

**Հեղուկների և գազերի  
կիրառական մեխանիկա**

# Դասախոսություն 1

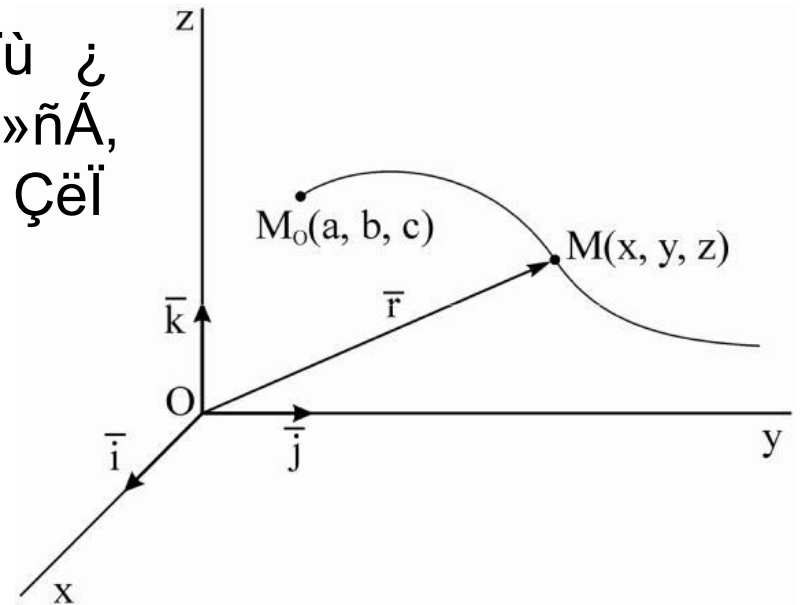
Թ<sup>o</sup> Ȫ à ô î Ø Æ æ<sup>2</sup> ì<sup>2</sup> Ú ð Æ  
î Æ Û<sup>o</sup> Ø<sup>2</sup> î Æ î<sup>2</sup>

# 1.1 $\vec{r} = \vec{r}(a, b, c, t)$ $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ ,  $\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$   $\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$   $\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$$\vec{r} = \vec{r}(a, b, c, t) \quad 1.1$$

$\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$   $\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$   $\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$



$\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$   $\vec{r} = \vec{r}(a, b, c, t)$   $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$$x = x(a, b, c, t), \quad y = y(a, b, c, t), \quad z = z(a, b, c, t): \quad 1.2$$

Đ³í³ë³ñáõÙÝ»ñÇ (1.2) Ñ³Ù³İ³ñ·Çó İ³ñáÕ »Ýù ëİ³Ý³É İÇÝ»Ù³İÇİ³ÛÇ ÙÛáõë μÝáõÃ³·ñÇãÝ»ñÁ

$$\begin{aligned}
 V_x &= \frac{dx}{dt} = \frac{\partial x}{\partial t} + \frac{\partial x}{\partial a} \cdot \frac{da}{dt} + \frac{\partial x}{\partial b} \cdot \frac{db}{dt} + \frac{\partial x}{\partial c} \cdot \frac{dc}{dt} = \frac{\partial x}{\partial t}, \\
 V_y &= \frac{dy}{dt} = \frac{\partial y}{\partial t} + \frac{\partial y}{\partial a} \cdot \frac{da}{dt} + \frac{\partial y}{\partial b} \cdot \frac{db}{dt} + \frac{\partial y}{\partial c} \cdot \frac{dc}{dt} = \frac{\partial y}{\partial t}, \\
 V_z &= \frac{dz}{dt} = \frac{\partial z}{\partial t} + \frac{\partial z}{\partial a} \cdot \frac{da}{dt} + \frac{\partial z}{\partial b} \cdot \frac{db}{dt} + \frac{\partial z}{\partial c} \cdot \frac{dc}{dt} = \frac{\partial z}{\partial t}.
 \end{aligned}
 \tag{1.3}$$

(1.3) μ³Ý³Ó³»ñÁ ëİ³Ý³ÉÇë Ñ³İİÇ ħ ³éÝİ»É, áñ a, b, c-Ý Ñ³ëİ³İáõÝ »Ý`

$$\frac{da}{dt} = \frac{db}{dt} = \frac{dc}{dt} = 0;
 \tag{1.4}$$

Đ»ÕáõİÇ Ù³ëÝÇİÝ»ñÇ ³ñ³.³óáõÙÁ İáñáİİÇ Ñ»İ³Û³É μ³Ý³Ó³»ñÁİ`

$$\begin{aligned}
 a_x &= \frac{dV_x}{dx} = \frac{\partial V_x}{\partial t} = \frac{\partial^2 x}{\partial t^2}, \\
 a_y &= \frac{dV_y}{dt} = \frac{\partial V_y}{\partial t} = \frac{\partial^2 y}{\partial t^2}, \\
 a_z &= \frac{dV_z}{dt} = \frac{\partial V_z}{\partial t} = \frac{\partial^2 z}{\partial t^2}.
 \end{aligned}
 \tag{1.5}$$

$\Phi = \Phi(x, y, z, t)$ 
 $\vec{v} = \vec{v}(x, y, z, t)$ 
 $\vec{v} = v_x \vec{e}_x + v_y \vec{e}_y + v_z \vec{e}_z$ 
 $\vec{v} = \vec{v}(x, y, z, t)$

$$\begin{aligned}
 v_x &= v_x(x, y, z, t), \\
 v_y &= v_y(x, y, z, t), \\
 v_z &= v_z(x, y, z, t):
 \end{aligned}
 \tag{1.6}$$

$\vec{v} = \vec{v}(x, y, z, t)$ 
 $\vec{v} = v_x \vec{e}_x + v_y \vec{e}_y + v_z \vec{e}_z$ 
 $\vec{v} = \vec{v}(x, y, z, t)$

$\vec{v} = \vec{v}(x, y, z, t)$ 
 $\vec{v} = v_x \vec{e}_x + v_y \vec{e}_y + v_z \vec{e}_z$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}, \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}, \quad a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2} :
 \tag{1.7}$$

Üİ³İÇ áõÝ»Ý³Éáí, áñ Ñ»Õáõİ Ù³ëÝÇİÇ x, y, z İááñ¹ÇÝ³İÝ»ñÁ ÷á÷áËİáõÙ »Ý Áëİ Å³Ù³Ý³İÇ, İáõÝ»Ý³Ýù

$$a_x = \frac{\partial V_x}{\partial t} + \frac{\partial V_x}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial V_x}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial V_x}{\partial z} \cdot \frac{dz}{dt}$$

Կամ

$$a_x = \frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z}, \quad 1.8$$

$$a_y = \frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z},$$

$$a_z = \frac{\partial V_z}{\partial t} + V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z} :$$

²ñ³.³óÙ³Ý í»İİáñÁ İÉÇÝÇ`

$$\bar{a} = \frac{d\bar{V}}{dt} = \frac{\partial \bar{V}}{\partial t} + V_x \frac{\partial \bar{V}}{\partial x} + V_y \frac{\partial \bar{V}}{\partial y} + V_z \frac{\partial \bar{V}}{\partial z} : \quad 1.9$$

(1.9)-Á ëÇÙİáÉÇİ Ó`áí İ³ñáÕ »Ýù ·ñ»É`

Կամ

$$\bar{a} = \frac{\partial \bar{V}}{\partial t} + (V_x \frac{\partial}{\partial x} + V_y \frac{\partial}{\partial y} + V_z \frac{\partial}{\partial z}) \bar{V} \quad 1.10$$

$$\bar{a} = \frac{\partial \bar{V}}{\partial t} + (\bar{V} \cdot \text{grad}) \bar{V} :$$

# Դասախոսություն 2





# 2.2 Flux of a Vector Field through a Surface

Let  $S$  be a surface in space. The flux of a vector field  $\mathbf{V}$  through  $S$  is defined as the surface integral of the normal component of  $\mathbf{V}$  over  $S$ .

$$\Phi = \iint_S \mathbf{V} \cdot \mathbf{n} \, ds \quad (2.2)$$

where  $\mathbf{n}$  is the unit normal vector to the surface  $S$  at each point. The flux is a scalar quantity. If  $\mathbf{V}$  is a vector field and  $\mathbf{n}$  is the unit normal vector to the surface  $S$ , then the flux is given by  $\Phi = \iint_S \mathbf{V} \cdot \mathbf{n} \, ds$ .

where

$$\Phi = \iint_S [V_x \cos(n_x) + V_y \cos(n_y) + V_z \cos(n_z)] \, ds \quad (2.3)$$

$$\Phi = \iint_S [V_x \, dydz + V_y \, dxdz + V_z \, dxdy] \quad (2.4)$$

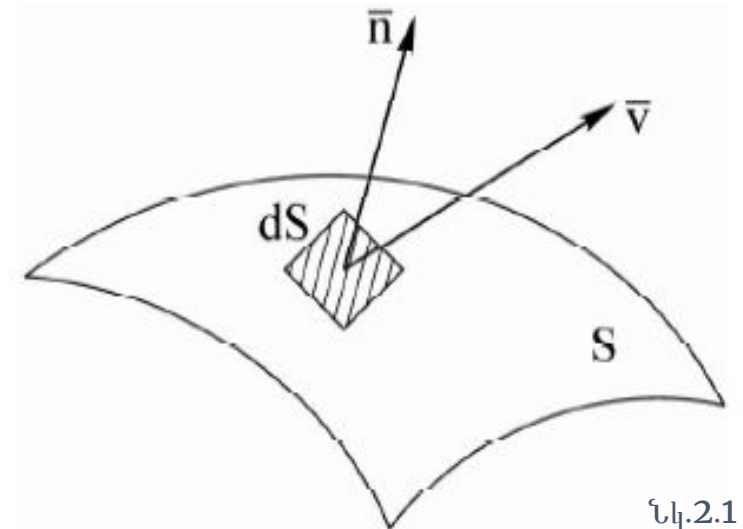


Fig. 2.1

2.3) Η συνθήκη  $\text{div } \vec{V} = 0$  (ή  $\nabla \cdot \vec{V} = 0$ ) είναι ισοδύναμη με την εξίσωση Laplace  $\nabla^2 \phi = 0$  για το δυναμικό  $\phi$  που ορίζεται από  $\vec{V} = \nabla \phi$ .

$$\oiint_S [V_x \cos(nx) + V_y \cos(ny) + V_z \cos(nz)] ds = \iiint_{\Delta\tau} \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right) d\tau \quad 2.5$$

2.4) Η συνθήκη  $\text{div } \vec{V} = 0$  (ή  $\nabla \cdot \vec{V} = 0$ ) είναι ισοδύναμη με την εξίσωση Laplace  $\nabla^2 \phi = 0$  για το δυναμικό  $\phi$  που ορίζεται από  $\vec{V} = \nabla \phi$ .

