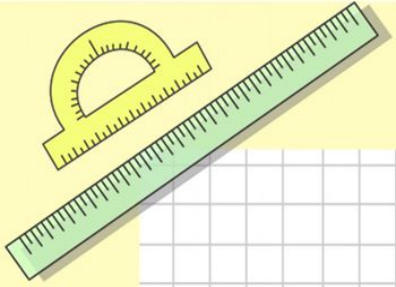
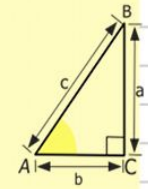
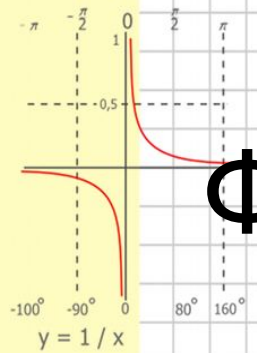
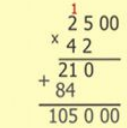
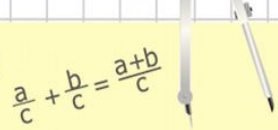
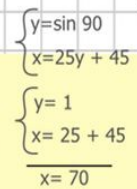
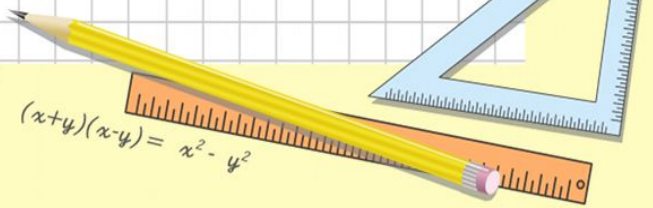
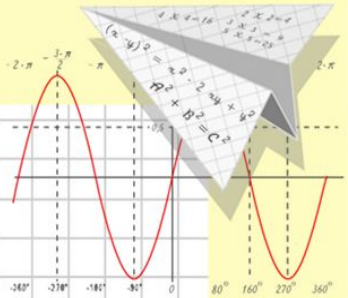
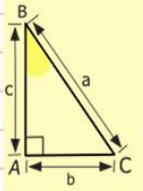
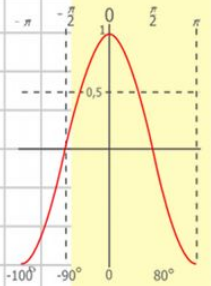
Математик
а

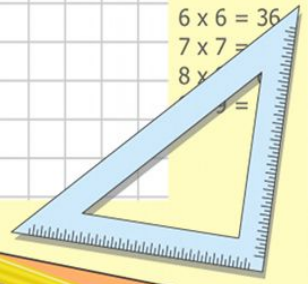
Занятие 45.
Формулы двойного угла.
Формулы половинного угла.





Формулы двойного угла

Если в формулах сложения принять β = α , то получим:

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\sin(\alpha+\alpha)=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha$$

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

Аналогично: $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$

$$tg(\alpha+\beta)=\frac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}$$

$$tg2\alpha=\frac{2tg\alpha}{1-tg^2\alpha}$$

Формулы двойного угла

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$tg\,2\alpha = \frac{2tg\alpha}{1-tg^2\alpha}$$

$$\sin 20^\circ = 2\sin 10^\circ\cos 10^\circ$$

$$\cos 20^\circ = \cos^2 10^\circ - \sin^2 10^\circ$$

$$\sin 6x = 2\sin 3x\cos 3x$$

$$\cos 6x = \cos^2 3x - \sin^2 3x$$

$$\sin\frac{2\pi}{5} = 2\sin\frac{\pi}{5}\cos\frac{\pi}{5}$$

$$\cos\frac{2\pi}{5} = \cos^2\frac{\pi}{5} - \sin^2\frac{\pi}{5}$$

$$tg\,20^\circ = \frac{2tg10^\circ}{1-tg^2 10^\circ}$$

$$tg\,\frac{2\pi}{5} = \frac{2tg\frac{\pi}{5}}{1-tg^2\frac{\pi}{5}}$$

$$tg\,6x = \frac{2tg3x}{1-tg^2 3x}$$

$$ctg\,6x = \text{???}$$

Формулы двойного угла

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$tg2\alpha = \frac{2tg\alpha}{1-tg^2\alpha}$$

