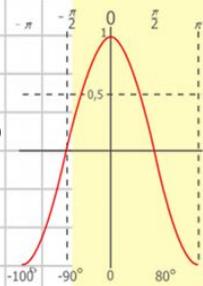
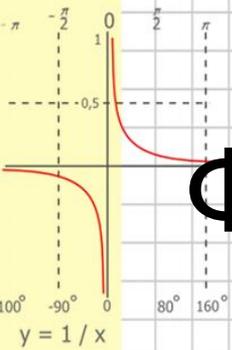
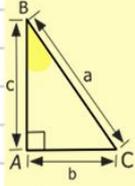
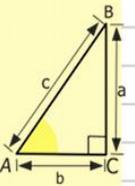
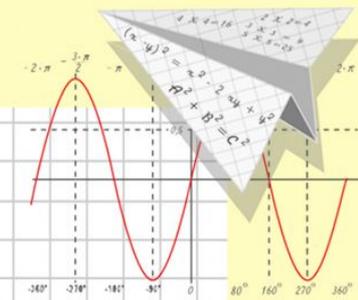
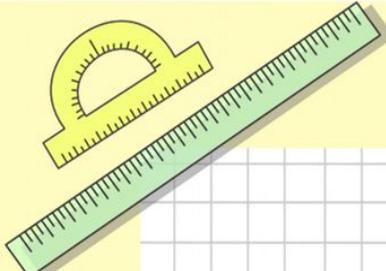


# Математик

## а

### Занятие 45.

# Формулы двойного угла. Формулы половинного угла.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \end{array}$$

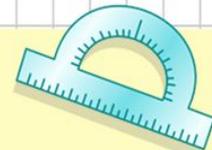


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

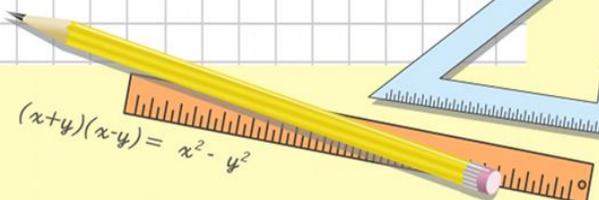
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$



$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$



$$(x+y)(x-y) = x^2 - y^2$$

# Формулы двойного угла

Если в формулах сложения принять  $\beta = \alpha$ , то получим:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha + \alpha) = \sin \alpha \cos \alpha + \cos \alpha \sin \alpha$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

Аналогично:  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \cdot \operatorname{tg} \beta}$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

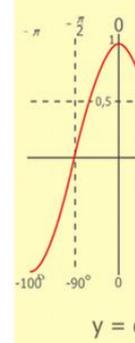
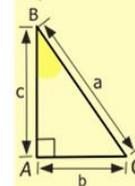
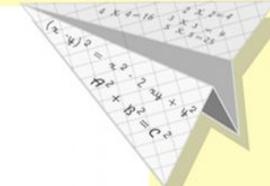
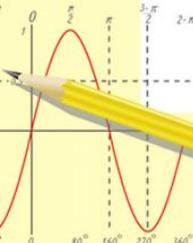
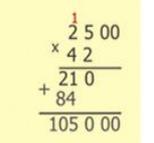
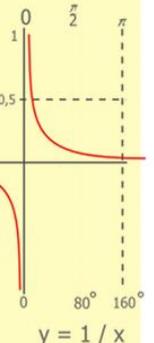
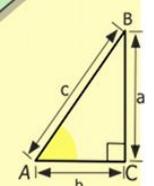
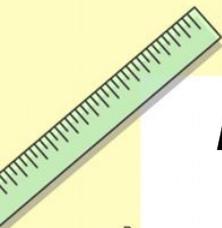
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

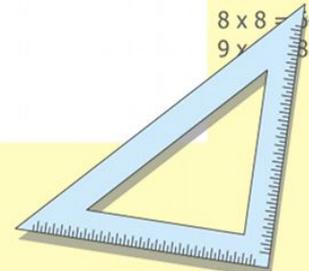
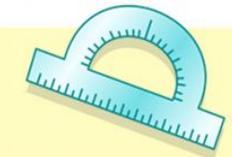
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



