Հեղուկների և գազերի կիրառական մեխանիկա

Դասախոսություն 1

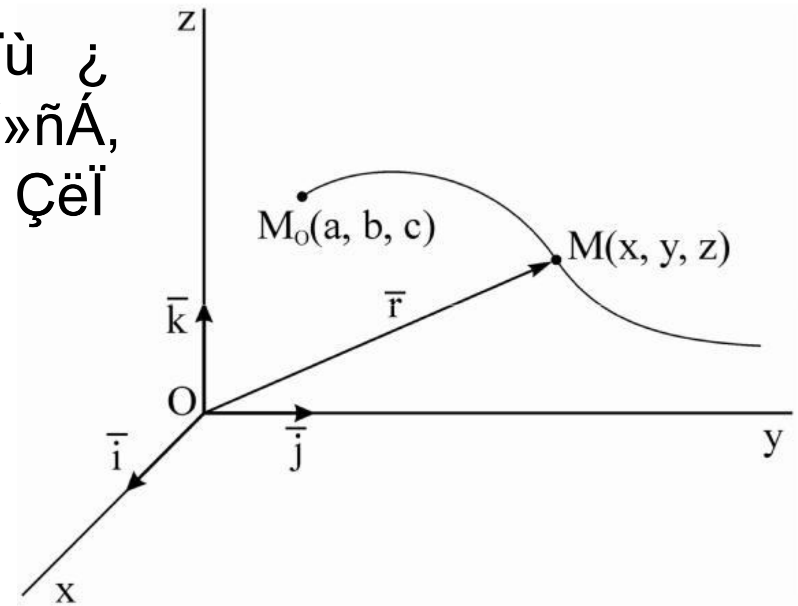
ÐºÔàôÎ ØÆæ²ì²ÚðÆ ÎÆÜºØ²îÆÎ²

1.1 Ð»ÕáõÏÇ ß³ñÅÙ³Ý ÝÏ³ñ³·ñáõÃÛ³Ý È³·ñ³ÝÅÇ ¨ ¾ÛÉ»ñÇ Ù»Ãá¹Ý»ñÁ

È³·ñ³ÝÅÇ Ù»Ãá¹Ç ¿áõÃÛáõÝÝ ³ÛÝ ¿, áñ Ñ»ÕáõÏÇ ß³ñÅáõÙÁ µ»ñíáõÙ ¿ Ýñ³ ³é³ÝÓÇÝ Ù³ëÝÇÏÝ»ñÇ ß³ñÅÙ³Ý áõëáõÙÝ³ëÇñáõÃÛ³ÝÁ: È³·ñ³ÝÅÇ Ù»Ãá¹Ç Ñ³Ù³Ó³ÛÝ ÁÝïñíáõÙ ¿ Ñ»ÕáõÏÇ áñ¨¿ Ï»ï, Ù³ëÝÇÏ Ï³Ù ïÇñáõÛÃ, Ñ»ï¨áõÙ Ýñ³ ß³ñÅÙ³Ý µÝáõÃ³·ñÇã Ù» ÍáõÃÛáõÝÝ»ñÇ ÷á÷ËáõÃÛ³ÝÁ:

$$\bar{r} = \bar{r}(a,b,c,t) \qquad 1.1$$

Ղ»ÕáõÏ ïÇñáõÛÃÇ ß³ñÅÙ³Ý ûñ»ÝùÁ ëï³Ý³Éáõ Ñ³Ù³ñ å»ïù ¿ ÙÇ³Å³Ù³Ý³Ï ÷á÷áË»É Ýßí³Í µáÉáñ a, b, c ¨ t ÷á÷áËÑ³Ï³ÝÝ»ñÁ, áñáÝù ÏáãíáõÙ »Ý È³·ñ³ÝÅÇ ÷á÷áËÑ³Ï³ÝÝ»ñ, ÇëÏ áõëáõÙÝ³ëÇñíáÕ Ù»Ãá¹Á` È³·ñ³ÝÅÇ Ù»Ãá¹:



Նկ.1.1

àôÕÕ³ÝÏÛáõÝ ¹»Ï³ñïÛ³Ý Ïááñ¹ÇÝ³ï³ÛÇÝ Ñ³Ù³Ï³ñ·áõÙ ß³ñÅÙ³Ý (1.1) ûñ»ÝùÝ áõÝÇ Ñ»ï¨Û³É ï»ëùÁ`

$$x = x\,(a,b,c,t), \quad y = y\,(a,b,c,t), \quad z = z\,(a,b,c,t): \qquad 1.2$$

Ð³í³ë³ñáõÙÝ»ñÇ (1.2) Ñ³Ù³Ï³ñ·Çó Ï³ñáÕ »Ýù ëï³Ý³É ÏÇÝ»Ù³ïÇÏ³ÛÇ ÙÛáõë µÝáõÃ³·ñÇãÝ»ñÁ

$$V_x = \frac{dx}{dt} = \frac{\partial x}{\partial t} + \frac{\partial x}{\partial a}\cdot\frac{da}{dt} + \frac{\partial x}{\partial b}\cdot\frac{db}{dt} + \frac{\partial x}{\partial c}\cdot\frac{dc}{dt} = \frac{\partial x}{\partial t},$$

$$V_y = \frac{dy}{dt} = \frac{\partial y}{\partial t} + \frac{\partial y}{\partial a}\cdot\frac{da}{dt} + \frac{\partial y}{\partial b}\cdot\frac{db}{dt} + \frac{\partial y}{\partial c}\cdot\frac{dc}{dt} = \frac{\partial y}{\partial t}, \qquad 1.3$$

$$V_z = \frac{dz}{dt} = \frac{\partial z}{\partial t} + \frac{\partial z}{\partial a}\cdot\frac{da}{dt} + \frac{\partial z}{\partial b}\cdot\frac{db}{dt} + \frac{\partial z}{\partial c}\cdot\frac{dc}{dt} = \frac{\partial z}{\partial t}:$$

(1.3) µ³Ý³Ó¨»ñÁ ëï³Ý³ÉÇë Ñ³ßíÇ ¿ ³éÝí»É, áñ a, b, c-Ý Ñ³ëï³ïáõÝ »Ý`

$$\frac{da}{dt} = \frac{db}{dt} = \frac{dc}{dt} = 0: \qquad 1.4$$

Ð»ÕáõÏÇ Ù³ëÝÇÏÝ»ñÇ ³ñ³·³óáõÙÁ ÏáñáßíÇ Ñ»ï¨Û³É µ³Ý³Ó¨»ñáí`

$$a_x = \frac{dV_x}{dx} = \frac{\partial V_x}{\partial t} = \frac{\partial^2 x}{\partial t^2},$$

$$a_y = \frac{dV_y}{dt} = \frac{\partial V_y}{\partial t} = \frac{\partial^2 y}{\partial t^2}, \qquad 1.5$$

$$a_z = \frac{dV_z}{dt} = \frac{\partial V_z}{\partial t} = \frac{\partial^2 z}{\partial t^2}:$$

Ð»ÕáõÏÝ»ñÇ ß³ñÅÙ³Ý áõëáõÙÝ³ëÇñáõÃÛ³Ý ¾ÛÉ»ñÇ Ù»Ãá¹áõÙ ÁÝïñíáõÙ ¿ ³Ýß³ñÅ ï³ñ³ÍáõÃÛ³Ý Ù»ç r ß³é³íÇÕ í»Ïïáñáí Ï³Ù x, y, z Ïááñ¹ÇÝ³ïÝ»ñáí áñáßíáÕ ÙÇ Ï»ï, áñáí Å³Ù³Ý³ÏÇ ÁÝÃ³óùáõÙ ³ÝóÝáõÙ »Ý Ñ»ÕáõÏÇ Ñ³çáñ¹Ï³Ý Ù³ëÝÇÏÝ»ñÁ: â³÷»Éáí ³Û¹ Ï»ïáí ³ÝóÝáÕ Ù³ëÝÇÏÝ»ñÇ ³ñ³·áõÃÛáõÝÝ»ñÁ Å³Ù³Ý³ÏÇ ï³ñµ»ñ å³Ñ»ñÇ, ÏáõÝ»Ý³Ýù`

$$V_x=V_x(x.\ y,\ z,\ t),\quad V_y=V_y(x,\ y,\ z,\ t),\quad V_z=V_z(x,\ y,\ z,\ t): \qquad 1.6$$

Ð»ÕáõÏ ïÇñáõÛÃÇ µáÉáñ Ù³ëÝÇÏÝ»ñÇ ³ñ³·áõÃÛáõÝÝ»ñÇ ¹ßïÁ Å³Ù³Ý³ÏÇ ó³ÝÏ³ó³Í å³ÑÇÝ ëï³Ý³Éáõ Ñ³Ù³ñ å»ïù ¿ ÷á÷áË»Ýù x, y, z ¨ t Ù»ÍáõÃÛáõÝÝ»ñÁ: ²Ûë x, y, z ¨ t ÷á÷áË³Ï³ÝÝ»ñÁ ÏáãíáõÙ »Ý ¾ÛÉ»ñÇ ÷á÷áË³Ï³ÝÝ»ñ, ÇëÏ áõëáõÙÝ³ëÇñÙ³Ý ³Ûë Ù»Ãá¹Á` ¾ÛÉ»ñÇ Ù»Ãá¹:

àôÝ»Ý³Éáí ³ñ³·áõÃÛáõÝÝ»ñÇ ¹ßïÁ, Ï³ñáÕ »Ýù áñáß»É ³ñ³·³óáõÙÝ»ñÇ ¹ßïÁ .

$$a_x=\frac{dV_x}{dt}=\frac{d^2x}{dt^2},\quad a_y=\frac{dV_y}{dt}=\frac{d^2y}{dt^2},\quad a_z=\frac{dV_z}{dt}=\frac{d^2z}{dt^2}: \qquad 1.7$$

ÜÏ³ïÇ áõÝ»Ý³Éáí, áñ Ñ»Õáõï Ù³ëÝÇÏÇ x, y, z Ïááñ¹ÇÝ³ïÝ»ñÁ ÷á÷áËíáõÙ »Ý Áëï Å³Ù³Ý³ÏÇ, ÏáõÝ»Ý³Ýù

$$a_x = \frac{\partial V_x}{\partial t} + \frac{\partial V_x}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial V_x}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial V_x}{\partial z}\cdot\frac{dz}{dt}$$

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$$a_x = \frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z}, \tag{1.8}$$

$$a_y = \frac{\partial V_y}{\partial t} + V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z},$$

$$a_z = \frac{\partial V_z}{\partial t} + V_x\frac{\partial V_z}{\partial x} + V_y\frac{\partial V_z}{\partial y} + V_z\frac{\partial V_z}{\partial z}:$$

²ñ³·³óÙ³Ý í»ÏïáñÁ ÏÉÇÝÇ`

$$\bar{a} = \frac{d\bar{V}}{dt} = \frac{\partial\bar{V}}{\partial t} + V_x\frac{\partial\bar{V}}{\partial x} + V_y\frac{\partial\bar{V}}{\partial y} + V_z\frac{\partial\bar{V}}{\partial z}: \tag{1.9}$$

(1.9)-Á ëÇÙíáÉÇÏ Ó¨áí Ï³ñáÕ »Ýù ·ñ»É`

$$\bar{a} = \frac{\partial\bar{V}}{\partial t} + \left(V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y} + V_z\frac{\partial}{\partial z}\right)\bar{V} \tag{1.10}$$

կամ

$$\bar{a} = \frac{\partial\bar{V}}{\partial t} + (\bar{V}\cdot \mathrm{grad})\bar{V}:$$

Դասախոսություն 2

2.1 **Ð»ï³·Í»ñ ¨ ÑáëùÇ ·Í»ñ Ñ»ÕáõÏ ïÇñáõÛÃáõÙ**

Ð»ÕáõÏ ïÇñáõÛÃáõÙ ÑáëùÇ ·ÇÍ Ïáãíáõ٠¿ ³ÛÝ ·ÇÍÁ, áñÇ Ûáõñ³ù³ÝãÛáõñ Ï»ïáõÙ ³ñ³·áõÃÛ³Ý í»ÏïáñÇ áõÕÕáõÃÛáõÝÁ Ñ³ÙÁÝÏÝáõ٠¿ ·ÍÇ ïíÛ³É Ï» ïáõÙ ßáß³÷áÕÇ áõÕÕáõÃÛ³Ý Ñ»ï:

$$\frac{dx}{V_x(x,y,z,t)} = \frac{dy}{V_y(x,y,z,t)} = \frac{dz}{V_z(x,y,z,t)}, \qquad 2.1$$

Ʒ»ï³·ÇÍÁ ÙÇ¨ÝáõÛÝ Ù³ëÝÇÏÇ Å³Ù³Ý³ÏÇ ÁÝÃ³óùáõÙ ·Í³Í ·ÇÍÝ ¿, Çëï ÑáëùÇ ·ÇÍÁ` ï³ñբեñ Ù³ëÝÇÏÝ»ñÇ ³ñ³·áõÃÛáõÝÝ»ñÇ í»ÏïáñÝ»ñÇ áõÕÕáõÃÛáõÝÝ»ñáí áõÕÕí³Í, ï³ññ³Ï³Ý Ñ³ïí³ÍÝ»ñÇó բաÕÏ³ó³Í ·ÇÍ ¿ Å³Ù³Ý³ÏÇ ïíÛ³É å³ÑÇÝ:²ÛÉ Ï»ñå ³ë³Í, Ñ»ï³·ÇÍÁ ïíÛ³É Ù³ëÝÇÏÇ Å³Ù³Ý³ÏÇ ï³ñբեñ å³Ñ»ñÇÝ ³ñ³·áõÃÛáõÝÝ»ñÇ å³ñáõñÇãÝ ¿, Çëï ÑáëùÇ ·ÇÍÁ`Å³Ù³Ý³ÏÇ ïíÛ³É å³ÑÇÝ ³ñ³·áõÃÛáõÝÝ»ñÇ å³ñáõñÇãÁ:

2.2 ²ñ³·áõÃÛ³Ý í»ÏïáñÇ Ñáëù, ¹Çí»ñ·»ÝóÇ³, ßñç³åïáõÛïÇ ¨ éáïáñ

îñí³Í S ³Ýß³ñÅ Ù³Ï»ñ¨áõÛÃáí ³ñ³·áõÃÛ³Ý í»ÏïáñÇ Ñáëù Ïáãíáõ٠¿

Ù $$Q = \iint_S V_n ds$$: 2.2

Üß³Ý³Ï»Éáí n^0-áí ÝáñÙ³ÉÇ ÙÇ³íáñ í»ÏïáñÁ ¨ Ñ³ßíÇ ³éÝ»Éáí, áñ $V_n = \bar{V} \cdot \bar{n}^0$,

³å³ (2.1)-Çó ÏáõÝ»Ý³Ýù

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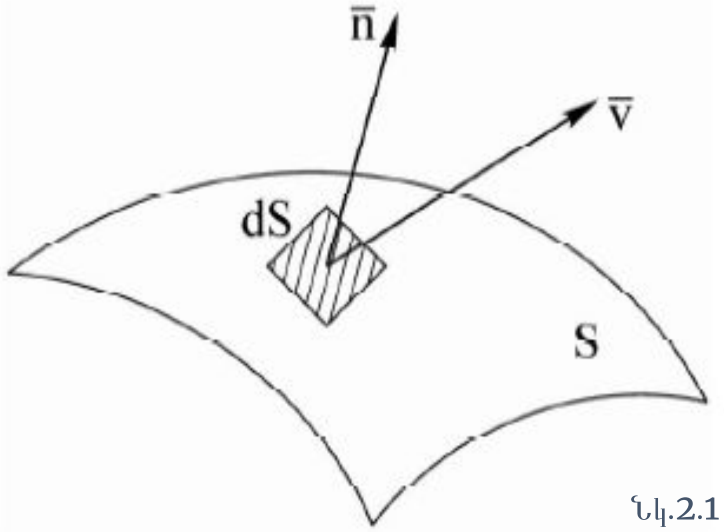
$$Q = \iint_S \left[V_x \cos(nx) + V_y \cos(ny) + V_z \cos(nz)\right] ds$$ 2.3



$$Q = \iint_S \left[V_x dydz + V_y dxdz + V_z dxdy\right]:$$ 2.4

Նկ.2.1

²ÛÅÙ Ù³Ï»ñ¨áõÛÃ³ÛÇÝ ÇÝï»·ñ³ÉÇó ¶³áõë-úëïñá·ñ³¹ëÏáõ բաÝ³Ó¨áí ³ÝóÝ»Éáí Í³í³É³ÛÇÝ ÇÝï»·ñ³ÉÇ

$$\oiint\limits_S \left[V_x \cos(nx) + V_y \cos(ny) + V_z \cos(nz) \right] ds = \iiint\limits_{\Delta\tau} \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right) d\tau \qquad 2.5$$

¨ ÏÇñ³é»Éáí ÙÇçÇÝ ³ñÅ»ùÇ Ã»áñ»ÙÁ

$$\iiint\limits_{\Delta\tau} \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right) d\tau = \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right)_{M_0} \cdot \Delta\tau , \qquad 2.6$$

ÏáõÝ»Ý³Ýù

$$\operatorname{div} \overline{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} : \qquad 2.7$$

(2.3), (2.5) և (2.7) բաÝ³Ó¨»ñÇó Ñ»ï¨áõÙ ¿, áñ

$$\oiint\limits_S V_n ds = \iiint\limits_{\tau} \operatorname{div} \overline{V} d\tau : \qquad 2.8$$

²ñ³·áõÃÛ³Ý í»ÏïáñÇ ßñç³åïáõÛï (óÇñÏáõÉÛ³óÇ³) Ïáãíáõ ¿ Ñ»ï¨Û³É Ù»ÍáõÃÛáõÝÁ

$$\Gamma = \oint\limits_L \overline{V} d\bar{r} = \oint\limits_L \left(V_x dx + V_y dy + V_z dz \right), \qquad 2.9$$

ú·ïí»Éáí Ïáñ³·ÇÍ ÇÝï»·ñ³ÉÇó Ù³Ï»ñ¨áõÛÃ³ÛÇÝÇÝ ³ÝóÝ»Éáõ êïáùëÇ բաÝ³Ó¨Çó ÏáõÝ»Ý³Ýù`

$$\oint_L \left(V_x d_x + V_y d_y + V_z d_z\right) =$$

$$= \iint_S \left[\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)dydz + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)dxdz + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)dxdy\right], \tag{2.10}$$