$$2\sin 25^\circ\cos 25^\circ = \sin 50^\circ$$

$$\cos^2 15^\circ - \sin^2 15^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x$$

$$\cos^2 5x - \sin^2 5x = \cos 10x$$

$$2\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \sin\frac{2\pi}{8} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos^2\frac{2\pi}{9} - \sin^2\frac{2\pi}{9} = \cos\frac{4\pi}{9}$$

$$\frac{2tg35^\circ}{1-tg^2 35^\circ} = tg70^\circ$$

$$\frac{2tg0{,}3x}{1-tg^2 0{,}3x} = tg0{,}6x$$

$$\frac{1-tg^2 4x}{2tg4x} = \text{???}$$

$$\frac{2tg\frac{3\pi}{14}}{1-tg^2\frac{3\pi}{14}} = tg\frac{6\pi}{14} = tg\frac{3\pi}{7}$$

Формулы двойного угла

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$tg2\alpha = \dfrac{2tg\alpha}{1-tg^2\alpha}$

$$\sin 15^\circ\cos 15^\circ = \frac{1}{2}\cdot 2\sin 15^\circ\cos 15^\circ = \frac{1}{2}\cdot\sin 30^\circ = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

$$\sin^2 5x - \cos^2 5x = -\cos 10x$$

$$\frac{10tg18^\circ}{1-tg^2 18^\circ} = 5\cdot\frac{2tg18^\circ}{1-tg^2 18^\circ} = 5tg36^\circ$$

$$\cos^2 50^\circ - \cos^2 40^\circ = \left|\cos 40^\circ = \sin(90^\circ - 40^\circ) = \sin 50^\circ\right| =$$

$$= \cos^2 50^\circ - \sin^2 50^\circ = \cos 100^\circ = \cos(90^\circ + 10^\circ) = -\sin 10^\circ$$

$$4\sin^2\frac{x}{2}\cdot\cos^2\frac{x}{2} = \left(2\sin\frac{x}{2}\cdot\cos\frac{x}{2}\right)^2 = \left(\sin 2\cdot\frac{x}{2}\right)^2 = \sin^2 x$$

Формулы двойного угла

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$tg\,2\alpha = \frac{2tg\alpha}{1-tg^2\alpha}$$

Упростить:

$$\frac{(\sin\alpha+\cos\alpha)^2-\sin 2\alpha}{\cos 2\alpha+2\sin^2\alpha} = \frac{\sin^2\alpha+2\sin\alpha\cos\alpha+\cos^2\alpha-2\sin\alpha\cos\alpha}{\cos^2\alpha-\sin^2\alpha+2\sin^2\alpha} =$$

$$= \frac{\sin^2\alpha+\cos^2\alpha}{\cos^2\alpha+\sin^2\alpha} = 1$$

$$\frac{10\sin 40^\circ\sin 50^\circ}{\cos 10^\circ} = \left|\sin 50^\circ = \cos(90^\circ-40^\circ) = \cos 40^\circ\right| =$$

$$= 5\cdot\frac{2\sin 40^\circ\cos 40^\circ}{\cos 10^\circ} = 5\cdot\frac{\sin 80^\circ}{\cos 10^\circ} = \left|\sin 80^\circ = \cos 10^\circ\right| =$$

$$= 5\cdot\frac{\cos 10^\circ}{\cos 10^\circ} = 5$$

Формулы половинного угла

$$\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$$

$$\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$$

$$tg^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\sin^2 x=\frac{1-\cos 2x}{2}$$