# Формулы двойного угла

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\sin 20^\circ = 2 \sin 10^\circ \cos 10^\circ$$

$$\cos 20^\circ = \cos^2 10^\circ - \sin^2 10^\circ$$

$$\sin 6x = 2 \sin 3x \cos 3x$$

$$\cos 6x = \cos^2 3x - \sin^2 3x$$

$$\sin \frac{2\pi}{5} = 2 \sin \frac{\pi}{5} \cos \frac{\pi}{5}$$

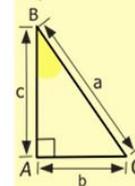
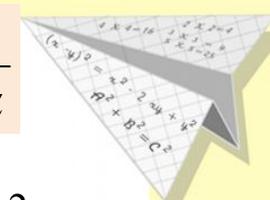
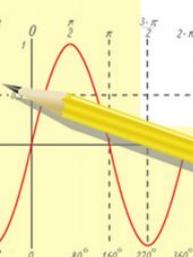
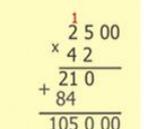
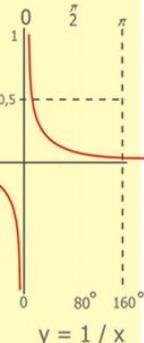
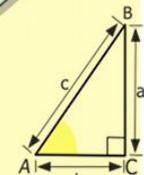
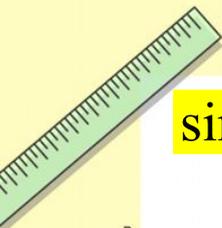
$$\cos \frac{2\pi}{5} = \cos^2 \frac{\pi}{5} - \sin^2 \frac{\pi}{5}$$

$$\operatorname{tg} 20^\circ = \frac{2 \operatorname{tg} 10^\circ}{1 - \operatorname{tg}^2 10^\circ}$$

$$\operatorname{tg} \frac{2\pi}{5} = \frac{2 \operatorname{tg} \frac{\pi}{5}}{1 - \operatorname{tg}^2 \frac{\pi}{5}}$$

$$\operatorname{tg} 6x = \frac{2 \operatorname{tg} 3x}{1 - \operatorname{tg}^2 3x}$$

$$\operatorname{ctg} 6x = ???$$

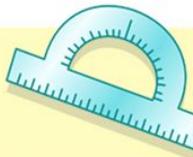


- 2 x 2 = 4
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$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

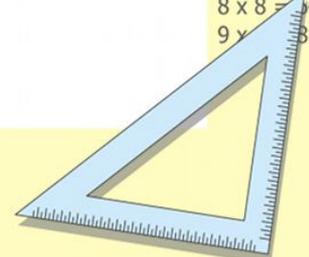
$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



# Формулы двойного угла

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$



$$2 \sin 25^\circ \cos 25^\circ = \sin 50^\circ$$

$$\cos^2 15^\circ - \sin^2 15^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$2 \sin \frac{x}{2} \cos \frac{x}{2} = \sin x$$

$$\cos^2 5x - \sin^2 5x = \cos 10x$$

$$2 \sin \frac{\pi}{8} \cos \frac{\pi}{8} = \sin \frac{2\pi}{8} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

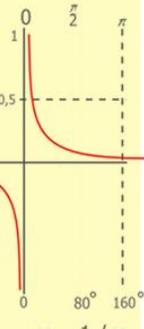
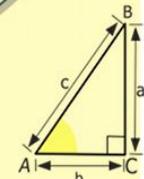
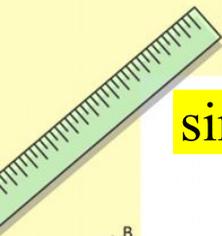
$$\cos^2 \frac{2\pi}{9} - \sin^2 \frac{2\pi}{9} = \cos \frac{4\pi}{9}$$

$$\frac{2 \operatorname{tg} 35^\circ}{1 - \operatorname{tg}^2 35^\circ} = \operatorname{tg} 70^\circ$$

