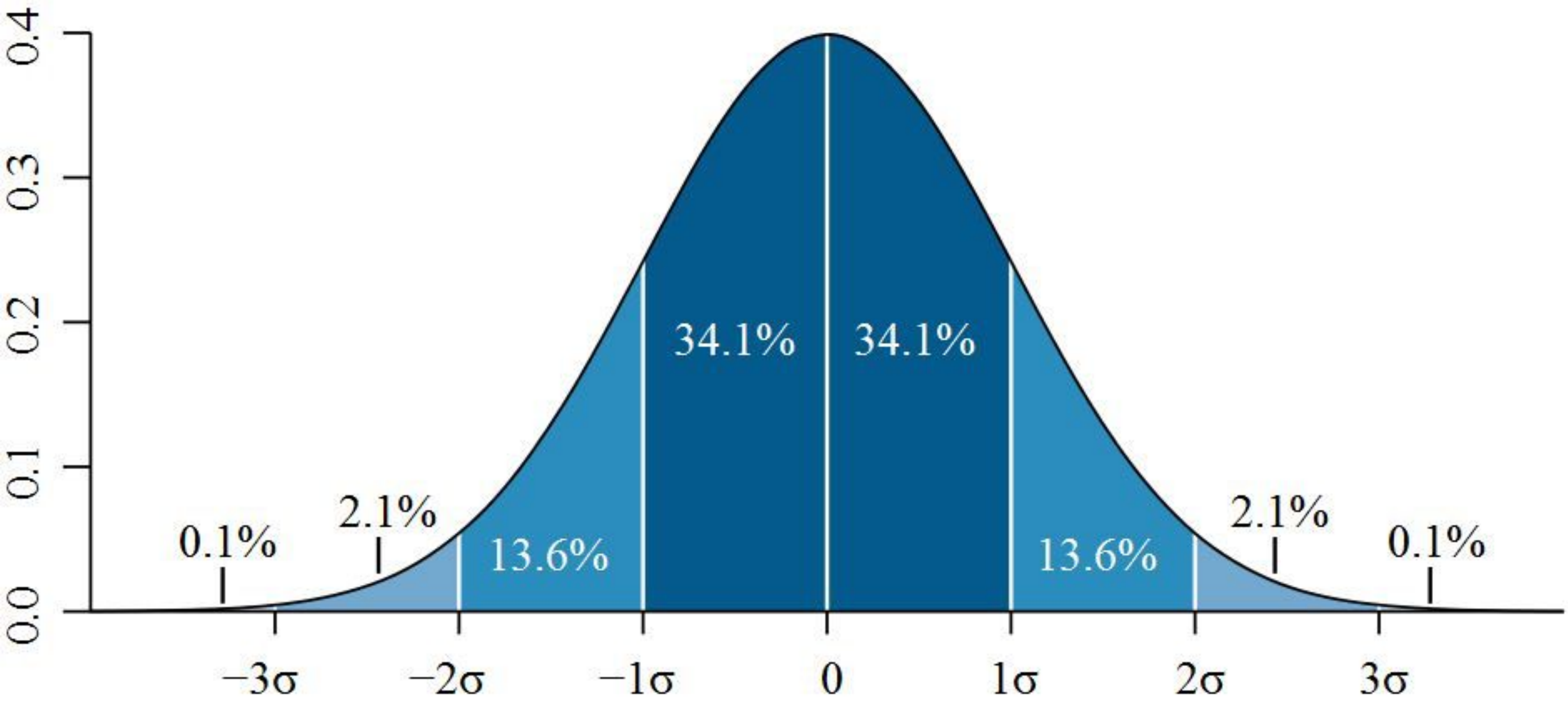
$$X_1, \ldots, X_n \sim \textit{i.i.d.} \ (\mu, \sigma^2)$$

$$\begin{aligned}
E\hat{\sigma}^2 &= E\Big(\frac{1}{n}\sum_i (X_i - \bar{X}_n)^2\Big) \\
&= \frac{1}{n} \cdot \sum_i (EX_i^2 - 2EX_i\bar{X}_n + E\bar{X}_n^2) \\
&= \frac{1}{n} \cdot n \left[(\mu^2 + \sigma^2) - 2(\mu^2 + \frac{\sigma^2}{n}) + \frac{\sigma^2}{n} + \mu^2\right] \\
&= \frac{n-1}{n}\sigma^2.
\end{aligned}$$