$$\iiint_{\Delta\tau} \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right) d\tau = \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right)_{M_0} \cdot \Delta\tau, \quad 2.6$$

2.5) Η συνθήκη  $\text{div } \vec{V} = 0$  (ή  $\nabla \cdot \vec{V} = 0$ ) είναι ισοδύναμη με την εξίσωση Laplace  $\nabla^2 \phi = 0$  για το δυναμικό  $\phi$  που ορίζεται από  $\vec{V} = \nabla \phi$ .

$$\text{div } \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} : \quad 2.7$$

2.6) Η συνθήκη  $\text{div } \vec{V} = 0$  (ή  $\nabla \cdot \vec{V} = 0$ ) είναι ισοδύναμη με την εξίσωση Laplace  $\nabla^2 \phi = 0$  για το δυναμικό  $\phi$  που ορίζεται από  $\vec{V} = \nabla \phi$ .

$$\oiint_S V_n ds = \iiint_{\tau} \text{div } \vec{V} d\tau : \quad 2.8$$

2.7) Η συνθήκη  $\text{div } \vec{V} = 0$  (ή  $\nabla \cdot \vec{V} = 0$ ) είναι ισοδύναμη με την εξίσωση Laplace  $\nabla^2 \phi = 0$  για το δυναμικό  $\phi$  που ορίζεται από  $\vec{V} = \nabla \phi$ .

$$\Gamma = \oint_L \vec{V} d\vec{r} = \oint_L (V_x dx + V_y dy + V_z dz), \quad 2.9$$

ú·íí»Éáí ĩáñ³·Çí ÇÝí»·ñ³ÉÇó Ù³İ»ñ̄·áõÛÃ³ÛÇÝÇÝ ³ÝóÝ»Éáõ êĩáùëÇ ρωÝ³Ó̄̄Çó ĩáõÝ»Ý³Ýù`

$$\oint_L (V_x dx + V_y dy + V_z dz) = \iint_S \left[ \left( \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right) dy dz + \left( \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right) dx dz + \left( \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) dx dy \right], \quad 2.10$$

²ñ³·áõÃÛ³Ý í»İĩáñÇ éáĩáñ ĩáĩíaõÙ ħ ³ÛÝ Ω í»İĩáñÁ, áñÇ áñáÛ»İóÇ³Ý»ñÁ úx, úy, úz ³é³ÝóùÝ»ñÇ íñ³ Ñ³Û³á³ĩ³ëË³Ý³ρωñ Ñ³ĩ³ë³ñ »Ý

$$\Omega_x = \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}, \Omega_y = \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}, \Omega_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \quad 2.11$$

ωϋηλρλ

$$\text{rot } \bar{V} = \bar{\Omega} = \left( \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right) \bar{i} + \left( \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right) \bar{j} + \left( \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \bar{k} : \quad 2.12$$

Ð³βίÇ ³éÝ»Éáí (2.9), (2.10) " (2.11) μ³Ý³Ó̄̄»ñÁ, ĩáõÝ»Ý³Ýù`

$$\Gamma = \oint_L \bar{V} d\bar{r} = \iint_S \bar{\Omega} n^0 ds = \iint_S \Omega_n ds, \quad 2.13$$

# Դասախոսություն 3

# 3.1 $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$ $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$

$\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$   $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$   $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$   $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$

$\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$   $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$   $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$   $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$

1.  $\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$

$\frac{1}{2} \rho_0 \frac{dV}{dt} = \int_V \rho \frac{dV}{dt}$

$$\int_{\tau_0} \rho_0(a,b,c,t_0) da db dc = \int_{\tau} \rho(x,y,z,t) dx dy dz \quad 3.1$$

u hwzqh wnlutlnd, nη x = x(a,b,c,t), y = y(a,b,c,t), z = z(a,b,c,t), (3.1)-nlu x, y, z ÷á÷áË³İ³ÝÝ»ñÇó ³ÝóÝ»Éáí a, b, c ÷á÷áË³İ³ÝÝ»ñÇÝ, İëİ³Ý³Ýù`

$$\int_{\tau_0} (\rho_0 - \rho \cdot l) da db dc = 0, \quad 3.2$$

áñi»Õ I -Ý Ó³÷áËáõÃÛ³Ý áñáßÇãÝ ¿ (Ú³İáµÇ³ÝÁ)`

$$I = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} & \frac{\partial z}{\partial a} \\ \frac{\partial x}{\partial b} & \frac{\partial y}{\partial b} & \frac{\partial z}{\partial b} \\ \frac{\partial x}{\partial c} & \frac{\partial y}{\partial c} & \frac{\partial z}{\partial c} \end{vmatrix} : \quad 3.3$$

ø³ÝÇ áñ İ³İ³ÉÇ dadbdc İ³ññÁ İ³Û³íañ ¿, " »ÝÃ³ÇÝi»·ñ³É ýáõÝİóÇ³Ý ³ÝÁÝ¹Ñ³i, ³ã³ (3.2)-Çó İëİ³Ý³Ýù`

$$\rho_0 = \rho \cdot I, \quad 3.4$$

áñÁ Ñ³Ý¹Çë³ÝáõÛ ¿ ³ÝË½»ÉÇáõÃÛ³Ý Ñ³İ³ë³ñáõÛÁ È³·ñ³ÝÅÇ ÷á÷áË³İ³ÝÝ»ñái:

2. ²ÝË½»ÉÇáõÃÛ³Ý Ñ³İ³ë³ñáõÛÁ Էլլերի փոփոխականներով

Համաձայն

բանաձևի կստանանք`

$$Q = \iint_S V_n ds$$

$$Q = \iint_S \rho V_n ds : \quad 3.3$$

Ομογενή (2.5) ρωύ³Ό΄ζό`

$$\begin{aligned} \iint_S \rho V_n ds &= \iint_S [\rho V_x \cos(nx) + \rho V_y \cos(ny) + \rho V_z \cos(nz)] ds = \\ &= \iiint_\tau \left[ \frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} \right] d\tau : \end{aligned} \quad 3.4$$

Υποθέτουμε

$$\iiint_\tau \left[ \frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} \right] d\tau = - \iiint_\tau \frac{\partial \rho}{\partial t} d\tau : \quad 3.5$$

²Ûëï»Õζό, Ñ³βίζ ³έÝ»Εάί, άñ τ-Υ Ι³U³U³Ι³Υ ζ, Ιëï³Υ³Υù`

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} = 0 \quad 3.6$$

άñÁ Ñ»Õάõïζ ³ÝË½»ÉÇάõÃÛ³Ý Ñ³í³ë³ñάõÛÝ ζ ¾ÛÉ»ñÇ ÷ά÷άË³í³ÝÝ»ñάί:

àñάβ Ó³÷άËάõÃÛάõÝÝ»ñζό Ñ»íά, (3.6)-Á Ι³ñ»ÉÇ ζ ·ñ»É Ñ»íÛ³É ï»ëùάί`

$$\frac{d\rho}{dt} + \rho \cdot \text{div} \bar{V} = 0 : \quad 3.7$$

2. Έξω από τον όγκο  $\rho = \text{const}$  (3.7) - Άρα  $\nabla \cdot \bar{V} = 0$

$$\text{div } \bar{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \quad (3.8)$$

2. Έξω από τον όγκο (3.4) - Στόμα  $\nabla \cdot \bar{V} = 0$

$$\rho \oint_S V_n ds = \rho \iiint_V \text{div } \bar{V} d\tau = 0, \quad (3.9)$$

3. Έξω από τον όγκο  $\rho = \text{const}$  (3.7) - Άρα  $\nabla \cdot (\rho \bar{V}) = 0$

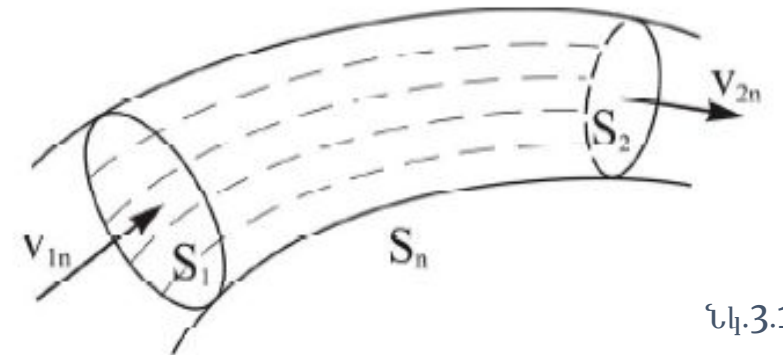
$$\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} = 0 \quad (3.10)$$

ή

$$\text{div}(\rho \bar{V}) = 0 \quad (3.11)$$

3. Έξω από τον όγκο (3.9)  $\rho = \text{const}$ ,  $\nabla \cdot \bar{V} = 0$

$$Q = \rho \oint_S V_n ds = \rho \iint_{S_1} V_{1n} ds + \rho \iint_{S_2} V_{2n} ds + \rho \iint_{S_n} V_n ds = 0, \quad (3.12)$$



3.1



$\rho \frac{d}{dt} \left( \frac{1}{r} \frac{d}{dr} \left( r^2 \frac{dV_r}{dr} \right) + \frac{1}{r \sin \vartheta} \frac{d}{d\vartheta} \left( \sin \vartheta \frac{dV_\vartheta}{d\vartheta} \right) + \frac{1}{r \sin \vartheta} \frac{d}{d\psi} \left( \sin \vartheta \frac{dV_\psi}{d\psi} \right) \right) = 0$

$\rho \frac{d}{dt} \left( \frac{1}{r} \frac{d}{dr} \left( r^2 \frac{dV_r}{dr} \right) + \frac{1}{r \sin \vartheta} \frac{d}{d\vartheta} \left( \sin \vartheta \frac{dV_\vartheta}{d\vartheta} \right) + \frac{1}{r \sin \vartheta} \frac{d}{d\psi} \left( \sin \vartheta \frac{dV_\psi}{d\psi} \right) \right) = 0$

$$-V_1 S_1 + V_2 S_2 = 0 \quad 3.13$$

$$V_1 S_1 = V_2 S_2 = \text{const} \quad 3.14$$

$\rho \frac{d}{dt} \left( \frac{1}{r} \frac{d}{dr} \left( r^2 \frac{dV_r}{dr} \right) + \frac{1}{r \sin \vartheta} \frac{d}{d\vartheta} \left( \sin \vartheta \frac{dV_\vartheta}{d\vartheta} \right) + \frac{1}{r \sin \vartheta} \frac{d}{d\psi} \left( \sin \vartheta \frac{dV_\psi}{d\psi} \right) \right) = 0$

$\rho \frac{d}{dt} \left( \frac{1}{r} \frac{d}{dr} \left( r^2 \frac{dV_r}{dr} \right) + \frac{1}{r \sin \vartheta} \frac{d}{d\vartheta} \left( \sin \vartheta \frac{dV_\vartheta}{d\vartheta} \right) + \frac{1}{r \sin \vartheta} \frac{d}{d\psi} \left( \sin \vartheta \frac{dV_\psi}{d\psi} \right) \right) = 0$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (\rho V_r r^2)}{\partial r} + \frac{1}{r \sin \vartheta} \frac{\partial (\rho V_\vartheta \sin \vartheta)}{\partial \vartheta} + \frac{1}{r \sin \vartheta} \frac{\partial (\rho V_\psi \sin \vartheta)}{\partial \psi} = 0 \quad 3.16$$

### 3.2 «ñáñÛ³óÇ³ÛÇ³ñ³·áõÃÛ³Ý Ã»Ý½áñ

Đ»Õáõİ ĩÇñáõÛÃÇ ¹»ýáñÛ³óÛ³Ý Ñ»ĩ³Ýùáí, ĩÇñáõÛÃÇ Û³ëÝÇİÝ»ñÁ ëĩ³ÝáõÛ »Ý Éñ³óáõóÇã ³ñ³·áõÃÛáõÝ: ²Û¹ V\_D ³ñ³·áõÃÛáõÝÁ á³ÛÛ³Ý³íañí³İ ħ ε₁, ε₂, ε₃, θ₁, θ₂, θ₃ Û» ÍáõÃÛáõÝÝ»ñáí, ÁÝ¹ áñáõÛ

$$\begin{aligned}
 V_{D\xi} &= \frac{\partial\phi}{\partial\xi} = \varepsilon_1\xi + \frac{1}{2}\theta_2\zeta + \frac{1}{2}\theta_3\eta, \\
 V_{D\eta} &= \frac{\partial\phi}{\partial\eta} = \varepsilon_2\eta + \frac{1}{2}\theta_3\xi + \frac{1}{2}\theta_1\zeta, \\
 V_{D\zeta} &= \frac{\partial\phi}{\partial\zeta} = \varepsilon_3\zeta + \frac{1}{2}\theta_1\eta + \frac{1}{2}\theta_2\xi:
 \end{aligned}
 \tag{3.17}$$

ε₁ -Ç ýÇ½Çİ³İ³Ý ÇÛ³ëİÁ á³ñ½»Éáõ Ñ³Û³ñ »ÝÃ³ñ»Ýù, áñ ĩ»ÕÇ ħ áõÝ»ó»É ³ÛÝáÇëÇ ¹» ýáñÛ³óÇ³, áñÇ Á³Û³Ý³İ, ε₁ ≠ 0, Çëİ ε₂=ε₃=θ₁=θ₂=θ₃=0: ²Ûë ¹»áùáõÛ (3.17)-Çó ÍáõÝ» Ý³Ýù

$$V_{D\xi} = \varepsilon_1\xi, \quad V_{D\eta} = 0, \quad V_{D\zeta} = 0:
 \tag{3.18}$$

³óÇ ³Û¹,

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = \text{div } \bar{V}
 \tag{3.19}$$

(3.19)Û»ÍáõÃÛáõÝÁ Ñ³Ý¹Çë³ÝáõÛ ħ ÁÝĩñí³İ Í³³ÉÇ Ñ³ñ³ϱñ³İ³Ý ÷á÷áË³İ³ÝáõÃÛáõÝÁ ÛÇ³íañ Á³Û³Ý³İ³ÛÇçáóáõÛ:

$\theta_3 - \zeta \gamma \zeta^{1/2} \zeta \dot{\Gamma}^3 \dot{\Gamma}^3 \dot{\Upsilon} \zeta \dot{\Upsilon}^3 \ddot{e} \dot{i} \dot{A} \dot{a}^3 \dot{n}^{1/2} \gg \dot{E} \dot{a} \dot{o} \dot{N}^3 \dot{U}^3 \dot{n} \dot{A} \dot{\Upsilon}^1 \dot{a} \dot{o} \dot{\Upsilon} \gg \dot{E} \dot{a} \dot{i} \quad (3.17) - \dot{a} \dot{o} \dot{U} \quad \theta_3 \neq 0, \zeta \dot{e} \dot{i} \quad \varepsilon_1 =$   
 $\varepsilon_2 = \varepsilon_3 = \theta_1 = \theta_2 = 0 \quad \dot{I} \dot{a} \dot{o} \dot{\Upsilon} \gg \dot{\Upsilon}^3 \dot{\Upsilon} \dot{u}$

$$V_{D\zeta} = \frac{1}{2} \theta_3 \eta, \quad V_{D\eta} = \frac{1}{2} \theta_3 \zeta, \quad V_{D\zeta} = 0: \quad 3.20$$

$\dot{U} \dot{e} \dot{a} \zeta \dot{e} \dot{a} \dot{i} \quad \varepsilon_1, \varepsilon_2, \varepsilon_3, \theta_1, \theta_2, \theta_3 \quad \dot{U} \gg \dot{I} \dot{a} \dot{o} \dot{A} \dot{U} \dot{a} \dot{o} \dot{\Upsilon} \dot{\Upsilon} \gg \dot{n} \dot{A} \quad \dot{A} \dot{\Upsilon}^1 \dot{N}^3 \dot{\Upsilon} \dot{a} \dot{o} \dot{n} \quad \dot{U} \gg \dot{a} \dot{u} \dot{a} \dot{o} \dot{U} \quad \rho \dot{\Upsilon} \dot{a} \dot{o} \dot{A}^3 \cdot \dot{n} \dot{a} \dot{o} \dot{U} \gg \dot{\Upsilon} \quad \dot{A} \dot{\Upsilon} \dot{i} \dot{n} \dot{i}^3 \dot{I} \quad \dot{N} \gg \dot{O} \dot{a} \dot{o} \dot{I} \quad \dot{i} \zeta \dot{n} \dot{a} \dot{o} \dot{U} \dot{A} \zeta \quad \dot{U} \gg \dot{y} \dot{a} \dot{n} \dot{U}^3 \dot{o} \zeta^3 \dot{\Upsilon} \quad \dot{U} \gg \dot{n}^3 \dot{\Upsilon} \dot{o} \quad \dot{U} \gg \dot{\Upsilon} \dot{i}^3 \dot{\Upsilon} \dot{a} \dot{o} \dot{U} \gg \dot{\Upsilon} \quad \dot{U} \gg \dot{y} \dot{a} \dot{n} \dot{U}^3 \dot{o} \zeta^3 \dot{U} \zeta \quad \dot{U} \gg \dot{n}^3 \cdot \dot{a} \dot{o} \dot{A} \dot{U}^3 \dot{\Upsilon} \quad \dot{U} \gg \dot{a} \dot{n} \dot{i}^3 \dot{I} \zeta \dot{o} \dot{\Upsilon} \gg \dot{n}: \quad \dot{U} \gg \dot{\beta}^3 \dot{\Upsilon}^3 \dot{I} \dot{a} \dot{o} \dot{U} \gg \dot{\Upsilon}$

$$\phi = \begin{pmatrix} \varepsilon_1 & \frac{1}{2} \theta_3 & \frac{1}{2} \theta_2 \\ \frac{1}{2} \theta_3 & \varepsilon_2 & \frac{1}{2} \theta_1 \\ \frac{1}{2} \theta_2 & \frac{1}{2} \theta_1 & \varepsilon_3 \end{pmatrix} \quad 3.21$$

$\dot{U} \gg \dot{y} \dot{a} \dot{n} \dot{U}^3 \dot{o} \zeta^3 \dot{U} \zeta \quad \dot{U} \gg \dot{n}^3 \cdot \dot{a} \dot{o} \dot{A} \dot{U}^3 \dot{\Upsilon} \quad \dot{U} \gg \dot{Y} \dot{1} \dot{2} \dot{a} \dot{n}: \quad \dot{U} \gg \dot{U} \dot{e} \quad \dot{U} \gg \dot{a} \dot{u} \dot{a} \dot{o} \dot{U} \quad \dot{U} \gg \dot{Y} \dot{1} \dot{2} \dot{a} \dot{n} \dot{A} \quad \dot{U} \gg \dot{N}^3 \dot{U}^3 \dot{a}^3 \dot{U} \quad \dot{U} \gg \rho \dot{\Upsilon} \dot{a} \dot{n} \dot{a} \dot{\beta} \dot{i} \dot{a} \dot{o} \dot{U} \quad \dot{U} \gg \dot{i} \dot{U} \gg \dot{I} \dot{a} \dot{o} \dot{A} \dot{U} \dot{a} \dot{o} \dot{\Upsilon} \dot{\Upsilon} \gg \dot{n} \dot{a} \dot{i}:$

# Դասախոսություն 4

***Æ, °² È² Î² Ü Ð° Ô à ô Í Æ, Æ Ü² Ø Æ Í²***

# 4.1 Đ»ŌáõĪ ĩÇñáõŪÃáõŪ ·áñláŌ áõÄ»ñÁ: Æ¹»³É³Ī³Ý Ñ» ŌáõĪ

Đ»ŌáõĪ τ ĩÇñáõŪÃáõŪ ·áñláŌ áõÄ»ñÁ  ${}^{13}ë{}^3Ī{}^3ñ·íaõŪ$  »Ý  $\frac{1}{2}{}^3Ý·í{}^3Ī{}^3ŪÇÝ$  áõÄ»ñÇ "  $Ū{}^3Ī$ »ñ"áõŪÃ ${}^3ŪÇÝ$  áõÄ»ñÇ:

$\frac{1}{4}{}^3Ý·í{}^3Ī{}^3ŪÇÝ$  ĩáãíaõŪ »Ý  ${}^3ŪÝ$  áõÄ»ñÁ, áñáÝù ĩÇñ ${}^3éí{}^3Ī$  »Ý ĩÇñáõŪÃÇ Δτ  $Í{}^3í{}^3É$ áõŪ  $ã{}^3ñ÷{}^3Īí{}^3Ī$   $\frac{1}{2}{}^3Ý·í{}^3ĪÇ$  íñ ${}^3$ , " áñÇ  ${}^3\frac{1}{2}{}^1$ »óáõÃŪáõÝÁ  $Í{}^3Ēí{}^3Ī$  ãç, Ã»  ${}^3Ū{}^1$  Δτ  $Í{}^3í{}^3É$ Á áõÝÇ± Çñ  $Ñ{}^3ñ$ " ${}^3Ý$ áõÃŪ ${}^3Ý$  Ū»ç  ${}^3ŪÉ$   $Í{}^3í{}^3ÉÝ$ »ñ, Ã»` áã:

$Ø{}^3Ī$ »ñ"áõÃ ${}^3ŪÇÝ$  »Ý ĩáãíaõŪ  ${}^3ŪÝ$  áõÄ»ñÁ, áñáÝù ĩÇñ ${}^3éí$ áõŪ »Ý Ñ»ŌáõĪÇ Ū ${}^3ëÝÇĪÇ$  (ĩÇñáõŪÃÇ) Ū ${}^3Ī$ »ñ"áõŪÃÇÝ " ç ${}^3ã$ »ë  $Í{}^3Ēí{}^3Ī$  »Ý Ýñ ${}^3$  Ñ ${}^3ñ$ " ${}^3Ý$ áõÃŪ ${}^3Ý$  Ū»ç ·íÝíaŌ  $Ū{}^3ëÝÇĪÝ$ »ñÇó (ĩÇñáõŪÃÝ»ñÇó),  ${}^3ŪëÇÝùÝ$   ${}^3ŪÝ$  áõÄ»ñÁ, áñáÝóái ÷á ${}^3\frac{1}{2}{}^1$ áõŪ »Ý  $Ñ{}^3ñ$ " ${}^3Ý$  Ñ»ŌáõĪ ĩÇñáõŪÃÝ»ñÁ " ĩÇñ ${}^3éí$ »Ý Ýñ ${}^3Ý$ ó Ū ${}^3Ī$ »ñ"áõŪÃÝ»ñÇÝ:

## 4.2 È³ñí³ÍáõÃÛ³Ý Ã»Ý½áñ

Đ»ÕáõİÇ İ³Û³Û³İ³Ý τ İÇñáõÛÃÁ ¹Çİ³ñİ»Éáí áñ»ë Û»Ë³ÝÇİ³İ³Ý Ñ³Û³İ³ñ. " Ýñ³ Ýİ³İÛ³Û³ İÇñ³é» Éáí , ³É³Û³ρτñÇ ëİ½ρáõÝùÁ, İáõÝ»Ý³Ýù

$$\iiint_{\tau} (\bar{F} \cdot \rho) d\tau + \iint_S \bar{P}_n ds - \iiint_{\tau} \bar{a} \rho d\tau = 0, \quad 4.1$$

ø³é³ÝÇëİÇ ÛÛáõë ÝÇëİ»ñÇ ³ñİ³ùÇÝ ÝáñÛ³ÉÝ»ñÁ İÉÇÝ»Ý -i, -j, -k í»İİáñÝ»ñÁ, Çëİ Û³İ»ñ» ëÝ»ñÁ Ñ³Û³á³İ³ëË³Ý³ρωñ Δ·α, ΔS·β, ΔS·γ: (4.1)-Çó İáõÝ»Ý³Ýù`

$$\bar{F} \cdot \rho \cdot \Delta\tau + \bar{P}_n \Delta S + \bar{P}_{-x} \cdot \Delta S \cdot \alpha + \bar{P}_{-y} \cdot \Delta S \cdot \beta + \bar{P}_{-z} \cdot \Delta S \cdot \gamma - \bar{a} \rho \Delta\tau = 0, \quad 4.2$$

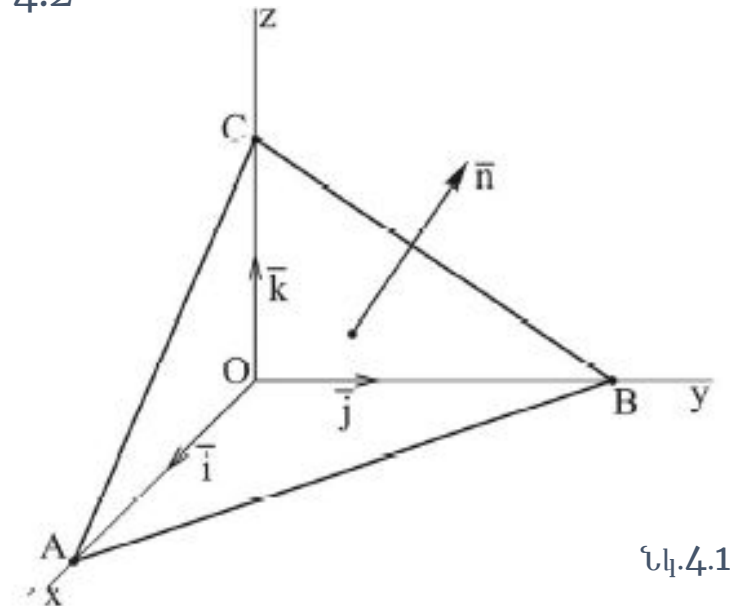
(4.2) Ñ³İ³ë³ñÛ³Ý Û»ç Ñ³İ³Ç ³éÝ»Éáí Δτ-Ç ³ñÁ»ùÁ " Ýİ³İÇ

áõÝ»Ý³Éáí, áñ  $\bar{P}_n = -\bar{P}_n$ , İáõÝ»Ý³Ýù

$$\frac{1}{3} (\bar{F} - \bar{a}) \rho h + \bar{P}_n - \alpha \bar{P}_x - \beta \bar{P}_y - \gamma \bar{P}_z = 0: \quad 4.3$$

(2.3)-áõÛ³ÝóÝ»Éáí ë³ÑÛ³ÝÇ »ñρ Ñ→0, İëİ³Ý³Ýù

$$\bar{P}_n = \alpha \bar{P}_x + \beta \bar{P}_y + \gamma \bar{P}_z : \quad 4.4$$



İ.4.1

ñáÛ»İ»Éáí (4.4)-Á Oxyz İááñ¹ÇÝ³İ³ÛÇÝ ³é³ÝóùÝ»ñÇ İñ³, İáõÝ»Ý³Ýù`

$$\begin{aligned}
 P_{nx} &= \alpha P_{xx} + \beta P_{yx} + \gamma P_{zx} , \\
 P_{ny} &= \alpha P_{xy} + \beta P_{yy} + \gamma P_{zy} , \\
 P_{nz} &= \alpha P_{xz} + \beta P_{yz} + \gamma P_{zz} :
 \end{aligned}
 \tag{4.5}$$

(4.5)-Á óáõÛó ĸ İ³ÉÇë, áñ É³ñí³İáõÃÛáõÝÁ Ñ»Õáõİ İÇñáõÛÃáõÛ »ñİñáñ¹ İ³ñ·Ç Ã»Ý½áñ³İ³Ý Û»İáõÃÛáõÝ ĸ, áñÁ áñáßİáõÛ ĸ 9 Û»İáõÃÛáõÝ»ñáí`

$$\Pi = \begin{pmatrix} P_{xx} & P_{yx} & P_{zx} \\ P_{xy} & P_{yy} & P_{zy} \\ P_{xz} & P_{yz} & P_{zz} \end{pmatrix} :
 \tag{4.6}$$

(4.6)-Á İáñİáõÛ ĸ É³ñí³İáõÃÛ³Ý Ã»Ý½áñ: ú·İİ»Éáí ¹³ë³İ³Ý Û»Ë³ÝÇİ³ÛÇ ûñ»ÝùÝ»ñÇó, İ³ñ»ÉÇ ĸ óáõÛó İ³É, áñ (4.6) Ã»Ý½áñÁ Ñ³Û³ã³÷ ĸ, ³ÛëÇÝùÝ`

$$P_{xy} = P_{yx}, \quad P_{yz} = P_{zy}, \quad P_{xz} = P_{zx} :
 \tag{4.7}$$

$\bar{P}_n = \bar{P}_{xx} = \bar{P}_{yy} = \bar{P}_{zz} = 0$ : à ô ë ï Ç (4.5) - Ç ó Í á õ Ý » Ý³ Ý ù`

$$\alpha P_n = \alpha P_{xx}, \quad \beta P_n = \beta P_{yy}, \quad \gamma P_n = \gamma P_{zz} \quad 4.8$$

Ч ѡ Ѣ

$$P_n = P_{xx} = P_{yy} = P_{zz}, \quad 4.9$$

Á ë ï á ñ Ç, Ç¹ »³ É³ Ì³ Ý Ñ » Õ á õ Ì á õ Ù Ý á ñ Ù³ É × Ý ß á õ Ù Ý » ñ Á ï Ì Ù³ É Ì » ï á õ Ù ð á É á ñ á õ Õ á õ Æ Ù á õ Ý Ý » ñ á Ì Ç ñ³ ñ Ñ³ Ì³ ë³ ñ » Ý " Ì³ È Ì³ Í ã » Ý Ñ³ ñ Æ³ Ì Ç ¹ Ç ñ ù á ñ á ß á õ Ù Ç ó (ä³ ë Ì³ É Ç û ñ » Ý ù Á):

² Ù ë á Ç ë á í, Ç¹ »³ É³ Ì³ Ý Ñ » Õ á õ Ì Ç Ù³ Ì » ñ " á õ Æ³ Ù Ç Ý É³ ñ á õ Ù Ý » ñ Ý á õ Ý » Ý Ñ » Ì " Ù³ É ï » ë ù Á`

$$\bar{P}_n = -P \cdot \bar{n}^0, \quad 4.10$$

á ñ ï » Õ P-Ý ï Ì Ù³ É Ì » ï á õ Ù × Ý ß á õ Ù Ý ç, n<sup>0</sup> -³ ñ ï³ ù Ç Ý Ý á ñ Ù³ É Ç Ù Ç³ í á ñ í » Ì í á ñ Á:



# Դասախոսություն 5

# 5.1 $\mathbf{AE}^1 \gg \mathbf{E}^3 \mathbf{I}^3 \mathbf{Y} \tilde{\mathbf{N}} \gg \mathbf{O} \mathbf{a} \mathbf{o} \mathbf{I} \mathbf{C} \beta^3 \mathbf{n} \mathbf{A} \mathbf{U}^3 \mathbf{Y} \mathbf{1} \mathbf{C} \mathbf{y} \gg \mathbf{n} \gg \mathbf{Y} \mathbf{o} \mathbf{C}^3 \mathbf{E}$ $\tilde{\mathbf{N}}^3 \mathbf{i}^3 \mathbf{e}^3 \mathbf{n} \mathbf{a} \mathbf{o} \mathbf{U} \mathbf{Y} \gg \mathbf{n} \mathbf{A}$

(4.1)  $\tilde{\mathbf{N}}^3 \mathbf{i}^3 \mathbf{e}^3 \mathbf{n} \mathbf{U}^3 \mathbf{Y} \mathbf{U} \gg \mathbf{C} \tilde{\mathbf{N}}^3 \mathbf{B} \mathbf{i} \mathbf{C} \mathbf{3} \mathbf{e} \mathbf{Y} \gg \mathbf{E} \mathbf{a} \mathbf{i}$ ,  $\mathbf{a} \mathbf{n} \mathbf{C} \mathbf{1} \gg \mathbf{E}^3 \mathbf{I}^3 \mathbf{Y} \tilde{\mathbf{N}} \gg \mathbf{O} \mathbf{a} \mathbf{o} \mathbf{I} \mathbf{C} \mathbf{U} \mathbf{C}^3 \mathbf{i} \mathbf{a} \mathbf{n} \mathbf{U}^3 \mathbf{I} \gg \mathbf{n} \gg \mathbf{e} \mathbf{C} \mathbf{i} \mathbf{n}^3 \mathbf{3} \mathbf{1} \mathbf{2} \mathbf{1} \mathbf{a} \mathbf{O}$   
 $\mathbf{U}^3 \mathbf{I} \gg \mathbf{n} \mathbf{a} \mathbf{o} \mathbf{A}^3 \mathbf{U} \mathbf{C} \mathbf{Y} \mathbf{a} \mathbf{o} \mathbf{A} \gg \mathbf{n} \mathbf{C} \cdot \mathbf{E} \mathbf{E}^3 \mathbf{i} \mathbf{a} \mathbf{n} \mathbf{i} \gg \mathbf{I} \mathbf{i} \mathbf{a} \mathbf{n} \mathbf{A} \mathbf{a} \mathbf{n} \mathbf{a} \mathbf{B} \mathbf{i} \mathbf{a} \mathbf{o} \mathbf{U} \mathbf{;}$  (4.10)  $\rho \omega \mathbf{Y}^3 \mathbf{O} \mathbf{a} \mathbf{i} \mathbf{;}$  (4.1)  $\mathbf{C} \mathbf{U}^3 \mathbf{I} \gg$   
 $\mathbf{n} \mathbf{a} \mathbf{o} \mathbf{A}^3 \mathbf{U} \mathbf{C} \mathbf{Y} \mathbf{C} \mathbf{Y} \mathbf{i} \gg \mathbf{n}^3 \mathbf{E} \mathbf{a} \mathbf{o} \mathbf{U} \mathbf{I}^3 \mathbf{i}^3 \mathbf{n} \gg \mathbf{E} \mathbf{a} \mathbf{i} \mathbf{O}^3 \mathbf{a} \mathbf{E} \mathbf{a} \mathbf{o} \mathbf{A} \mathbf{U} \mathbf{a} \mathbf{o} \mathbf{Y} \tilde{\mathbf{N}}^3 \mathbf{U}^3 \mathbf{O}^3 \mathbf{U} \mathbf{Y} \mathbf{1} \mathbf{a} \mathbf{o} \mathbf{e} \mathbf{-} \mathbf{u} \mathbf{e} \mathbf{i} \mathbf{n} \mathbf{a} \cdot \mathbf{n}^3 \mathbf{1} \mathbf{e} \mathbf{l} \mathbf{a} \mathbf{o}$   
 $\rho \omega \mathbf{Y}^3 \mathbf{O} \mathbf{C} \mathbf{,} \mathbf{I} \mathbf{e} \mathbf{i}^3 \mathbf{Y}^3 \mathbf{Y} \mathbf{u} \mathbf{;}$

$$\iint_S \bar{\rho}_n ds = - \iint_S P n^0 ds = - \iint_S P [\cos(nx) \bar{i} + \cos(ny) \bar{j} + \cos(nz) \bar{k}] ds = - \iiint_\tau \left( \frac{\partial P}{\partial x} \bar{i} + \frac{\partial P}{\partial y} \bar{j} + \frac{\partial P}{\partial z} \bar{k} \right) d\tau = - \iiint_\tau \text{grad} P d\tau : \quad 5.1$$

$\mathbf{2} \mathbf{U} \mathbf{A} \mathbf{U} \mathbf{i} \gg \mathbf{O}^3 \mathbf{1} \mathbf{n} \gg \mathbf{E} \mathbf{a} \mathbf{i}$  (5.1)  $\mathbf{-} \mathbf{A}$  (4.1)  $\mathbf{-} \mathbf{a} \mathbf{o} \mathbf{U}$ ,  $\mathbf{I} \mathbf{a} \mathbf{o} \mathbf{Y} \gg \mathbf{Y}^3 \mathbf{Y} \mathbf{u} \mathbf{;}$

$$\iiint (\bar{\mathbf{F}} - \bar{\mathbf{a}}) \rho d\tau - \iiint \text{grad} P d\tau = 0 .$$

$\mathbf{I}^3 \mathbf{U} \tilde{\mathbf{N}}^3 \mathbf{B} \mathbf{i} \mathbf{C} \mathbf{3} \mathbf{e} \mathbf{Y} \gg \mathbf{E} \mathbf{a} \mathbf{i}$ ,  $\mathbf{a} \mathbf{n} \mathbf{\tau} \mathbf{-} \mathbf{Y} \mathbf{I}^3 \mathbf{U}^3 \mathbf{U}^3 \mathbf{I}^3 \mathbf{Y} \mathbf{a} \mathbf{u} \mathbf{n} \mathbf{i} \mathbf{C} \mathbf{n} \mathbf{a} \mathbf{o} \mathbf{U} \mathbf{A} \mathbf{;}$

$$\bar{\mathbf{a}} = \bar{\mathbf{F}} - \frac{1}{\rho} \text{grad} P : \quad 5.2$$

añáÛ»İóÇ³Ý»ñáí ¹ñ³Ýù İáõÝ»Ý³Ý Ñ»İÛ³É ï»ëùÁ`

$$\frac{dV_x}{dt} = F_x - \frac{1}{\rho} \frac{\partial P}{\partial x}, \quad \frac{dV_y}{dt} = F_y - \frac{\partial P}{\partial y}, \quad \frac{dV_z}{dt} = F_z - \frac{\partial P}{\partial z} \quad 5.3$$

4uuı

$$\begin{aligned} \frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} &= F_x - \frac{1}{\rho} \frac{\partial P}{\partial x}, \\ \frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} &= F_y - \frac{1}{\rho} \frac{\partial P}{\partial y}, \\ \frac{\partial V_z}{\partial t} + V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z} &= F_z - \frac{1}{\rho} \frac{\partial P}{\partial z}. \end{aligned} \quad 5.4$$

(5.4)-Á Ç¹»³É³İ³Ý Ñ»ÕáõİÇ ß³ñÅÛ³Ý ¹Çý»ñ»ÝóÇ³É Ñ³í³ë³ñáõÛÝ»ñÝ »Ý İ İáñíaõÛ »Ý ¾ÛÉ» ñÇ Ñ³í³ë³ñáõÛÝ»ñ:

Æ¹»³É³İ³Ý Ñ»ÕáõİÇ ß³ñÅÜ³Ý ¹hy»ñ»ÝóÇ³É Ñ³İ³ë³ñáõÜÝ»ñÄ ·É³Ý³İ³Ý r,φ,z İááñ¹ÇÝ³İ³ÛÇÝ Ñ³Ü³İ³ñ·áõÜ ·ñíaõÜ »Ý Ñ»İ³Û³É İ»ëùáı̇

$$\begin{aligned}
 \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_r}{\partial \varphi} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\varphi^2}{r} &= F_r - \frac{1}{\rho} \frac{\partial P}{\partial r}, \\
 \frac{\partial V_\varphi}{\partial t} + V_r \frac{\partial V_\varphi}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_\varphi}{\partial \varphi} + V_z \frac{\partial V_\varphi}{\partial z} + \frac{V_r V_\varphi}{r} &= F_\varphi - \frac{1}{\rho r} \frac{\partial P}{\partial \varphi}, \\
 \frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_z}{\partial \varphi} + V_z \frac{\partial V_z}{\partial z} &= F_z - \frac{1}{\rho} \frac{\partial P}{\partial z},
 \end{aligned}
 \tag{5.5}$$

Çëİ ·Ý¹³ÛÇÝ r, θ, ψ İááñ¹ÇÝ³İ³ÛÇÝ Ñ³Ü³İ³ñ·áõÜ`

$$\begin{aligned}
 \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_r}{\partial \theta} + \frac{V_\psi}{r \sin \theta} \frac{\partial V_r}{\partial \psi} - \frac{V_\theta^2 + V_\psi^2}{r} &= F_r - \frac{1}{\rho} \frac{\partial P}{\partial r}, \\
 \frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} + \frac{V_\psi}{r \sin \theta} \frac{\partial V_\theta}{\partial \psi} + \frac{V_r V_\theta}{r} - \frac{\text{ctg} \theta}{r} V_\psi^2 &= F_\theta - \frac{1}{\rho r} \frac{\partial P}{\partial \theta}, \\
 \frac{\partial V_\psi}{\partial t} + V_r \frac{\partial V_\psi}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\psi}{\partial \theta} + \frac{V_\psi}{r \sin \theta} \frac{\partial V_\psi}{\partial \psi} + \frac{V_r V_\psi}{r} + \frac{\text{ctg} \theta}{r} V_\theta V_\psi &= \\
 = F_\psi - \frac{1}{\rho r \sin \theta} \frac{\partial P}{\partial \psi} &
 \end{aligned}
 \tag{5.6}$$

# 5.2 $\mathcal{A}^1 \gg \mathcal{E}^3 \mathcal{I}^3 \mathcal{Y} \tilde{\mathcal{N}} \gg \mathcal{O} \acute{\mathcal{a}} \mathcal{o} \mathcal{I} \mathcal{C} \quad \mathcal{1} \mathcal{C} \mathcal{Y}^3 \mathcal{U} \mathcal{C} \mathcal{I}^3 \mathcal{U} \mathcal{C} \quad \tilde{\mathcal{N}} \mathcal{C} \mathcal{U} \mathcal{Y}^3 \mathcal{I}^3 \mathcal{Y} \mathcal{E} \mathcal{Y}^1 \mathcal{C} \mathcal{n} \mathcal{Y} \quad \mathcal{I} \quad \mathcal{C} \mathcal{Y}^1 \mathcal{I} \gg \cdot \tilde{\mathcal{n}} \gg \mathcal{E} \acute{\mathcal{a}} \mathcal{I} \tilde{\mathcal{N}} \gg \mathcal{O} \acute{\mathcal{a}} \mathcal{o} \mathcal{I} \mathcal{C} \quad \mathcal{B}^3 \mathcal{n} \mathcal{A} \mathcal{U}^3 \mathcal{Y} \quad \mathcal{1} \mathcal{C} \mathcal{Y} \gg$

