The game “Towers of Hanoi” consists of three rods, on the first of them a pyramid of n disks is installed, the radius of which decreases from the lower disk to the upper one. It is required to transfer disks to the third core, using the second core as an auxiliary one and following the following rules.

a) In one operation, you can transfer only one disk.

b) You can not put a larger disc diameter on a smaller disc diameter.

Problems

1. Transfer 5 disks
2. Prove that N disks can be moved.

Recursion

```
procedure Solve(n: integer; a,b,c: Char);
begin
 if n > 0 then
 begin
  Solve(n-1, a, c, b);
  Writeln(‘transfer', a, ‘to rod',b);
  Solve(n-1, c, b, a);
 end;
end;
begin
 Solve(4, '1','2','3');
end.
```

Mathematical induction method

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

$$11^{n+2} + 12^{2n+1} \vdots 133.$$

$$1^4 + 2^4 + \ldots + n^4 = ?$$

Mathematical induction in geometry

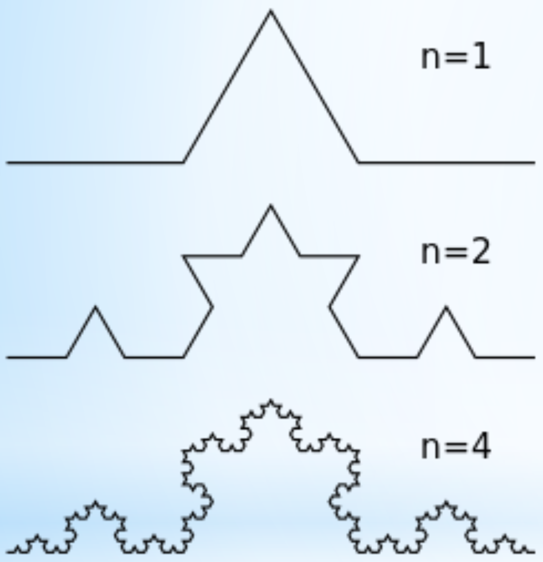
1. Several straight lines were drawn on the plane. Prove that it is possible to color the plane in two colors so that the two areas that have a common part of the border have a different color. Areas that have only one common vertex may be of the same color.

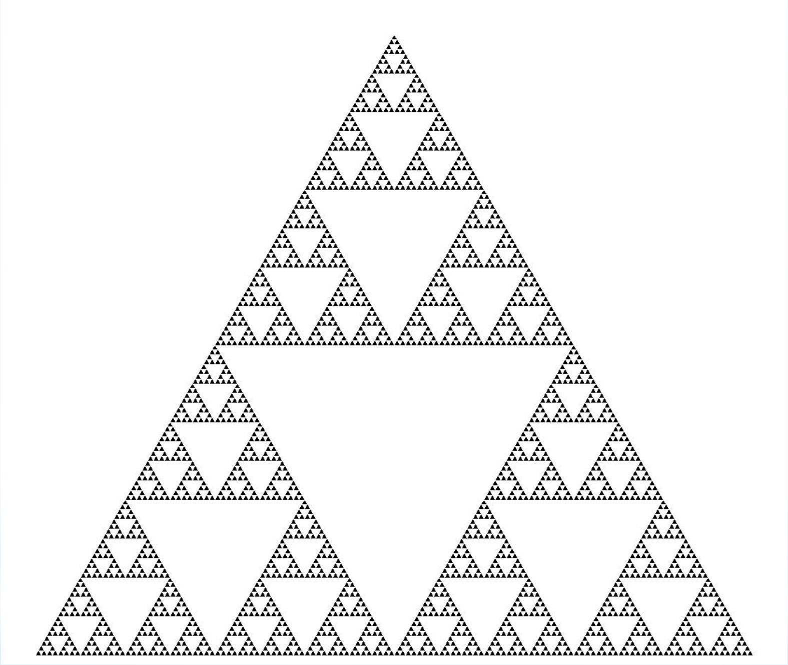
2. Prove that a square can be cut into any number of squares, starting with 6.

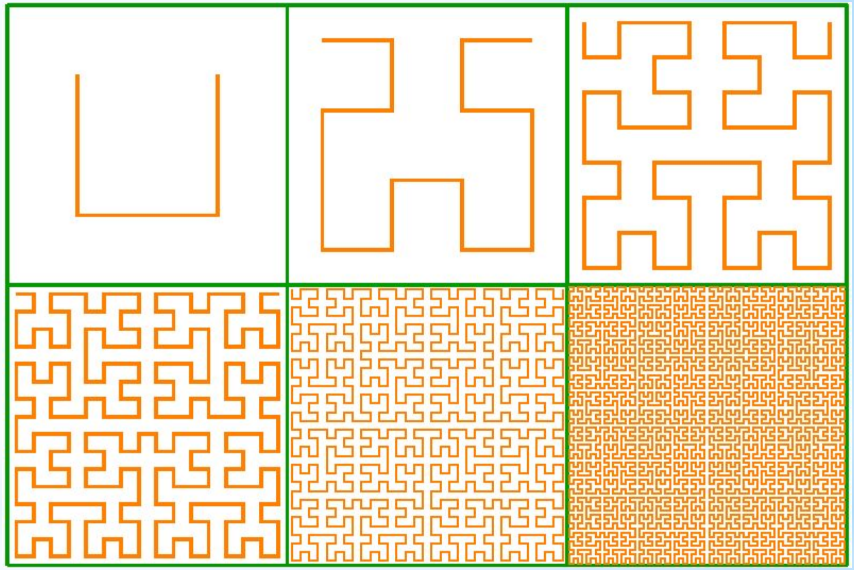
3. Prove that for every $N > 2$ exists N–gon with three acute angles.

4. Prove that the square $2^N \times 2^N$, from which one cell was cut can be cut into “corners” of three cells.

Recursion in geometry
Fractals







1. In the company of $2n + 1$ people for any n people there is a different person from them who is familiar with each of them. Prove that in this company there is a person who knows everyone.

From n to $(n + 1)$

2. Among the participants of the conference, everyone has at least one friend. Prove that the participants can be distributed in two rooms so that each participant has a friend in the other room.

We can reduce the problem for example with a tree

Calculation of the determinants + **verifiation**

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{12}a_{32}a_{13} - a_{31}a_{22}a_{13} - a_{21}a_{12}a_{33} - a_{11}a_{32}a_{23}$$

$$\det A = \sum_{k=1}^{n} a_{ik} \cdot (-1)^{i+k} \det A^{(i,k)}$$

Calculation of the determinants + **verifiation**

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$\begin{vmatrix} 8 & 7 & 2 & 10 \\ -8 & 2 & 7 & 10 \\ 4 & 4 & 4 & 5 \\ 0 & 4 & -3 & 2 \end{vmatrix}$$

Solve the equation $$\begin{vmatrix} 3 & x & -x \\ 2 & -1 & 3 \\ x+10 & 1 & 1 \end{vmatrix} = 0$$

Optimization

$$\begin{vmatrix} 8 & 7 & 2 & 10 \\ -8 & 2 & 7 & 10 \\ 4 & 4 & 4 & 5 \\ 0 & 4 & -3 & 2 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 6 & 0 \\ 0 & 10 & 15 & 20 \\ 4 & 4 & 4 & 5 \\ 0 & 4 & -3 & 2 \end{vmatrix} = (-1)^{1+3} 4 \begin{vmatrix} -1 & -6 & 0 \\ 10 & 15 & 20 \\ 4 & -3 & 2 \end{vmatrix} = 4 \begin{vmatrix} -1 & -6 & 0 \\ 0 & -45 & 20 \\ 0 & -27 & 2 \end{vmatrix} =$$

$$= 4(-1)^{1+1}(-1)\begin{vmatrix} -45 & 20 \\ -27 & 2 \end{vmatrix} = -4\left(-90 + 540\right) = -1800.$$

Inverse matrix

$$A \cdot A^{-1} = A^{-1} \cdot A = E$$

$$A^{*} = \begin{pmatrix} \det A^{(1,1)} & -\det A^{(2,1)} & \det A^{(3,1)} & \dots & (-1)^{n+1} \det A^{(n,1)} \\ \dots & \dots & \dots & \dots & \dots \\ (-1)^{n+1} \det A^{(1,n)} & (-1)^{n+2} \det A^{(2,n)} & (-1)^{n+3} \det A^{(3,n)} & \dots & \det A^{(n,n)} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & -2 & 5 \end{pmatrix} \qquad \begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 2 \\ 2 & 1 & 4 \end{pmatrix}$$

Solve the equation $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$

Interpolation and verification

Lagrange interpolation polynomial

Given a set of $k + 1$ data points

$$(x_0, y_0), \ldots, (x_j, y_j), \ldots, (x_k, y_k)$$

where no two x_j are the same, the **interpolation polynomial in the Lagrange form** is a linear combination

$$L(x) := \sum_{j=0}^{k} y_j \ell_j(x)$$

of Lagrange basis polynomials

$$\ell_j(x) := \prod_{\substack{0 \le m \le k \\ m \ne j}} \frac{x - x_m}{x_j - x_m} = \frac{(x - x_0)}{(x_j - x_0)} \cdots \frac{(x - x_{j-1})}{(x_j - x_{j-1})} \frac{(x - x_{j+1})}{(x_j - x_{j+1})} \cdots \frac{(x - x_k)}{(x_j - x_k)},$$

$p(-4) = -3; \quad p(2) = -9; \quad p(-2) = 19; \quad p(-1) = 3;$
$p(1) = 7;$

In algebra, the **rational root theorem** (or **rational root test**, **rational zero theorem**, **rational zero test** or ***p*/*q* theorem**) states a constraint on rational solutions of a polynomial equation

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$$

with integer coefficients. Solutions of the equation are roots (equivalently, zeroes) of the polynomial on the left side of the equation.

The theorem states that if a_0 and a_n are nonzero, then each rational solution x, when written as a fraction $x = p/q$ in lowest terms (i.e., the greatest common divisor of p and q is 1), satisfies

- p is an integer factor of the constant term a_0, and
- q is an integer factor of the leading coefficient a_n.

$$6x^4 - 29x^3 + 29x^2 + 16x - 12$$

Prove that the number is composite

1. $1 + 2^{3456789}$.
2. $2^8 + 2^5 \cdot 5^6 + 5^{12}$.
3. $2^{10} + 5^{12}$.
4. 99…9919 (100 number).
5. 100..0027 (61 number).

6. Is it true that $n^2 + n + 41$ is a prime number for all positive integers n?