conditional semantics

What is a conditional?

A *conditional* is two propositions related by some “if...then...” construction.

“If it is Monday, then I have class.”

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"If it is Monday, then I have class."

M: it is Monday

C: I have class

What is a conditional?

“If it is Monday, then I have class.”

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C: I have class

“If’ and “then” are not part of the propositions; they are “connectives”.

What is a conditional?

"If it is Monday, then I have class."

M: it is Monday

C: I have class

The proposition to the left of "then" is the antecedent, and the proposition to the right of "then" is the consequent.

A big project in philosophy is to give a correct account of the semantics of conditionals.

(I assigned the von Fintel reading to give a sense of the project's influence on linguistics.)

When are they true? When are they false? When (if ever) are they meaningless?

today we'll talk about

- standard ways to understand the semantics of conditionals
  - the material conditional
  - the strict conditional
  - Stalnaker-Lewis semantics
- and how this work connects to more general topics in cognitive science
  - pretense and imagination
  - scientific reasoning
  - the role of formal logic in human thought

material conditional

In most introductory logic classes, you're taught that the "material conditional", which I'll indicate with "□", is the appropriate way to think about the truth-value of a conditional in a natural language.

quick note...

The “truth-value” of a proposition or sentence just means whether the sentence is true or false.

E.g., the truth-value of “Moscow is in Russia” is “true,” whereas the truth-value of “Barcelona is in France” is “false”.

“If it is Monday, then I have class.”

M: it is Monday

H: I have class

M □ C

In general, a material conditional of the form "A □ B" will be false if and only if *A* is true and *B* is false.

A	□	B
T	T	T
F	T	T
T	**F**	F
F	T	F

"If it is Monday, then I have class."

M	□	B
T	**T**	T
F	**T**	T
T	**F**	F
F	**T**	F

M	□	B
T	T	T
F	T	T
T	**F**	**F**
F	T	F

The only time this conditional is false is if it is Monday and you don't have class.

M	□	B
T	T	T
F	T	T
T	**F**	**F**
F	T	F

When a conditional is true only because its antecedent is false, we'll say the conditional is "vacuously true".

three problems with the material conditional

(1) the material conditional generates some inferences that seem wrong

(2) the material conditional doesn’t handle conditionals with false antecedents very well

(3) the material conditional does pretty bad with counterfactual/subjunctive conditionals

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“It is not the case that, if there is a God, then the moon is made of cheese. Hence, there is a God.”

If we interpret the conditional as a material conditional, then that inference is valid.

That’s a bit strange...

proof

"It is not the case that, if there is a God, then the moon is made of cheese. Hence, there is a God."

G: there is a God

C: the moon is made of cheese

<u>~(G □ C)</u>

G

proof

<u>~(G □ C)</u>

G

1. ~(G □ C) assumption
2. ~(~G v C)   1, implication
3. G & ~C   2, De Morgan’s
4. G   3, & elimination

Or here’s another strange inference:

Imagine that a light will go on if and only if you flip up both the left switch and the right switch.

Then we can say “the light will go on if and only if both the left switch is up and the right switch is up”.

Then we can say “the light will go on if and only if both the left switch is up and the right switch is up”.

But if we treat the conditional here as a material conditional, it follows that either if you flip up the left switch the light will go on or if you flip up the right switch the light will go on. But this conclusion is just wrong.

L: Left switch is up

R: Right switch is up

O: Light is on

“the light will go on if and only if both the left switch is up and the right switch is up”.