$\tilde{\mathcal{n}} \gg \mathcal{Y} \acute{\mathcal{o}} \mathcal{C}^3 \mathcal{E} \tilde{\mathcal{N}}^3 \mathcal{I}^3 \mathcal{e}^3 \mathcal{n} \acute{\mathcal{a}} \mathcal{o} \mathcal{U} \mathcal{Y} \gg \mathcal{n} \mathcal{A} \quad \mathcal{I} \mathcal{n} \mathcal{I}^3 \mathcal{I} \quad \mathcal{3} \mathcal{n} \mathcal{I}^3 \mathcal{U} \mathcal{C} \mathcal{Y} \quad \acute{\mathcal{a}} \mathcal{o} \mathcal{A} \gg \mathcal{n} \mathcal{C} \quad \cdot \quad \mathcal{e} \mathcal{I} \frac{1}{2} \rho \mathcal{Y}^3 \mathcal{I}^3 \mathcal{Y} \quad \acute{\mathcal{a}} \mathcal{o} \quad \gg \frac{1}{2} \mathcal{n}^3 \mathcal{U} \mathcal{C} \mathcal{Y} \quad \acute{\mathcal{a}}^3 \mathcal{U} \mathcal{U}^3 \mathcal{Y} \mathcal{Y} \gg \mathcal{n} \mathcal{C} \quad \mathcal{1} \gg$   
 $\acute{\mathcal{a}} \mathcal{u} \acute{\mathcal{a}} \mathcal{o} \mathcal{U}, \quad \cdot \mathcal{I} \mathcal{Y} \gg \mathcal{E} \tilde{\mathcal{N}} \gg \mathcal{O} \acute{\mathcal{a}} \mathcal{o} \mathcal{I} \mathcal{C} \quad \mathcal{3} \mathcal{n}^3 \cdot \acute{\mathcal{a}} \mathcal{o} \mathcal{A} \mathcal{U}^3 \mathcal{Y}, \quad \times \mathcal{Y} \mathcal{B} \mathcal{U}^3 \mathcal{Y}, \quad \mathcal{E} \mathcal{I} \acute{\mathcal{a}} \mathcal{o} \mathcal{A} \mathcal{U}^3 \mathcal{Y}, \quad \mathcal{C} \mathcal{e} \mathcal{I} \quad \acute{\mathcal{a}} \mathcal{a} \quad \mathcal{C} \frac{1}{2} \acute{\mathcal{a}} \mathcal{A} \gg \mathcal{n} \mathcal{U} \quad \mathcal{B}^3 \mathcal{n} \mathcal{A} \acute{\mathcal{a}} \mathcal{o} \mathcal{U} \mathcal{Y} \gg \mathcal{n} \mathcal{C} \quad \mathcal{1} \gg$   
 $\acute{\mathcal{a}} \mathcal{u} \acute{\mathcal{a}} \mathcal{o} \mathcal{U} \quad \mathcal{Y}^3 \quad \mathcal{C} \gg \mathcal{n} \mathcal{U}^3 \mathcal{U} \mathcal{C} \mathcal{Y} \quad \mathcal{1}^3 \mathcal{B} \mathcal{I} \gg \mathcal{n} \mathcal{A} \quad \mathcal{I}^3 \mathcal{E} \mathcal{I}^3 \mathcal{I} \gg \mathcal{I} \mathcal{C} \quad \mathcal{I} \acute{\mathcal{a}} \mathcal{a} \mathcal{n}^1 \mathcal{C} \mathcal{Y}^3 \mathcal{I} \mathcal{Y} \gg \mathcal{n} \mathcal{C} \acute{\mathcal{o}} \quad \cdot \quad \mathcal{A}^3 \mathcal{U}^3 \mathcal{Y}^3 \mathcal{I} \mathcal{C} \acute{\mathcal{o}}: \quad \mathcal{I} \mathcal{n} \mathcal{I}^3 \mathcal{I} \quad \mathcal{E} \mathcal{Y}^1 \mathcal{C} \mathcal{n} \mathcal{A} \quad \mathcal{E} \acute{\mathcal{a}} \mathcal{o} \mathcal{I} \gg$   
 $\mathcal{E} \acute{\mathcal{a}} \mathcal{o} \quad \tilde{\mathcal{N}}^3 \mathcal{U}^3 \mathcal{n} \quad \mathcal{B}^3 \mathcal{n} \mathcal{A} \mathcal{U}^3 \mathcal{Y} \quad \mathcal{1} \mathcal{C} \quad \mathcal{y} \gg \mathcal{n} \gg \mathcal{Y} \acute{\mathcal{o}} \mathcal{C}^3 \mathcal{E} \tilde{\mathcal{N}}^3 \mathcal{I}^3 \mathcal{e}^3 \mathcal{n} \acute{\mathcal{a}} \mathcal{o} \mathcal{U} \mathcal{Y} \gg \mathcal{n} \mathcal{C} \mathcal{Y} \quad \acute{\mathcal{a}} \gg \mathcal{i} \mathcal{u} \quad \mathcal{I} \quad \mathcal{U} \mathcal{C}^3 \acute{\mathcal{o}} \mathcal{Y} \gg \mathcal{E} \quad \mathcal{3} \mathcal{Y} \mathcal{E} \frac{1}{2} \gg \mathcal{E} \mathcal{C} \acute{\mathcal{a}} \mathcal{o} \mathcal{A} \mathcal{U}^3 \mathcal{Y}$   
 $\tilde{\mathcal{N}}^3 \mathcal{I}^3 \mathcal{e}^3 \mathcal{n} \acute{\mathcal{a}} \mathcal{o} \mathcal{U} \mathcal{A}: \quad \acute{\mathcal{a}} \mathcal{o} \mathcal{O} \mathcal{O}^3 \mathcal{Y} \mathcal{I} \mathcal{U} \acute{\mathcal{a}} \mathcal{o} \mathcal{Y} \quad \mathcal{1} \gg \mathcal{I}^3 \mathcal{n} \mathcal{I} \mathcal{U}^3 \mathcal{Y} \quad \mathcal{I} \acute{\mathcal{a}} \mathcal{a} \mathcal{n}^1 \mathcal{C} \mathcal{Y}^3 \mathcal{I}^3 \mathcal{U} \mathcal{C} \mathcal{Y} \quad \tilde{\mathcal{N}}^3 \mathcal{U}^3 \mathcal{I}^3 \mathcal{n} \cdot \acute{\mathcal{a}} \mathcal{o} \mathcal{U} \quad \mathcal{1} \mathcal{n}^3 \mathcal{Y} \mathcal{u} \quad \gg \mathcal{Y}$

$$\begin{aligned} \frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} &= F_x - \frac{1}{\rho} \frac{\partial P}{\partial x}, \\ \frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} &= F_y - \frac{1}{\rho} \frac{\partial P}{\partial y}, \\ \frac{\partial V_z}{\partial t} + V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z} &= F_z - \frac{1}{\rho} \frac{\partial P}{\partial z}. \end{aligned}$$

5.7

$\acute{\mathcal{a}} \mathcal{n} \acute{\mathcal{a}} \gg \mathcal{e} \quad \mathcal{3} \mathcal{Y} \mathcal{I}^3 \mathcal{E} \quad \tilde{\mathcal{N}} \mathcal{C} \mathcal{Y} \cdot \gg \mathcal{n} \acute{\mathcal{a}} \mathcal{n}^1 \quad \frac{dp}{dt} + \rho \left( \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right) = 0:$   
 $\mathcal{I} \acute{\mathcal{a}} \mathcal{U} \acute{\mathcal{a}} \mathcal{o} \mathcal{A} \mathcal{U} \acute{\mathcal{a}} \mathcal{o} \mathcal{Y} \quad \acute{\mathcal{a}} \mathcal{o} \mathcal{Y} \mathcal{C} \quad \tilde{\mathcal{N}} \gg \mathcal{O} \acute{\mathcal{a}} \mathcal{o} \mathcal{I} \mathcal{Y} \gg \mathcal{n} \mathcal{C} \quad \mathcal{U} \mathcal{C} \quad \mathcal{1}^3 \mathcal{e}, \quad \acute{\mathcal{a}} \mathcal{n} \mathcal{C} \quad \tilde{\mathcal{N}}^3 \mathcal{U}^3 \mathcal{n} \quad \mathcal{E} \mathcal{I} \acute{\mathcal{a}} \mathcal{o} \mathcal{A} \mathcal{U} \acute{\mathcal{a}} \mathcal{o} \mathcal{Y} \mathcal{A} \quad \mathcal{y} \acute{\mathcal{a}} \mathcal{o} \mathcal{Y} \mathcal{I} \acute{\mathcal{o}} \mathcal{C}^3 \quad \mathcal{I} \quad \mathcal{U} \mathcal{C}^3 \mathcal{U} \mathcal{Y}$   
 $\times \mathcal{Y} \mathcal{B} \acute{\mathcal{a}} \mathcal{o} \mathcal{U} \mathcal{C} \acute{\mathcal{o}} \quad \rho = f(P):$

$^2\hat{U}^1 \gg \text{à} \text{ã} \text{ü}, \gg \text{ñ} \mu \tilde{N} \gg \text{Õ} \text{á} \text{õ} \text{İ} \text{Ç} \text{È} \text{í} \text{á} \text{õ} \text{Ã} \text{Û} \text{á} \text{õ} \text{Ý} \text{Á} \text{ý} \text{á} \text{õ} \text{Ý} \text{İ} \text{ó} \text{Ç}^3 \text{¿} \text{á} \text{ã} \text{Ü} \text{Ç}^3 \text{Û} \text{Ý} \times \text{Ý} \text{ß} \text{á} \text{õ} \text{Ü} \text{Ç} \text{ó}, \text{³} \hat{U} \text{É} \text{Ç} \gg$   
 $\tilde{N} \text{Ü}^3 \text{è} \text{İ} \text{Ç} \times \text{³} \text{Ý} \text{Ç} \text{ó}, \text{İ} \text{Ç} \times \text{³} \text{İ} \text{Ç} \tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{Á} \text{á} \text{õ} \text{Ý} \text{Ç} \text{³} \text{í} \gg \text{É} \text{Ç} \mu^3 \text{ñ}^1 \text{İ} \gg \text{è} \text{ü}, \text{á} \text{ñ} \text{á} \text{Ý} \text{ó} \text{Ç} \text{ó} \text{á}^3 \text{ñ}^{\frac{1}{2}} \cdot \text{á} \text{õ} \text{Û} \text{Ý} \text{Á} \text{Ø} \gg \text{Ý}^1 \gg \text{É} \gg \text{''} -$   
 $\hat{I} \text{É}^3 \text{á} \gg \text{Û} \text{ñ} \text{á} \text{Ý} \text{Ç} \tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{Ý} \text{¿}$

