WESTMINSTER
INTERNATIONAL UNIVERSITY IN TASHKENT

An Accredited Institution of the University of Westminster (UK)

LECTURE 9

CALCULUS

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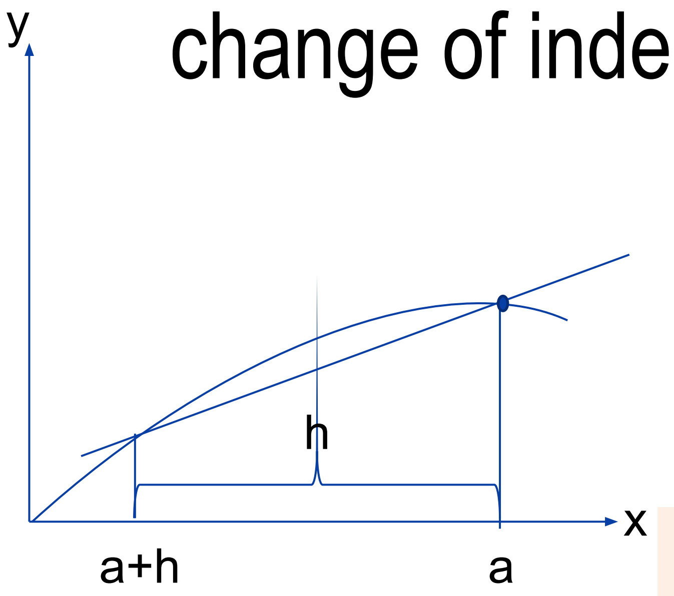
Lecture outline

- What is the derivative?
- The derivatives of functions and their use
- Applications of functions to revenue, cost, profit and to the demand and supply theory

Derivatives

Definition: Derivative line is simply a slope of the function. it shows the rate of change of dependent variable resulted from a change of independent variable.

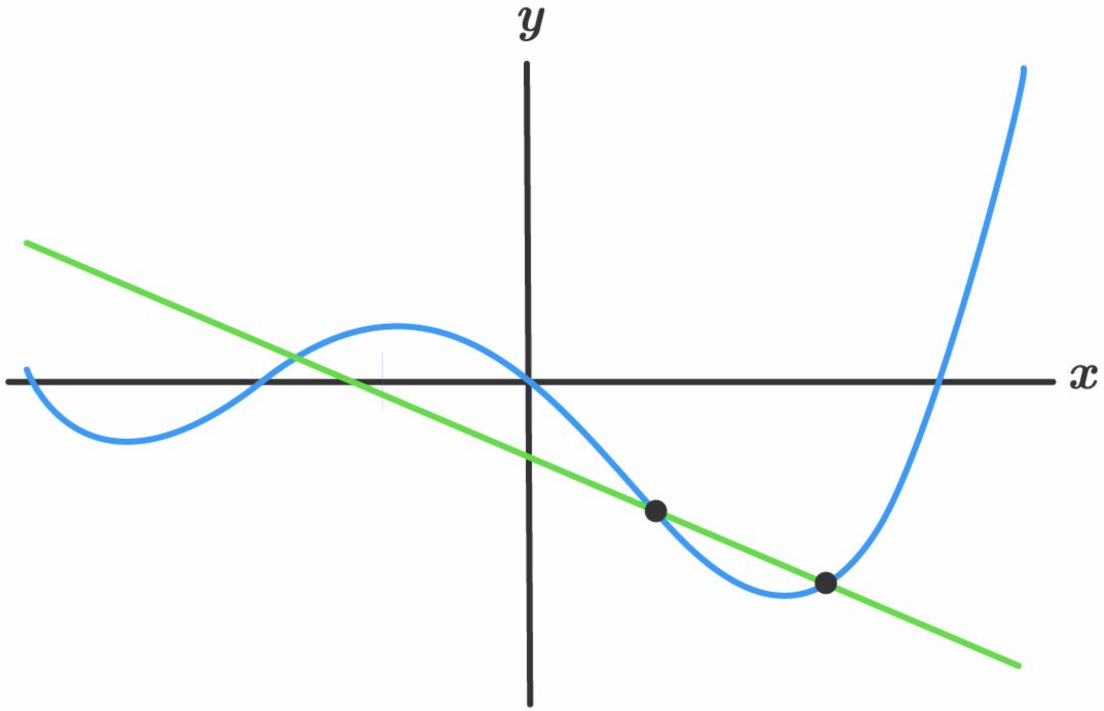


$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx} \Rightarrow dy \approx f'(x_0)dx$$

Or, simply

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Derivatives



The slope of Tangent line to the point is derivative of the function.
Tangent means that the line touches the curve at one point only!

Limit

For $y = f(x)$, we say that $\lim\limits_{x\to a} f(x) = A$ if the value of $f(x)$ gets closer to A as x gets closer to a.

Examples:

1) $\lim\limits_{x\to 1}(2x) = 2$

2) $\lim\limits_{x\to 2} x^2 = 2^2 = 4$

3) $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1} = \lim\limits_{x\to 1}\dfrac{(x-1)(x+1)}{x-1} = \lim\limits_{x\to 1}(x+1) = 2$

4) $\lim\limits_{x\to +\infty}\dfrac{x+1}{2x-1} = \lim\limits_{x\to +\infty}\dfrac{x\left(1+\dfrac{1}{x}\right)}{x\left(2-\dfrac{1}{x}\right)} = \lim\limits_{x\to +\infty}\dfrac{1+\dfrac{1}{x}}{2-\dfrac{1}{x}} = \dfrac{1+0}{2-0} = \dfrac{1}{2}$

5) $\lim\limits_{x\to 0}\dfrac{1}{x} = \infty \quad (no\ limit)$

Table of Derivatives

1) $y = C$ (C – const),
$y' = (C)' = 0$ *Ex:* $(5)' = 0$; $(-13)' = 0$.

2) $y = ax + b$ (a, b – const),
$y' = (ax + b)' = a$ *Ex:* $(12x + 3)' = 12$.

3) $y = ax^n$ (a, n – const),
$y' = (ax^n)' = nax^{n-1}$ *Ex:* $(4x^5)' = 5*4x^{5-1}$

4) $y = a^x$ ($a > 0$, $a \neq 1$),
$y' = (a^x)' = a^x \ln a$ *Ex:* $(7^x)' = 7^x \ln 7$.
$y' = (e^x)' = e^x$

5) $y = \log_a x$ ($a > 0$, $a \neq 1$),
$y' = (\log_a x)' = 1/(x \ln a)$ *Ex:* $(\log_4 x)' = 1/(x \ln 4)$
$y' = (\ln x)' = 1/x$

Definitions of key economic terms

Revenue is the amount of money received by selling products

Cost of production is the amount of money spent to make the products

Quantity demanded is the number of products demanded by customers at the market

Quantity supplied is the number of products delivered to the market for sales

Economic application of linear function

A firm sells pens for \$2 each. Hence, the revenue function is

$$R(Q) = 2Q$$

A firm has \$150 dollar *fixed cost** and \$1.5 *variable costs**. Hence, the cost function is

$$C(Q) = \$1.5Q+150$$

A firm produces books. Books supply and demand depend upon the price. Therefore, the supply and demand function are:

$$Q_D(P) = 100 - 0.5P$$
$$Q_S(P) = 80 + 0.5P$$

Rule of Differentiation (1)

1. The constant rule.

If $f(x) = Cg(x)$ (C is constant),

$$f'(x) = (Cg(x))' = Cg'(x)$$

Ex: For $f(x) = 3x^2$,

$$f'(x) = (3x^2)' = 3(x^2)' = 3(2x) = 6x$$

Rule of Differentiation (2)

2. The sum and difference rule.

If $f(x) = g(x) \pm h(x)$,

$$f'(x) = (g(x) \pm h(x))' =$$

$$= g'(x) \pm h'(x)$$

Ex: For $f(x) = 3x^3 + x^2$,

$$f'(x) = (3x^3 + x^2)' =$$

$$=(3x^3)' + (x^2)' = 9x^2 + 2x$$

Rule of Differentiation (3)

3. The product rule.

If $f(x) = g(x)\,h(x)$,

$$f'(x) = (g(x)\,h(x))' =$$

$$= g'(x)\,h(x) + g(x)\,h'(x)$$

Ex: If $f(x) = xe^x$, then

$$f'(x) = [(x)(e^x)]' =$$

$$(x)'(e^x) + (x)(e^x)' =$$

$$= e^x + 2xe^x.$$

Rule of Differentiation (4)

4. The quotient rule.

If $f(x)=\dfrac{g(x)}{h(x)}$

$$f'(x)=\frac{g'(x)h(x)-g(x)h'(x)}{h^2(x)}$$

Ex: If $f(x)=\dfrac{x-1}{x^2}$,

then $f'(x)=\left(\dfrac{x-1}{x^2}\right)'=$

$$=\frac{(x-1)'(x^2)-(x-1)(x^2)'}{(x^2)^2}=$$

$$=\frac{x^2-(x-1)2x}{x^4}=\frac{-x^2+2x}{x^4}$$

Rule of Differentiation (5)

5. The chain rule

$$\text{if} \quad f(x) = g(h(x)),$$

$$f'(x) = g'(h(x)) = g'(h(x)) * h'(x)$$

Example:

$$f(x) = g(h(x)) = (2e^x - 1)^2$$

Where, $g(x) = x^2$ and $h(x) = (2e^x - 1)$

$$f'(x) = [(2e^x - 1)^2]' =$$

$$= 2(2e^x - 1) * (2e^x - 1)' =$$

$$= 2(2e^x - 1) * 2e^x =$$

$$= 8e^{2x} - 4e^x$$

Concluding remarks

Today, you learnt

- Definition of derivatives
- Limits and derivation of derivatives
- Derivatives and their usage in real world

Essential readings

1. Jon Curwin…, "Quantitative methods…", Ch 23
2. Glyn Burton…, "Quantitative methods…", Ch 12
4. Mik Wisniewski…, "Foundation Quantitative…", Ch 10-11
5. Clare Morris, "Quantitative Approaches…", Ch 1
6. Louise Swift "Quantitative methods…", Ch EM3, MM2.