²ñ³·áõÃÛ³Ý í»ÏïáñÇ éáïáñ Ïáãíáõ٠¿ ³ÛÝ Ω í»ÏïáñÁ, áñÇ åñáÛ»ÏóÇ³Ý»ñÁ úx, úy, úz ³é³ÝóùÝ»ñÇ íñ³ Ñ³Ù³å³ï³ëË³Ý³բաñ Ñ³í³ë³ñ »Ý

$$\Omega_x = \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}, \Omega_y = \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}, \Omega_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \tag{2.11}$$

այսինքն

$$\mathrm{rot}\ \overline{V} = \overline{\Omega} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\overline{i} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\overline{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\overline{k}: \tag{2.12}$$

Ð³ßíÇ ³éÝ»Éáí (2.9), (2.10) ¨ (2.11) µ³Ý³Ó¨»ñÁ, ÏáõÝ»Ý³Ýù`

$$\Gamma = \oint_L \overline{V} d\overline{r} = \iint_S \overline{\Omega}\,\overline{n^0} ds = \iint_S \Omega_n ds, \tag{2.13}$$

Դասախոսություն 3

3.1 ²ÝË½»ÉÇáõÃÛ³Ý Ñ³í³ë³ñáõÙÁ: ÐáëùÇ ËáÕáí³ÏÇ ³ßË³ï³ÝùÇ ëÏ½µáõÝùÁ

Ð»ÕáõÏÇ Ù³ëÝÇÏÝ»ñÇ ¨ ½³Ý·í³ÍÇ µ³ßËÙ³Ý ûñ»ÝùÝ»ñÇ Ï³Ëí³ÍáõÃÛáõÝÁ ³ñï³Ñ³Ûïíáõ٠¿ ÙÇ Ù³Ã»Ù³ïÇÏ³Ï³Ý ³éÝãáõÃÛ³Ùµ, áñÁ Ïáãíáõ٠¿ ³ÝË½»ÉÇáõÃÛ³Ý Ï³Ù ³ÝÁÝ¹Ñ³ïáõÃÛ³Ý Ñ³í³ë³ñáõÙ:

²ÝË½»ÉÇáõÃÛ³Ý Ñ³í³ë³ñáõÙÁ Ï³ñ»ÉÇ ¿ ³ñï³Í»É ÇÝãå»ë È³·ñ³ÝÅÇ, ³ÛÝå»ë ¿É ¾ÛÉ»ñÇ ÷á÷áË³Ï³ÝÝ»ñáí:

1. <u>²ÝË½»ÉÇáõÃÛ³Ý Ñ³í³ë³ñáõÙÁ Լանգրանժի փոփոխականներով</u>

¼³Ý·í³ÍÇ å³Ñå³ÝÙ³Ý ûñ»ÝùÇ Ñ³Ù³Ó³ÛÝ

$$\iiint_{\tau_0} \rho_0(a,b,c,t_0)\,da\,db\,dc = \iiint_{\tau} \rho(x,y,z,t)\,dx\,dy\,dz \qquad 3.1$$

և հաշվի առնելով, որ x = x(a,b,c,t), y = y(a,b,c,t), z = z(a,b,c,t), (3.1)-ում x, y, z ÷á÷áË³Ï³ÝÝ»ñÇó ³ÝóÝ»Éáí a, b, c ÷á÷áË³Ï³ÝÝ»ñÇÝ, Ïëï³Ý³Ýù`

$$\iiint_{\tau_0} (\rho_0 - \rho \cdot I)\,da\,db\,dc = 0\,, \qquad 3.2$$

áñï»Õ I -Ý Ó¨³÷áËáõÃÛ³Ý áñáßÇãÝ ¿ (Ú³ÏáµÇ³ÝÁ)`

$$I = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} & \frac{\partial z}{\partial a} \\ \frac{\partial x}{\partial b} & \frac{\partial y}{\partial b} & \frac{\partial z}{\partial b} \\ \frac{\partial x}{\partial c} & \frac{\partial y}{\partial c} & \frac{\partial z}{\partial c} \end{vmatrix} : \qquad 3.3$$

ø³ÝÇ áñ Í³í³ÉÇ dadbdc ï³ññÁ Ï³Ù³íáñ ¿, ¨ »ÝÃ³ÇÝï»·ñ³É ýáõÝÏóÇ³Ý ³ÝÁÝ¹Ñ³ï, ³å³ (3.2)-Çó Ïëï³Ý³Ýù`

$$\rho_0 = \rho \cdot I, \qquad 3.4$$

áñÁ Ñ³Ý¹Çë³ÝáõÙ ¿ ³ÝË½»ÉÇáõÃÛ³Ý Ñ³í³ë³ñáõÙÁ È³·ñ³ÝÅÇ ÷á÷áË³Ï³ÝÝ»ñáí:

2. <u>²ÝË½»ÉÇáõÃÛ³Ý Ñ³í³ë³ñáõÙÁ Էյլերի փոփոխականներով</u>

Համաձայն բանաձևի կստանանք`

$$Q = \iint_S V_n ds$$

$$Q = \oiint_S \rho V_n ds : \qquad 3.3$$

Օգտվելով ¶³áõë-úëïñá·ñ³¹ëÏáõ (2.5) բաÝ³Ó¨Çó`

$$\oiint_S \rho V_n ds = \oiint_S \left[\rho V_x \cos(nx) + \rho V_y \cos(ny) + \rho V_z \cos(nz)\right] ds = \tag{3.4}$$

$$= \iiint_\tau \left[\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z}\right] d\tau :$$

Կստանանք

$$\iiint_\tau \left[\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z}\right] d\tau = -\iiint_\tau \frac{\partial \rho}{\partial t} d\tau : \tag{3.5}$$

²Ûëï»ÕÇó, Ñ³ßíÇ ³éÝ»Eáí, áñ τ-Y I³U³U³I³Y ¿, Iëï³Y³Yù`

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} = 0 \tag{3.6}$$

áñÁ Ñ»ÕáõÏÇ ³ÝË½»ÉÇáõÃÛ³Ý Ñ³í³ë³ñáõÙÝ ¿ ¾ÛÉ»ñÇ ÷á÷áËÇ³Ï³ÝÝ»ñáí:

àñáß Ó¨³÷áËáõÃÛáõÝÝ»ñÇó Ñ»ïá, (3.6)-Á Ï³ñ»ÉÇ ¿ ·ñ»É Ñ»ï¨Û³É ï»ëùáí`

$$\frac{d\rho}{dt} + \rho \cdot \operatorname{div}\overline{V} = 0: \tag{3.7}$$

²Ýë»ÕÙ»ÉÇ Ñ»ÕáõÏÇ Ñ³Ù³ñ ρ = const ¨ (3.7)-Á ÏÁÝ¹áõÝÇ Ñ»ï¨Û³É ï»ëùÁ`

$$\operatorname{div}\overline{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0: \qquad 3.8$$

²Ûë ¹»åùáõÙ (3.4)-Çó ÏáõY»Y³Yù

$$\rho\oiint_S V_n ds = \rho\iiint_\tau \operatorname{div}\overline{V} d\tau = 0, \qquad 3.9$$

Ð»ÕáõÏÇ Ñ³ëï³ïí³Í ß³ñÅÙ³Ý ¹»åùáõÙ ³ÝË½»ÉÇáõÃÛ³Ý Ñ³í³ë³ñáõÙÁ ÏÉÇÝÇ`

$$\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} = 0 \qquad 3.10$$

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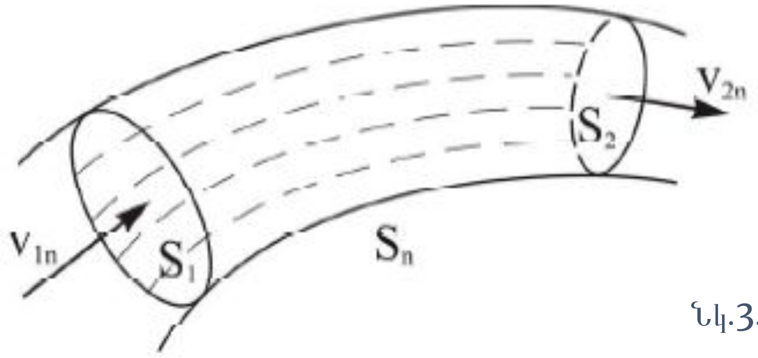
$$\operatorname{div}(\rho\overline{V}) = 0: \qquad 3.11$$

Յամաձայն (3.9) բաÝ³Ó¨Ç, ³Ýë»ÕÙ»ÉÇ Ñ»ÕáõÏÇ Ñ³Ù³ñ ÏáõÝ»Ý³Ýù`

$$Q = \rho\oiint_S V_n ds = \rho\iint_{S_1} V_{1n} ds + \rho\iint_{S_2} V_{2n} ds + \rho\iint_{S_n} V_n ds = 0, \qquad 3.12$$