$$P = \rho RT, \quad 5.8$$

$\text{Ð} \gg \text{Õ} \text{á} \text{õ} \text{İ} \text{Ý} \gg \text{ñ} \text{Ç} \text{³} \hat{U} \text{ë} \text{İ} \gg \text{è}^3 \text{İ} \text{Á} \text{Á} \text{Ý}^1 \text{á} \text{õ} \text{Ý} \text{í} \text{¿} \text{³} \text{Ý} \text{í}^3 \text{Ý} \gg \text{É} \rho \omega \text{ñ} \text{á} \text{İ} \text{É} \text{Ç} \text{Ý} \tilde{N} \gg \text{Õ} \text{á} \text{õ} \text{İ} \text{Ý} \gg \text{ñ} \text{:} \text{°} \text{ñ} \rho \text{ß}^3 \text{ñ} \text{Á} \text{á} \text{õ} \text{Ü} \text{Á} \text{Ç}^{\frac{1}{2}} \text{á} \text{Ã} \gg \text{ñ} \text{Ü}$   
 $\text{¿} \text{`} \text{T} = \text{const}, \text{³} \text{á}^3 \text{(5.8)} \tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{Á} \text{(5.7)} \tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{Ý} \gg \text{ñ} \text{Ç} \tilde{N} \gg \text{İ} \text{İ}^{\frac{1}{2}} \text{Ü} \text{á} \text{õ} \text{Ü} \text{¿} \tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{Ý} \gg \text{ñ} \text{Ç} \text{÷}^3 \text{İ}$   
 $\tilde{N}^3 \text{Ü}^3 \text{İ}^3 \text{ñ} \cdot V_x, V_y, V_z, P, \rho \text{³} \text{Ý} \tilde{N}^3 \text{Û} \text{İ} \text{Ý} \gg \text{ñ} \text{Ç} \text{Ý} \text{İ}^3 \text{İ} \text{Ü}^3 \text{Ü} \rho \text{:} \text{²} \hat{U} \text{Ý} \text{¹} \gg \text{á} \text{ü} \text{á} \text{õ} \text{Ü}, \gg \text{ñ} \rho \text{ß}^3 \text{ñ} \text{Á} \text{á} \text{õ} \text{Ü} \text{Á} \text{Ç}^{\frac{1}{2}} \text{á} \text{Ã} \gg \text{ñ} \text{Ü} \text{ã} \text{¿},$   
 $\text{³} \hat{U} \text{ë} \text{Ç} \text{Ý} \text{ü} \text{Ý} \text{İ} - \text{Ý}^y \text{÷} \text{á} \text{÷} \text{á} \text{È} \text{í} \text{á} \text{õ} \text{Ü} \text{¿}, \tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{Ý} \gg \text{ñ} \text{Ç} \text{(5.7)} \text{''} \text{(5.8)} \tilde{N}^3 \text{Ü}^3 \text{İ}^3 \text{ñ} \cdot \text{Á} \text{÷}^3 \text{İ} \text{ã} \text{¿}, \text{ù}^3 \text{Ý} \text{Ç} \text{á} \text{ñ} \text{á} \text{õ} \text{Ý} \gg \text{Ý} \text{ü}$   
 $\tilde{N} \text{Ç} \text{Ý} \cdot \tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{í} \gg \text{ó}^3 \text{Ý} \tilde{N}^3 \text{Û} \text{İ} \text{Ý} \gg \text{ñ} \text{á} \text{İ} \text{`} V_x, V_y, V_z, P, \rho, T: \text{Ü} \text{á} \text{õ} \text{Û} \text{Ý} \text{÷} \text{á} \text{÷} \text{á} \text{È}^3 \text{İ}^3 \text{Ý} \text{Ý} \gg \text{ñ} \text{á} \text{İ} \text{·} \text{ñ} \text{í}^3 \text{İ} \text{í} \gg \text{ó} \gg \text{ñ} \text{á} \text{ñ}^1$   
 $\tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{Ý} \text{¿} \text{¿} \text{Ý} \gg \text{ñ} \cdot \text{Ç}^3 \text{Û} \text{Ç} \tilde{N} \text{á} \text{è} \text{ü} \text{Ç} \tilde{N}^3 \text{í}^3 \text{ë}^3 \text{ñ} \text{á} \text{õ} \text{Ü} \text{Á}$

$$\varepsilon = C_{v\rho} \frac{dT}{dt} - \frac{jP}{\rho} \frac{d\rho}{dt}, \quad 5.9$$

# Դասախոսություն 6

# 6.1 $\mathcal{A}^1 \gg \mathcal{E}^3 \mathcal{I}^3 \mathcal{Y} \mathcal{N} \gg \mathcal{O} \mathcal{a} \mathcal{o} \mathcal{I} \mathcal{C} \mathcal{B}^3 \mathcal{n} \mathcal{A} \mathcal{U}^3 \mathcal{Y}^1 \mathcal{C} \mathcal{y} \gg \mathcal{n} \gg \mathcal{Y} \mathcal{o} \mathcal{C}^3 \mathcal{E} \mathcal{N}^3 \mathcal{i}^3 \mathcal{e}^3 \mathcal{n} \mathcal{a} \mathcal{o} \mathcal{U} \mathcal{Y} \gg \mathcal{n} \mathcal{C}$ $\mathcal{C} \mathcal{Y} \mathcal{i} \gg \cdot \mathcal{n} \mathcal{a} \mathcal{o} \mathcal{U} \mathcal{A}$

$\mathcal{N}^3 \mathcal{n}^{1/2} \cdot \mathcal{a} \mathcal{o} \mathcal{U} \mathcal{Y} \mathcal{C} \mathcal{Y} \mathcal{i} \gg \cdot \mathcal{n}^3 \mathcal{E} \mathcal{Y} \gg \mathcal{n}$

3)  $\mathcal{C} \mathcal{Y} \mathcal{i} \gg \mathcal{n} \cdot \mathcal{C}^3 \mathcal{U} \mathcal{C} \mathcal{C} \mathcal{Y} \mathcal{i} \gg \cdot \mathcal{n}^3 \mathcal{E} \mathcal{A}$ ,  $\mathcal{a} \mathcal{n} \mathcal{A} \mathcal{e} \mathcal{i}^3 \mathcal{o} \mathcal{i} \mathcal{a} \mathcal{o} \mathcal{U} \mathcal{C} \mathcal{C}^1 \gg \mathcal{E}^3 \mathcal{I}^3 \mathcal{Y} \mathcal{N} \gg \mathcal{O} \mathcal{a} \mathcal{o} \mathcal{I} \mathcal{C} \mathcal{i} \mathcal{n}^3 \mathcal{n}^3 \mathcal{i}^3 \mathcal{N} \gg \mathcal{i}^3 \mathcal{U}^3 \mathcal{E} \mathcal{e}^3 \mathcal{N} \mathcal{U}^3 \mathcal{Y}^3 \mathcal{I} \mathcal{a} \mathcal{o} \mathcal{U} \mathcal{Y} \gg \mathcal{n} \mathcal{C}$   
 $^1 \gg \mathcal{a} \mathcal{u} \mathcal{a} \mathcal{o} \mathcal{U}$

1.  $\mathcal{B}^3 \mathcal{n} \mathcal{A} \mathcal{a} \mathcal{o} \mathcal{U} \mathcal{A} \mathcal{e} \mathcal{i}^3 \mathcal{o} \mathcal{C} \mathcal{a} \mathcal{Y}^3 \mathcal{n} \mathcal{C}$

2.  $^3 \mathcal{n} \mathcal{i}^3 \mathcal{u} \mathcal{C} \mathcal{Y} \mathcal{a} \mathcal{o} \mathcal{A}^3 \mathcal{U} \mathcal{C} \mathcal{Y} \mathcal{B}^3 \mathcal{i}^3 \mathcal{Y} \mathcal{a} \mathcal{o} \mathcal{Y} \mathcal{C} \mathcal{a} \mathcal{a} \mathcal{i} \gg \mathcal{Y} \mathcal{o} \mathcal{C}^3 \mathcal{E}$ ,

$$\bar{\mathbf{F}} = -\text{grad} \Pi$$

$\mathcal{a} \mathcal{n} \mathcal{i} \gg \mathcal{O} \mathcal{N} \mathcal{A} \mathcal{N} \gg \mathcal{O} \mathcal{a} \mathcal{o} \mathcal{I} \mathcal{C} \mathcal{U} \mathcal{C}^3 \mathcal{i} \mathcal{a} \mathcal{n}$

$^2 \mathcal{U} \mathcal{e} \mathcal{e}^3 \mathcal{N} \mathcal{U}^3 \mathcal{Y}^3 \mathcal{I} \mathcal{a} \mathcal{o} \mathcal{U} \mathcal{Y} \gg \mathcal{n} \mathcal{C} \mathcal{C}^1 \gg \mathcal{a} \mathcal{u} \mathcal{a} \mathcal{o} \mathcal{U}$   $\bar{\mathbf{a}} = \bar{\mathbf{F}} - \frac{1}{\rho} \text{grad} P$   $\mathcal{B}^3 \mathcal{n} \mathcal{A} \mathcal{U}^3 \mathcal{Y}^1 \mathcal{C} \mathcal{y} \gg \mathcal{n} \gg \mathcal{Y} \mathcal{o} \mathcal{C}^3 \mathcal{E}$   
 $\mathcal{N}^3 \mathcal{i}^3 \mathcal{e}^3 \mathcal{n} \mathcal{U}^3 \mathcal{Y} \gg \mathcal{n} \mathcal{i} \mathcal{a} \mathcal{o} \mathcal{I} \mathcal{a} \mathcal{O} \mathcal{U} \gg \mathcal{n} \mathcal{A} \mathcal{e} \mathcal{i}^3 \mathcal{E} \mathcal{U}^3 \mathcal{n}^3 \mathcal{a} \gg \mathcal{e} \mathcal{p} \mathcal{w} \mathcal{q} \mathcal{u}^3 \mathcal{a} \mathcal{o} \mathcal{I} \mathcal{I} \gg \mathcal{E} \mathcal{a} \mathcal{i}$

$\mathcal{C} \mathcal{Y} \mathcal{i} \gg \cdot \mathcal{n} \gg \mathcal{E} \mathcal{a} \mathcal{i} \mathcal{N} \gg \mathcal{O} \mathcal{a} \mathcal{o} \mathcal{I} \mathcal{C} \mathcal{T} \mathcal{i} \mathcal{C} \mathcal{n} \mathcal{a} \mathcal{o} \mathcal{U} \mathcal{A} \mathcal{a} \mathcal{i}$ ,  $\mathcal{N}^3 \mathcal{B} \mathcal{i} \mathcal{C} \mathcal{e} \mathcal{Y} \gg \mathcal{E} \mathcal{a} \mathcal{i}$ ,  $\mathcal{a} \mathcal{n}$   $\bar{\mathbf{a}} d\mathbf{r} = \left( \frac{d\bar{V}}{dt} \right) d\mathbf{r} = \bar{V} d\bar{V} = d\left( \frac{V^2}{2} \right)$

