



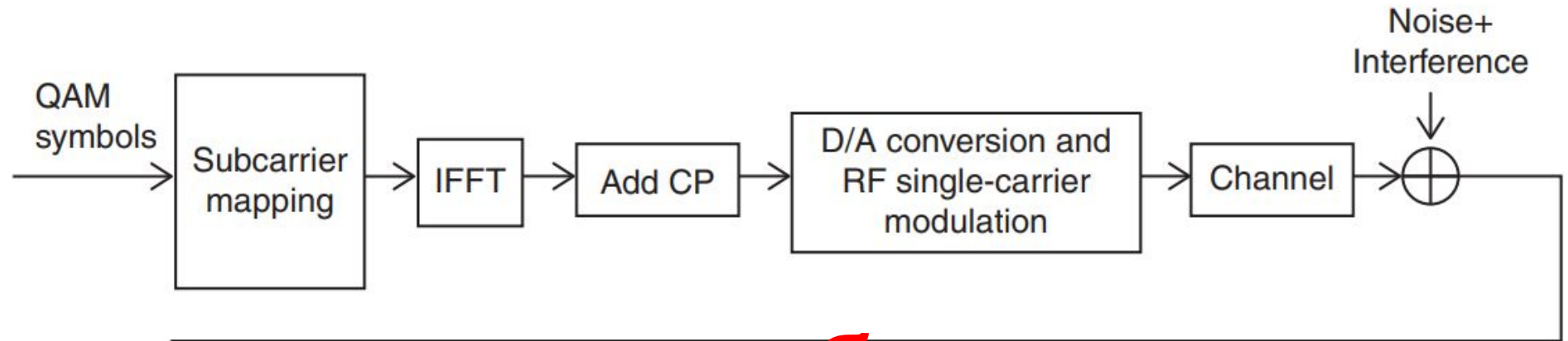
# Методы обработки сигналов в телекоммуникационных системах

Тимошенко Александр Геннадиевич

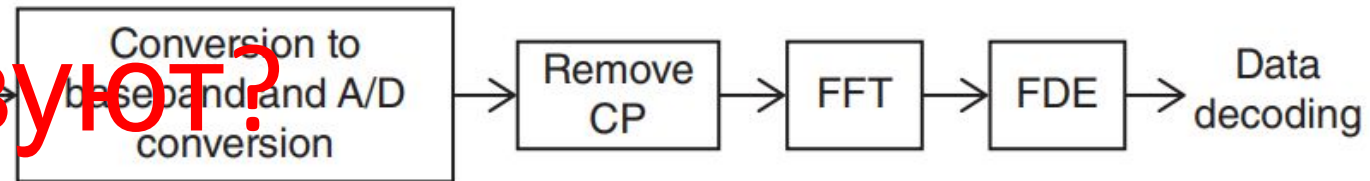
4226а

Некоторые слайды на английском  
языке

**Какие методы необходимо применять в предлагаемой схеме?**



**А какие методы вообще существуют?**





## **А какие методы вообще существуют?**

- А какие сигналы мы обрабатываем?

## Analog signals

Infinite precision in amplitude

Continuous signal

Continuous mathematics



## Digital signals

Finite precision in amplitude

Only present at certain points

Discrete mathematics





## Analog signals

восстановление

разделение информационных  
потоков

подавление шумов

сжатие данных

фильтрация

усиление сигналов

## Digital signals

спектральный анализ

линейная фильтрация

свертка традиционных типов

частотно-временной анализ

нелинейная обработка

адаптивная фильтрация

многоскоростная обработка

секционная свертка



## Базовые операции

задержка сигналов

сложение сигналов

вычитание сигналов

умножение сигналов

деление сигналов

интегрирование

дифференцирование

фильтрация

спектральный анализ

линейная фильтрация

свертка традиционных типов

частотно-временной анализ

нелинейная обработка

адаптивная фильтрация

многоскоростная обработка

секционная свертка

## Какие ещё виды обработки сигналов возможны?

- Статистическая обработка сигналов
- Обработка звука
  - Распознавание речи
- Обработка изображений
  - Обработка видео
  
- Обработка сигналов нужна для обработки информации
  - Включая экспериментальные данные



## Signal processing methods

- Today:
- Part 1
  - Digital signals
- Part 2
  - Basic programming in Python
  - Data types (in Python)
  - Correct notations in coding
  - Visualizing your signals



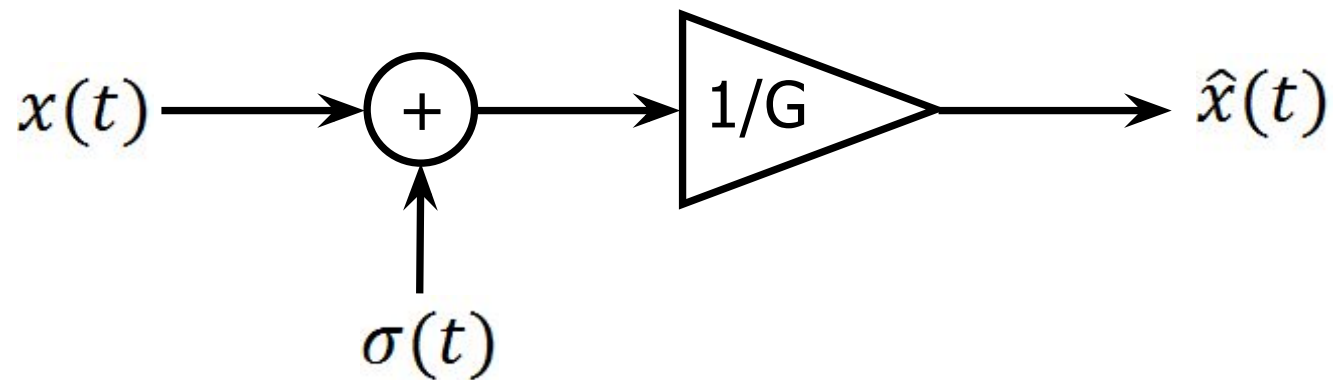


## Why digital signals are useful?

- Storage (generalized computer memory, compatibility)
- Processing (coding, no mechanical or physical interactions)
- Transmission(signal regeneration)

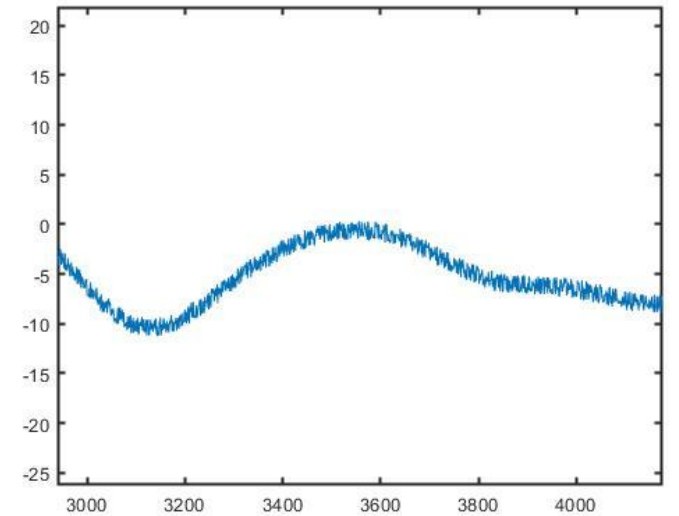
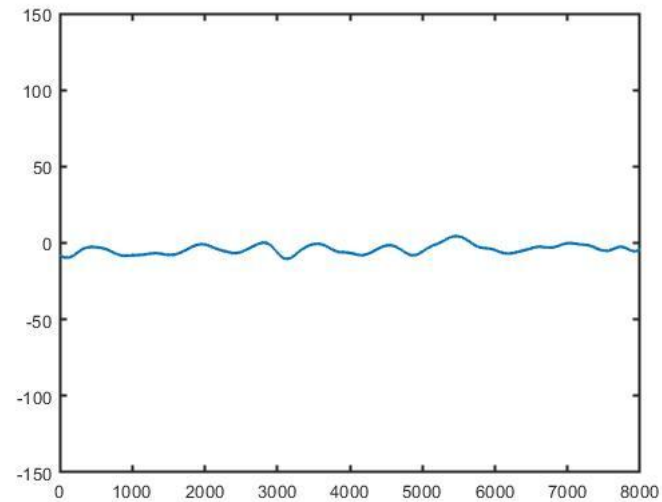
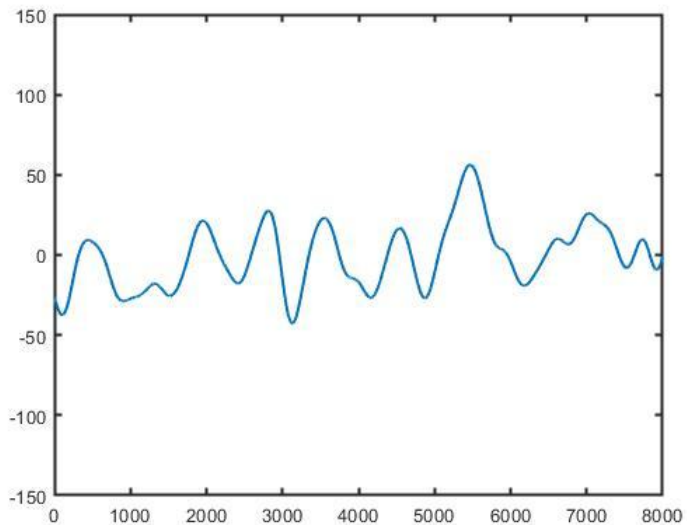
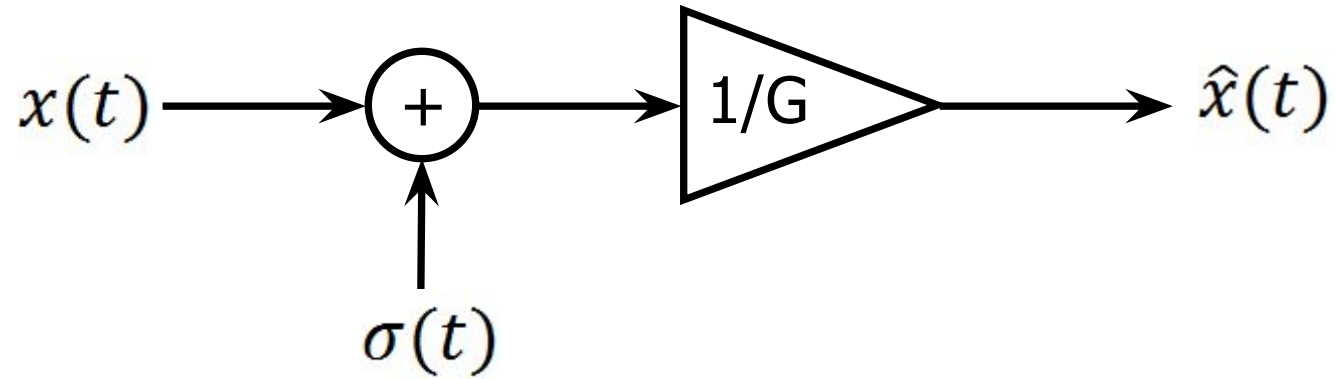


## Transmission - signal regeneration



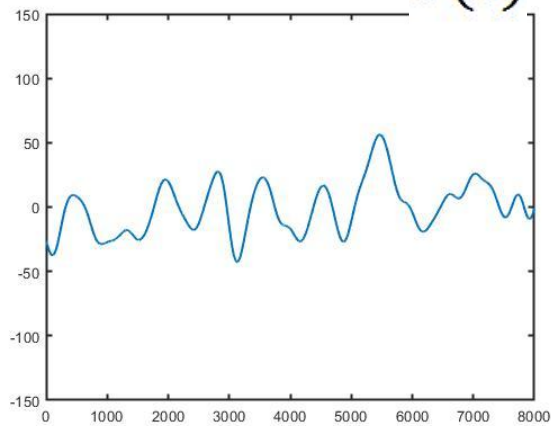
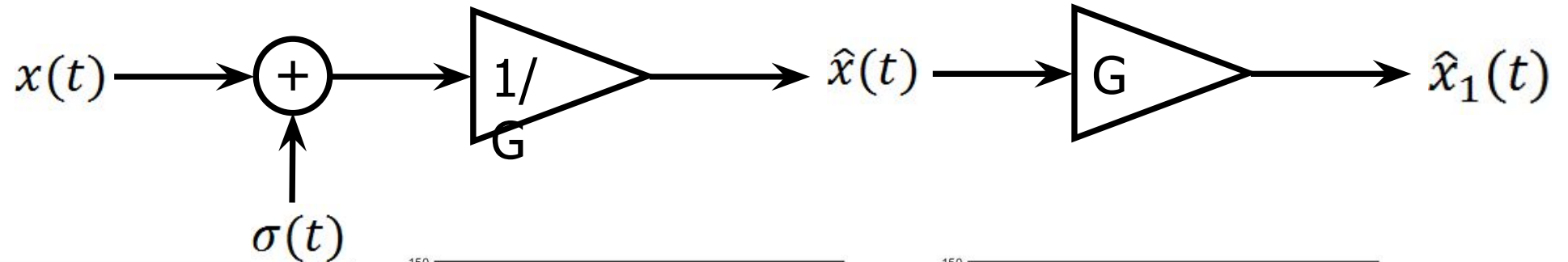