(L & R) □ □ O

“either if you flip up the left switch the light will go on or if you flip up the right switch the light will go on”

(L □ O) v (R □ O)

(L & R) □□ O

(L □ O) v (R □ O)

1.	(L & R) □□ O	Assumption
2.	[(L & R) □ O] & [O □ (L &R)]	1, bicondit.
3.	(L & R) □ O	2, & elimin.
4.	~(L & R) v O	3, impl.
5.	(~L v ~R) v O	4, De Morgan's
6.	~L v (~R v O)	4, paren. dist.
7.	(~L v O) v (~R v O)	6, v intro.
8.	(L □ O) v (R □ O)	7, impl. (x2)

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(a) "If Moscow is in New Zealand, then 2 + 2 = 4"

(b) "If Moscow is in New Zealand, then 2 + 2 = 5"

(c) "If Moscow is in New Zealand, then Red Square is in New Zealand"

If we treat these as material conditionals, they all turn out to be vacuously true, because the antecedent in each is false.

but this is a bit weird...

(a) “If Moscow is in New Zealand, then 2 + 2 = 4”

(b) “If Moscow is in New Zealand, then 2 + 2 = 5”

But how could (a) and (b) *both* be true?

(c) “If Moscow is in New Zealand, then Red Square is in New Zealand”

Moreover, it seems (c) is true, but not just *vacuously* true—i.e., it is true not merely because its antecedent is false.

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(d) “If I were living in Paris, then I would try to learn French.”

This conditional is a counterfactual in the subjunctive mood. (More on what this means in a minute.)

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This creates two problems.

(d) “If I were living in Paris, then I would try to learn French.”

First, can we even assign a truth-value to the antecedent? What is the truth-value of “I were living in Paris”?

(d) “If I were living in Paris, then I would try to learn French.”

Second, the antecedent is false (I guess), so the whole conditional is vacuously true. But I assure you the conditional is not just vacuously true. I would try to learn French if I lived in Paris!

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the strict conditional

Before looking at the Stalnaker-Lewis approach to counterfactusl, I want to say a bit about the "strict conditional".

This was developed by C.I. Lewis (1883-1964)

C.I. Lewis

quick modal logic lesson

L means "necessarily" and *M* means "possibly"

(A box is often used instead of *L*, and a diamond instead of *M*).

So *L*(P) means "necessarily *P*", while *M*(P) means "possibly *P*".

~*L*(P) means "not necessarily *P*" which is equivalent to *M~(P*) or "possibly not *P*".

quick modal logic lesson

What do “necessarily” and “possibly” mean?

There are different types of necessity/possibility. Three common types are logical/mathematical, nomological, and metaphysical.

(Further distinctions are often made, but these will suffice for our purposes.)

quick modal logic lesson

"Possibly, I could jump high enough to land on the moon."

If "possibly" is read as *nomological*, then this sentence is false. But if it is read as metaphysical, it is true.

quick modal logic lesson

When people talk about “possible worlds,” they generally mean worlds where the laws of physics or other arguably “contingent facts” are different.

For instance, there is a possible world at which objects move faster than the speed of light, or in which I am named “Randall,” rather than “Brian.”

quick modal logic lesson

On the other hand, there is no possible world at which “2 + 2 = 5”, for that would be a violation of logic/math, or where an animal is both alive and not alive, for that would be a violation of metaphysics (perhaps).

Logical/mathematical and metaphysical truths are taken to hold *across all possible worlds*.

Lewis was unhappy with the material conditional.

He said we should interpret conditionals as claims about what is necessarily true.

For Lewis, “If A, then B” is true if and only if *L*(A□B) is true

We use the material conditional within the parentheses, but the *L* indicates “necessity”.

So in words, “If A, then B” is true if and only if “Necessarily, if A then B”.

the strict conditional

One very nice feature of treating conditional statements as “strict” in this sense is that the problematic inferences we saw above are *invalid*

For instance, the following is invalid when the "if...then" part is read as "strict":

~(If there is a God, then the moon is made of cheese)
There is a God

That's (arguably) good!

~(If there is a God, then the moon is made of cheese)

There is a God

This becomes:

~[*L*(G□C)]

G

...which is invalid.

In case you're interested, here's a quick illustration of why the argument's invalid, using the tableaux method.

Showing why the light switch argument is invalid will take too long, so feel free to try it on your own as an exercise.