$$\cos^2 3x=\frac{1+\cos 6x}{2}$$

$$tg^2\frac{x}{4}=\frac{1-\cos\frac{x}{2}}{1+\cos\frac{x}{2}}$$

$$\sin^2 10^\circ=\frac{1-\cos 20^\circ}{2}$$

$$\cos^2 40^\circ=\frac{1+\cos 80^\circ}{2}$$

$$tg^2 32^\circ=\frac{1-\cos 64^\circ}{1+\cos 64^\circ}$$

$$\sin^2\frac{2\pi}{7}=\frac{1-\cos\frac{4\pi}{7}}{2}$$

$$\cos^2\frac{3\pi}{14}=\frac{1+\cos\frac{3\pi}{7}}{2}$$

$$tg^2 0{,}2\pi=\frac{1-\cos 0{,}4\pi}{1+\cos 0{,}4\pi}$$

Формулы половинного угла

$$\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$$

$$\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$$

$$tg^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{1+\cos\alpha}$$

Вычислить:

$$\cos\frac{\pi}{8}=\sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}=\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}=\sqrt{\frac{\left(1+\frac{\sqrt{2}}{2}\right)\cdot 2}{2\cdot 2}}=\sqrt{\frac{2+\sqrt{2}}{4}}=\frac{\sqrt{2+\sqrt{2}}}{2}$$

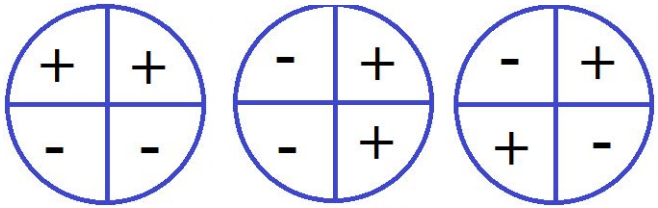
Упростить:

$$\frac{1-2\cos 6x+\cos^2 6x}{4}=\frac{(1-\cos 6x)^2}{4}=\left(\frac{1-\cos 6x}{2}\right)^2=$$

$$=\left(\sin^2 3x\right)^2=\sin^4 3x$$

Справочные материалы

α	0°	30°	45°	60°	90°	180°	270°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin α	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cos α	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
tg α	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—	0	—	0
ctg α	—	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	—	0	—



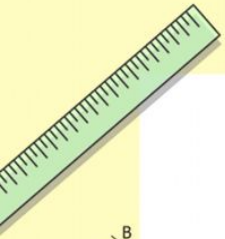
$\sin\alpha$ $\cos\alpha$ $tg\alpha$ $ctg\alpha$

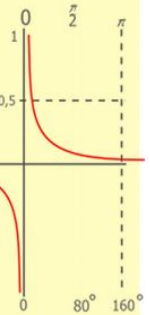
$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$
$$tg(-\alpha) = -tg\alpha$$
$$ctg(-\alpha) = -ctg\alpha$$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ $\sin 90° = 1$

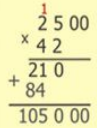
$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$ $\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$ $x = 70$

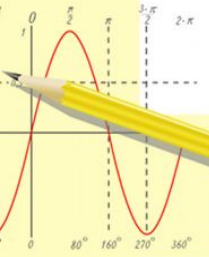
$(x+y)(x-y) = x^2 - y^2$





y = 1 / x





$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

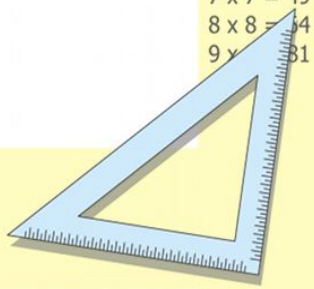
$\sin 90° = 1$

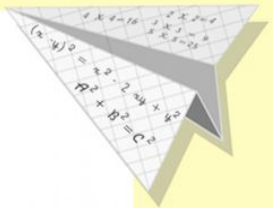
$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$

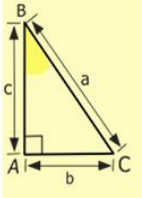
$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$

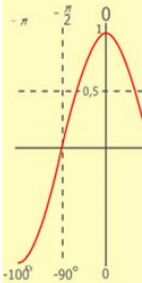
$x = 70$

$(x+y)(x-y) = x^2 - y^2$









y = co

2 x 2 = 4
3 x 3 = 9
4 x 4 = 16
5 x 5 = 25
6 x 6 = 36
7 x 7 = 49
8 x 8 = 64
9 x 9 = 81

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