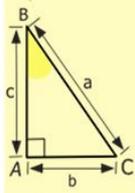
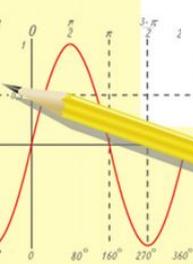
$$\frac{2 \operatorname{tg} 0,3x}{1 - \operatorname{tg}^2 0,3x} = \operatorname{tg} 0,6x$$

$$\frac{1 - \operatorname{tg}^2 4x}{2 \operatorname{tg} 4x} = ???$$

$$\frac{2 \operatorname{tg} \frac{3\pi}{14}}{1 - \operatorname{tg}^2 \frac{3\pi}{14}} = \operatorname{tg} \frac{6\pi}{14} = \operatorname{tg} \frac{3\pi}{7}$$



$\frac{1}{2} 500$   
 $\times 42$   
 $\hline 210$   
 $+ 84$   
 $\hline 105000$



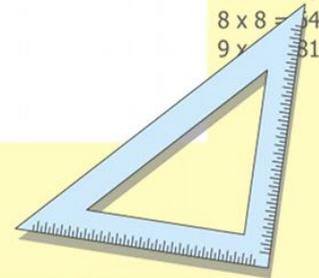
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$\sin 90^\circ = 1$



$$\begin{cases} x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



# Формулы двойного угла

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

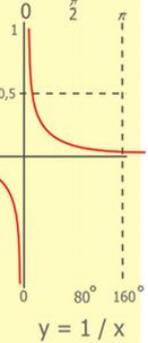
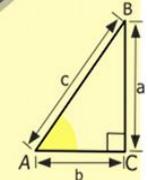
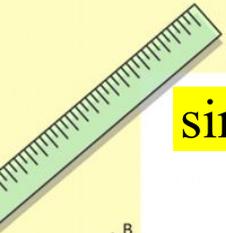
$$\sin 15^\circ \cos 15^\circ = \frac{1}{2} \cdot 2 \sin 15^\circ \cos 15^\circ = \frac{1}{2} \cdot \sin 30^\circ = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\sin^2 5x - \cos^2 5x = -\cos 10x$$

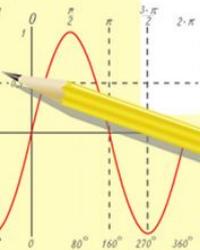
$$\frac{10 \operatorname{tg} 18^\circ}{1 - \operatorname{tg}^2 18^\circ} = 5 \cdot \frac{2 \operatorname{tg} 18^\circ}{1 - \operatorname{tg}^2 18^\circ} = 5 \operatorname{tg} 36^\circ$$

$$\begin{aligned} \cos^2 50^\circ - \cos^2 40^\circ &= |\cos 40^\circ = \sin(90^\circ - 40^\circ) = \sin 50^\circ| = \\ &= \cos^2 50^\circ - \sin^2 50^\circ = \cos 100^\circ = \cos(90^\circ + 10^\circ) = -\sin 10^\circ \end{aligned}$$

$$4 \sin^2 \frac{x}{2} \cdot \cos^2 \frac{x}{2} = \left( 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} \right)^2 = \left( \sin 2 \cdot \frac{x}{2} \right)^2 = \sin^2 x$$



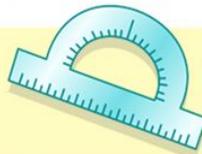
$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 2500 \\ + 210 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

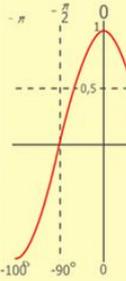
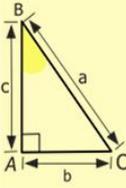
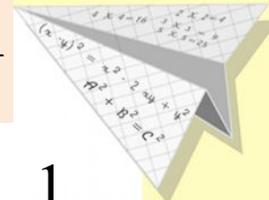