## Transmission - signal regeneration

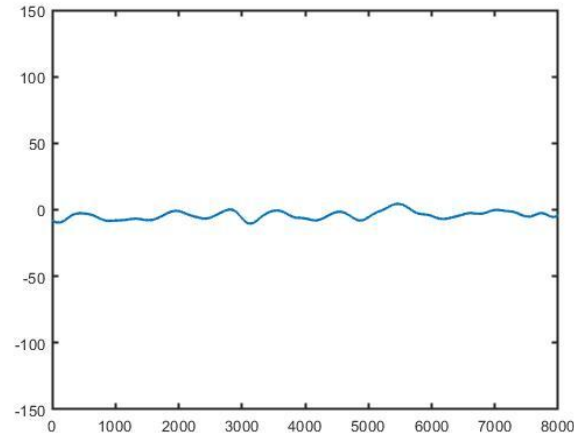




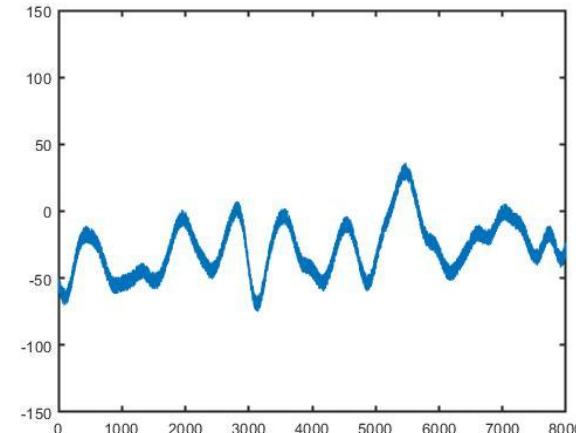
## Transmission - signal regeneration



$x(t)$



$\hat{x}(t) = x(t)/G + \sigma(t)$

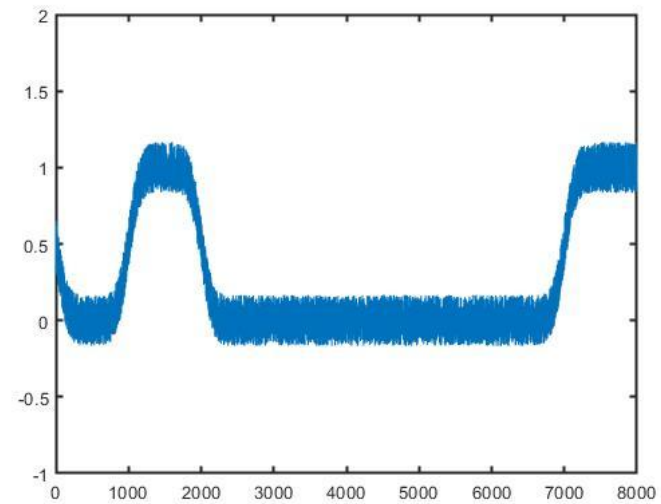
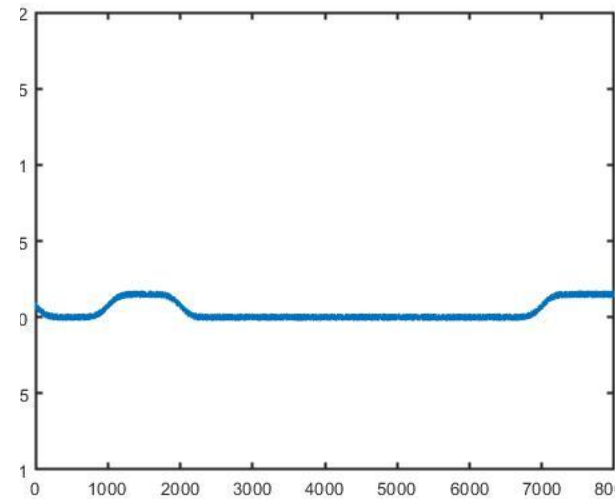
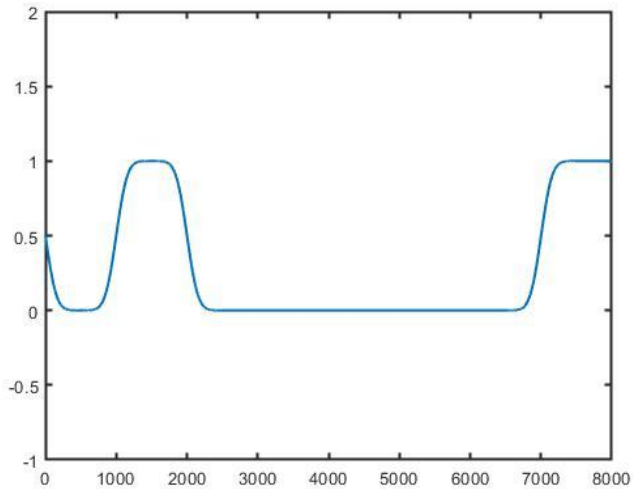
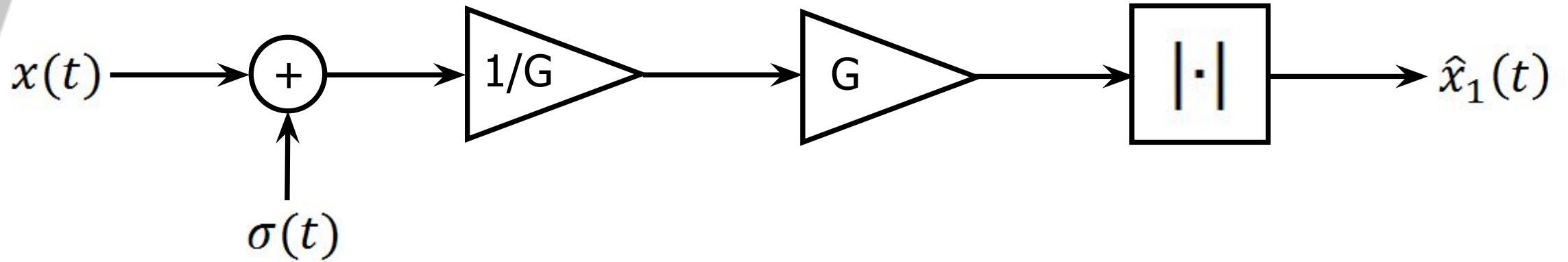


$$\hat{x}_1(t) = [x(t)/G + \sigma(t)]G$$

$$\hat{x}_1(t) = x(t) + G\sigma(t)$$

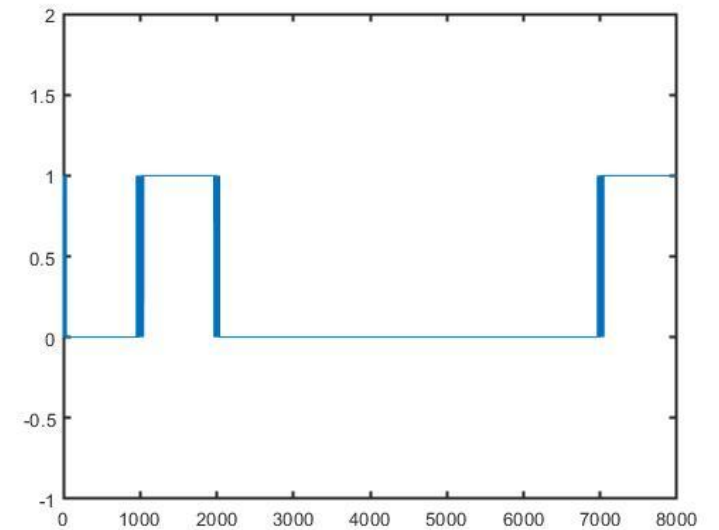
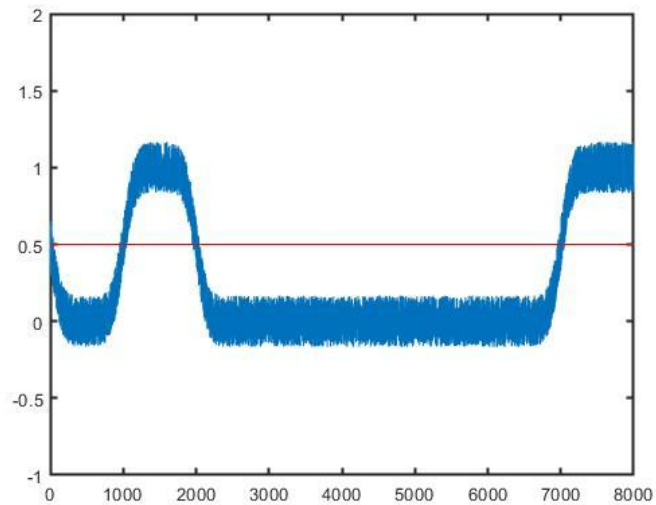
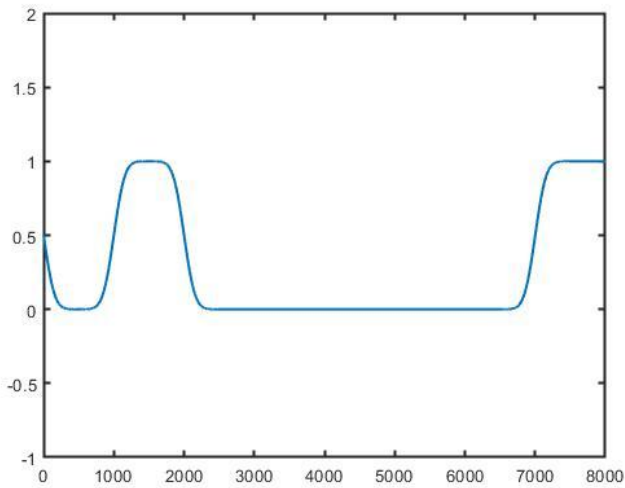
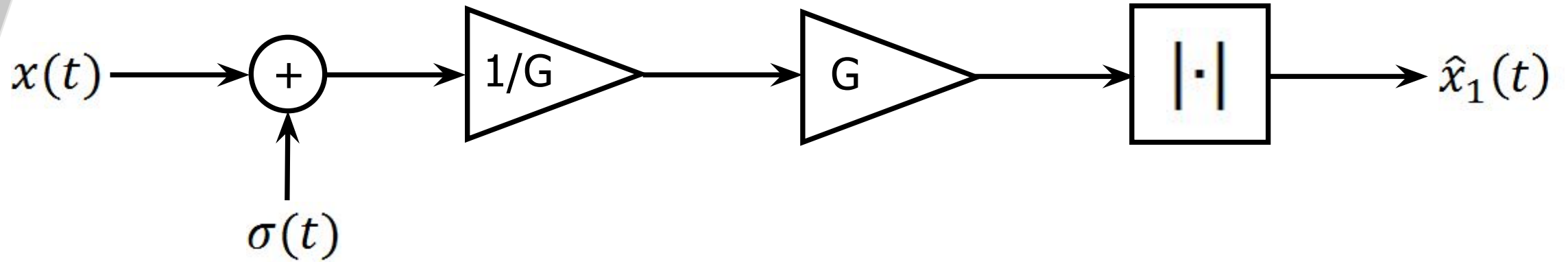


## Transmission - signal regeneration





## Transmission - signal regeneration





## Why digital signals are useful?

- Storage (generalized computer memory, compatibility)
- Processing (coding, no mechanical or physical interactions)
- Transmission(signal regeneration)



## Digital signals

- One dimensional (for now)
- Notation  $x[n]$  where «n» is an integer
- Two sides sequences  $x : \mathbb{Z} \rightarrow \mathbb{C}$
- n is a-dimensional
  
- Analysis (by taking periodic measurements)
- Synthesis (program that generates a stream of samples)

$\mathbb{Z}$  (*set of integers*)

$\mathbb{C}$  (*set of complex numbers*)



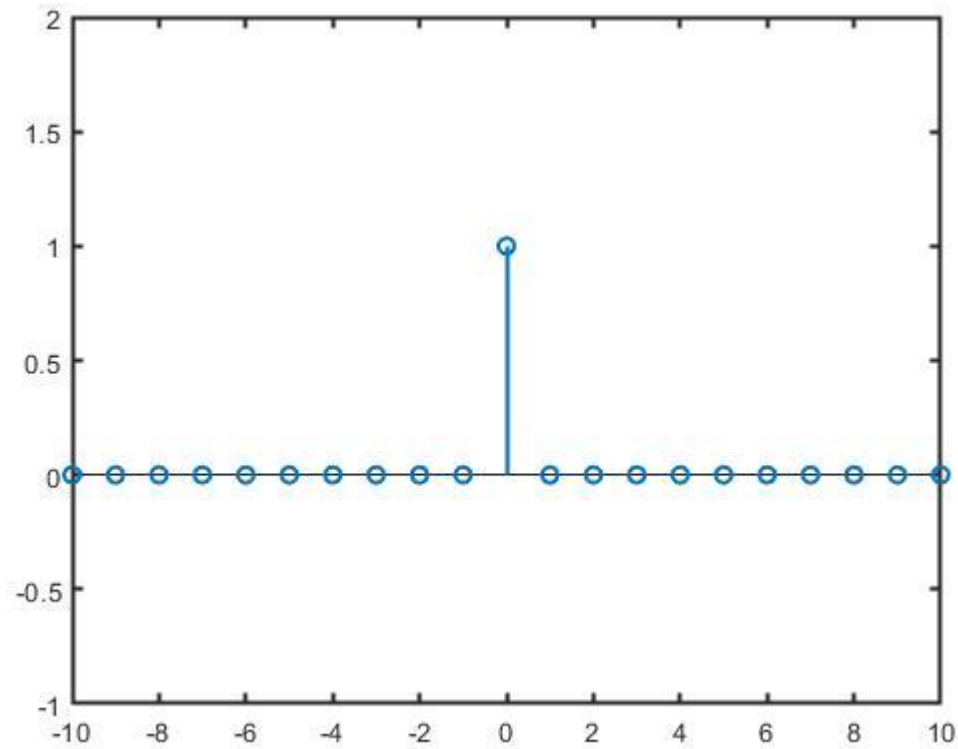


## **Digital Signals – common signals**

- Delta Dirac function
- Unit step function
- Exponential decay
- Sinusoidal signals