Նկ.3.1

ºÃ» í»ñóÝ»Ýù S_1 ¨ S_2-Á ß³ï ÷áùñ, ³å³ ¹ñ³ÝóáõÙ Ï³ñ»ÉÇ ¿ ÁÝ¹áõÝ»É V_{1n} ¨ V_{2n}-Á Ñ³Ù³å³ï³ëË³Ý³բաñ V_1 ¨ V_2 Ñ³ëï³ïáõÝ ³ñÅ»ùÝ»ñ, Ñ»ï¨³բաñ (3.12)-Çó Ïëï³Ý³Ýù`

$$-V_1S_1+V_2S_2=0 \quad 3.13$$

կամ

$$V_1S_1=V_2S_2=const: \quad 3.14$$

¶É³Ý³ÛÇÝ Ïááñ¹ÇÝ³ï³ÛÇÝ r, φ, z Ñ³Ù³Ï³ñ·áõÙ (x= r ·cosφ ,y= r ·sinφ, z=z), ³ÝË½»ÉÇáõÃÛ³Ý (3.6) Ñ³í³ë³ñáõÙÝ áõÝÇ Ñ»ï¨Û³É ï»ëùÁ`

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\cdot\frac{\partial(\rho V_r r)}{\partial r}+\frac{1}{r}\cdot\frac{\partial(\rho V_\varphi)}{\partial\varphi}+\frac{\partial(\rho V_z)}{\partial z}=0, \quad 3.15$$

ÇëÏ ·Ý¹³ÛÇÝ Ïááñ¹ÇÝ³ï³ÛÇÝ r, ϑ, ψ Ñ³Ù³Ï³ñ·áõÙ (x=r·sinϑ cosψ, y=r·sinϑ sinψ , z=r·cosϑ) ÏÉÇÝÇ

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\cdot\frac{\partial(\rho V_r r^2)}{\partial r}+\frac{1}{r\sin\vartheta}\cdot\frac{\partial(\rho V_\vartheta \sin\vartheta)}{\partial\vartheta}+\frac{1}{r\sin\vartheta}\cdot\frac{\partial(\rho V_\psi)}{\partial\psi}=0: \quad 3.16$$

3.2 ¸»ýáñÙ³óÇ³ÛÇ ³ñ³·áõÃÛ³Ý Ã»Ý½áñ

Ð»ÕáõÏ ïÇñáõÛÃÇ ¹»ýáñÙ³óÙ³Ý Ñ»ï¨³Ýùáí, ïÇñáõÛÃÇ Ù³ëÝÇÏÝ»ñÁ ëï³ÝáõÙ »Ý Éñ³óáõóÇã ³ñ³·áõÃÛáõÝ: ²Û¹ V_D ³ñ³·áõÃÛáõÝÁ å³ÛÙ³Ý³íáñí³Í ¿ $\varepsilon_1, \varepsilon_2, \varepsilon_3, \theta_1, \theta_2, \theta_3$ Ù»ÍáõÃÛáõÝÝ»ñáí, ÁÝ¹ áñáõÙ`

$$V_{D\xi} = \frac{\partial\phi}{\partial\xi} = \varepsilon_1\xi + \frac{1}{2}\theta_2\zeta + \frac{1}{2}\theta_3\eta\,,$$

$$V_{D\eta} = \frac{\partial\phi}{\partial\eta} = \varepsilon_2\eta + \frac{1}{2}\theta_3\xi + \frac{1}{2}\theta_1\zeta\,, \qquad 3.17$$

$$V_{D\zeta} = \frac{\partial\phi}{\partial\zeta} = \varepsilon_3\zeta + \frac{1}{2}\theta_1\eta + \frac{1}{2}\theta_2\xi :$$

ε_1 -Ç ýÇ½ÇÏ³Ï³Ý ÇÙ³ëïÁ å³ñ½»Éáõ Ñ³Ù³ñ »ÝÃ³¹ñ»Ýù, áñ ï»ÕÇ ¿ áõÝ»ó»É ³ÛÝåÇëÇ ¹»ýáñÙ³óÇ³, áñÇ Å³Ù³Ý³Ï, $\varepsilon_1 \neq 0$, ÇëÏ $\varepsilon_2=\varepsilon_3=\theta_1=\theta_2=\theta_3=0$: ²Ûë ¹»åùáõÙ (3.17)-Çó ÏáõÝ»Ý³Ýù

$$V_{D\xi} = \varepsilon_1\xi\,, \quad V_{D\eta} = 0, \quad V_{D\xi} = 0: \qquad 3.18$$

´³óÇ ³Û¹,

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = \mathrm{div}\,\overline{V} \qquad 3.19$$

(3.19)Ù»ÍáõÃÛáõÝÁ Ñ³Ý¹Çë³ÝáõÙ ¿ ÁÝïñí³Í Í³í³ÉÇ Ñ³ñ³µ»ñ³Ï³Ý ÷á÷áËáõÃÛáõÝÁ ÙÇ³íáñ Å³Ù³Ý³Ï³ÙÇçáóáõÙ:

θ_3 -Ç ýÇ½ÇÏ³Ï³Ý ÇÙ³ëïÁ å³ñ½»Éáõ Ñ³Ù³ñ ÁÝ¹áõÝ»Éáí (3.17)-áõÙ $\theta_3 \neq 0$, Çëï $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \theta_1 = \theta_2 = 0$ ÏáõÝ»Ý³Ýù`

$$V_{D\xi} = \frac{1}{2}\theta_3\eta, \quad V_{D\eta} = \frac{1}{2}\theta_3\xi, \quad V_{D\zeta} = 0 : \qquad 3.20$$

²ÛëåÇëáí ε_1, ε_2, ε_3, θ_1, θ_2, θ_3 Ù»ÍáõÃÛáõÝÝ»ñÁ ÁÝ¹Ñ³Ýáõñ ¹»åùáõÙ µÝáõÃ³·ñáõÙ »Ý ÁÝïñí³Í Ñ»Õáõï ïÇñáõÛÃÇ ¹»ýáñÙ³óÇ³Ý` ¨ ¹ñ³Ýó ³Ýí³ÝáõÙ »Ý ¹»ýáñÙ³óÇ³ÛÇ ³ñ³·áõÃÛ³Ý ·áñÍ³ÏÇóÝ»ñ: Üß³Ý³ÏáõÙ »Ý`

$$\phi = \begin{pmatrix} \varepsilon_1 & \frac{1}{2}\theta_3 & \frac{1}{2}\theta_2 \\ \frac{1}{2}\theta_3 & \varepsilon_2 & \frac{1}{2}\theta_1 \\ \frac{1}{2}\theta_2 & \frac{1}{2}\theta_1 & \varepsilon_3 \end{pmatrix} \qquad 3.21$$

¨ ³Ýí³ÝáõÙ ¹» ýáñÙ³óÇ³ÛÇ ³ñ³·áõÃÛ³Ý Ã»Ý½áñ: ²Ûë ¹»åùáõÙ Ã»Ý½áñÁ Ñ³Ù³ã³÷ ¿ ¨ µÝáñáßíáõÙ ¿ í»ó Ù»ÍáõÃÛáõÝÝ»ñáí:

Դասախոսություն 4

Æ¸º²È²Î²Ü ÐºÔàôÎÆ ¸ÆÜ²ØÆÎ²

4.1 Ð»ÕáõÏ ïÇñáõÛÃáõÙ ·áñÍáÕ áõÅ»ñÁ: Æ¹»³É³Ï³Ý Ñ» ÕáõÏ

Ð»ÕáõÏ т ïÇñáõÛÃáõÙ ·áñÍáÕ áõÅ»ñÁ ¹³ë³Ï³ñ·íáõÙ »Ý` ½³Ý·í³Í³ÛÇÝ áõÅ»ñÇ ¨ Ù³Ï»ñ¨áõÛÃ³ÛÇÝ áõÅ»ñÇ:

¼³Ý·í³Í³ÛÇÝ ÏáãíáõÙ »Ý ³ÛÝ áõÅ»ñÁ, áñáÝù ÏÇñ³éí³Í »Ý ïÇñáõÛÃÇ Δт Í³í³ÉáõÙ å³ñ÷³Ïí³Í ½³Ý·í³ÍÇ íñ³, ¨ áñÇ ³½¹»óáõÃÛáõÝÁ Ï³Ëí³Í ã¿, Ã» ³Û¹ Δт Í³í³ÉÁ áõÝÇ± Çñ Ñ³ñ¨³ÝáõÃÛ³Ý Ù»ç ³ÛÉ Í³í³ÉÝ»ñ, Ã»` áã:

Ø³Ï»ñ¨áõÃ³ÛÇÝ »Ý ÏáãíáõÙ ³ÛÝ áõÅ»ñÁ, áñáÝù ÏÇñ³éíáõÙ »Ý Ñ»ÕáõÏÇ Ù³ëÝÇÏÇ (ïÇñáõÛÃÇ) Ù³Ï»ñ¨áõÛÃÇÝ ¨ ¿³å»ë Ï³Ëí³Í »Ý Ýñ³ Ñ³ñ¨³ÝáõÃÛ³Ý Ù»ç ·ïÝíáÕ Ù³ëÝÇÏÝ»ñÇó (ïÇñáõÛÃÝ»ñÇó), ³ÛëÇÝùÝ ³ÛÝ áõÅ»ñÁ, áñáÝóáí ÷áË³½¹áõÙ »Ý Ñ³ñ¨³Ý Ñ»ÕáõÏÇ ïÇñáõÛÃÝ»ñÁ ¨ ÏÇñ³éí³Í »Ý Ýñ³Ýó Ù³Ï»ñ¨áõÛÃÝ»ñÇÝ:

4.2 È³ñí³ÍáõÃÛ³Ý Ã»Ý½áñ

Ð»ÕáõÏÇ Ï³Ù³Û³Ï³Ý τ ïÇñáõÛÃÁ ¹Çï³ñÏ»Éáí áñå»ë Ù»Ë³ÝÇÏ³Ï³Ý Ñ³Ù³Ï³ñ· ¨ Ýñ³ ÝÏ³ïÙ³Ùµ ÏÇñ³é»Éáí ¸³É³ÙµtñÇ ëÏ½µáõÝùÁ, ÏáõÝ»Ý³Ýù

$$\iiint_{\tau} \left(\bar{F} \cdot \rho\right) d\tau + \iint_{S} \overline{P_n} ds - \iiint_{\tau} \bar{a} \rho d\tau = 0 ,$$ 4.1

ø³é³ÝÇëïÇ ÙÛáõë ÝÇëï»ñÇ ³ñï³ùÇÝ ÝáñÙ³ÉÝ»ñÁ ÏÉÇÝ»Ý -i, -j, -k í»Ïïáñ»ñÁ, ÇëÏ Ù³Ï»ñ» ëÝ»ñÁ` Ñ³Ù³å³ï³ëË³Ý³µ³ñ ∆·α, ∆S ·β, ∆S · γ: (4.1)-Çó ÏáõÝ»Ý³Ýù`

$$\bar{F} \cdot \rho \cdot \Delta\tau + \overline{P_n} \Delta S + \bar{P}_{-x} \cdot \Delta S \cdot \alpha + \bar{P}_{-y} \cdot \Delta S \cdot \beta + \bar{P}_{-z} \cdot \Delta S \cdot \gamma - \bar{a} \rho \Delta\tau = 0 ,$$ 4.2

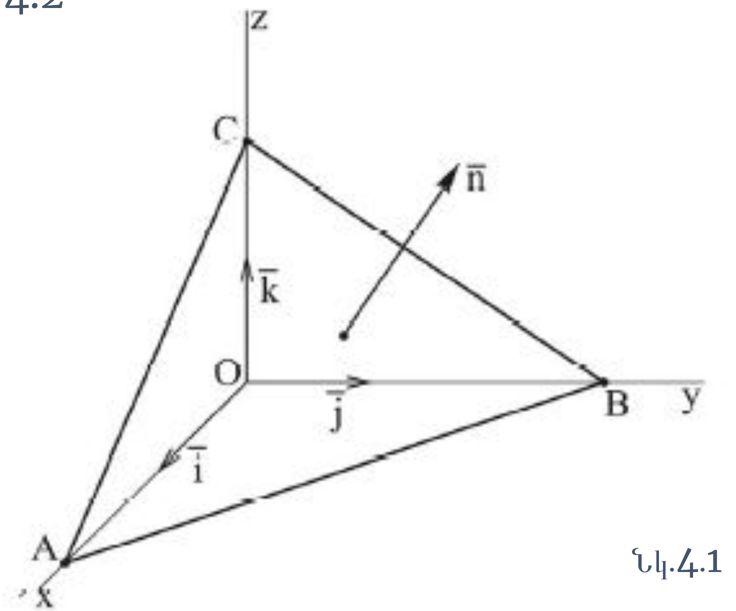
(4.2) Ñ³í³ë³ñÙ³Ý Ù»ç Ñ³ßíÇ ³éÝ»Éáí Δτ-Ç ³ñÅ»ùÁ ¨ ÝÏ³ïÇ

áõÝ»Ý³Éáí, áñ $\overline{P}_{-n} = -\overline{P}_n$, ÏáõÝ»Ý³Ýù

$$\frac{1}{3}\left(\bar{F} - \bar{a}\right)\rho h + \overline{P_n} - \alpha \overline{P_x} - \beta \overline{P_y} - \gamma \overline{P_z} = 0 :$$ 4.3

(2.3)-áõÙ ³ÝóÝ»Éáí ë³ÑÙ³ÝÇ »ñµ Ñ→0 , Ïëï³Ý³Ýù

$$\overline{P_n} = \alpha \overline{P_x} + \beta \overline{P_y} + \gamma \overline{P_z} :$$ 4.4



Նկ.4.1

äñáÛ»Ïï»Éáí (4.4)-Á Oxyz Ïááñ¹ÇÝ³ï³ÛÇÝ ³é³ÝóùÝ»ñÇ íñ³, ÏáõÝ»Ý³Ýù`

$$
\begin{aligned}
P_{nx} &= \alpha P_{xx} + \beta P_{yx} + \gamma P_{zx}\,, \\
P_{ny} &= \alpha P_{xy} + \beta P_{yy} + \gamma P_{zy}\,, \\
P_{nz} &= \alpha P_{xz} + \beta P_{yz} + \gamma P_{zz}\,:
\end{aligned}
\tag{4.5}
$$

(4.5)-Á óáõÛó ¿ ï³ÉÇë, áñ É³ñí³ÍáõÃÛáõÝÁ Ñ»ÕáõÏ ïÇñáõÛÃáõÙ »ñÏñáñ¹ Ï³ñ·Ç Ã»Ý½áñ³Ï³Ý Ù»ÍáõÃÛáõÝ ¿, áñÁ áñáßíáõÙ ¿ 9 Ù»ÍáõÃÛáõÝÝ»ñáí`

$$
\Pi = \begin{pmatrix} P_{xx} & P_{yx} & P_{zx} \\ P_{xy} & P_{yy} & P_{zy} \\ P_{xz} & P_{yz} & P_{zz} \end{pmatrix}: \tag{4.6}
$$

(4.6)-Á ÏáãíáõÙ ¿ É³ñí³ÍáõÃÛ³Ý Ã»Ý½áñ: ú·ïí»Éáí ¹³ë³Ï³Ý Ù»Ë³ÝÇÏ³ÛÇ ûñ»ÝùÝ»ñÇó, Ï³ñ»ÉÇ ¿ óáõÛó ï³É, áñ (4.6) Ã»Ý½áñÁ Ñ³Ù³ã³÷ ¿, ³ÛëÇÝùÝ`

$$
P_{xy}=P_{yx}, \quad P_{yz}=P_{zy}, \quad P_{xz}=P_{zx}: \tag{4.7}
$$

Æ¹»³É³Ï³Ý Ñ»ÕáõÏÇ Ñ³Ù³ñ $\overline{P_n}$, $\overline{P_x}$, $\overline{P_y}$, $\overline{P_z}$ í»ÏïáñÝ»ñÁ å»ïù ¿ áõÕÕí³Í ÉÇÝ»Ý Ñ³Ù³å³ï³ëË³Ý³բաñ n, O_x, O_y, O_z ¹ñ³Ï³Ý áõÕÕáõÃÛáõÝÝ»ñÇÝ Ñ³Ï³é³Ï, ÁÝ¹ áñáõÙ` $P_{xy}=P_{yz}=P_{xz}=0$: àôëïÇ (4.5)-Çó ÏáõÝ»Ý³Ýù`

$$\alpha P_n = \alpha P_{xx}, \ \beta P_n = \beta P_{yy}, \ \gamma P_n = \gamma P_{zz} \qquad 4.8$$

Կամ

$$P_n = P_{xx} = P_{yy} = P_{zz}, \qquad 4.9$$

Áëï áñÇ, Ç¹»³É³Ï³Ý Ñ»ÕáõÏáõÙ ÝáñÙ³É ×ÝßáõÙÝ»ñÁ ïíÛ³É Ï»ïáõÙ µáÉáñ áõÕÕáõÃÛáõÝÝ»ñáí Çñ³ñ Ñ³í³ë³ñ »Ý ¨ Ï³Ëí³Í ã»Ý Ñ³ñÃ³ÏÇ ¹ÇñùáñáßáõÙÇó (ä³ëÏ³ÉÇ û»ÝùÁ):

²ÛëåÇëáí, Ç¹»³É³Ï³Ý Ñ»ÕáõÏÇ Ù³Ï»ñ¨áõÃ³ÛÇÝ É³ñáõÙÝ»ñÝ áõÝ»Ý Ñ»ï¨Û³É ï»ëùÁ`

$$\overline{P_n} = -P \cdot \overline{n^0}, \qquad 4.10$$

áñï»Õ P-Ý ïíÛ³É Ï»ïáõÙ ×ÝßáõÙÝ ¿, n^0 -³ñï³ùÇÝ ÝáñÙ³ÉÇ ÙÇ³íáñ í»ÏïáñÁ:

Դասախոսություն 5

5.1 Æ¹»³É³Ï³Ý Ñ»ÕáõÏÇ ß³ñÅÙ³Ý ¹Çý»ñ»ÝóÇ³É Ñ³í³ë³ñáõÙÝ»ñÁ

(4.1) Ñ³í³ë³ñÙ³Ý Ù»ç Ñ³ßíÇ ³éÝ»Éáí, áñ Ç¹»³É³Ï³Ý Ñ»ÕáõÏÇ ÙÇ³íáñ Ù³Ï»ñ»ëÇ íñ³ ³½¹áÕ Ù³Ï»ñ¨áõÃ³ÛÇÝ áõÅ»ñÇ ·ÉË³íáñ í»ÏïáñÁ áñáßíáõÙ ¿ (4.10) ƀɯÝ³Ó¨áí ¨ (4.1)-Ç Ù³Ï» ñ¨áõÃ³ÛÇÝ ÇÝï»·ñ³ÉáõÙ Ï³ï³ñ»Éáí Ó¨³÷áËáõÃÛáõÝ Ñ³Ù³Ó³ÛÝ ¶³áõë-úëïñá·ñ³¹ëÏáõ ƀɯÝ³Ó¨Ç, Ïëï³Ý³Ýù`

$$\iint_S \overline{p_n}\,ds = -\iint_S P\overline{n^0}\,ds = -\iint_S P\left[\cos(nx)\bar{i} + \cos(ny)\bar{j} + \cos(nz)\bar{k}\right]ds = -\iiint_\tau \left(\frac{\partial P}{\partial x}\bar{i} + \frac{\partial P}{\partial y}\bar{j} + \frac{\partial P}{\partial z}\bar{k}\right)d\tau = -\iiint_\tau \operatorname{grad}P\,d\tau \,; \quad 5.1$$