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

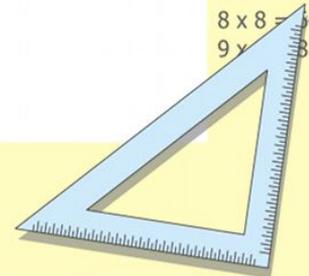
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

$$\begin{aligned} 2 \times 2 &= 4 \\ 3 \times 3 &= 9 \\ 4 \times 4 &= 16 \\ 5 \times 5 &= 25 \\ 6 \times 6 &= 36 \\ 7 \times 7 &= 49 \\ 8 \times 8 &= 64 \\ 9 \times 9 &= 81 \end{aligned}$$



# Формулы двойного угла

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

Упростить:

$$\frac{(\sin \alpha + \cos \alpha)^2 - \sin 2\alpha}{\cos 2\alpha + 2 \sin^2 \alpha} = \frac{\sin^2 \alpha + 2 \sin \alpha \cos \alpha + \cos^2 \alpha - 2 \sin \alpha \cos \alpha}{\cos^2 \alpha - \sin^2 \alpha + 2 \sin^2 \alpha} =$$

$$= \frac{\sin^2 \alpha + \cos^2 \alpha}{\cos^2 \alpha + \sin^2 \alpha} = 1$$

$$\frac{10 \sin 40^\circ \sin 50^\circ}{\cos 10^\circ} = |\sin 50^\circ = \cos(90^\circ - 40^\circ) = \cos 40^\circ| =$$

$$= 5 \cdot \frac{2 \sin 40^\circ \cos 40^\circ}{\cos 10^\circ} = 5 \cdot \frac{\sin 80^\circ}{\cos 10^\circ} = |\sin 80^\circ = \cos 10^\circ| =$$

$$= 5 \cdot \frac{\cos 10^\circ}{\cos 10^\circ} = 5$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

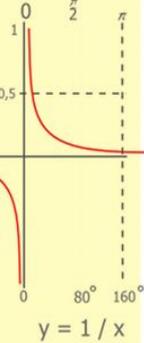
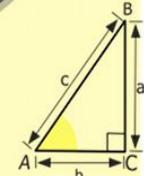
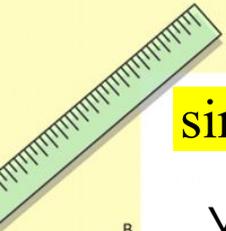
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

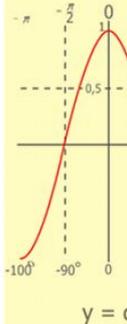
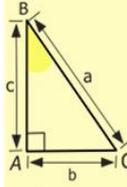
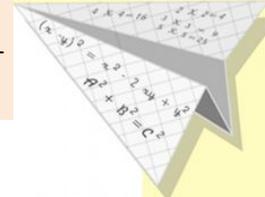
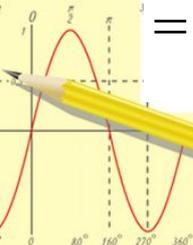
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

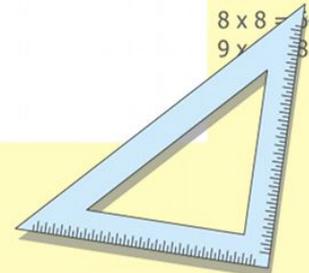
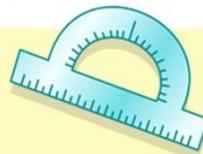
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \ 5 \ 00 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105 \ 000 \end{array}$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