## Delta signal

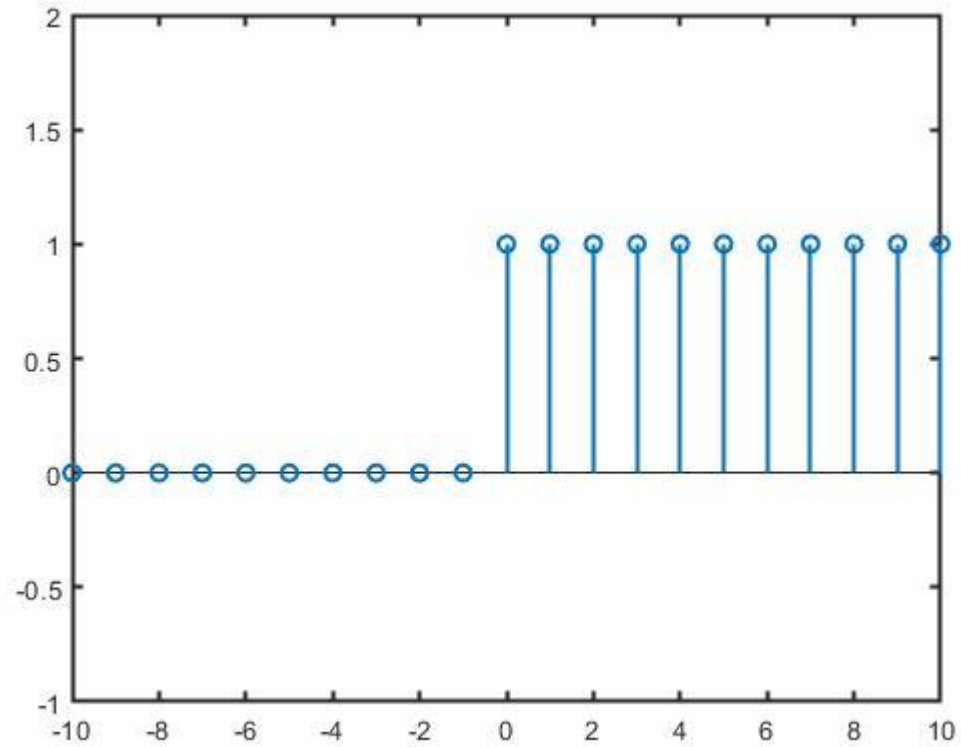


$$x[n] = \delta[n]$$

$$x[n] = \begin{cases} 0, & \text{when } n \neq 0 \\ 1, & \text{when } n = 0 \end{cases}$$



## Unit Step function



$$\begin{matrix} \square & \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \end{matrix} = \begin{matrix} \square & \square \\ \hline \square & \square \\ \hline \square & \square \end{matrix}$$

$$\begin{matrix} \square & \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \end{matrix} = \begin{matrix} \square & \square \\ \hline \square & \square \\ \hline \square & \square \end{matrix}$$

$$\begin{matrix} \square & \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \end{matrix} = \begin{matrix} \square & \square \\ \hline \square & \square \\ \hline \square & \square \end{matrix}$$

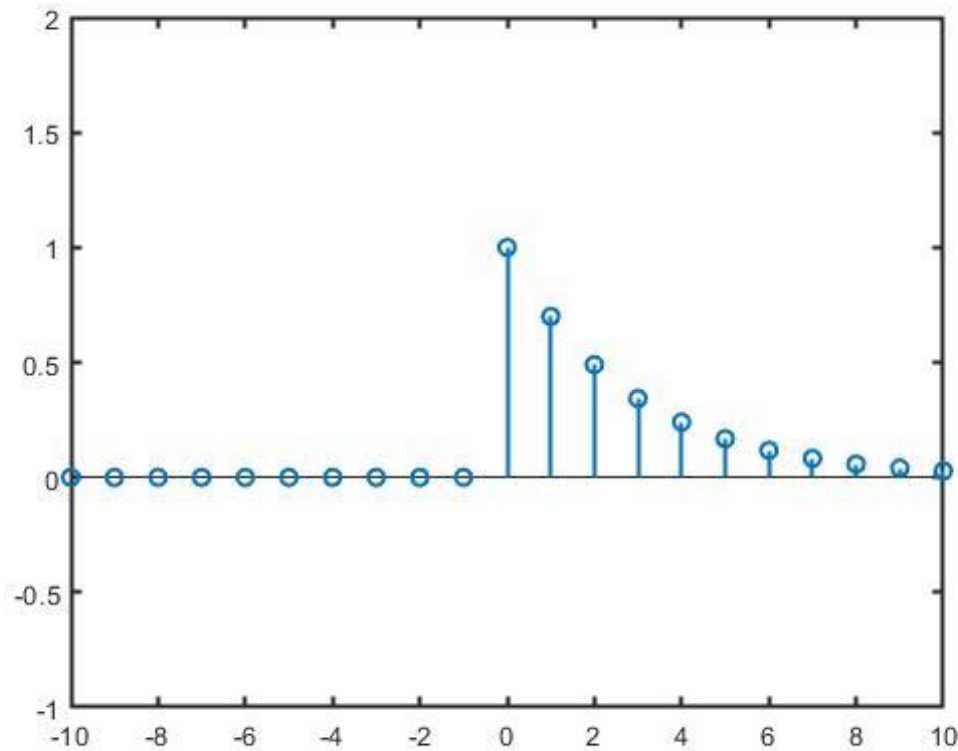
$$\begin{matrix} \square & \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \end{matrix} = 1 \begin{matrix} \square & \square \\ \hline \square & \square \\ \hline \square & \square \end{matrix}$$

$$\begin{matrix} \square & \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \end{matrix} = \begin{matrix} \square & \square \\ \hline \square & \square \\ \hline \square & \square \end{matrix}$$

$$x[n] = \begin{cases} 0, & \text{when } n < 0 \\ 1, & \text{when } n \geq 0 \end{cases}$$



## Exponential functions

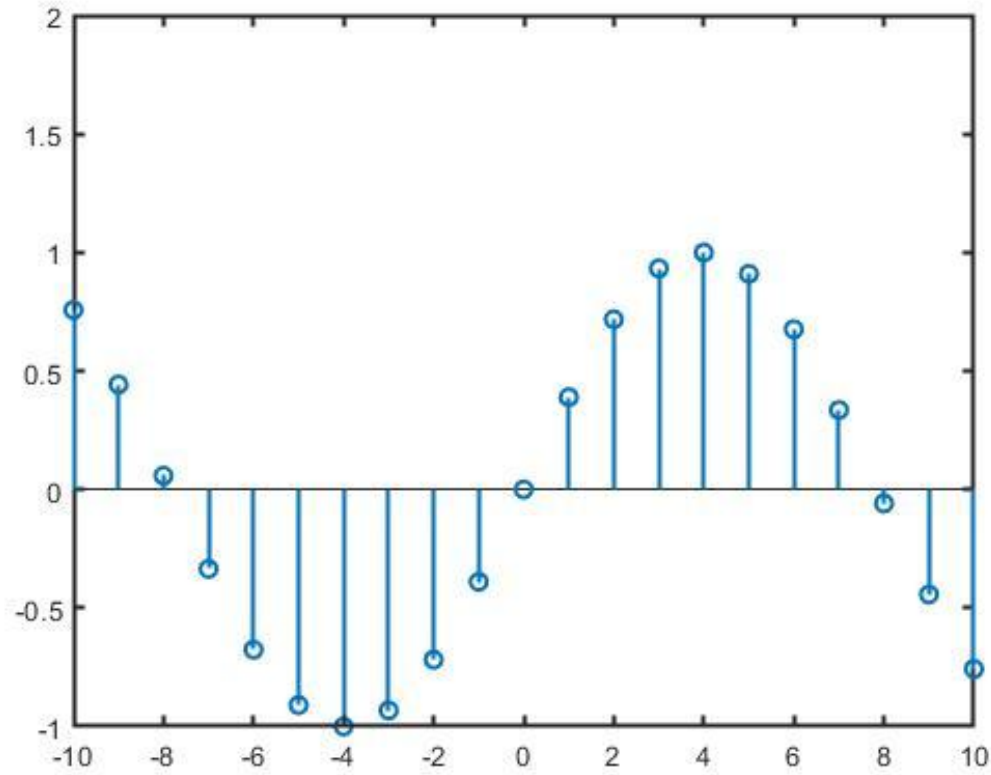


$$x[n] = \begin{cases} 0, & \text{when } n < 0 \\ a^n, & \text{when } n \geq 0 \end{cases}$$

*when  $a < 1$  converges to 0*



## Sinusoidal functions



$$x[n] = \sin(w_0 n + \theta)$$



## Signal classes

- Finite length  $x[n]$  for  $n = 0, 1, \dots, N-1$
- Infinite length
- Periodic
- Finite-support



## Finite length

□ Sequence notation

$$x[n] = 0, 1, \dots, N - 1$$

□ Vector notation

$$x = [x_0 \ x_1 \ \dots \ x_{N-1}]^T$$

□ Easy to process in numerical packages (numPy)



## Infinite length

- Sequence notation
- Good for theorems

$$x[n], n \in \mathbb{Z}$$





## Periodic

- N-periodic Sequence  $\tilde{x}[n] = \tilde{x}[n + kN], \quad n, k, N \in \mathbb{Z}$
- The same information as finite signals with length N
- Useful as a bridge between finite and infinite signals



## Finite support signals

$$x[n] = \begin{cases} x[n], & \text{if } 0 \leq n < N \\ 0, & \text{otherwise} \end{cases} \quad n \in \mathbb{Z}$$

- The same information as finite signals of length  $N$
- Useful as a bridge between finite and infinite signals



## Energy and Power

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P_x = \lim_{N \rightarrow \infty} \frac{1}{2N + 1} \sum_{n=-N}^N |x[n]|^2$$



## Elementary operators

□ Delay  $y[n] = x[n - k] \quad k \in \mathbb{Z}$

□ Scaling  $y[n] = ax[n]$

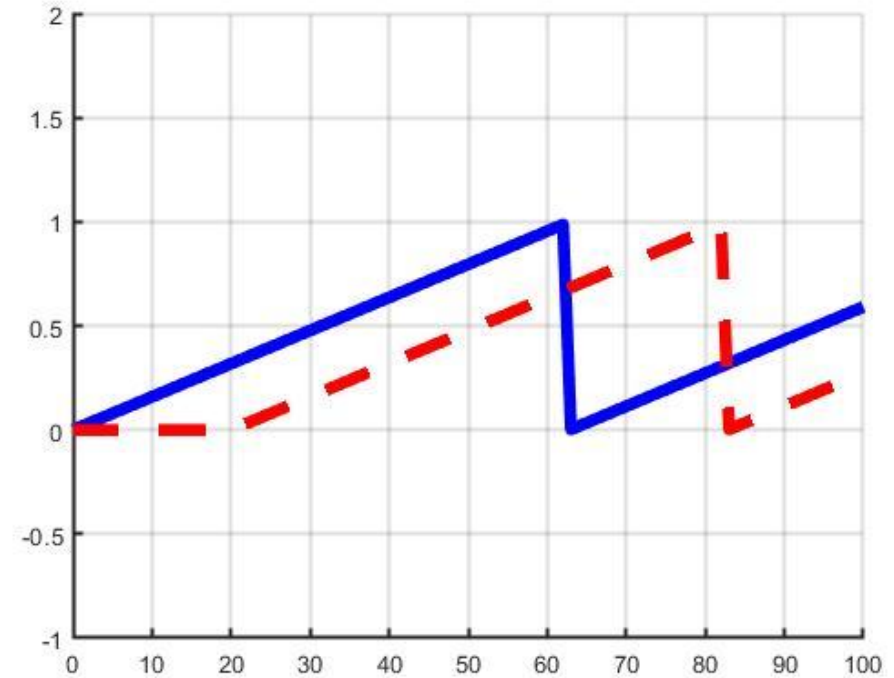
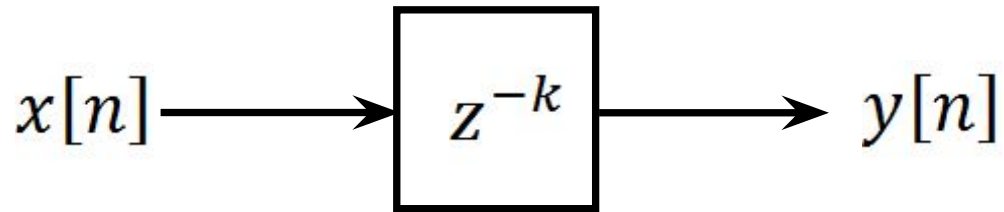
□ Summation  $y[n] = x[n] + f[n]$



## Elementary operators

□ Delay

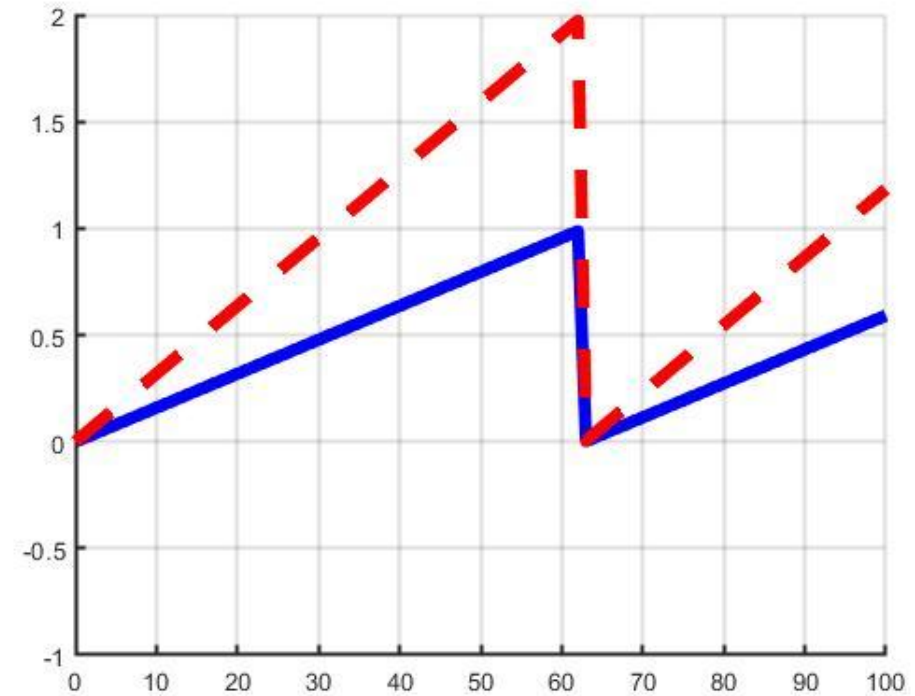
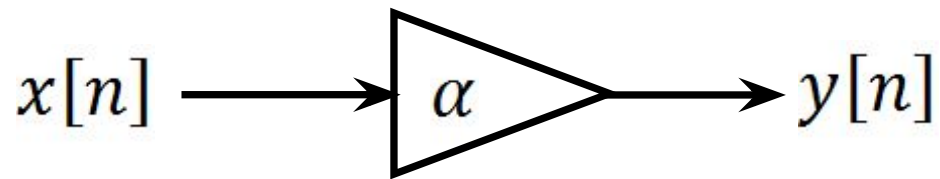
$$y[n] = x[n - k] \quad k \in \mathbb{Z}$$