1.	~[*L*(G □ C)]	Assumption
2.	~G	negated conclusion
3.	~[*L*(~G v C)]	1, implication
4.	M~(~G v C)	3, ~*L* rule
5.	M(G & ~C)	4, De Moran’s
6.	G & ~C, 1	*M* rule
7.	G, 1	6, & elim.
8.	~C, 1	6, & elim.
	open	

Unfortunately, the strict conditional has problems too.

These are called the “paradoxes” of strict implication.

paradoxes of strict implication

(1) If *B* is necessarily true, then *L*(A □ B) will be true.

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Why is this weird?

(1) If *B* is necessarily true, then *L*(A □ B) will be true.

Why is this weird?

Well, let *B* be the proposition “7 is a prime number.” Most would say 7 is prime as a matter of mathematical necessity.

That means…

(e) “If Moscow is in Russian, then 7 is a prime number.”

(f) “If Moscow is in France, then 7 is a prime number.”

are both true, if we interpret these conditionals as strict conditionals.

paradoxes of strict implication

(1) If *B* is necessarily true, then *L*(A □ B) will be true.

(2) If *A* is necessarily false, then *L*(A □ B) will be true.

So then these sentences turn out true:

(g) “If 2 + 2 = 5, then Moscow is in Russia.”

(h) “If 2 + 2 = 5, then Moscow is in France.”

since most would say “2 + 2 = 5” is necessarily false.

That’s enough of strict conditionals.

Now we’re going to move on to the semantics for counterfactuals/subjunctives that Robert Stalnaker and David Lewis put forward.

indicative vs. subjunctive

(i) “If Oswald didn’t shoot Kennedy, then someone else did.” (indicative)

(j) “If Oswald hadn’t shot Kennedy, then someone else would have.” (subjunctive)

One way to think about the difference is that an indicative conditional attempts to describe the way the world is, whereas a subjunctive attempts to describe the way the world could have been (or would be like) if something had (or does) happen.

Generally, indicatives have antecedents with verbs in the simple present or simple past and no modal in the consequent. (A modal is a word like “would”, “could,” “should”).

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In contrast, subjunctives have verbs in the past perfect or the word “were” and a modal in the consequent.

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(j) “If Oswald hadn’t shot Kennedy, then someone else would have.”

For the purposes of this discussion, we’ll say a *counterfactual* is a subjunctive conditional with a false antecedent. E.g.,

(d) “If I were living in Paris, then I would try to learn French.”

I am not living in Paris, so the conditional is a counterfactual—i.e., it is counter to fact. It is also clearly subjunctive.

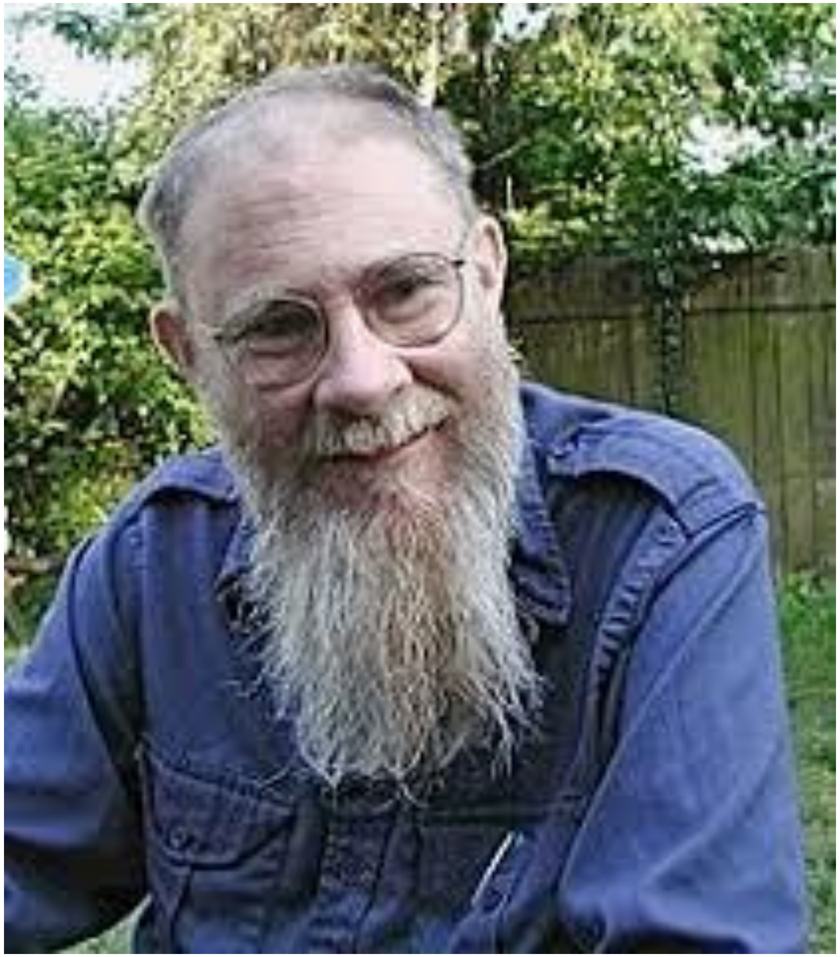
Note, not all subjunctive conditionals have a false antecedent:

(k) “If Jones had taken the arsenic, he would have just exactly those symptoms which he does in fact show.”

But for the purposes of this discussion, we’ll just be concerned with subjunctive counterfactuals, and I’ll just say “counterfactuals” from here on.

David Lewis (and, before that, Robert Stalnaker) came up with a framework for assigning a truth-value to a counterfactual that involves (a) possible worlds and (b) a “similarity relation” between possible worlds and the actual world.

David Lewis (1941-2001)



Take the following counterfactual:

(I) “If kangaroos had no tails, they’d topple over.”

How do we assign a truth-value to this counterfactual?

Roughly, the Stalnaker-Lewis approach is that we go to the nearest possible world about which the antecedent is true, then see if the consequent is true at that world too. If it is, the conditional is itself true. If not, it is false.

more precisely...

Bjerring's formulation (2017, 330):

(SL) A counterfactual of the form "If *P*, then *Q*" is true in the actual world if and only if some possible world in which *P* and *Q* are true is closer to the actual world than any possible world in which *P* is true and *Q* is false.

The "SL" refers to Stalnaker and Lewis.

So if the world at which kangaroos lack tails and topple over is closer to the actual world than any world in which kangaroos lack tails and don’t topple over, then the conditional is true. If not, it is false.

The way Lewis was thinking about this is that you imagine some “small miracle” occurs at a world that changes the world in a surgical way from the actual world to make the antecedent true

(m) “If I had struck this match, it would have lit.”

Again, this will be true precisely when the closest world to ours at which we the match is struck and it catches on fire is closer to our world than is any world at which the match is struck and it doesn’t catch on fire.

antecedent strengthening

I didn’t mention this above, but yet another problem with the material conditional and the strict conditional is called “antecedent strengthening.”

antecedent strengthening

If “A □ B” is true, then so too is “(A & C) □ B”

and

If “*L*(A□B)” is true, then so too “*L*([A & C] □ B)”

But natural languages don’t seem to work this way:

(m) “If I had struck this match, it would have lit.”

(n) “If I had struck this match **and** the room had no oxygen, it would have lit.”

Above, (m) seems correct, whereas (n) seems false.

Fortunately, with SL we can say (m) is true and (n) is false.

We'd analyze the first conditional differently than the second. With the first, we go to a world where the world is like ours but the match is struck. (So, if the room has oxygen in the actual world, it would there too.) In the second, we go to a world in which we strike the match and we remove oxygen from the room.

A similar story can be told about:

(l) “If kangaroos had no tails, they’d topple over.”

(o) “If kangaroos had no tails **and** used crutches, they’d topple over.”

some issues

(1) How do we determine the "similarity" or "nearness" of worlds?

(2) What *are* these "worlds"?

(3) Counterfactuals with impossible antecedents

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Quine’s example (about Douglas MacArthur during the Korean War):

(p) “If Caesar were in command, he would use the atom bomb.”

(q) “If Caesar were in command, he would use catapults.”

(Both seem true, or at least plausible.)

What world are we talking about: a world very much like ours (e.g., 1953), but Caesar is the general?

Or a world circa 2000 years ago and Caesar is the general?

(Conversational context is relevant here.)

The “uniqueness assumption” was endorsed by Stalnaker but not Lewis.

It says that, for each antecedent that is not impossible, there is a world that is most similar to ours at which the antecedent is true.

again from Quine...

(r) "If Bizet and Verdi had been compatriots, Bizet would have been Italian."

(s) "If Bizet and Verdi had been compatriots, Verdi would have been French."

If the uniqueness assumption is correct, only one of (r) and (s) is true, but it's not clear which.

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some issues

(1) ~~How do we determine the "similarity" or "nearness" of worlds?~~

(2) ~~What *are* these "worlds"?~~ (We'll skip this.)

(3) Counterfactuals with impossible antecedents

some issues

(1) ~~How do we determine the “similarity” or “nearness” of worlds?~~

(2) ~~What *are* these “worlds”?~~ (We’ll skip this.)

(3) Counterfactuals with impossible antecedents

squaring the circle

- To “square a circle” is to use only a compass and a ruler to construct a square that has the same area as a circle
- Thomas Hobbes (1588-1679) believed he had squared a circle.
- Apparently, it is in fact mathematically possible to do this.

(t) “If Hobbes had squared the circle, he would have been a famous mathematician.

(u) “If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would have cared.”

- Because squaring the circle is mathematically impossible, and because possible worlds must obey the rules of math, the above sentences are not just counterfactual, but also *counterpossible*—i.e., they are counterfactuals with an impossible antecedent.

- So how do we check to see whether these counterfactuals are true, given the Stalnaker and Lewis approach?
- We can’t go to the nearest possible world in which the antecedent is true and check to see whether the consequent is true there too—there are no such possible worlds.

Lewis (and others) thought counterpossibles are all vacuously true.

A lot of people think this is an unsatisfying response.

Why?

(t) “If Hobbes had squared the circle, he would have been a famous mathematician.

(u) “If Hobbes had (secretly) squared the circle, sick children in the mountains of South America at the time would have cared.”

To many, (t) seems true, but not vacuously so, while (u) seems false.

So there are a number of people in philosophy who are trying to extend counterfactual semantics by incorporating “impossible worlds”—i.e., worlds about which impossible propositions or sentences are true.

conditionals and pretense

A lot of reasoning that we engage in occurs when we act “as if” something were true, then infer the consequences

This is often referred to as “pretense”, “supposition”, “make-believe”, or “imagination”

This plays an important role in everyday life (from quite early on), as well as in science

"How is it possible for a child to think of a banana as if it were a telephone, a lump of plastic as if it were alive, or an empty dish as if it contained soap? If a representational system is developing, how can its semantic relations tolerate distortion in these more or less arbitrary ways?...Why does pretending not undermine their representation system and bring it crashing down?" (Leslie 1987, 412)

Effectively, Leslie is asking how counterfactual reasoning in possible in young children?

Consider Galileo...

How would a ball roll down this inclined plane *if there were no friction*?

Which object would hit the ground first *were I to drop them at the same time from a large tower*?

Or Newton...

What would an object do *were there no forces* acting on the object at all?

Another contemporary topic in philosophy (and cognitive science) is whether scientific reasoning is just an outgrowth and self-conscious application and modification of the sort of counterfactual reasoning even young children can engage in

If you'd like to learn more about the logic I discuss above, I recommend:

Priest, G. (2001) *An Introduction to Non-Classical Logic*. Cambridge University Press.