# Формулы половинного угла

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

$$\operatorname{tg}^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 3x = \frac{1 + \cos 6x}{2}$$

$$\operatorname{tg}^2 \frac{x}{4} = \frac{1 - \cos \frac{x}{2}}{1 + \cos \frac{x}{2}}$$

$$\sin^2 10^\circ = \frac{1 - \cos 20^\circ}{2}$$

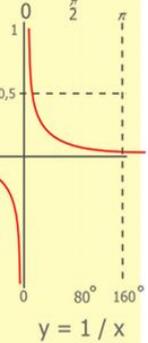
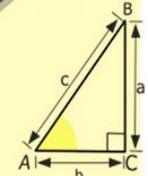
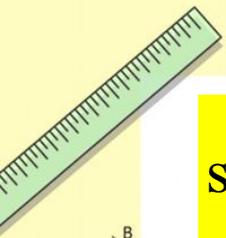
$$\cos^2 40^\circ = \frac{1 + \cos 80^\circ}{2}$$

$$\operatorname{tg}^2 32^\circ = \frac{1 - \cos 64^\circ}{1 + \cos 64^\circ}$$

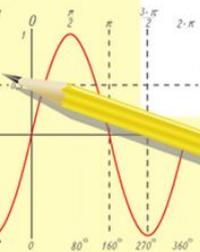
$$\sin^2 \frac{2\pi}{7} = \frac{1 - \cos \frac{4\pi}{7}}{2}$$

$$\cos^2 \frac{3\pi}{14} = \frac{1 + \cos \frac{3\pi}{7}}{2}$$

$$\operatorname{tg}^2 0,2\pi = \frac{1 - \cos 0,4\pi}{1 + \cos 0,4\pi}$$



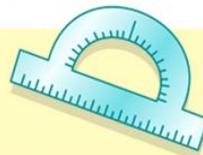
$\frac{1}{2} \begin{array}{r} 500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

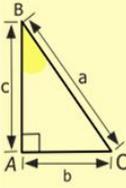
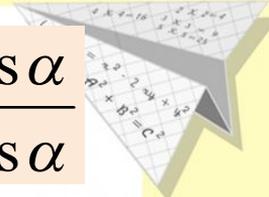


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

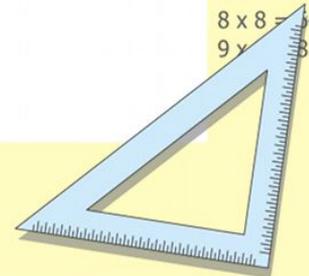
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



y = cos

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



# Формулы половинного угла

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

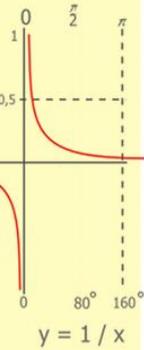
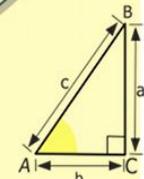
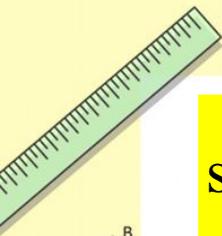
$$\operatorname{tg}^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

Вычислить:

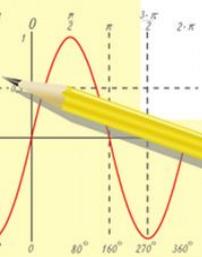
$$\cos \frac{\pi}{8} = \sqrt{\frac{1 + \cos \frac{\pi}{4}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\left(1 + \frac{\sqrt{2}}{2}\right) \cdot 2}{2 \cdot 2}} = \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

Упростить:

$$\frac{1 - 2 \cos 6x + \cos^2 6x}{4} = \frac{(1 - \cos 6x)^2}{4} = \left(\frac{1 - \cos 6x}{2}\right)^2 = (\sin^2 3x)^2 = \sin^4 3x$$



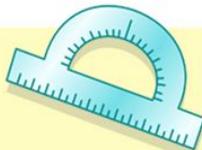
$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

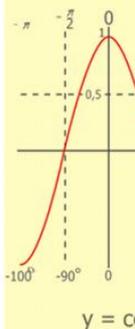
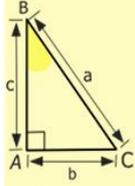
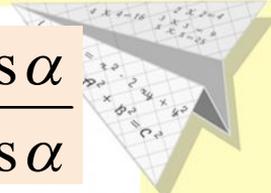