## Elementary operators

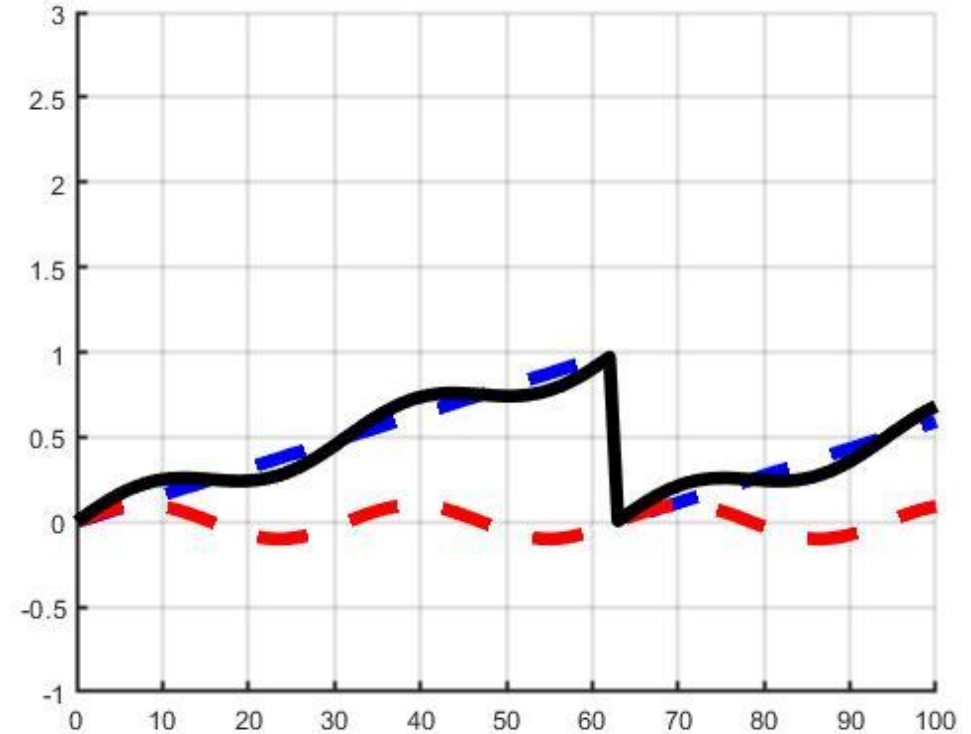
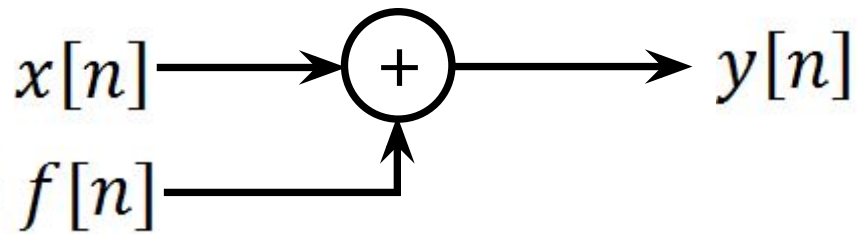
□ Scaling  $y[n] = ax[n]$

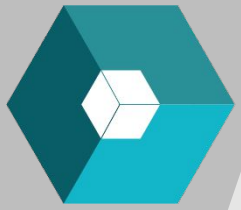




## Elementary operators

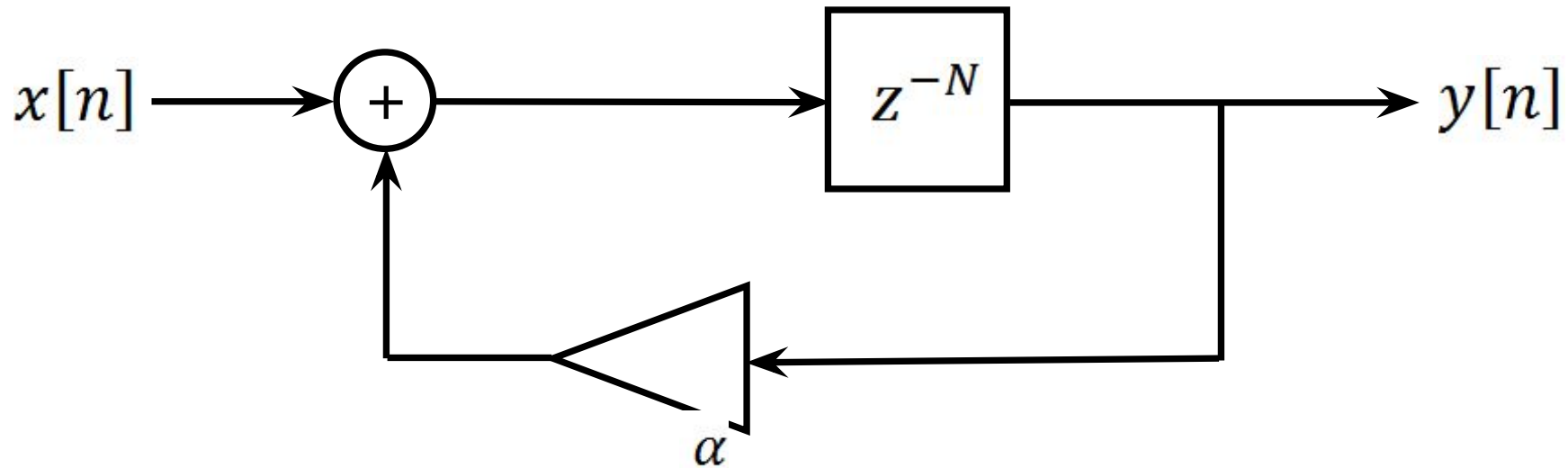
□ Summation  $y[n] = x[n] + f[n]$





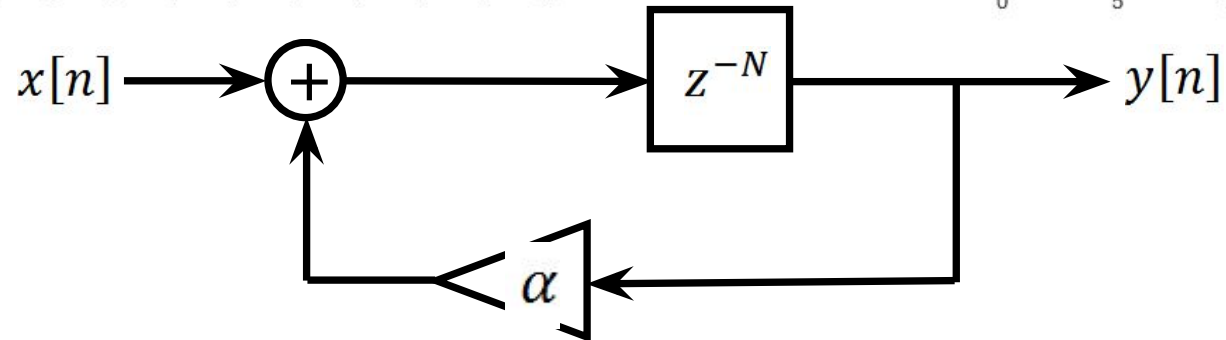
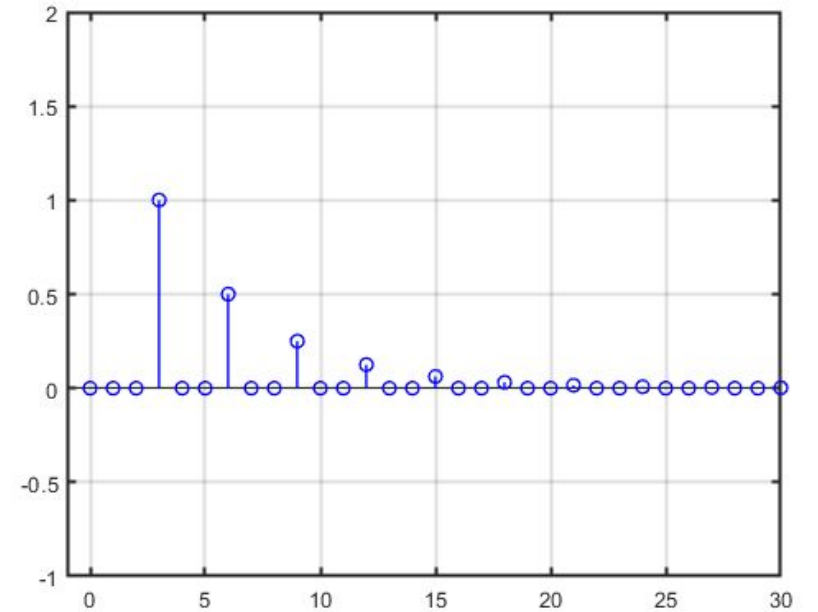
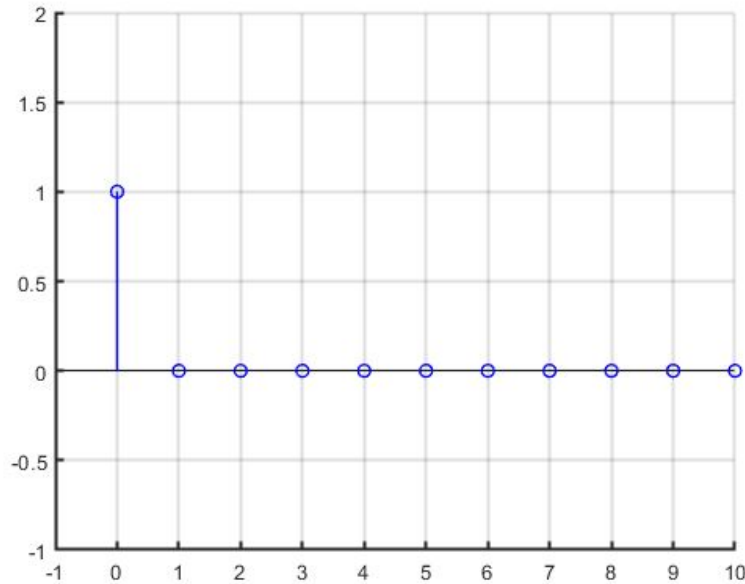
## Karplus-Strong algorithm - simplified

$$y[n] = x[n - N] + \alpha \cdot y[n - N]$$





## Karplus-Strong algorithm - simplified

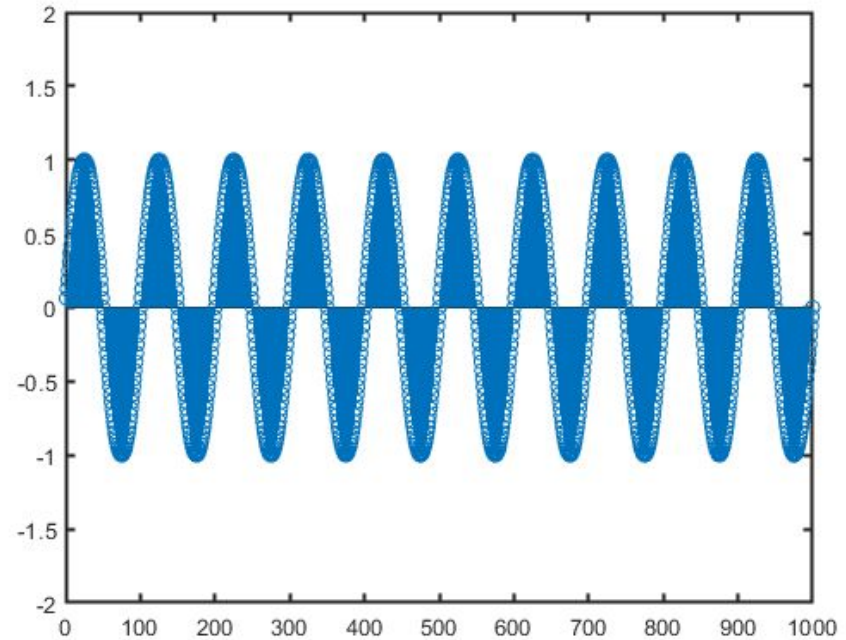
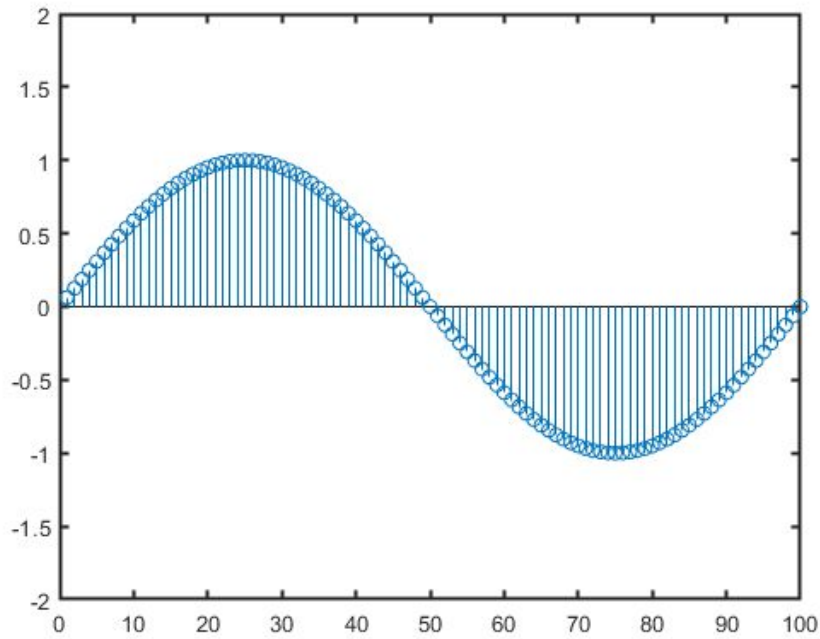