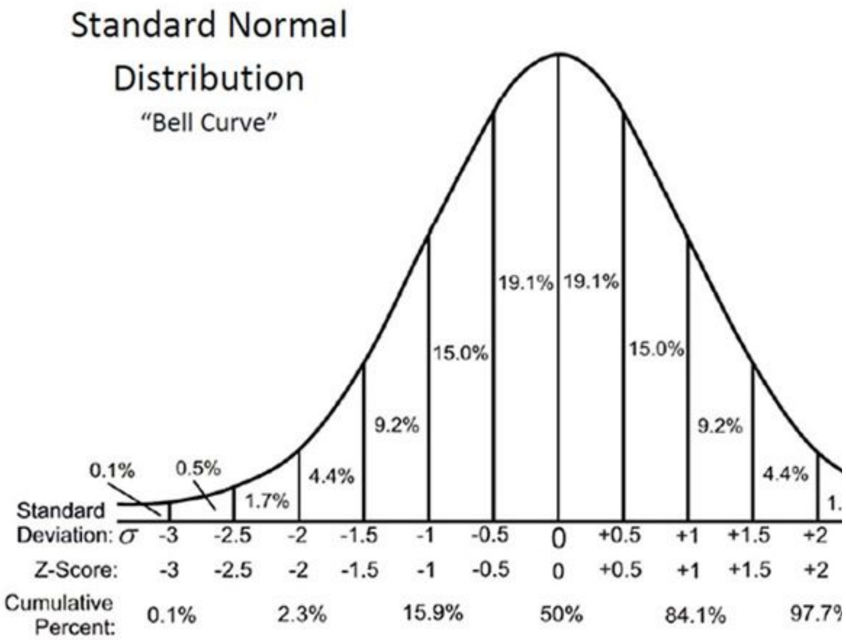
Point Estimators

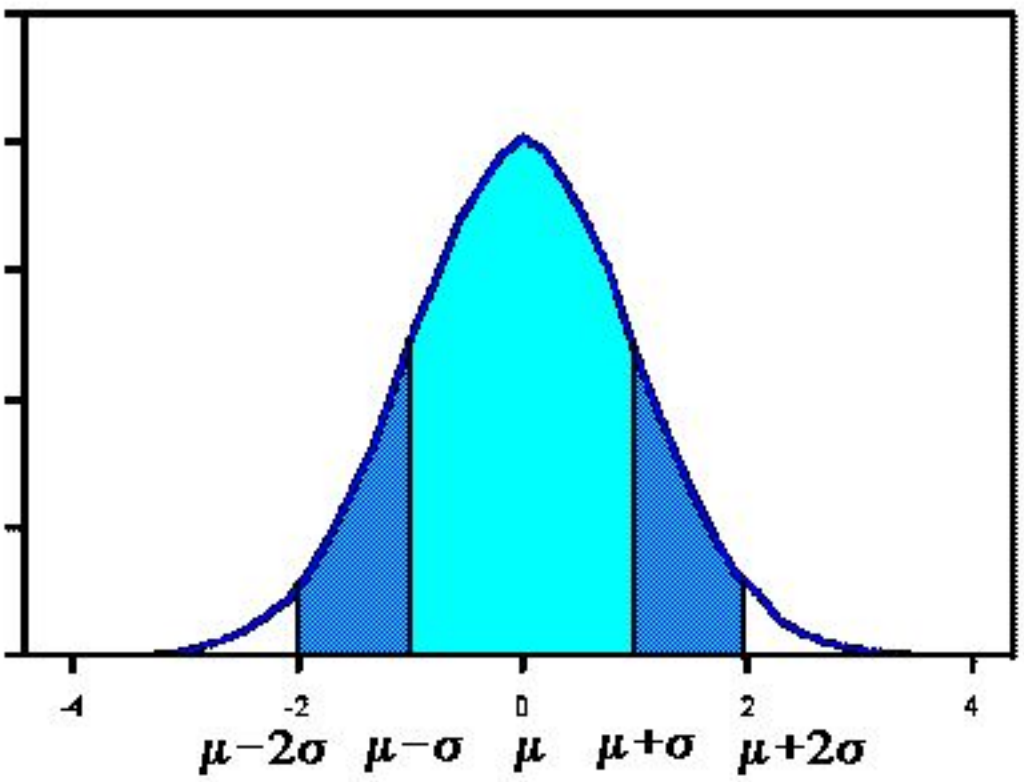


Point Estimators



Point Estimators





Свойства

$$Bias\left(\hat{\theta}\right) = E\left(\hat{\theta}\right) - \theta$$

$$MSE(W) = VW + (EW - \theta)^2 = \text{Variance} + (\text{Bias})^2$$

СВОЙСТВА

1. несмещённость (отсутствие систематической ошибки при оценивании),
2. состоятельность (увеличение точности оценивания с ростом числа наблюдений),
3. эффективность (самая высокая точность по сравнению со всеми другими оценками)

