

# Simulation

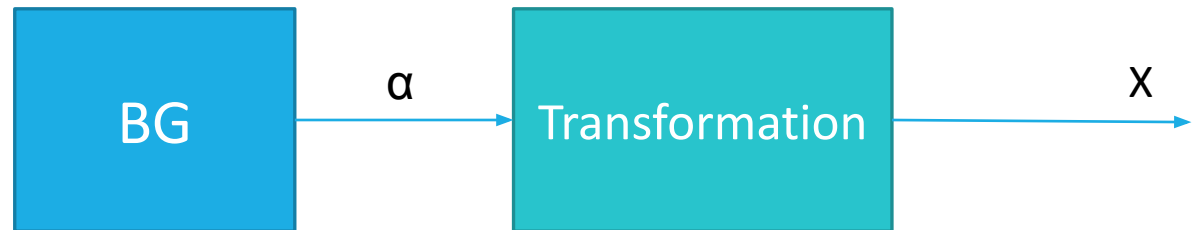
---

STOCHASTIC MODELING

# Random objects and base generator

---

1. Random events
2. Random variables
3. Random processes
4. Random point processes



# Discrete RV

---

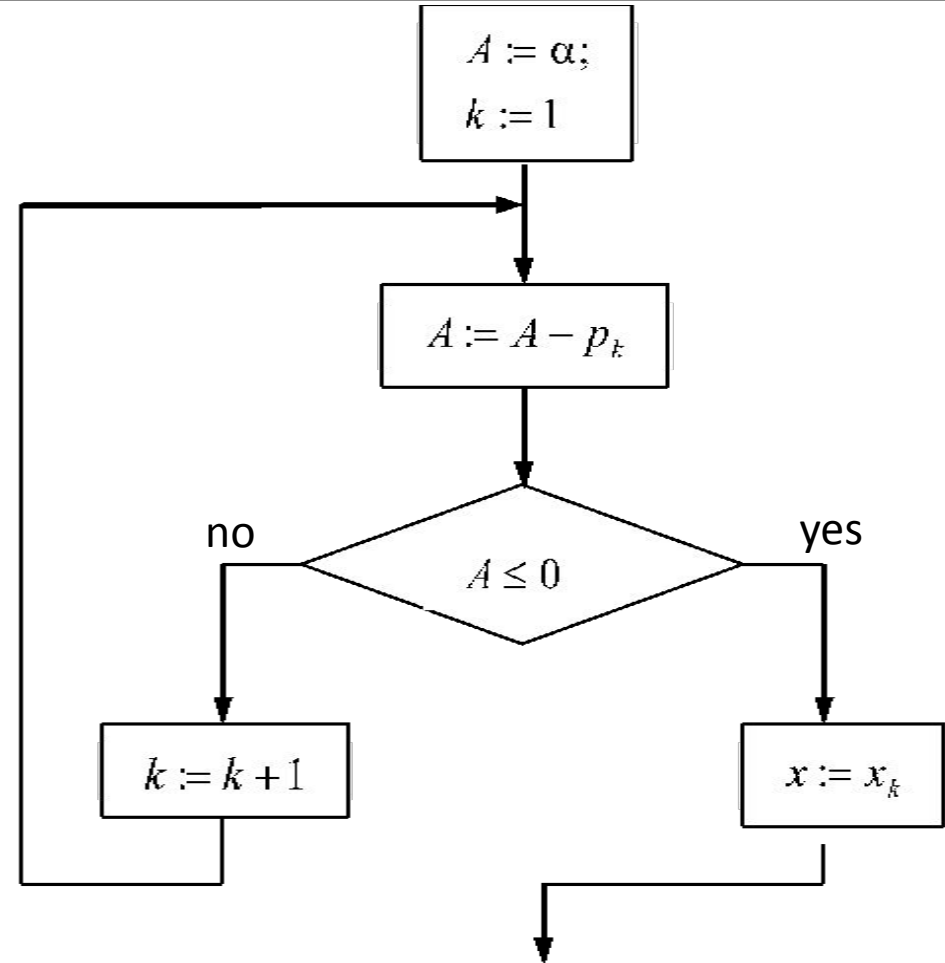
METHODS OF GENERATION

# Discrete RV given by distribution

$x$	$x_1$	$x_2$	$\dots$	$x_m$
$p$	$p_1$	$p_2$	$\dots$	$p_m$

$$p_i = P\{x = x_i\}$$

$$\sum_{i=1}^m p_i = 1$$

 $A_1$  $A_2$  $\dots$  $A_m$ 

# Statistical processing for discrete RVs

$x$	$x_1$	$x_2$	$\dots$	$x_m$
$p$	$p_1$	$p_2$	$\dots$	$p_m$

Mathematical expectation (mean):  $E x = \sum_{i=1}^m p_i x_i$

Variance:  $\text{Var } x = D x = \sum_{i=1}^m p_i (x_i - E x)^2 = \sum_{i=1}^m p_i x_i^2 - (E x)^2$

Relative frequencies

$$\hat{p}_i = \frac{n_i}{N}$$

$n_i$  is the number of appearances of value  $i$   
(they are named as frequencies),  
 $N$  is the total number of trials

Empiric expectation (average):

$$\hat{E} x = \sum_{i=1}^m \hat{p}_i x_i$$

Empiric variance:

$$\hat{D} x = \sum_{i=1}^m \hat{p}_i x_i^2 - (\hat{E} x)^2$$

Absolute errors:  $\Delta_E = |\hat{E} x - E x|$   
 $\Delta_D = |\hat{D} x - D x|$

Relative errors:  $\delta_E = \frac{\Delta_E}{|E x|}$   $\delta_D = \frac{\Delta_D}{|D x|}$

# Statistical processing for discrete RVs

## Chi-squared test

$$\hat{X}_N^2 = \sum_{i=1}^m \frac{n_i^2}{Np_i} - N$$

Hypothesis that empiric distribution corresponds to the theoretical one is not true if and only if

$$\hat{X}_N^2 > \chi_{1-\alpha, m-1}^2$$