## Karplus-Strong algorithm - simplified

$$y[n] = x[n - N] + \alpha \cdot y[n - N]$$

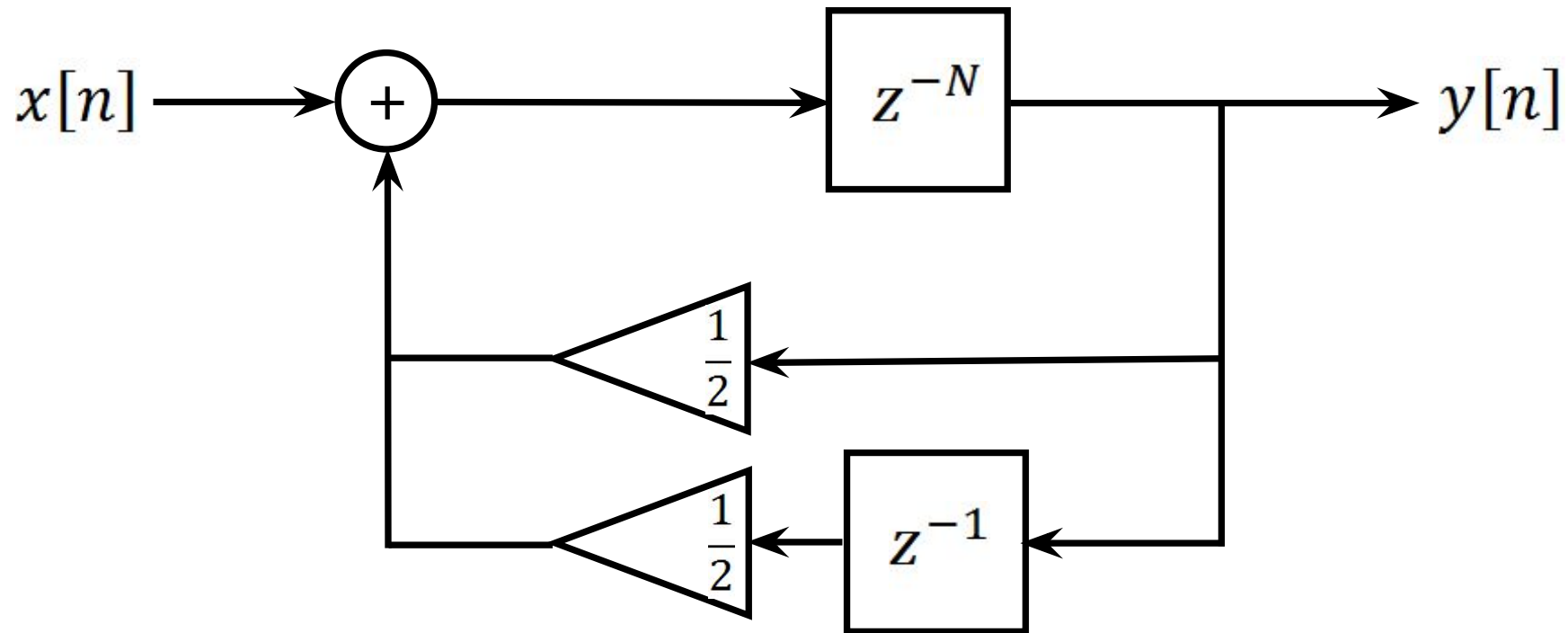
$$\bar{x}(n) = \sin\left(2\pi \frac{n}{100}\right)$$

$$N = 100 \quad \alpha = 1$$

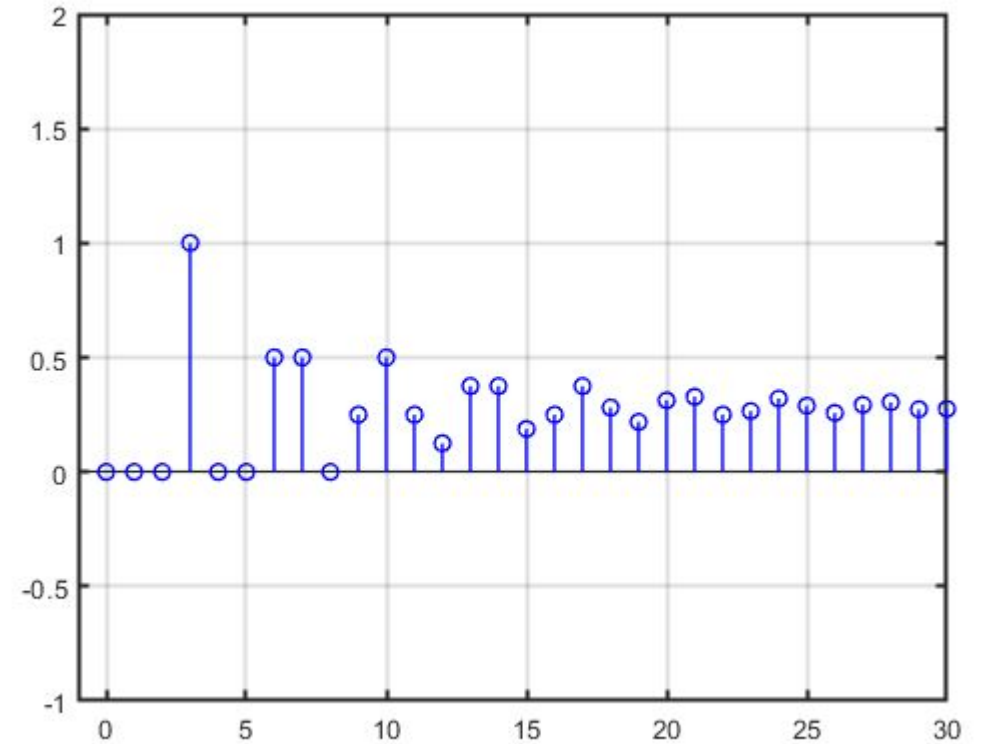
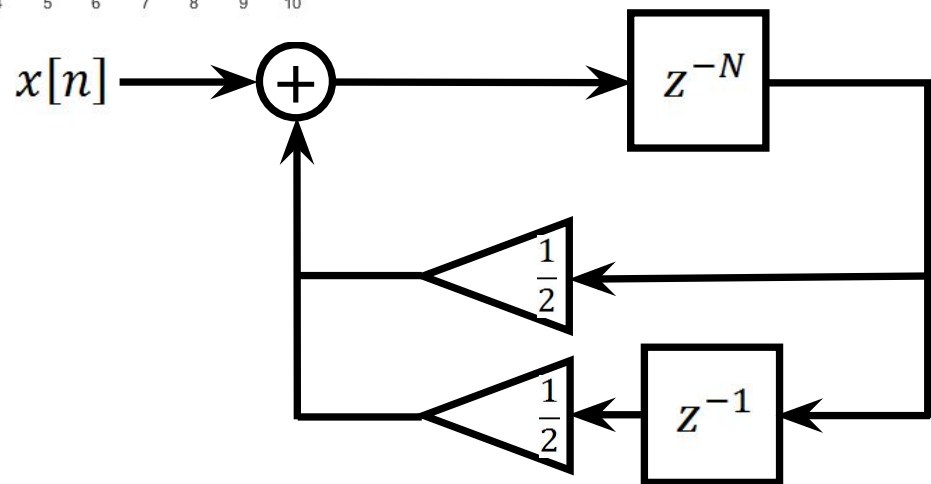
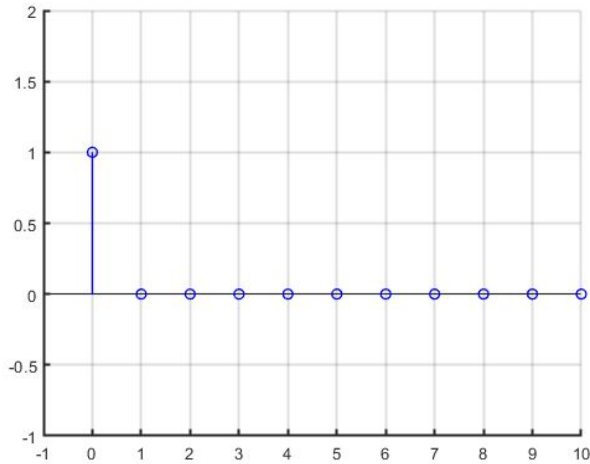


## Karplus-Strong algorithm

$$y[n] = x[n - N] + \frac{y[n - N] + y[n - (N + 1)]}{2}$$

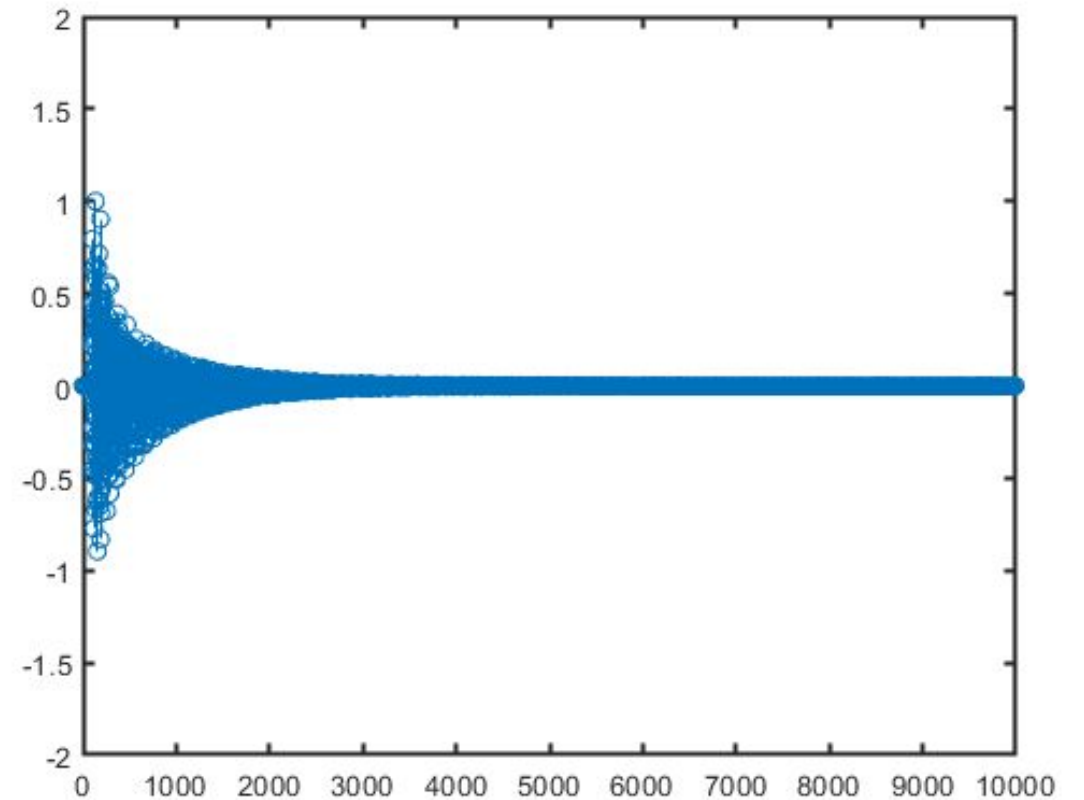
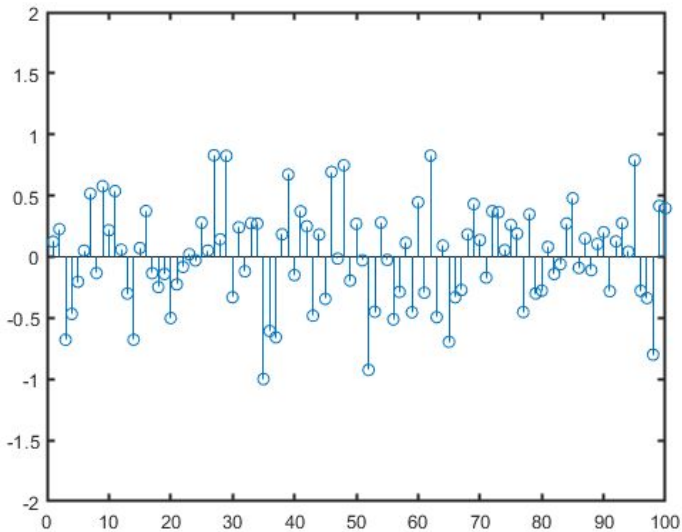