Таблица нормального распределения

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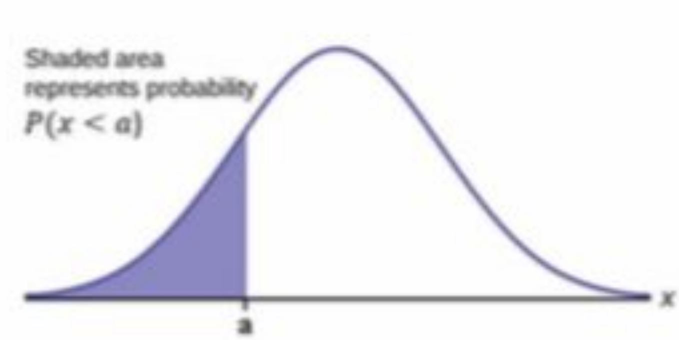
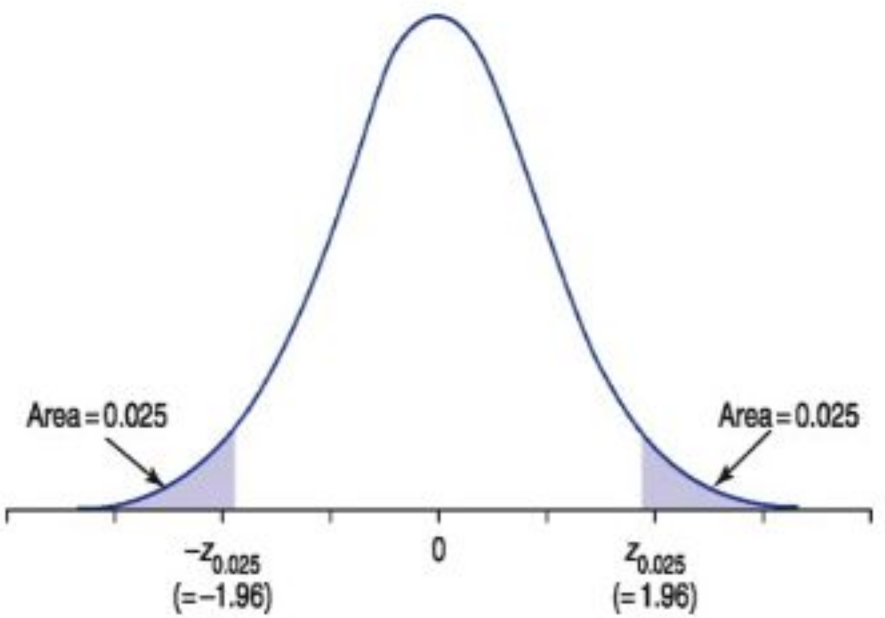


a	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

a	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Интервальное оценивание

$$X_1, \ldots, X_n \sim \textit{i.i.d.}\ (\mu, \sigma^2)$$



$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \sqrt{n}\frac{\overline{X} - \mu}{\sigma}$$