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

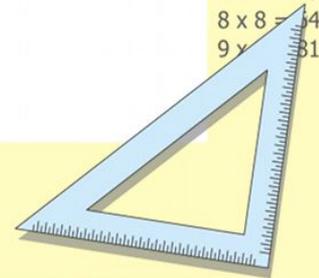
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

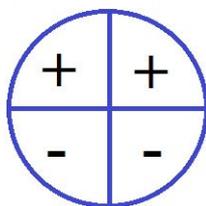


- 2 x 2 = 4
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- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
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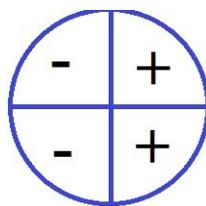


# Справочные материалы

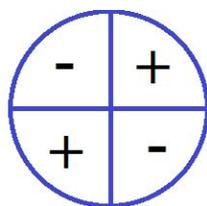
$\alpha$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\operatorname{tg} \alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—	0	—	0
$\operatorname{ctg} \alpha$	—	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	—	0	—



$\sin \alpha$



$\cos \alpha$



$\operatorname{tg} \alpha \quad \operatorname{ctg} \alpha$

$$\sin(-\alpha) = -\sin \alpha$$

$$\cos(-\alpha) = \cos \alpha$$

$$\operatorname{tg}(-\alpha) = -\operatorname{tg} \alpha$$

$$\operatorname{ctg}(-\alpha) = -\operatorname{ctg} \alpha$$

$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

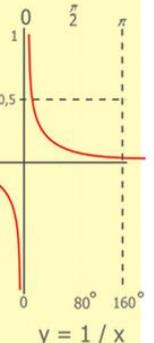
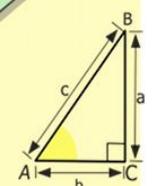
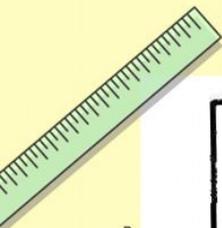
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

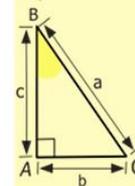
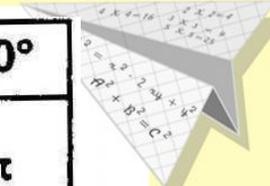
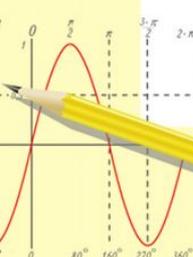
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

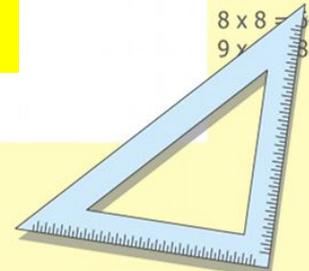
$$(x+y)(x-y) = x^2 - y^2$$



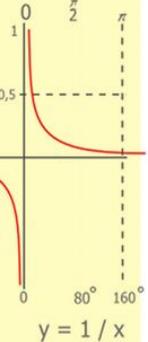
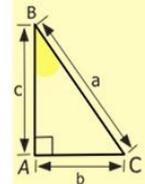
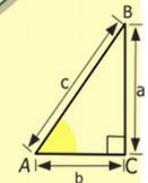
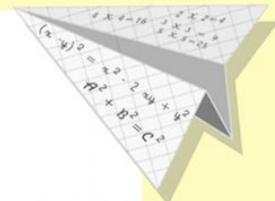
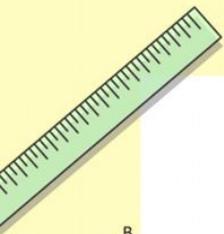
$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$

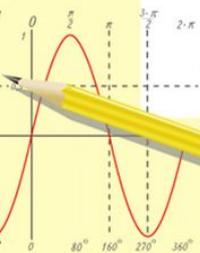


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Юрьевич  
преподаватель математики  
ГБПОУ ЗКНО  
Москва, 2020г.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

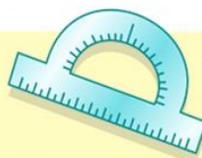
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