## Karplus-Strong algorithm

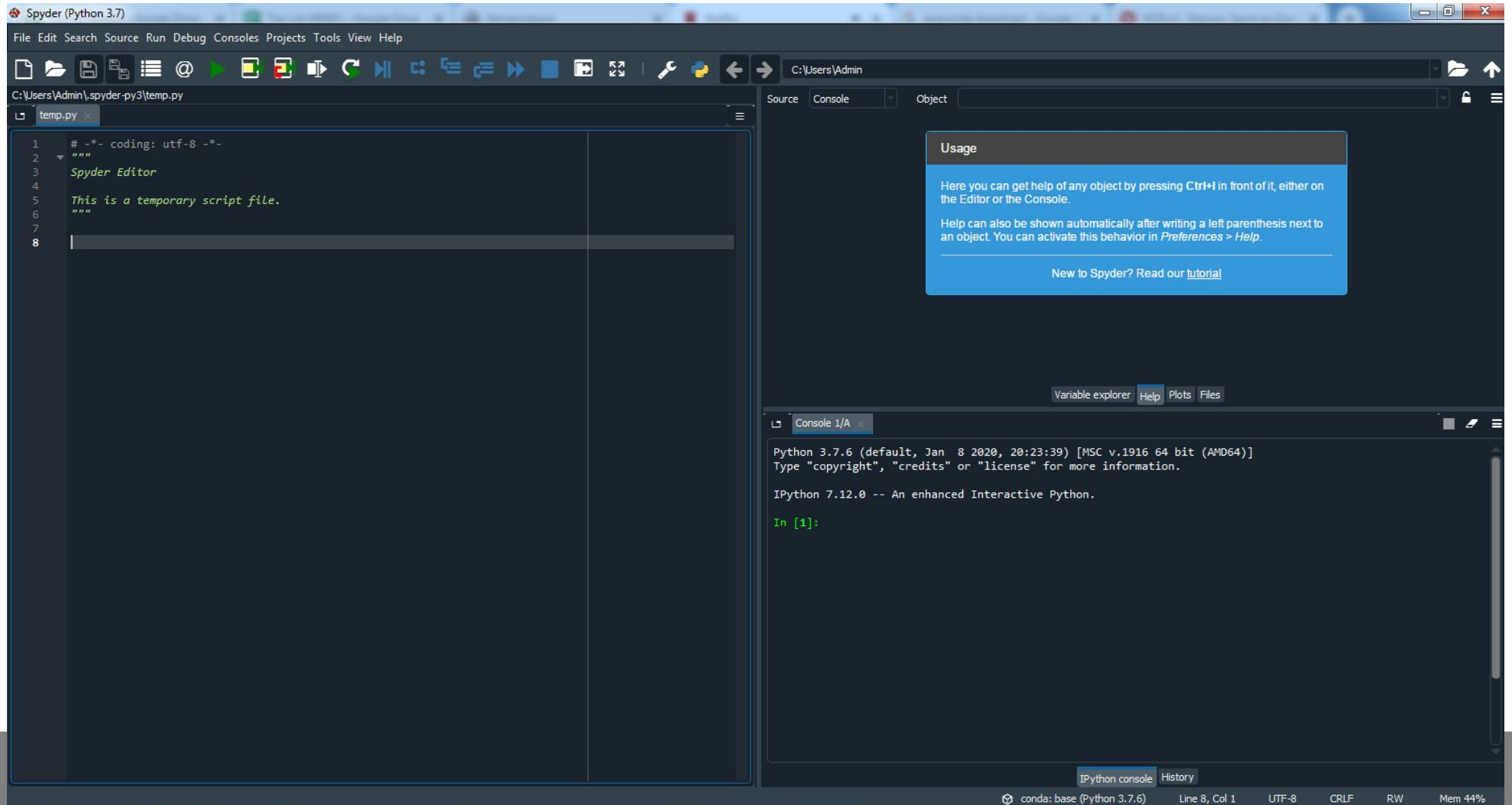


## Karplus-Strong algorithm

*$N$  controls the frequency (pitch)  
 $a$  controls the envelope (decay)  
 $\bar{x}(n)$  controls the color (Timbre)*



# Python 3 Anaconda - Spyder





## Data types

Text Type: str

Numeric Types: int, float, complex

Sequence Types: list, tuple, range

Mapping Type: dict

Set Types: set, frozenset

Boolean Type: bool

Binary Types: bytes, bytearray, memoryview

## Data types

You can assign the datatype manually (like programming in C)

OR

You can assign the datatype automatically

```
Console 1/A  
Python 3.7.6 (default, Jan 8 2020, 20:23:39) [MSC v.1916 64 bit (AMD64)]  
Type "copyright", "credits" or "license" for more information.  
  
IPython 7.12.0 -- An enhanced Interactive Python.  
  
In [1]: x = 5  
...: print(type(x))  
<class 'int'>  
  
In [2]: x = float(5)  
...: print(type(x))  
<class 'float'>
```



## Data types

Some data types can interact

Try assigning the INTEGER 5 to variable 'x'

Try assigning the float 7 to variable 'y'

Multiply 'x' and 'y' and assign the result to 'z' (you type 'z=x\*y')

Determine the data type of 'z'

```
In [8]: x=5
In [9]: y=float(7)
In [10]: z=x*y
In [11]: print(type(z))
```



## Importing libraries

- For example try to print the number  $\pi$

```
In [1]: pi
Traceback (most recent call last):

  File "<ipython-input-1-f84ab820532c>", line 1, in <module>
    pi
NameError: name 'pi' is not defined

In [2]: math.pi
Traceback (most recent call last):

  File "<ipython-input-2-c49acc181da4>", line 1, in <module>
    math.pi
NameError: name 'math' is not defined
```



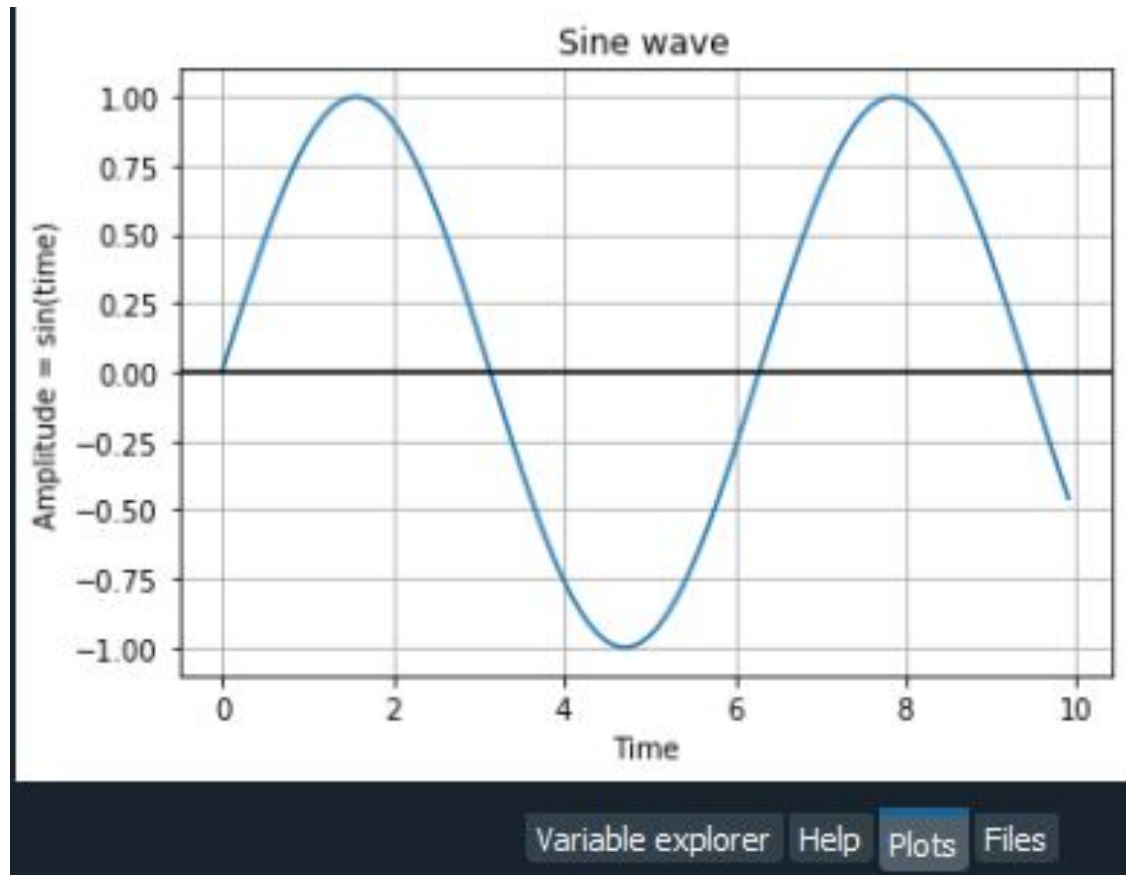
## Importing functions

- For example try to print the number  $\pi$

```
In [3]: import math
...: print(math.pi)
3.141592653589793
```



## Visualizing numbers – plotting a sine wave





## Visualizing numbers – plotting a sine wave

Import module numpy

Import the function pyplot from the module matplotlib

```
temp.py x
1 # -*- coding: utf-8 -*-
2 import numpy as np
3 import matplotlib.pyplot as plot
4
```



## Visualizing numbers – plotting a sine wave

Create time points for the x-axis

Numbers 0 until 10 with step size 0.01

(so you will get a range from [0 0.01 0.02 0.03 ... 1] )

```
# Get x values of the sine wave  
time = np.arange(0, 10, 0.1);
```



## Visualizing numbers – plotting a sine wave

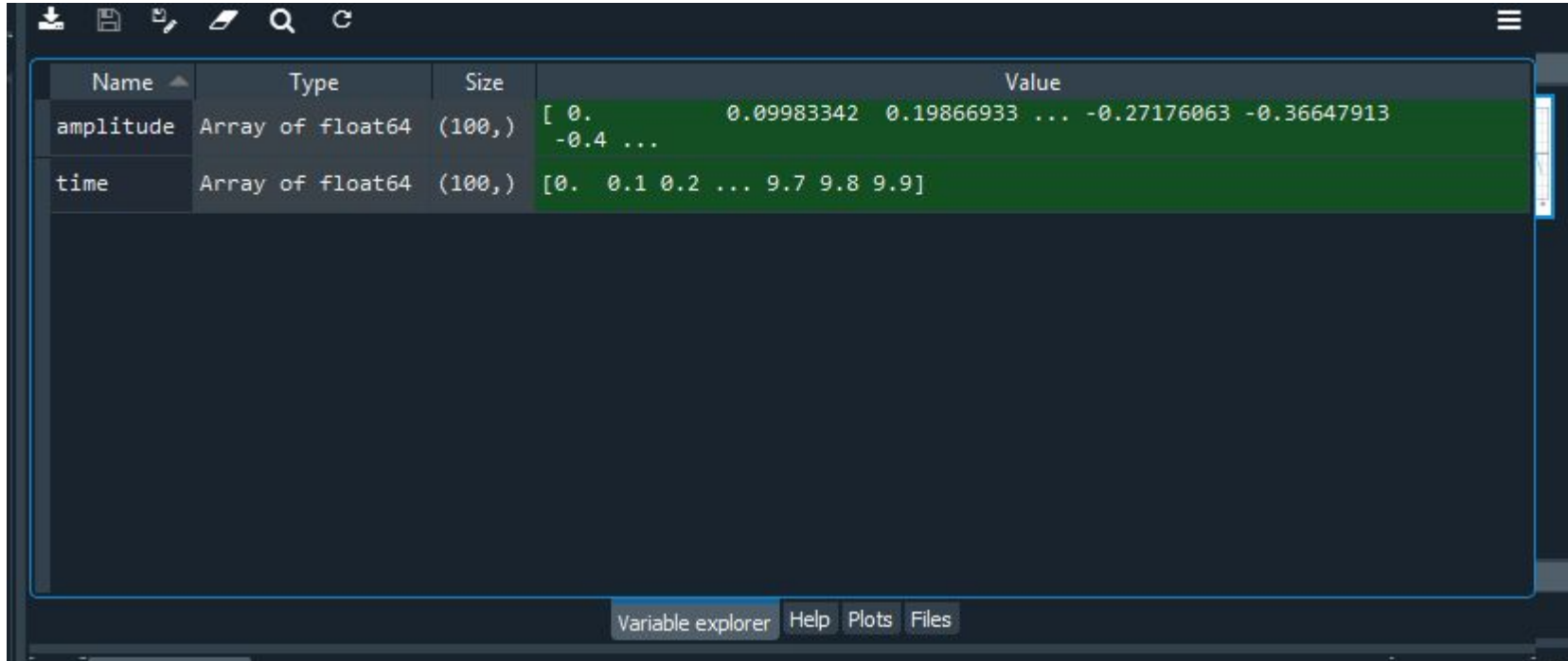
Create the amplitude points for each point in time

Use the sin function of the numpy module

(which you loaded and renamed as 'np')

```
# Amplitude of the sine wave is sine of a variable like time  
amplitude = np.sin(time)
```

## Visualizing numbers – plotting a sine wave



The screenshot shows a software interface with a variable explorer window. The window displays two variables: 'amplitude' and 'time'. The 'amplitude' variable is an array of float64 with a size of (100,). The 'time' variable is also an array of float64 with a size of (100,). The values for 'amplitude' are shown as [ 0. 0.09983342 0.19866933 ... -0.27176063 -0.36647913 -0.4 ...]. The values for 'time' are shown as [0. 0.1 0.2 ... 9.7 9.8 9.9].

Name	Type	Size	Value
amplitude	Array of float64	(100,)	[ 0. 0.09983342 0.19866933 ... -0.27176063 -0.36647913 -0.4 ...]
time	Array of float64	(100,)	[0. 0.1 0.2 ... 9.7 9.8 9.9]



## Visualizing numbers – plotting a sine wave

Create the plot.

```
# Plot a sine wave using time and amplitude obtained for the sine wave  
plot.plot(time, amplitude)
```

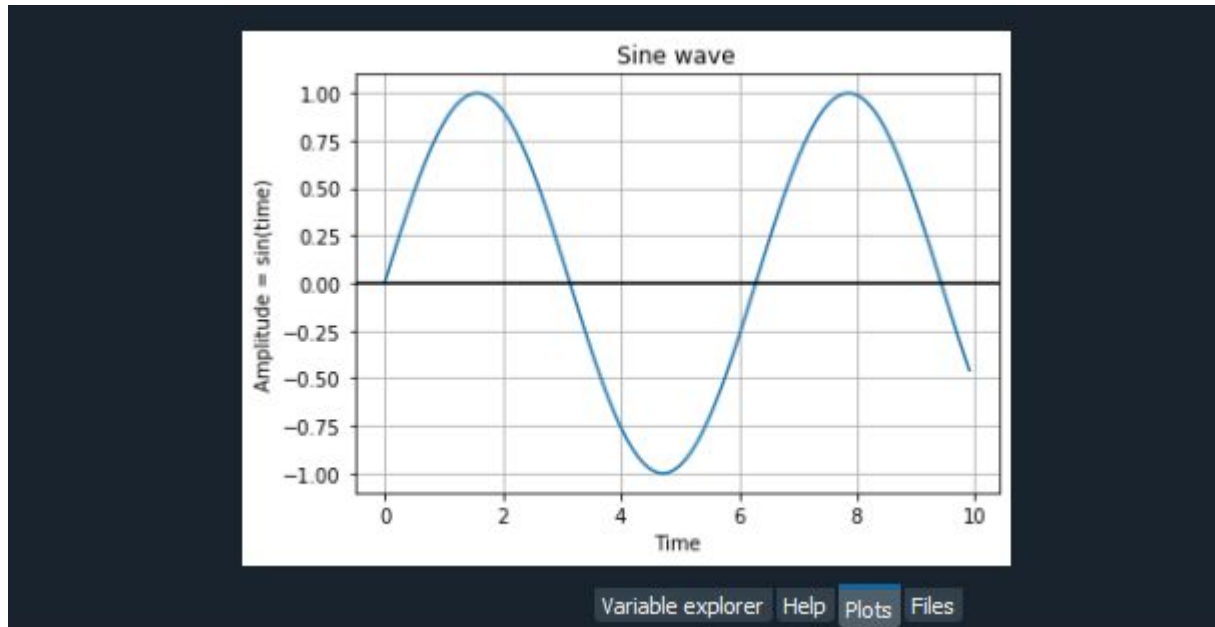
Create the mark up for the plot

```
# Give a title for the sine wave plot  
plot.title('Sine wave')  
# Give x axis label for the sine wave plot  
plot.xlabel('Time')  
# Give y axis label for the sine wave plot  
plot.ylabel('Amplitude = sin(time)')  
plot.grid(True, which='both')  
plot.axhline(y=0, color='k')
```

## Visualizing numbers – plotting a sine wave

Display the plot.

```
22  
23 plot.show()
```





## Python

Try to adhere to good practice!!