Интервальное оценивание

Suppose $X \sim N(\mu, \sigma^2)$ and μ is unknown, σ is known. Then we know $\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$.
Hence for any value of μ we have

$$\mathrm{P}\left(\frac{|\overline{X} - \mu|}{\sigma\sqrt{n}} \leq 1.96\right) = 0.95 \implies P\left(\mu \in \left[\overline{X} - 1.96\frac{\sigma}{\sqrt{n}},\ \overline{X} + 1.96\frac{\sigma}{\sqrt{n}}\right]\right) = 0.95.$$

$$\mu \in \left[\overline{x} - 1.96\frac{\sigma}{\sqrt{n}},\ \overline{x} + 1.96\frac{\sigma}{\sqrt{n}}\right].$$

Instead of writing $\mu \in [\overline{X} - m,\ \overline{X} + m]$, where m can be, for example, $1.96\sigma/\sqrt{n}$, we will write

$$\mu = \overline{X} \pm m \text{ (at confidence level p)},$$

and m is called the margin of error (at a given confidence level p).

Confidence intervals for population means and population proportions

(a) F is a normal distribution with known σ:

$$\mu = \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \text{ (at confidence level } p).$$

The sample size n can be any.

(b) F is any population distribution with known σ:

$$\mu = \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \text{ (at confidence level } p).$$

Condition: the sample size is large ($n \geq 30$ or 40).

(c) F is any population distribution with unknown σ:

$$\mu = \overline{X} + z_{\alpha/2}\frac{s}{\sqrt{n}} \text{ (at confidence level } p).$$

Condition: sample size is large ($n \geq 30$ or 40).

(d) F is the Bernoulli distribution, with unknown population proportion p:

$$p = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \text{ (at confidence level } p).$$

Интервальное оценивание

Two-sided C.I.	Lower C.I.	Upper C.I.
$\mu = \overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$	$\mu \geq \overline{x} - z_{\alpha}\frac{\sigma}{\sqrt{n}}$	$\mu \leq \overline{x} + z_{\alpha}\frac{\sigma}{\sqrt{n}}$
$\mu = \overline{x} \pm t_{\alpha/2}(n-1)\frac{s}{\sqrt{n}}$	$\mu \geq \overline{x} - t_{\alpha}(n-1)\frac{s}{\sqrt{n}}$	$\mu \leq \overline{x} + t_{\alpha}(n-1)\frac{s}{\sqrt{n}}$
$\mu_x - \mu_y = \overline{x} - \overline{y} \pm z_{\alpha/2}\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$	$\mu_x - \mu_y \geq \overline{x} - \overline{y} - z_{\alpha}\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$	$\mu_x - \mu_y \leq \overline{x} - \overline{y} + z_{\alpha}\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$
$\mu_x - \mu_y = \overline{x} - \overline{y} \pm t_{\alpha/2}(k)\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}$	$\mu_x - \mu_y \geq \overline{x} - \overline{y} - t_{\alpha}(k)\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}$	$\mu_x - \mu_y \leq \overline{x} - \overline{y} + t_{\alpha}(k)\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}$
$\mu_x - \mu_y = \overline{x} - \overline{y} \pm t_{\alpha/2}(k)\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$	$\mu_x - \mu_y \geq \overline{x} - \overline{y} - t_{\alpha}(k)\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$	$\mu_x - \mu_y \leq \overline{x} - \overline{y} + t_{\alpha}(k)\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$

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Есть вопросы или замечания?

Напишите в чат свои вопросы и замечания!

Ставьте + если все понятно

Спасибо за внимание!