$\alpha$  is a significance level

Critical values of the Chi-square distribution with  $d$  degrees of freedom

Probability of exceeding the critical value							
$m$	0.05	0.01	0.001	$m$	0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	7.815	11.345	16.266	13	22.362	27.688	34.528
4	9.488	13.277	18.467	14	23.685	29.141	36.123
5	11.070	15.086	20.515	15	24.996	30.578	37.697
6	12.592	16.812	22.458	16	26.296	32.000	39.252
7	14.067	18.475	24.322	17	27.587	33.409	40.790
8	15.507	20.090	26.125	18	28.869	34.805	42.312
9	16.919	21.666	27.877	19	30.144	36.191	43.820
10	18.307	23.209	29.588	20	31.410	37.566	45.315

# Laboratory #9

---

## ASSIGNMENT:

### Simulation of discrete random variable

- Implement an algorithm for conducting a series of experiments to simulate a discrete random variable specified by the distribution
- Calculate empirical probabilities, sample mean and variance, their relative errors
- Calculate the chi-squared statistic and apply the chi-squared test for different values of  $N$  ( $N = 10, 100, 1,000, 10,000$ )
- Draw a conclusion

# Laboratory #9

---

## ЗАДАНИЕ:

### Имитационное моделирование дискретных случайных величин

- Реализовать алгоритм проведения серии экспериментов по генерации дискретной случайной величины, заданной рядом распределения
- Вычислить эмпирические вероятности, выборочные среднее и дисперсию, их относительные погрешности
- Вычисление статистику хи-квадрат и применить критерий хи-квадрат при разных значениях  $N$  ( $N = 10, 100, 1\ 000, 10\ 000$ )
- Сделать вывод