²ÛÅÙ ï»Õ³¹ñ»Éáí (5.1)-Á (4.1)-áõÙ, ÏáõÝ»Ý³Ýù`

$$\iiint\left(\bar{F} - \bar{a}\right)\rho\,d\tau - \iiint \operatorname{grad}P\,d\tau = 0\,,$$

Ï³Ù Ñ³ßíÇ ³éÝ»Éáí, áñ τ -Ý Ï³Ù³Û³Ï³Ý ÷áùñ ïÇñáõÛÃ ¿`

$$\bar{a} = \bar{F} - \frac{1}{\rho}\operatorname{grad}P \,; \quad 5.2$$

äñáÛ»ÏóÇ³Ý»ñáí ¹ñ³Ýù ÏáõÝ»Ý³Ý Ñ»ï¨Û³É ï»ëùÁ`

$$\frac{dV_x}{dt} = F_x - \frac{1}{\rho}\frac{\partial P}{\partial x}, \quad \frac{dV_y}{dt} = F_y - \frac{\partial P}{\partial y}, \quad \frac{dV_z}{dt} = F_z - \frac{\partial P}{\partial z} \qquad 5.3$$

Կամ

$$\begin{aligned}
&\frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z} = F_x - \frac{1}{\rho}\frac{\partial P}{\partial x},\\
&\frac{\partial V_y}{\partial t} + V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z} = F_y - \frac{1}{\rho}\frac{\partial P}{\partial y},\\
&\frac{\partial V_z}{\partial t} + V_x\frac{\partial V_z}{\partial x} + V_y\frac{\partial V_z}{\partial y} + V_z\frac{\partial V_z}{\partial z} = F_z - \frac{1}{\rho}\frac{\partial P}{\partial z}:
\end{aligned} \qquad 5.4$$

(5.4)-Á Ç¹»³É³Ï³Ý Ñ»ÕáõÏÇ ß³ñÅÙ³Ý ¹Çý»ñ»ÝóÇ³É Ñ³í³ë³ñáõÙÝ»ñÝ »Ý ¨ ÏáãíáõÙ »Ý ¾ÛÉ»ñÇ Ñ³í³ë³ñáõÙÝ»ñ:

Æ¹»³É³Ï³Ý Ñ»ÕáõÏÇ ß³ñÅÙ³Ý ¹Çý»ñ»ÝóÇ³É Ñ³í³ë³ñáõÙÝ»ñÁ ·É³Ý³Ï³Ý r,φ,z Ïááñ¹ÇÝ³ï³ÛÇÝ Ñ³Ù³Ï³ñ·áõÙ ·ñíáõÙ »Ý Ñ»ï¨Û³É ï»ëùáí`

$$
\begin{aligned}
&\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_r}{\partial \varphi}+V_z\frac{\partial V_r}{\partial z}-\frac{V^2{}_\varphi}{r}=F_r-\frac{1}{\rho}\frac{\partial P}{\partial r},\\
&\frac{\partial V_\varphi}{\partial t}+V_r\frac{\partial V_\varphi}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_\varphi}{\partial \varphi}+V_z\frac{\partial V_\varphi}{\partial z}+\frac{V_rV_\varphi}{r}=F_\varphi-\frac{1}{\rho r}\frac{\partial P}{\partial \varphi},\\
&\frac{\partial V_z}{\partial t}+V_r\frac{\partial V_z}{\partial r}+\frac{V_\varphi}{r}\frac{\partial V_z}{\partial \varphi}+V_z\frac{\partial V_z}{\partial z}=F_z-\frac{1}{\rho}\frac{\partial P}{\partial z},
\end{aligned}
\tag{5.5}
$$

ÇëÏ ·Ý¹³ÛÇÝ r, θ, ψ Ïááñ¹ÇÝ³ï³ÛÇÝ Ñ³Ù³Ï³ñ·áõÙ`

$$
\begin{aligned}
&\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}+\frac{V_\psi}{r\sin\theta}\frac{\partial V_r}{\partial \psi}-\frac{V^2{}_\theta+V^2{}_\psi}{r}=F_r-\frac{1}{\rho}\frac{\partial P}{\partial r},\\
&\frac{\partial V_\theta}{\partial t}+V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_\psi}{r\sin\theta}\frac{\partial V_\theta}{\partial \psi}+\frac{V_rV_\theta}{r}-\frac{\mathrm{ctg}\theta}{r}V^2{}_\psi=F_\theta-\frac{1}{\rho r}\frac{\partial P}{\partial \theta},\\
&\frac{\partial V_\psi}{\partial t}+V_r\frac{\partial V_\psi}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\psi}{\partial \theta}+\frac{V_\psi}{r\sin\theta}\frac{\partial V_\psi}{\partial \psi}+\frac{V_rV_\psi}{r}+\frac{\mathrm{ctg}\theta}{r}V_\theta V_\psi=\\
&=F_\psi-\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \psi};
\end{aligned}
\tag{5.6}
$$

5.2Æ¹»³É³Ï³Ý Ñ»ÕáõÏÇ ¹ÇÝ³ÙÇÏ³ÛÇ ÑÇÙÝ³Ï³Ý ËÝ¹ÇñÁ

Æ¹»³É³Ï³Ý Ñ»ÕáõÏÇ ¹ÇÝ³ÙÇÏ³ÛÇ ÑÇÙÝ³Ï³Ý ËÝ¹ÇñÝ ¿`ÇÝï»·ñ»Éáí Ñ»ÕáõÏÇ ß³ñÅÙ³Ý ¹Çý» ñ»ÝóÇ³É Ñ³í³ë³ñáõÙÝ»ñÁ ïñí³Í ³ñï³ùÇÝ áõÅ»ñÇ ¨ ëÏ½µÝ³Ï³Ý áõ »½ñ³ÛÇÝ å³ÛÙ³ÝÝ»ñÇ ¹» åùáõÙ, ·ïÝ»É Ñ»ÕáõÏÇ ³ñ³·áõÃÛ³Ý, ×ÝßÙ³Ý, ËïáõÃÛ³Ý, Çëï áã Ç½áÃ»ñÙ ß³ñÅáõÙÝ»ñÇ ¹» åùáõÙ Ý³¨ ç»ñÙ³ÛÇÝ ¹³ßï»ñÁ` Ï³Ëí³Í Ï»ïÇ Ïáá ñ¹ÇÝ³ïÝ»ñÇó ¨ Å³Ù³Ý³ÏÇó: îñí³Í ËÝ¹ÇñÁ Éáõí» Éáõ Ñ³Ù³ñ ß³ñÅÙ³Ý ¹Ç ý»ñ»ÝóÇ³É Ñ³í³ë³ñáõÙÝ»ñÇÝ å»ïù ¿ ÙÇ³óÝ»É ³ÝË½»ÉÇáõÃÛ³Ý Ñ³í³ë³ñáõÙÁ: àôÕÕ³ÝÏÛáõÝ ¹»Ï³ñïÛ³Ý Ïáá ñ¹ÇÝ³ï³ÛÇÝ Ñ³Ù³Ï³ñ·áõÙ ¹ñ³Ýù »Ý

$$\frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z} = F_x - \frac{1}{\rho}\frac{\partial P}{\partial x},$$
$$\frac{\partial V_y}{\partial t} + V_x\frac{\partial V_y}{\partial x} + V_y\frac{\partial V_y}{\partial y} + V_z\frac{\partial V_y}{\partial z} = F_y - \frac{1}{\rho}\frac{\partial P}{\partial y},\qquad 5.7$$
$$\frac{\partial V_z}{\partial t} + V_x\frac{\partial V_z}{\partial x} + V_y\frac{\partial V_z}{\partial y} + V_z\frac{\partial V_z}{\partial z} = F_z - \frac{1}{\rho}\frac{\partial P}{\partial z},$$

àñå»ë ³ÝÏ³Ë ÑÇÝ·»ñáñ¹ $\frac{d\rho}{dt} + \rho\left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right) = 0$: ÕáõÏÇ (·³½Ç) íÇ×³ÏÇ Ñ³í³ë³ñáõÙÁ: ¶áÛáõÃÛáõÝ áõÝÇ Ñ»ÕáõÏÝ»ñÇ ÙÇ ¹³ë, áñÇ Ñ³Ù³ñ ËïáõÃÛáõÝÁ ýáõÝÏóÇ³ ¿ ÙÇ³ÛÝ ×ÝßáõÙÇó $\rho = f(P)$:

²ÛÝ ¹»åùáõÙ, »ñµ Ñ»ÕáõÏÇ ËïáõÃÛáõÝÁ ýáõÝÏóÇ³ ¿ áã ÙÇ³ÛÝ ×ÝßáõÙÇó, ³ÛÉ¨ ç»ñÙ³ëïÇ×³ÝÇó, íÇ×³ÏÇ Ñ³í³ë³ñáõÙÁ áõÝÇ ³í»ÉÇ µ³ñ¹ ï»ëù, áñáÝóÇó å³ñ½³·áõÛÝÁ Ø»Ý¹»É»¨-ÎÉ³å»ÛñáÝÇ Ñ³í³ë³ñáõÙÝ ¿`

$$P = \rho RT, \qquad 5.8$$

Ð»ÕáõÏÝ»ñÇ ³Ûë ï»ë³ÏÁ ÁÝ¹áõÝí³Í ¿ ³Ýí³Ý»É բաñáÏÉÇÝ Ñ»ÕáõÏÝ»ñ: ºñբ ß³ñÅáõÙÁ Ç½áÃ»ñÙ ¿` T=const, ³å³ (5.8) Ñ³í³ë³ñáõÙÁ (5.7) Ñ³í³ë³ñáõÙÝ»ñÇ Ñ»ï Ï³½ÙáõÙ ¿ Ñ³í³ë³ñáõÙÝ»ñÇ ÷³Ï Ñ³Ù³Ï³ñ· $V_x$, $V_y$, $V_z$, P, ρ ³ÝÑ³ÛïÝ»ñÇ ÝÏ³ïÙ³Ùբ: ²ÛÝ ¹»åùáõÙ, »ñբ ß³ñÅáõÙÁ Ç½áÃ»ñÙ ã¿, ³ÛëÇÝùÝ T-Ý ÷á÷áËíáõÙ ¿, Ñ³í³ë³ñáõÙÝ»ñÇ (5.7) ¨ (5.8) Ñ³Ù³Ï³ñ·Á ÷³Ï ã¿, ù³ÝÇ áñ áõÝ»Ýù ÑÇÝ· Ñ³í³ë³ñáõÙ í»ó³ÝÑ³ÛïÝ»ñáí` V_x, V_y, V_z, P, ρ, T: ÜáõÛÝ ÷á÷áËÑ³Ï³ÝÝ»ñáí ·ñí³Í í»ó»ñáñ¹ Ñ³í³ë³ñáõÙÝ ¿ ¿Ý»ñ·Ç³ÛÇ ÑáëùÇ Ñ³í³ë³ñáõÙÁ

$$\varepsilon = C_v \rho \frac{dT}{dt} - \frac{jP}{\rho}\frac{d\rho}{dt}, \qquad 5.9$$

Դասախոսություն 6

6.1 Æ¹»³É³Ï³Ý Ñ»ÕáõÏÇ ß³ñÅÙ³Ý ¹Çý»ñ»ÝóÇ³É Ñ³í³ë³ñáõÙÝ»ñÇ ÇÝï»·ñáõÙÁ

¶³ñ½³·áõÛÝ ÇÝï»·ñ³ÉÝ»ñ`

³) ¿Ý»ñ·Ç³ÛÇ ÇÝï»·ñ³ÉÁ, áñÁ ëï³óíáõÙ ¿ Ç¹»³É³Ï³Ý Ñ»ÕáõÏÇ íñ³ ¹ñí³Í Ñ»ï¨Û³É ë³ÑÙ³Ý³÷³ÏáõÙÝ»ñÇ ¹»åùáõÙ`

1. ß³ñÅáõÙÁ ëï³óÇáÝ³ñ ¿

2. ³ñï³ùÇÝ áõÅ³ÛÇÝ ¹³ßïÝ áõÝÇ åáï»ÝóÇ³É`, $\bar{F} = -\mathrm{grad}\Pi$

áñï»Õ Π-Á Ñ»ÕáõÏÇ ÙÇ³íáñ

²Ûë ë³ÑÙ³Ý³÷³ÏáõÙÝ»ñÇ ¹»åùáõÙ $\bar{a} = \bar{F} - \frac{1}{\rho}\mathrm{grad}P$ ß³ñÅÙ³Ý ¹Çý»ñ»ÝóÇ³É

Ñ³í³ë³ñÙ³Ý »ñÏáõ ÏáÕÙ»ñÁ ëÏ³ÉÛ³ñ³å»ë µwqU³a³ïl»Eal $(\rho d\tau)d\bar{r}$ áõ

ÇÝï»·ñ»Éáí Ñ»ÕáõÏÇ τ ïÇñáõÛÃáí, Ñ³ßíÇ ³éÝ»Éáí, áñ $\bar{a}d\bar{r} = \left(d\bar{V}/_{dt}\right)d\bar{r} = \bar{V}d\bar{V} = d\left(V^2/_2\right)$

Áñáß Ó¨³÷áËáõÃÛáõÝÝ»ñÇó Ñ»ïá Ïëï³Ý³Ýù`

$$dE_կ + d\Pi_* = -\iiint_\tau \left(\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\nu z}\right)d\tau ; \quad 6.1$$

²Ûëï»Õ $E_ï$ ¨ Π_* -Ý ³ÙµáÕç т ïÇñáõÛÃáõÙ å³ñ÷³Ïí³Í Ñ»ÕáõÏÇ ÏÇÝ»ïÇÏ ¨ åáï»ÝóÇ³É ¿Ý»ñ·Ç³Ý»ñÝ »Ý`

$$E_ï=\frac{1}{2}\iiint_\tau \rho V^2 d\tau\,, \quad \Pi_* = \iiint_\tau \Pi\rho d\tau\,; \qquad 6.2$$

(6.1)-Ç »ñÏáõ Ù³ë»ñÁ µ³Å³Ý»Éáí dt-Ç ¨ Ñ³ßíÇ ³éÝ»Éáí, áñ

$$\frac{dx}{dt} = V_x\,, \quad \frac{\partial p}{\partial x}V_x = \frac{\partial}{\partial x}(PV_x) - P\frac{\partial V_x}{\partial x}$$

¨ ÏÇñ³é»Éáí ¶³áõë- úëïñá·ñ³

$$\frac{d}{dt}(E_ï + \Pi_*) = -\iint_S \rho V_n ds + \iiint_\tau P\,\mathrm{div}\overline{V} d\tau\,; \qquad 6.3$$

բ) ԲեéÝáõÉÇÇ ÇÝï»·ñ³ÉÁ, áñÁ ëï³óíáõÙ ¿ Ç¹»³É³Ï³Ý Ñ»ÕáõÏÇ ß³ñÅÙ³Ý íñ³ ¹ñí³Í Ñ»ï¨Û³É ë³ÑÙ³Ý³÷³ÏáõÙÝ»ñÇ ¹»åùáõÙ`

1. ß³ñÅáõÙÁ ëï³óÇáÝ³ñ ¿

2. ³ñï³ùÇÝ áõÅ»ñÝ áõÝ»Ý åáï»ÝóÇ³É` $\bar{F} = -\mathrm{grad}\Pi$ áñï»Õ Π-Ն ÙÇ³íáñ ½³Ý·í³ÍÇ åáï»ÝóÇ³É ¿Ý»ñ·Ç³Ý ¿

3. Ñ»ÕáõÏÁ բաñáïñáå ¿` $\rho=f(P)$ Ï³Ù $P=\varphi(\rho)$:

²Ûë ¹»åùáõÙ ß³ñÅÙ³Ý ¹Ç ý»ñ»ÝóÇ³É Ñ³í³ë³ñáõÙÝ»ñÁ ·ñ»Éáí È³ÙբÇ ï»ëùáí`

$$\frac{\partial \bar{V}}{\partial t} + \frac{1}{2}\mathrm{grad}V^2 - \bar{V}\times \mathrm{rot}\bar{V} = F - \frac{1}{\rho}\mathrm{grad}P, \qquad 6.4$$

Ýßí³Í å³ÛÙ³ÝÝ»ñÁ Ñ³ßíÇ ³éÝ»Éáí, (6.4)-Çó ÏáõÝ»Ý³Ýù`

$$\mathrm{grad}\left(\frac{V^2}{2} + \Pi + \Phi\right) = \bar{V}\times \mathrm{rot}\bar{V}, \qquad 6.5$$

որտեղ

$$\Phi = \int \frac{dP}{\rho}, \quad \frac{1}{\rho}\mathrm{grad}P = \mathrm{grad}\Phi : \qquad 6.6$$

(6.5)-Ç »ñÏáõ ÏáÕÙ»ñÁ ëÏ³É³ñ³å»ë բազմ³å³ïÏ»Ýù Ñáëù ·ÍÇ ßáß³÷áÕÇ e^0 ÙÇ³íáñ í»Ïïáñáí: Ð³í³ë³ñÙ³Ý ³ç Ù³ëÁ ÏÉÇÝÇ 0, ù³ÝÇ áñ $\bar{V} \times \mathrm{rot}\bar{V}$ -Ý áõÕÕ³Ñ³Û³ó ¿ $\bar{V}$ -ÇÝ, Çëï e^0 -Ý ¿É Ñ³Ù³·ÇÍ ¿ $\bar{V}$ -ÇÝ:

ÎáõÝ»Ý³Ýù`

$$\operatorname{grad}\left(\frac{V^2}{2}+\Pi+\Phi\right)\overline{e^0}=\frac{\partial}{\partial e}\left(\frac{V^2}{2}+\Pi+\Phi\right)=0 \qquad 6.7$$

այստեղից

$$\frac{V^2}{2}+\Pi+\Phi=\mathrm{const}=c$$

կամ

$$\frac{V^2}{2}+\Pi+\int\frac{dP}{\rho}=c: \qquad 6.8$$

²Ýë»ÕÙ»ÉÇ Ñ»ÕáõÏÇ ¹»åùáõÙ (6.8)-Çó ÏáõÝ»Ý³Ýù`

$$\frac{V^2}{2}+\Pi+\frac{P}{\rho}=C: \qquad 6.9$$

ºÃ» ³½¹áÕ ³ñï³ùÇÝ áõÅ»ñÁ ÙÇ³ÛÝ Í³ÝñáõÃÛ³Ý áõÅ»ñ »Ý, ³å³ $\Pi=g \cdot z$ ¨ (6.9)-Çó ÏáõÝ»Ý³Ýù`

$$\frac{V^2}{2g}+z+\frac{P}{\rho g}=C: \qquad 6.10$$

´»éÝáõÉÇÇ ÇÝï»·ñ³ÉÇ ¿Ý»ñ·»ïÇÏ ÇÙ³ëïÁ å³ñ½»Éáõ Ñ³Ù³ñ (6.10)-Á Ý»ñÏ³Û³óÝ»Ýù

$$\frac{V^2}{2}+zg+\frac{P}{\rho}=c \qquad 6.11$$

´»éÝáõÉÇÇ ÇÝï»·ñ³ÉÇ ¿Ý»ñ·»ïÇÏ ÇÙ³ëïÝ ³ÛÝ ¿, áñ ïíÛ³É ÑáëùÇ ·ÍÇ »ñÏ³ÛÝùáí ÙÇ³íáñ ½³Ý·í³Í áõÝ»óáÕ Ù³ëÝÇÏÇ ÏÇÝ»ïÇÏ, åáï»ÝóÇ³É ¨ ×ÝßÙ³Ý ¿Ý»ñ·Ç³Ý»ñÇ ·áõÙ³ñÁ Å³Ù³Ý³ÏÇ ó³ÝÏ³ó³Í å³ÑÇ Ñ³ëï³ïáõÝ ¿:

·) <u>ÎáßÇÇ</u> ÇÝï»·ñ³ÉÁ, áñÁ ëï³óíáõÙ ¿ Ç¹»³É³Ï³Ý Ñ»ÕáõÏÇ íñ³ ¹ñí³Í Ñ»ï¨Û³É ë³ÑÙ³Ý³÷³ÏáõÙÝ»ñÇ ¹»åùáõÙ

1. ß³ñÅáõÙÁ åáï»ÝóÇ³É ¿

2. ³½¹áÕ ³ñï³ùÇÝ áõÅ³ÛÇÝ ¹³ßïÝ áõÝÇ åáï»ÝóÇ³É`

$$\bar{F} = \text{grad}U = -\text{grad}\Pi$$

3. Ñ»ÕáõÏÁ µwñáïñáå ¿:

Þ³ñÅÙ³Ý ¹Çý»ñ»ÝóÇ³É Ñ³í³ë³ñáõÙÁ ·ñ»Éáí È³ÙµÇ (6.4) ï»ëùáí, Ýßí³Í å³ÛÙ³ÝÝ»ñÇ ¹»åùáõÙ ÏáõÝ»Ý³Ýù`

áñï»Õ Ñ³ßíÇ »Ý ³éÝí³Í Ա

$$\frac{\partial}{\partial t}\text{grad}\varphi + \frac{1}{2}\text{grad}V^2 = -\text{grad}\Pi - \text{grad}\Phi, \quad 6.12$$

Ýß³Ý³ÏáõÙÝ»ñÁ: (6.12)-Á Ý»ñÏ³Û³óÝ»Éáí

$$\bar{V} = \text{grad}\varphi \qquad \Phi = \int\frac{dP}{\rho}, \quad \frac{1}{\rho}\text{grad}P = \text{grad}\Phi$$

ï»ëùáí ¨ Ñ³ßíÇ ³éÝ»Éáí, áñ ß³ñ

$$\text{grad}\left(\frac{\partial\varphi}{\partial t} + \frac{V^2}{2} + \Pi + \Phi\right) = 0 \quad 6.13$$

³Ý å³ÛÙ³Ý ãÇ ¹ñí³Í, ÏáõÝ»Ý³Ýù

áñï»Õ f(t)-Ý Å³Ù³Ý³ÏÇó Ï³Ëí³Í áñ

$$\frac{\partial\varphi}{\partial t} + \frac{V^2}{2} + \Pi + \Phi = f(t), \quad 6.14$$

(6.14)-Á ÏáãíáõÙ ¿ ÎáßÇÇ ÇÝï»·ñ³É, Áëï áñÇ »Ã» Ç¹»³É³Ï³Ý Ñ»ÕáõÏÇ ß³ñÅÙ³Ý Å³Ù³Ý³Ï բաí³ñ³ñí³Í »Ý í»ñÁ Ýßí³Í å³ÛÙ³ÝÝ»ñÁ, ³å³ (6.14) Ñ³í³ë³ñáõÃÛ³Ý Ó³Ë Ù³ëÇ ãáñë ³Ý¹³ÙÝ»ñÇ ·áõÙ³ñÁ ïíÛ³É å³ÑÇÝ Ñ³ëï³ïáõÝ ¿ ïÇñáõÛÃÇ µáÉáñ Ï»ï»ñáõÙ: ºñբ Ñ»ÕáõÏÁ ³Ýë»ÕÙ»ÉÇ ¿ ¨ »ñբ ազդող áõÅ»ñÁ Í³ÝñáõÃÛ³Ý áõÅ»ñ »Ý, (6.14) –Á ÏÁÝ¹áõÝÇ Ñ»ï¨Û³É ï»ëùÁ

$$\frac{\partial\varphi}{\partial t}+\frac{V^2}{2}+gz+\frac{P}{\rho}=f(t): \qquad 6.15$$

Ä³Ù³Ý³ÏÇ ïíÛ³É å³ÑÇÝ f(t) ýáõÝÏóÇ³ÛÇ ³ñÅ»ùÁ Ïáñáßí³Ç, »Ã» ³Û¹ å³ÑÇÝ Ñ³ÛïÝÇ ÉÇÝÇ (6.14)-Ç Ó³Ë Ù³ëÇ ³ñÅ»ùÁ ïÇñáõÛÃÇ áñ¨¿ Ï»ïáõÙ: ²Ýë»ÕÙ»ÉÇ Ñ»ÕáõÏÇ ååï»ÝóÇ³É ß³ñÅÙ³Ý Å³Ù³Ý³Ï φ ýáõÝÏóÇ³Ý բաí³ñ³ñáõÙ ¿ È³åÉ³ëÇ

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=0 \qquad 6.16$$

ղ) ´»éÝáõÉÇ-¾ÛÉ»ñÇ ÇÝï»·ñ³ÉÁ, áñÁ ëï³óíáõÙ ¿ ÎáßÇÇ ÇÝï»·ñ³ÉÇ Ñ³Ù³ñ ³ÝÑñ³Å»ßï å³ÛÙ³ÝÝ»ñÇÝ ³í»É³óÝ»Éáí ß³ñÅÙ³Ý ëï³óÇáÝ³ñáõÃÛ³Ý å³ÛÙ³ÝÁ: ²Û¹ å³ÛÙ³ÝÝ»ñÇ ¹»åùáõÙ (6.14)-Çó ÏáõÝ»Ý³Ýù

$$\frac{V^2}{2} + \Pi + \int \frac{dP}{\rho} = c; \qquad 6.17$$

(6.17)-Á Ïáãíáõ٠¿ ´»éÝáõÉÇ-¾ÛÉ»ñÇ ÇÝï»·ñ³É: Ð³Ù³Ó³ÛÝ ³Û¹ ÇÝï»·ñ³ÉÇ, (6.17)-Ç Ó³Ë Ù³ëáõÙ ·ïÝíáÕ »ñ»ù Ù»ÍáõÃÛáõÝÝ»ñÇ ·áõÙ³ñÁ ÙÝáõÙ ¿ Ñ³ëï³ïáõÝ ïÇñáõÛÃÇ ó³ÝÏ³ó³Í Ï»ïáõÙ Å³Ù³Ý³ÏÇ ó³ÝÏ³ó³Í å³ÑÇÝ: ºñµ ωqηáÕ áõÅ»ñÁ Í³ÝñáõÃÛ³Ý áõÅ»ñ »Ý, ÇëÏ Ñ»ÕáõÏÁ ³Ýë»ÕÙ»ÉÇ ¿, (6.17)-Çó ÏáõÝ»Ý³Ýù

$$\frac{V^2}{2} + gz + \frac{P}{\rho} = c; \qquad 6.18$$

²ÝÑñ³Å»ßï ¿ Ýß»É, áñ (6.18)-Á ÙÇ³ÛÝ ï»ëùáí ¿ ÝÙ³Ý ´»éÝáõÉÇÇ (6.11) ÇÝï»·ñ³ÉÇÝ: ¸ñ³Ýù ¿³å»ë Çñ³ñÇó ËÇëï ï³ñµ»ñíáõÙ »Ý: