Homework

Exercise 1: For $A \in M_n$ proof:

$$(a) \; \||\| A^2 \||\| \leq \||\| A \||\|^2, \; \||\| A^p \||\| \leq \||\| A \||\|^p, \; p = 2,3,\ldots$$

$$(b) \; \text{if } A^2 = A \text{ then } \||\| A \||\| \geq 1$$

$$(c) \; \text{if } A \text{ is invertible, then } \||\| A^{-1} \||\| \geq \frac{\||\| I \||\|}{\||\| A \||\|}$$

$$(d) \; \||\| I \||\| \geq 1.$$

Homework

Exercise 2. Proof $n\|\cdot\|_\infty$ is a matrix norm, where n is the size of the matrix.

Exercise 3. The *spectral norm* $\||\cdot\||_2$

$$\||A\||_2 = \max\left\{\sqrt{\lambda} : \lambda \text{ is an eigenvalue of } A^*A\right\}$$

is deduced by the l_2 norm.