Code is written once, but read multiple times.

Find ground rules here:

<https://www.python.org/dev/peps/pep-0008/>



## Digital signals - limitations

$$1 + 1e20 - 1e20 = ?$$

$$1 + (1e20 - 1e20) = ?$$

Why?



- $(1e20 \cdot 1e20) \cdot 1e-20 = ?$
- $1e20 \cdot (1e20 \cdot 1e-20) = ?$
- $1e20 \cdot (1e20 - 1e20) = ?$
- $1e20 \cdot 1e20 - 1e20 \cdot 1e20 = ?$



## Data types - Integer (int)

```
In [16]: a = 100
```

```
In [17]: a.bit_length()  
Out[17]: 7
```

```
In [25]: bin(100)  
Out[25]: '0b1100100'
```

```
In [26]: bin(-4)  
Out[26]: '-0b100'
```

```
In [38]: a=1
```

```
In [39]: b=1
```

```
In [40]: a is b  
Out[40]: True
```

```
In [46]: a==b  
Out[46]: True
```

```
In [41]: a=257
```

```
In [42]: b=257
```

```
In [43]: a is b  
Out[43]: False
```

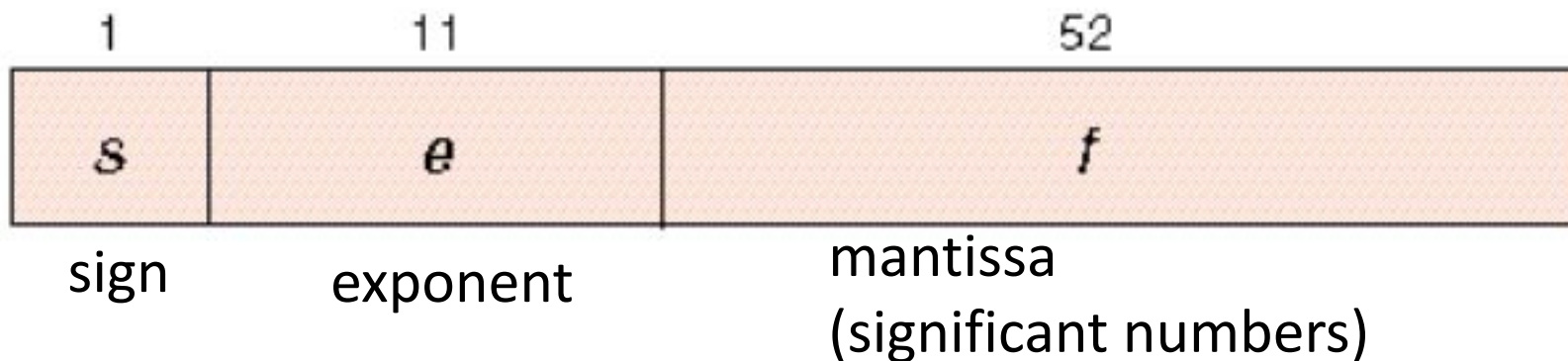
## Floating point - Single precision (single) / double precision (double)

double precision (64 bit) floating numbers

$$-123.4567890123 = -1.234567890123 e2$$

sign
mantissa
exponent

(significant numbers)



## Floating point - Single precision (single) / double precision (double)

double precision (64 bit) floating numbers



$$x = \pm (1 + f) \cdot 2^e$$

Giving the decimal interval for  $e$  as  
 $-1022 \leq e \leq 1023$

Using offset binary representation

With fraction  $f$

$$0 \leq f < 1$$

In the interval where  
 $2^{52} \cdot f$  is an integer

$$0 \leq 2^{52} \cdot f < 2^{52}$$



## Floating point - Single precision (single) / double precision (double)

double precision (64 bit) floating numbers



With exponent *e* as integer in the interval

$$-1022 \leq e \leq 1023$$

$$x = \pm (1 + f) \cdot 2^e$$

```
In [11]: a = float(2) ** 1023
```

```
In [12]: a = float(2) ** 1024
Traceback (most recent call last):
```

```
File "<ipython-input-12-a340186a386a>", line 1, in <module>
    a = float(2) ** 1024
```

```
OverflowError: (34, 'Result too large')
```

With fraction *f*

$$0 \leq f < 1$$

In the interval where  $2^{52} \cdot f$  is an integer

$$0 \leq 2^{52} \cdot f < 2^{52}$$

```
In [9]: a = float(10) ** 308
```

```
In [10]: a = float(10) ** 309
Traceback (most recent call last):
```

```
File "<ipython-input-10-92873524ff56>", line 1, in <module>
    a = float(10) ** 309
```

```
OverflowError: (34, 'Result too large')
```



## Не сравнивайте числа с плавающей запятой

```
float f = 0.1;
while (f != 1.0) {
    f += 0.1;
}
```

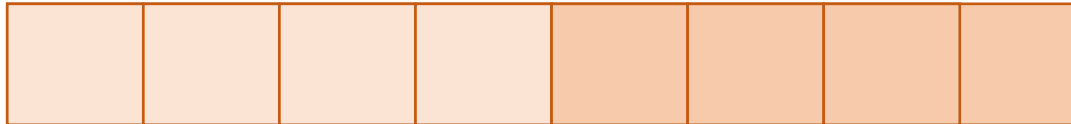
```
f=0.2000000030
f=0.3000000119
f=0.4000000060
f=0.5000000000
f=0.6000000238
f=0.7000000477
f=0.8000000715
f=0.9000000954
f=1.0000001192
f=1.1000001431
f=1.2000001669
f=1.3000001907
f=1.4000002146
f=1.5000002384
f=1.6000002623
```

## **Будьте аккуратны с числами с плавающей запятой**

- Точность уменьшается по мере роста величины
- Ошибки переполнения и округления
  - Ракета-носитель «Ариан-5» 4 июня 1996 была подорвана на 34-й секунде полёта
    - Конвертация данных из 64-разрядного числа с плавающей запятой в 16-разрядное привела к зависанию компьютера
  - Точность ракет Patriot в 1991 (Война в Персидском заливе) была ограничена ошибкой округления

## Data types - Fixed point

A fixed decimal point. Often used in low level devices.



example

10001011

$$2^3 + 2^{-1} + 2^{-3} + 2^{-4} = 8.675$$

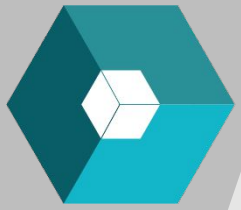
```
>>> from decimal import *
>>> getcontext()
Context(prec=28, rounding=ROUND_HALF_EVEN, Emin=-999999999, Emax=999999999,
        capitals=1, flags=[], traps=[Overflow, DivisionByZero,
        InvalidOperation])

>>> getcontext().prec = 7           # Set a new precision
```



## **5 steps of a method/function in any language**

- 1 Initialization
- 2 input
- 3 process
- 4 output
- 5 Termination



## 1) Define a function

```
def myFirstFunction(input):
```

```
    print("This will be the output, %s" %(input))
```

A screenshot of a code editor window showing a Python script named 'myFirstScript.py'. The code includes a docstring with creation and modification dates and an author name, followed by the definition of the 'myFirstFunction' which prints the input string.

```
1  # -*- coding: utf-8 -*-  
2  """  
3  Created on Sat Aug 29 18:53:26 2020  
4  @author: Admin  
5  
6  Last Modified on Sat Aug 29 18:53:26 2020  
7  @author: Admin  
8  
9  
10 """  
11  
12  
13 def myFirstFunction(input):  
14     print("This will be the output,%s" %(input))  
15
```



## Importing the function

```
In [53]: import myFirstScript
```

## Calling the function

```
In [55]: myFirstScript.myFirstFunction( 'aaaa' )  
This will be the output,aaaa
```



## Data types

Text Type: str

Numeric Types: int, float, complex

Sequence Types: list, tuple, range

Mapping Type: dict

Set Types: set, frozenset

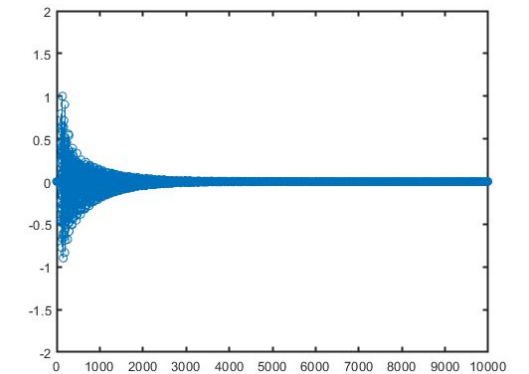
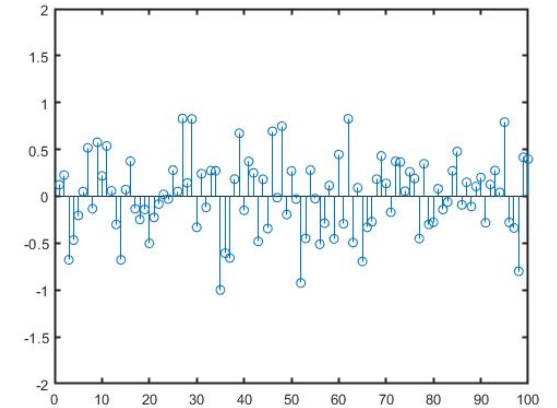
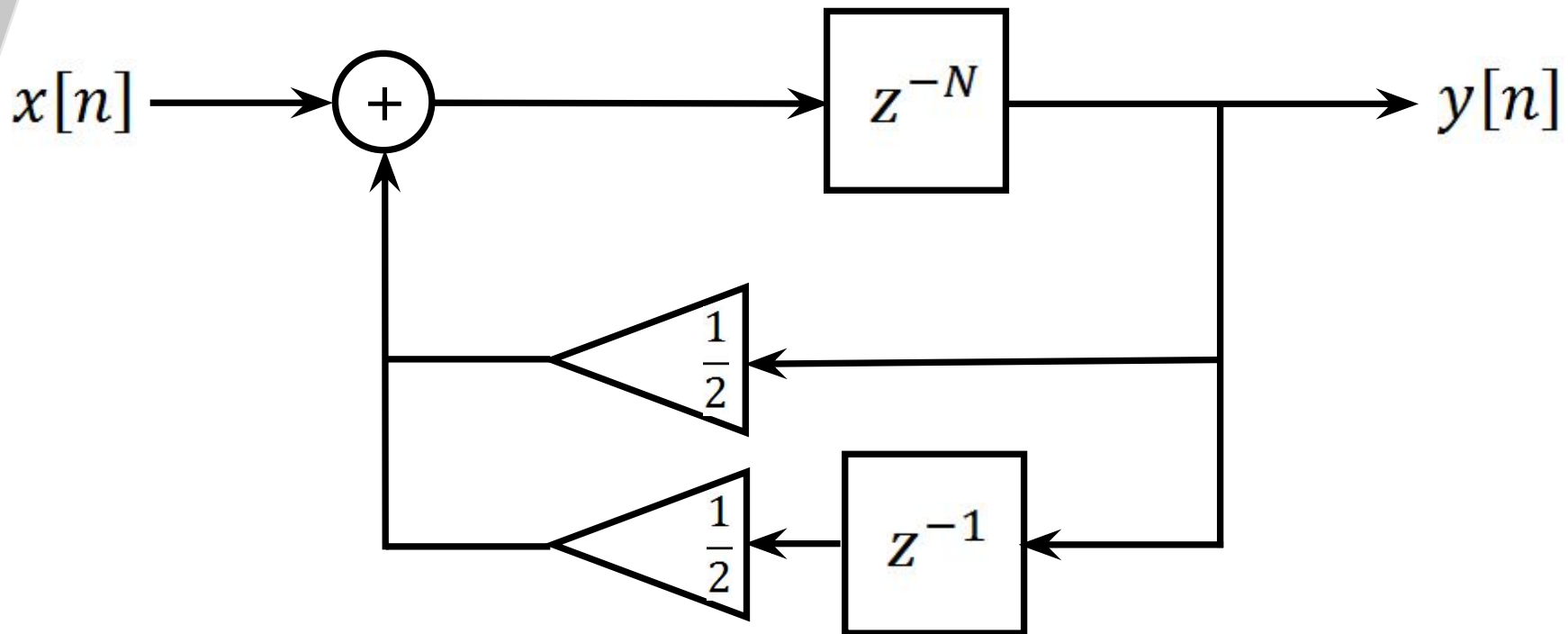
Boolean Type: bool