# Laboratory #9

## UI PROTOTYPE

Prob 1

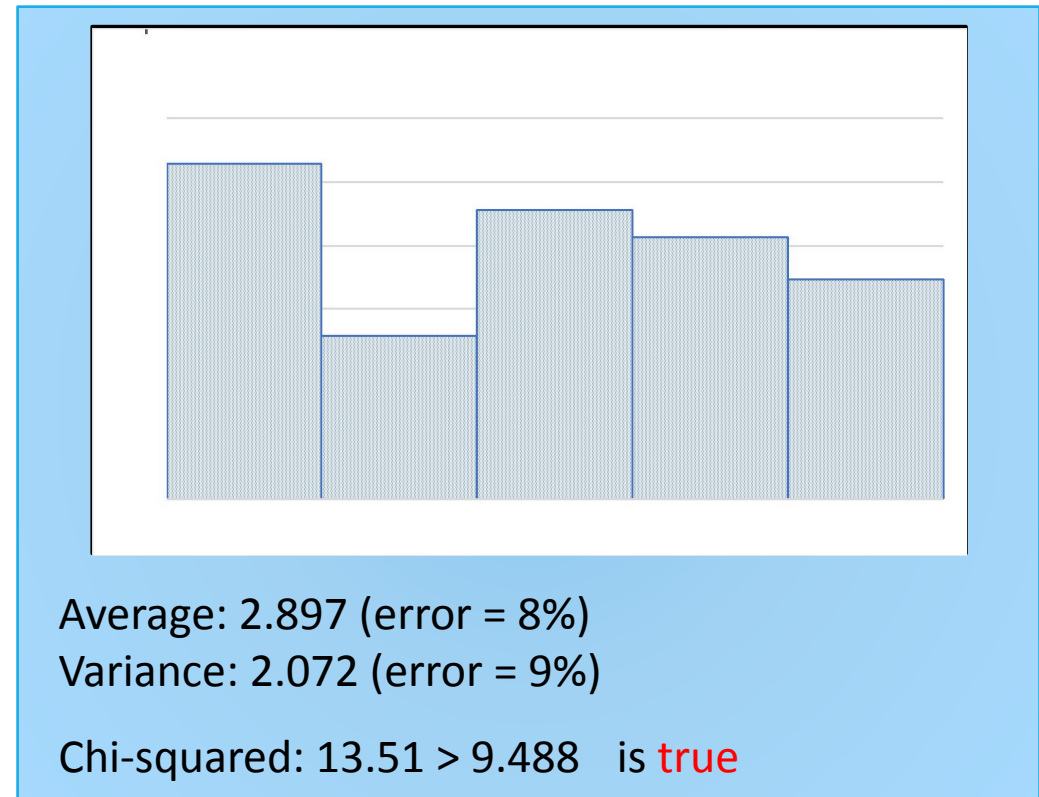
Prob 2

Prob 3

Prob 4

Prob 5

Number of experiments



# Uniform discrete distribution

FROM 0 TO  $n$ :

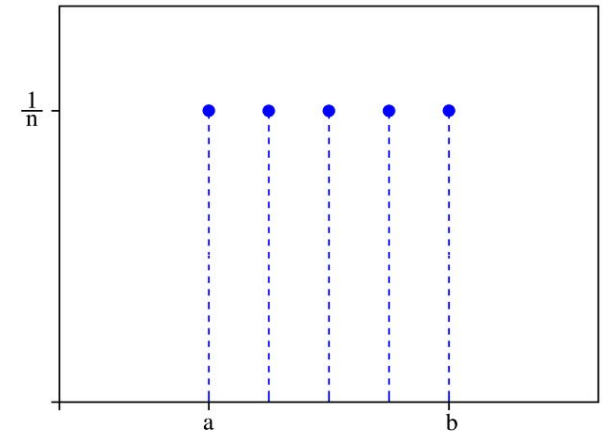
$x$	0	1	...	$n$
$p$	$\frac{1}{n+1}$	$\frac{1}{n+1}$	...	$\frac{1}{n+1}$

GENERATOR:

$$x = \text{Int}(\alpha \cdot (n + 1))$$

Int is a truncating operation

FROM  $a$  TO  $b$ :



1. Set  $n = b - a$
2. Use  $x = \text{Int}(\alpha \cdot (n + 1))$
3. Calculate  $x = x + a$

For example, if  $x$  from  $\{1, 2, \dots, n\}$  then use formula:  $x = \text{Int}(\alpha \cdot n) + 1$

# Geometric distribution

The probability distribution of  
**the number of failures before the first success**

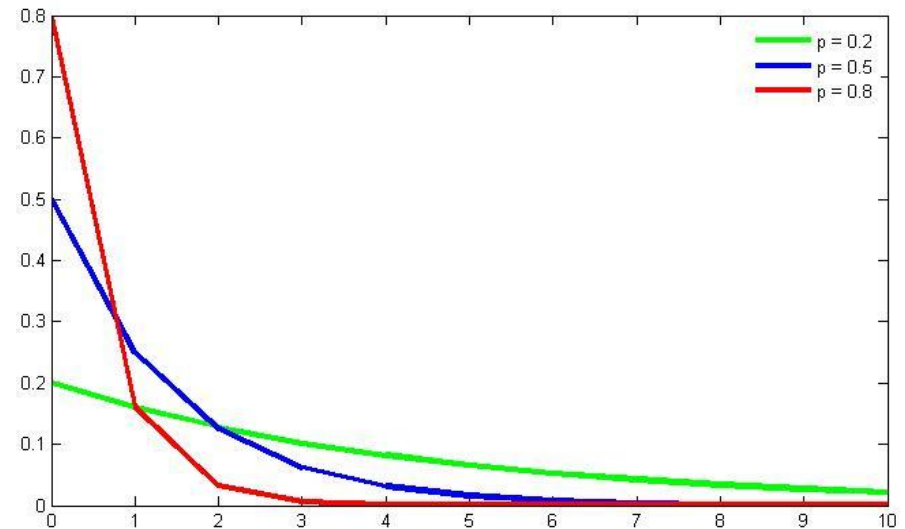
$$X \in \{0, 1, 2, 3, \dots\}$$

$p$  is a probability of success in one trial

$$P\{x = m\} = p(1 - p)^m$$

GENERATOR:

$$x = \text{Int}\left(\frac{\ln \alpha}{\ln(1 - p)}\right)$$



# Negative binomial distribution

The probability distribution of  
**the number of failures before the  $r$ -th success**

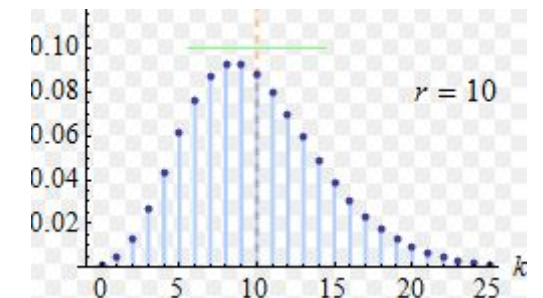
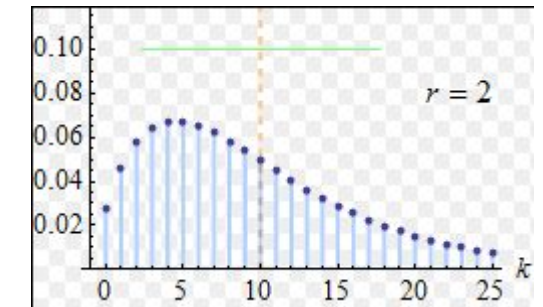
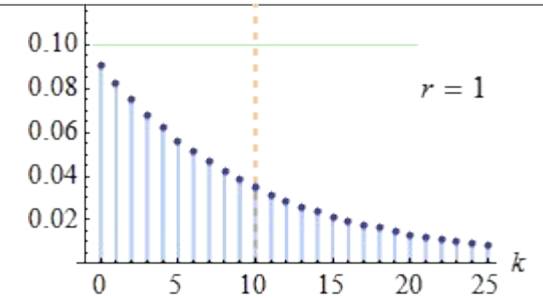
$$X \in \{0, 1, 2, 3, \dots\}$$

$p$  is a probability of success in one trial

$$P\{x = m\} = C_{m+r-1}^m \cdot p^r (1-p)^m$$

GENERATOR:

$$x = \sum_{i=1}^r \text{Int} \left( \frac{\ln \alpha_i}{\ln(1-p)} \right)$$



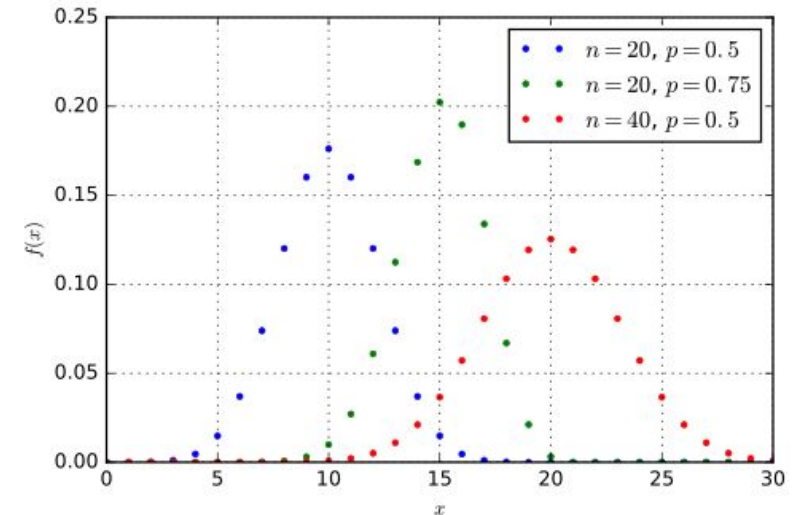
# Binomial distribution

The probability distribution of  
**the number of successes in  $n$  trials**

$$X \in \{0, 1, 2, \dots, n\}$$

$p$  is a probability of success in one trial

$$P\{x = m\} = C_n^m p^m (1-p)^{n-m}$$



GENERATOR:  $x = \sum_{k=1}^n \theta(p - \alpha_k)$  where  $\theta(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0. \end{cases}$  is a Heaviside step function

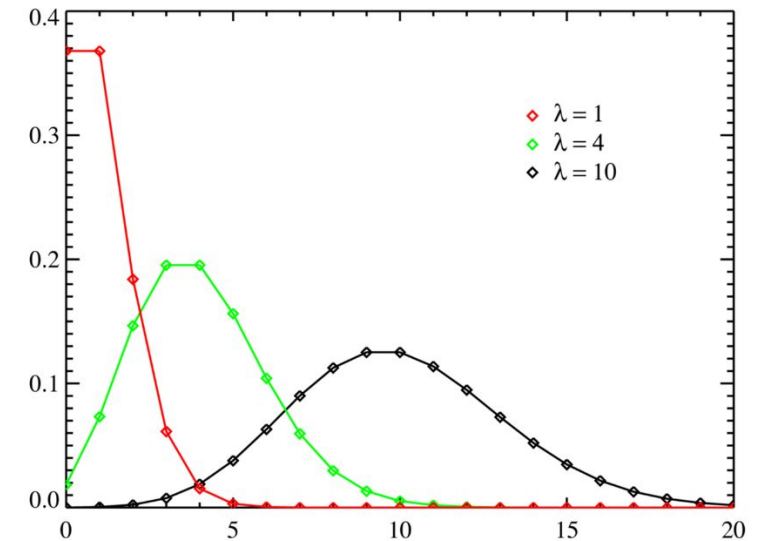
# Poisson distribution

The probability distribution of  
**the number of events occurred in a fixed interval  
of time if these events occur independently and  
with a known constant rate  $\lambda$**

$$X \in \{0, 1, 2, 3, \dots\}$$

$$P\{x = m\} = \frac{\lambda^m}{m!} e^{-\lambda}$$

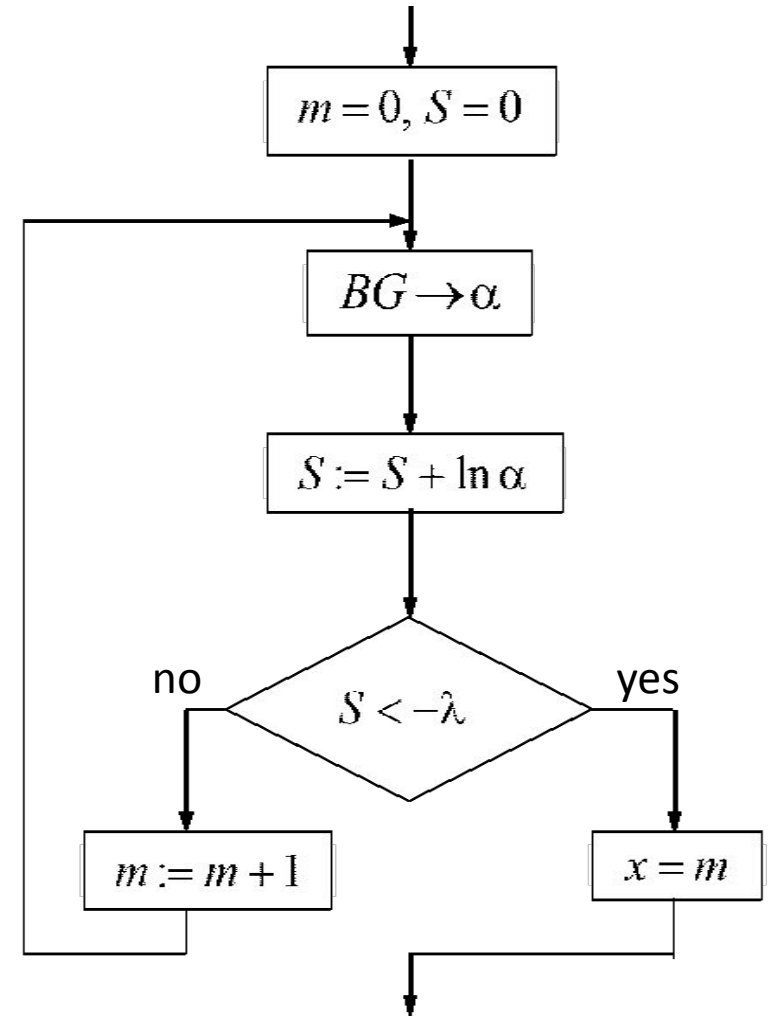
$$P\{x = m\} = \frac{(\lambda t)^m}{m!} e^{-\lambda t}$$



# Poisson distribution generator

GENERATOR:

$$x = \min \left\{ m : \sum_{i=0}^m \ln \alpha_i < -\lambda \right\}$$



# Laboratory #10

## ASSIGNMENT:

- Football manager game
- Basketball**

WATFORD 1 | 07:01 PAUSED | Stoke 0 | First Half | Second Half | Play

Pitch Tactics Analysis Updates

**WATFORD 1**

PKD	STATUS	NAME	CONDITL...	RAT	GLS	AST
GK		H. Gomes	98%	6.8	-	-
DR		K. Femenía	96%	6.8	-	-
DCR		S. Prödl	97%	6.8	-	-
DCL		M. Britos	93%	6.7	-	-
DL		J. Holebas	93%	6.7	-	-
MCR		N. Chalobah	92%	6.8	-	-
MCL		A. Doucouré	92%	6.7	-	-
AMR		A. Carrillo	92%	6.7	-	1
AMC		T. Cleverley	96%	7.3	-	-
AML		G. Deulofeu	91%	6.7	-	-
STC		A. Gray	93%	6.7	-	-
S1		O. Karnezis	100%	-	-	-
S2		D. Janmaat	93%	-	-	-
S3		M. Zeegelaar	97%	-	-	-
S4		S. Okaka	87%	-	-	-
S5		D. Ndong	95%	-	-	-

