

Homework

Exercise 1: For $A \in M_n$ proof:

$$(a) \quad \| \| A^2 \| \| \leq \| \| A \| \|^2, \quad \| \| A^p \| \| \leq \| \| A \| \|^p, \quad p = 2, 3, \dots$$

$$(b) \quad \text{if } A^2 = A \text{ then } \| \| A \| \| \geq 1$$

$$(c) \quad \text{if } A \text{ is invertible, then } \| \| A^{-1} \| \| \geq \frac{\| \| I \| \|}{\| \| A \| \|}$$

$$(d) \quad \| \| I \| \| \geq 1.$$

Homework

Exercise 2. Prove $n \|\cdot\|_\infty$ is a matrix norm, where n is the size of the matrix.

Exercise 3. The *spectral norm* $\|\cdot\|_2$

$$\|A\|_2 = \max \{ \sqrt{\lambda} : \lambda \text{ is an eigenvalue of } A^*A \}$$

is deduced by the l_2 norm.