Binary Types: bytes, bytearray, memoryview



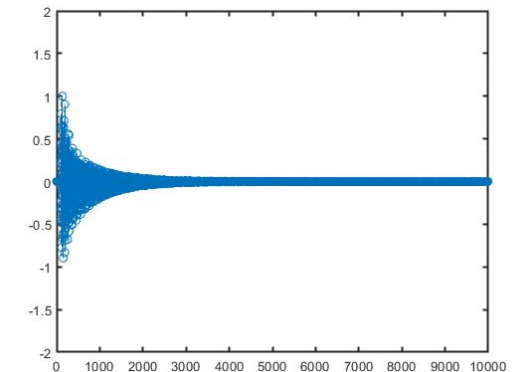
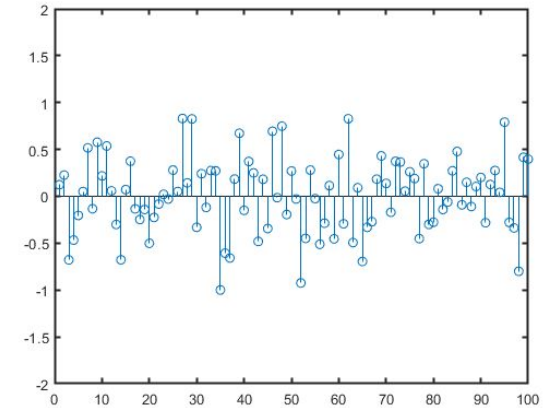
## Writing a Karplus-Strong algorithm in Python

$$y[n] = x[n - N] + \frac{y[n - N] + y[n - (N + 1)]}{2}$$



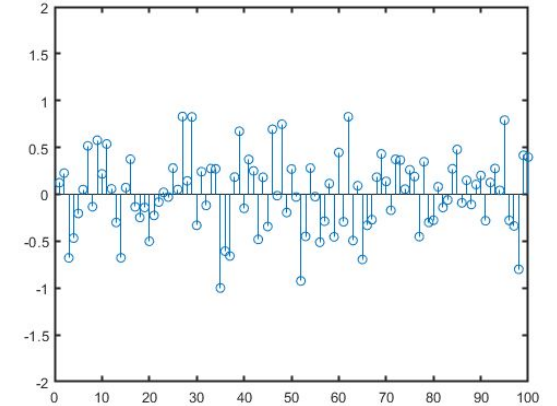
## Writing a Karplus-Strong algorithm in Python

- Generate a random input
- Process the input using the algorithm
- Show the Result
- Save the result



## Writing a Karplus-Strong algorithm in Python

- Generate a random input



[Understanding the Karplus-Strong with Python \(Synthetic Guitar Sounds Included\) | Frolian's blog \(flothsof.github.io\)](#)



## Data types

Text Type: str

Numeric Types: int, float, complex

Sequence Types: list, tuple, range

Mapping Type: dict

Set Types: set, frozenset

Boolean Type: bool

Binary Types: bytes, bytearray, memoryview

## Writing a Karplus-Strong algorithm in Python

- Generate a random input

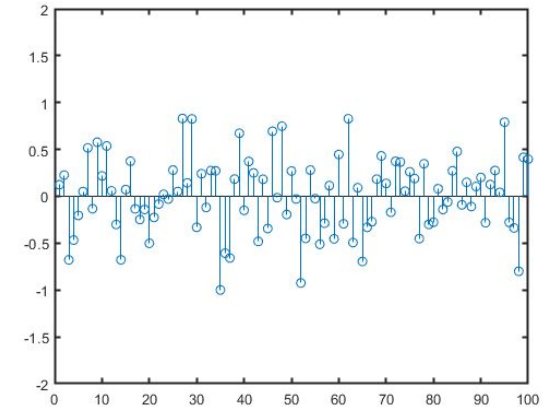
`numpy.random.uniform`

- Syntax

*`import numpy as np`*

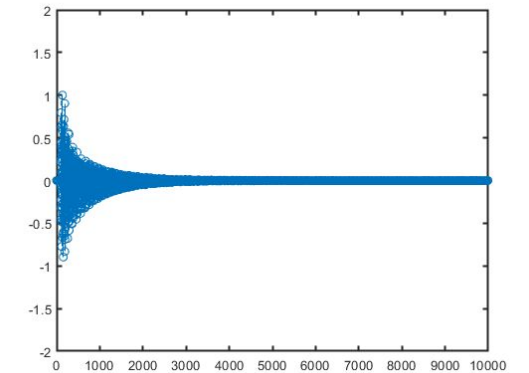
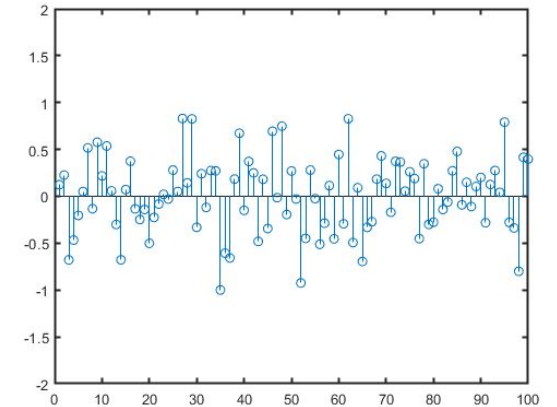
`np.random.uniform(low=0.0, high=1.0, size=None)`

`np.random.uniform(-1,1,1000)`



## Writing a Karplus-Strong algorithm in Python

- ~~Generate a random input~~
- Process the input using the algorithm
- Show the Result
- Save the result



## Writing a Karplus-Strong algorithm in Python

### First solution

$$y[n] = x[n - N] + \frac{y[n - N] + y[n - (N + 1)]}{2}$$

- 1) Create variable **y** as a copy of input **x**;  
Append **N** zeros in front of **y**

**OR**

Create **y** as zeros for a length of **x** plus **N**  
Replace the indexed values in **y** with values of **x**

- 2) Write a for-loop, which goes through all indexes of **y**  
Replace indexed values of **y**, with a sum of the current value plus scaled previous values



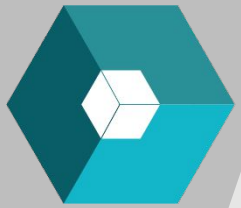
## Writing a Karplus-Strong algorithm in Python

### First solution

$$y[n] = x[n - N] + \frac{y[n - N] + y[n - (N + 1)]}{2}$$

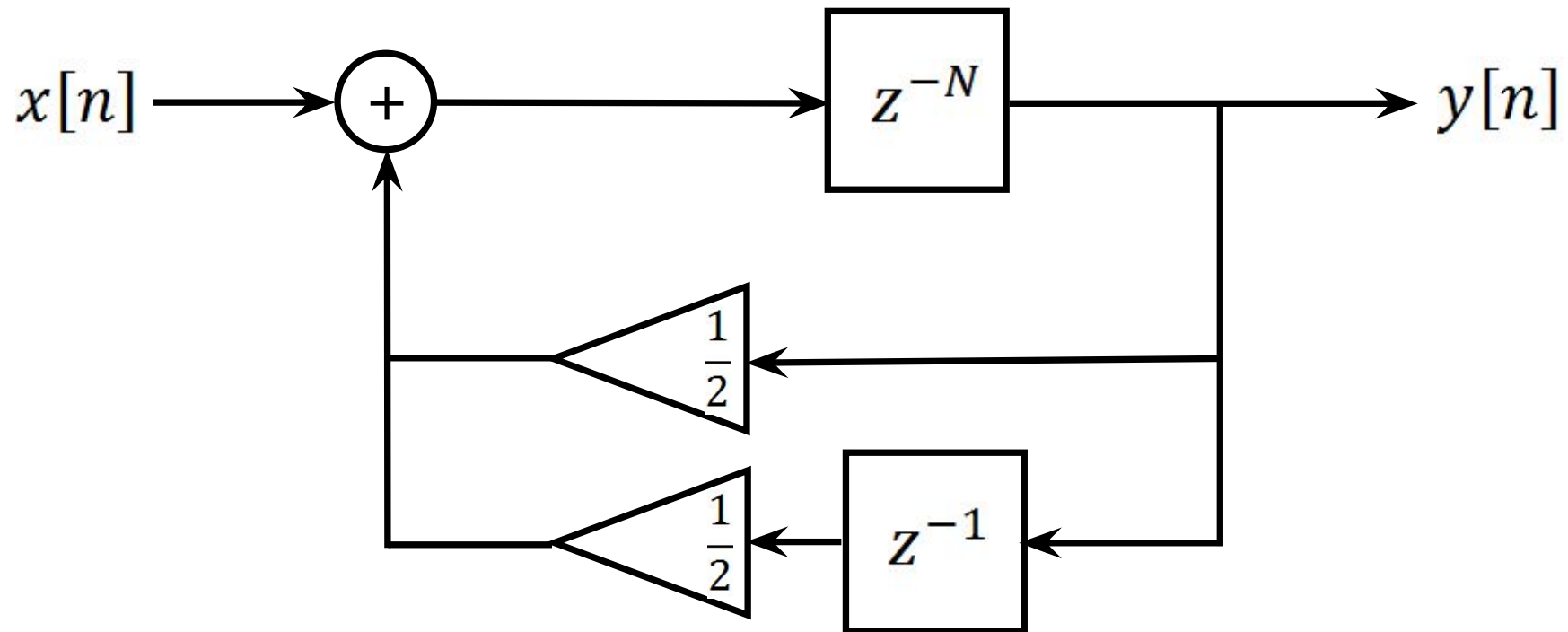
```
def KS1(signal, delayN, envelopeA, totalLength):  
    import numpy as np  
    # M = len(signal)  
    y = np.zeros((delayN))  
    y = np.append(y, signal)  
    y = np.append(y, np.zeros(totalLength - len(y)))  
  
    for index in range(delayN, totalLength):  
        y[index] = y[index] + envelopeA/2 * ( y[index - delayN] + y[index - (delayN+1)] )  
    return y
```





## Writing a Karplus-Strong algorithm in Python

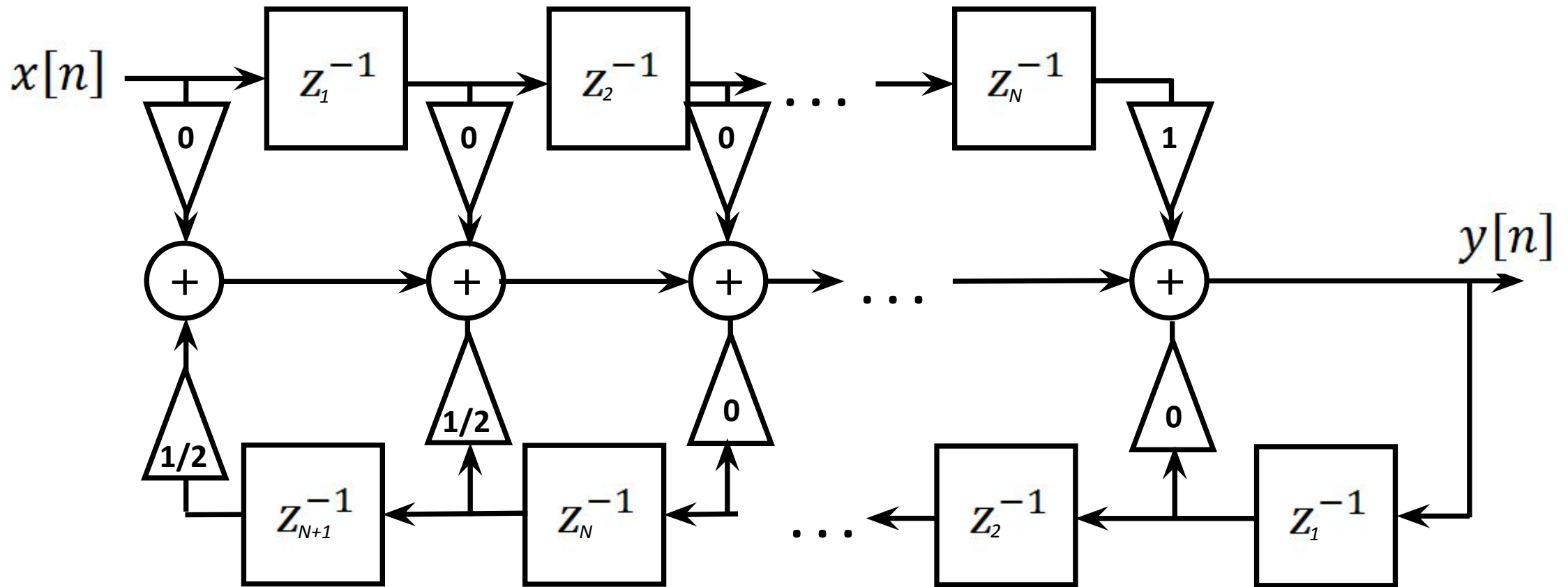
Second solution





## Writing a Karplus-Strong algorithm in Python

Second solution





## Writing a Karplus-Strong algorithm in Python

Second solution:

IIR Filter

**b-coefficients**

$b_0, b_1, \dots, b_{N-1}$  are 0

$b_N$  is 1

**a-coefficients**

$a_0$  is 1

$a_1 \dots a_{N-1}$  are 0

$a_N$  and  $a_{N+1}$  are 0.5



## Writing a Karplus-Strong algorithm in Python

```
#using a filter
randomInput=np.append(randomInput, np.zeros(totalLength-len(randomInput)))
b = np.append(np.zeros(delay), [1])
a = np.append([1], np.zeros(delay-1))
a = np.append(a, [-alpha/2, -alpha/2])
outputSignal = signal.lfilter(b, a, randomInput)
plot.plot(outputSignal)
plot.show()
```



## Save your output

```
fs = 44100 # 44100 samples per second
audio = outputSignal * (2**15 - 1) / np.max(np.abs(outputSignal))
audio = audio.astype(np.int16)
write('test.wav', 44100, audio)
```

## Homework Task 1

- 1) Write your own function and generate outputs (for example your own random number generator, a lookup table etc.)
- 2) Change the code to match «good practice»

<https://www.python.org/dev/peps/pep-0008/>

- 3) Create a function that has multiple outputs, and optimize the memory usage and processing speed.
- 4) Retrieve the outputs from the function and display them (or plot them)

## Полезная литература

- Методы и техника обработки сигналов при физических измерениях (2 тома)
- Signal processing for 5G
- Multi-Carrier Communication Systems with Examples in MATLAB: A New Perspective
- An Image Processing Tour of College Mathematics Yevgeniy V. Galperin
- Fundamentals of IMAGE, AUDIO, and VIDEO PROCESSING Using MATLAB: With Applications To PATTERN RECOGNITION
- Introduction to Modeling and Simulation with MATLAB and Python
- PySDR: A Guide to SDR and DSP using Python
- Python documentation