**Match Information**

RECENT EVENTS

MATCH STATS

Team	Shots	On Target	Fouls	Yellow Cards
WATFORD	1	0	1	0
STOKE	1	0	2	0

FOCUS OF ATTACKS

Team	Stoke	Watford
STOKE	25%	25%
WATFORD	75%	37%

**STOKE 0**

PKD	STATUS	NAME	CONDITL...	RAT	GLS	AST
GK		J. Butland	98%	6.8	-	-
DR		M. Bauer	93%	6.8	-	-
DCR		K. Zouma	94%	6.7	-	-
DCL		K. Wimmer	98%	6.7	-	-
DL		B. Martins Indi	93%	6.7	-	-
DM		J. Allen	93%	6.7	-	-
MCR		Badou Ndiaye	95%	6.7	-	-
MCL		D. Fletcher	93%	6.7	-	-
STCR		M. Choupo-Moting	93%	6.7	-	-
STC		S. Berahino	95%	6.7	-	-
STCL		Jesé	92%	6.7	-	-
S1		L. Grant	95%	-	-	-
S2		R. Shawcross	99%	-	-	-
S3		R. Sobhi	96%	-	-	-
S4		M. Diouf	98%	-	-	-
S5		K. Stafylidis	98%	-	-	-

Carrillo played a key role in the goal with a fine assist



# Laboratory #10

English Premier Division  
Holders - Leicester (ID:11)

16 Dec 2016  
Fri 0:00

Overview ▾ Matches ▾ News ▾ Stats ▾ Transfers ▾ History ▾

STAGES League Table ▾ Overall ▾

Current Season

POS	INF	TEAM	PLD	WON	DRN	LST	FOR	AG	GD	PTS	FORM
1st		Chelsea	16	13	1	2	34	11	23	40	
2nd		Liverpool	16	10	4	2	40	20	20	34	
3rd		Arsenal	16	10	4	2	37	17	20	34	
4th		Man City	16	10	3	3	34	19	15	33	
5th		Tottenham	16	8	6	2	27	11	16	30	
6th		Man Utd	16	7	6	3	22	17	5	27	
7th		West Brom	16	6	5	5	23	19	4	23	
8th		Everton	16	6	5	5	21	20	1	23	
9th		Southampton	16	5	6	5	14	15	-1	21	
10th		Bournemouth	16	6	3	7	22	25	-3	21	
11th		Watford	16	6	3	7	21	28	-7	21	
12th		Stoke	16	5	5	6	17	22	-5	20	
13th		Burnley	16	5	2	9	15	26	-11	17	
14th		Leicester	16	4	4	8	21	27	-6	16	
15th		West Ham	16	4	4	8	18	31	-13	16	
16th		Crystal Palace	16	4	3	9	28	31	-3	15	
17th		Middlesbrough	16	3	6	7	13	19	-6	15	
18th		Swansea	16	3	3	10	20	34	-14	12	
19th		Hull	16	3	3	10	14	35	-21	12	
20th		Sunderland	16	3	2	11	14	28	-14	11	



Use Poisson distribution for the number of goals in a match

# Statistical processing for discrete RVs with infinite number of values

---

only for chi-squared criteria!

$$X: \sum_{i=X}^{\infty} p_i \leq \min_{i < X} p_i$$

Example (geometric distribution  $p=0.8$ ):  $P\{x = m\} = p(1-p)^m$

$$p_0 = 0.8; \quad p_1 = 0.8 \times 0.2 = 0.16; \quad p_2 = 0.8 \times 0.2 \times 0.2 = 0.032; \quad \sum_{i=3}^{\infty} p_i = 0.008.$$

“Similar” distribution:

0	1	2	>2
0.8	0.16	0.032	0.008