$\mathcal{A} \mathcal{n} \mathcal{a} \mathcal{B} \mathcal{O}^3 \mathcal{a} \mathcal{E} \mathcal{a} \mathcal{o} \mathcal{A} \mathcal{U} \mathcal{a} \mathcal{o} \mathcal{Y} \mathcal{Y} \gg \mathcal{n} \mathcal{C} \mathcal{o} \mathcal{N} \gg \mathcal{i} \mathcal{a} \mathcal{I} \mathcal{e} \mathcal{i}^3 \mathcal{Y}^3 \mathcal{Y} \mathcal{u}$

$$dE_q + d\Pi_* = - \iiint_{\tau} \left( \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz \right) d\tau : \quad 6.1$$



2.  $E_4 = \frac{1}{2} \iiint_{\tau} \rho V^2 d\tau$ ,  $\Pi_* = \iiint_{\tau} P \rho d\tau$  : 6.2

(6.1) -  $\frac{dx}{dt} = V_x$ ,  $\frac{\partial p}{\partial x} V_x = \frac{\partial}{\partial x} (P V_x) - P \frac{\partial V_x}{\partial x}$

$$\frac{dx}{dt} = V_x, \quad \frac{\partial p}{\partial x} V_x = \frac{\partial}{\partial x} (P V_x) - P \frac{\partial V_x}{\partial x}$$

3.  $\frac{d}{dt} (E_4 + \Pi_*) = - \iint_S \rho V_n ds + \iiint_{\tau} P \operatorname{div} \bar{V} d\tau$  : 6.3

$$\frac{d}{dt} (E_4 + \Pi_*) = - \iint_S \rho V_n ds + \iiint_{\tau} P \operatorname{div} \bar{V} d\tau : 6.3$$

ρ)  $\frac{d}{dt} \int_{\Omega} \rho \mathbf{v} \cdot d\mathbf{V} = \int_{\Omega} \rho \mathbf{f} \cdot d\mathbf{V} + \int_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dA$ ,  $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$ ,  $\mathbf{n} = \mathbf{n}(\mathbf{x}, t)$

1.  $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$ ,  $\mathbf{n} = \mathbf{n}(\mathbf{x}, t)$
2.  $\mathbf{F} = -\text{grad}\Pi$ ,  $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$ ,  $\mathbf{n} = \mathbf{n}(\mathbf{x}, t)$
3.  $\rho = \rho(P)$ ,  $P = \varphi(\rho)$ :  
 $\frac{d}{dt} \int_{\Omega} \rho \mathbf{v} \cdot d\mathbf{V} = \int_{\Omega} \rho \mathbf{f} \cdot d\mathbf{V} + \int_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dA$

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \frac{1}{2} \text{grad} V^2 - \bar{\mathbf{v}} \times \text{rot} \bar{\mathbf{v}} = \mathbf{F} - \frac{1}{\rho} \text{grad} P, \quad 6.4$$

Υβίβί  $\frac{d}{dt} \int_{\Omega} \rho \mathbf{v} \cdot d\mathbf{V} = \int_{\Omega} \rho \mathbf{f} \cdot d\mathbf{V} + \int_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dA$ , (6.4)- $\int_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dA$

$$\text{grad} \left( \frac{V^2}{2} + \Pi + \Phi \right) = \bar{\mathbf{v}} \times \text{rot} \bar{\mathbf{v}}, \quad 6.5$$

ηηηη

$$\Phi = \int \frac{dP}{\rho}, \quad \frac{1}{\rho} \text{grad} P = \text{grad} \Phi : \quad 6.6$$

(6.5)- $\int_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dA = \int_{\Omega} \rho \mathbf{f} \cdot d\mathbf{V} + \int_{\partial\Omega} \rho \mathbf{v} \cdot \mathbf{n} dA$ ,  $\mathbf{v} = \mathbf{v}(\mathbf{x}, t)$ ,  $\mathbf{n} = \mathbf{n}(\mathbf{x}, t)$

ÍáõÝ»Ý³Ýù`

$$\text{grad} \left( \frac{V^2}{2} + \Pi + \Phi \right) \overline{e^0} = \frac{\partial}{\partial e} \left( \frac{V^2}{2} + \Pi + \Phi \right) = 0 \quad 6.7$$

ωjuuntηhg

$$\frac{V^2}{2} + \Pi + \Phi = \text{const} = c$$

κωδ

$$\frac{V^2}{2} + \Pi + \int \frac{dP}{\rho} = c : \quad 6.8$$

²Ýë»ÕÙ»ÉÇ Ñ»ÕáõİÇ ¹»åùáõÙ (6.8)-Çó ÍáõÝ»Ý³Ýù`

$$\frac{V^2}{2} + \Pi + \frac{P}{\rho} = C : \quad 6.9$$

°Ã» ³½¹áÕ ³ñĩ³ùÇÝ áõÅ»ñÁ ÙÇ³ÛÝ Í³ÝñáõÃÛ³Ý áõÅ»ñ »Ý, ³å³ Π=g · z `` (6.9)-Çó ÍáõÝ»Ý³Ýù`

$$\frac{V^2}{2g} + z + \frac{P}{\rho g} = C : \quad 6.10$$

´»éÝáõÉÇÇ ÇÝİ»·ñ³ÉÇ ζÝ»ñ·»İÇİ ÇÙ³ëİÁ å³ñ½»Éáõ Ñ³Û³ñ (6.10)-Á Ý»ñĩ³Û³óÝ»Ýù`

$$\frac{V^2}{2} + zg + \frac{P}{\rho} = c \quad 6.11$$

´»éÝáõÉÇÇ ÇÝİ»·ñ³ÉÇ ζÝ»ñ·»İÇİ ÇÙ³ëİÝ ³ÛÝ ζ, áñ İİÛ³É ÑáëùÇ ·ÍÇ »ñĩ³Û³ùáÍ ÙÇ³íañ ½³Ý·Í³Í áõÝ»  
óáÕ Ù³ëÝÇİÇ İÇÝ»İÇİ, ááİ»ÝóÇ³É `` ×ÝßÛ³Ý ζÝ»ñ·Ç³Ý»ñÇ ·áõÛ³ñÁ Å³Û³Ý³İÇ ó³Ýİ³óÍ å³ÑÇ Ñ³ëİ³íaõÝ  
ζ:



(6.14) - Á ĩáãíaõÛ ħ ĩáßçç çÝi»·ñ³É, Áëï áñç »Ã» ç¹»³É³ĩ³Ý Ñ»Õáõĩç ß³ñÅÛ³Ý Å³Û³Ý³ĩ ρωί³ñ³ĩ³ĩ »Ý í» ñÁ Ýβί³ĩ á³ÛÛ³ÝÝ»ñÁ, ³á³ (6.14) Ñ³í³ë³ñáõÃÛ³Ý Ó³Ë Û³ëç ãáñë ³Ý¹³ÛÝ»ñç ·áõÛ³ñÁ ĩĩÛ³É á³ÑçÝ Ñ³ëĩ³íaõÝ ħ ĩçñáõÛÃç μάÉáñ ĩ»ĩ»ñáõÛ: °ñρ Ñ»ÕáõĩÁ ³Ýë»ÕÛ»Éç ħ " »ñρ ωϩηηη áõÅ»ñÁ ĩ³ÝñáõÃÛ³Ý áõÅ»ñ »Ý, (6.14) -Á ĩÁÝ¹áõÝç Ñ»ĩÛ³É ĩ»ëùÁ

$$\frac{\partial \varphi}{\partial t} + \frac{V^2}{2} + gz + \frac{P}{\rho} = f(t): \quad 6.15$$

Ä³Û³Ý³ĩç ĩĩÛ³É á³ÑçÝ f(t) ýáõÝĩóç³Ûç ³ñÅ»ùÁ ĩáñáβίç, »Ã» ³Û¹ á³ÑçÝ Ñ³ÛĩÝç ÉçÝç (6.14)-ç Ó³Ë Û³ëç ³ñÅ»ùÁ ĩçñáõÛÃç áñ ħ ĩ»íaõÛ: ²Ýë»ÕÛ»Éç Ñ»Õáõĩç ááĩ»Ýóç³É ß³ñÅÛ³Ý Å³Û³Ý³ĩ φ ýáõÝĩóç³Ý ρωί³ñ³ñáõÛ ħ É³áÉ³ëç

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \quad 6.16$$

η) «éÝáõÉÇ-¾ÛÉ»ñÇ ÇÝí»·ñ³ÉÁ, áñÁ ëï³óíáõÙ ; ÍáβÇÇ ÇÝí»·ñ³ÉÇ Ñ³Û³ñ ³ÝÑñ³Å»βï å³ÛÛ³ÝÝ»ñÇÝ ³í»É³óÝ»Éáí β³ñÅÛ³Ý ëï³óÇáÝ³ñáõÃÛ³Ý å³ÛÛ³ÝÁ: ²Û¹ å³ÛÛ³ÝÝ»ñÇ ¹»åùáõÙ (6.14)-Çó ÍáõÝ»Ý³Ýù

$$\frac{V^2}{2} + \Pi + \int \frac{dP}{\rho} = c: \quad 6.17$$

(6.17)-Á ÍáãíáõÙ ; «éÝáõÉÇ-¾ÛÉ»ñÇ ÇÝí»·ñ³ÉÇ, (6.17)-Ç Ó³Ë Û³ëáõÙ ·íÝíáÕ »ñ»ù Û»ÍáõÃÛáõÝÝ»ñÇ ·áõÛ³ñÁ ÛÝáõÙ ; Ñ³ëï³íáõÝ íÇñáõÛÃÇ ó³Ýí³ó³í Í»íáõÙ Å³Û³Ý³íÇ ó³Ýí³ó³í å³ÑÇÝ: °ñμ ωqηάÕ áõÅ»ñÁ ÍÝñáõÃÛ³Ý áõÅ»ñ »Ý, Çëï Ñ»ÕáõÍÁ ³Ýë»ÕÛ»ÉÇ ;, (6.17)-Çó ÍáõÝ»Ý³Ýù

$$\frac{V^2}{2} + gz + \frac{P}{\rho} = c: \quad 6.18$$

²ÝÑñ³Å»βï ; Ýβ»É, áñ (6.18)-Á ÛÇ³ÛÝ í»ëùáí ; ÝÛ³Ý «éÝáõÉÇÇ (6.11) ÇÝí»·ñ³ÉÇÝ: ,ñ³Ýù ;³å»ë Çñ³ñÇó ËÇëï í³ñptñíáõÙ »Ý: