

WESTMINSTER

INTERNATIONAL UNIVERSITY IN TASHKENT

An Accredited Institution of the University of Westminster (UK)

# LECTURE 9

# CALCULUS



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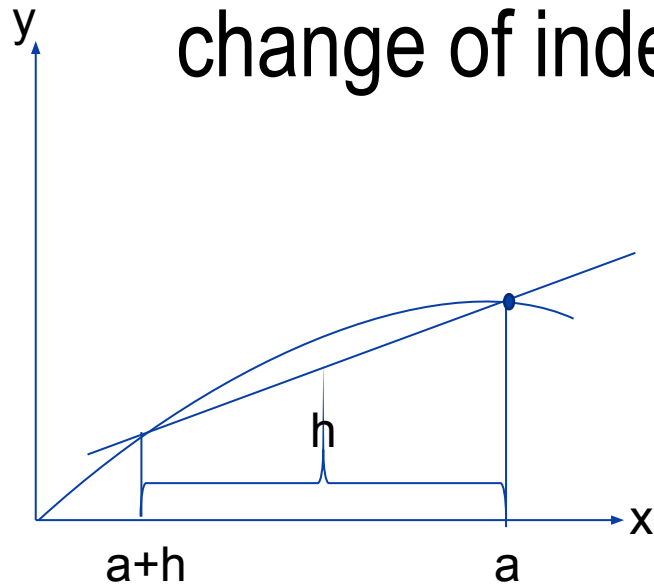
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- What is the derivative?
- The derivatives of functions and their use
- Applications of functions to revenue, cost, profit and to the demand and supply theory

**Definition:** Derivative line is simply a slope of the function. it shows the rate of change of dependent variable resulted from a change of independent variable.

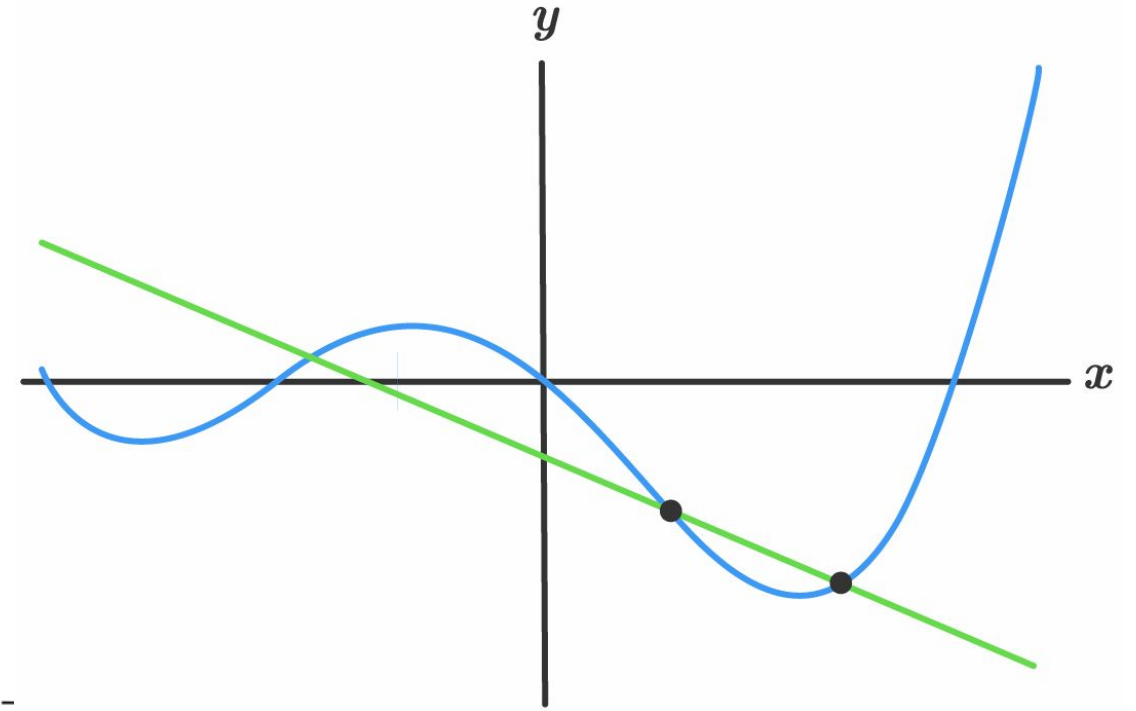
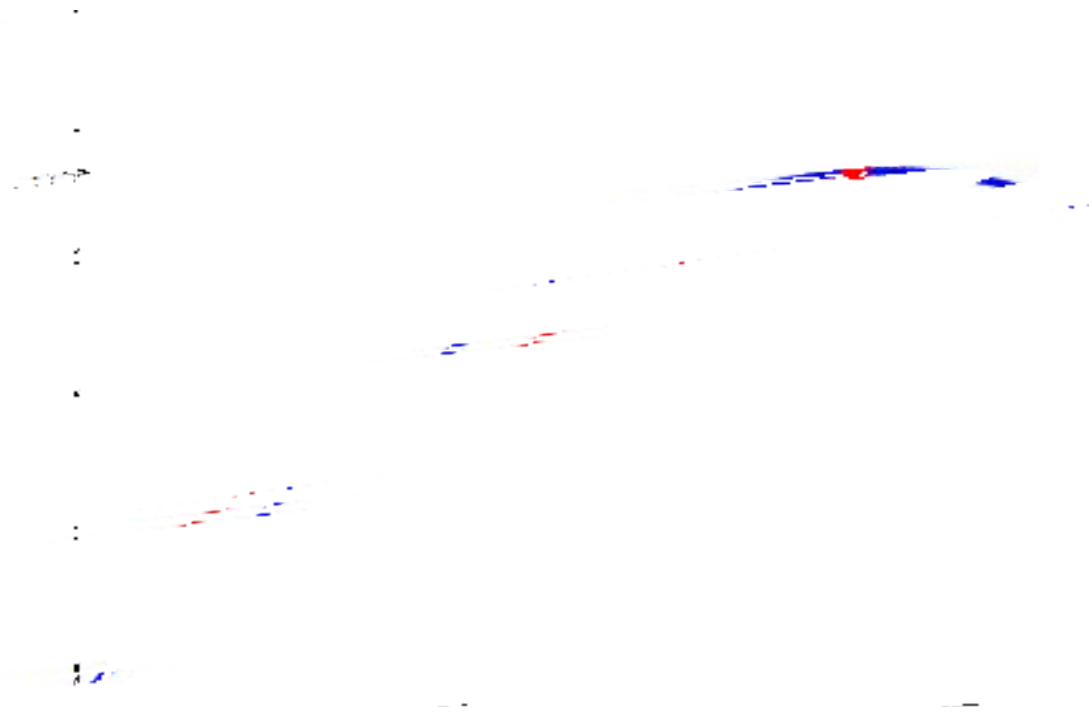


$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx} \Rightarrow dy \approx f'(x_0)dx$$

Or, simply

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

# Derivatives



The slope of Tangent line to the point is derivative of the function.  
Tangent means that the line touches the curve at one point only!

For  $y = f(x)$ , we say that  $\lim_{x \rightarrow a} f(x) = A$  if the value of  $f(x)$  gets closer to  $A$  as  $x$  gets closer to  $a$ .

## Examples:

$$1) \lim_{x \rightarrow 1} (2x) = 2 \qquad 2) \lim_{x \rightarrow 2} x^2 = 2^2 = 4$$

$$3) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x + 1)}{x - 1} = \lim_{x \rightarrow 1} (x + 1) = 2$$

$$4) \lim_{x \rightarrow +\infty} \frac{x + 1}{2x - 1} = \lim_{x \rightarrow +\infty} \frac{x \left( 1 + \frac{1}{x} \right)}{x \left( 2 - \frac{1}{x} \right)} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{1}{x}}{2 - \frac{1}{x}} = \frac{1 + 0}{2 - 0} = \frac{1}{2}$$

$$5) \lim_{x \rightarrow 0} \frac{1}{x} = \infty \text{ (no limit)}$$

# Table of Derivatives

1)  $y = C$  ( $C - \text{const}$ ),  
 $y' = (C)' = 0$       *Ex:*  $(5)' = 0$ ;  $(-13)' = 0$ .

2)  $y = ax + b$  ( $a, b - \text{const}$ ),  
 $y' = (ax + b)' = a$       *Ex:*  $(12x + 3)' = 12$ .

3)  $y = ax^n$  ( $a, n - \text{const}$ ),  
 $y' = (ax^n)' = nax^{n-1}$       *Ex:*  $(4x^5)' = 5 * 4x^{5-1}$

4)  $y = a^x$  ( $a > 0, a \neq 1$ ),  
 $y' = (a^x)' = a^x \ln a$       *Ex:*  $(7^x)' = 7^x \ln 7$ .  
 $y' = (e^x)' = e^x$

5)  $y = \log_a x$  ( $a > 0, a \neq 1$ ),  
 $y' = (\log_a x)' = 1/(x \ln a)$       *Ex:*  $(\log_4 x)' = 1/(x \ln 4)$   
 $y' = (\ln x)' = 1/x$



# Definitions of key economic terms

**Revenue** is the amount of money received by selling products

**Cost of production** is the amount of money spent to make the products

**Quantity demanded** is the number of products demanded by customers at the market

**Quantity supplied** is the number of products delivered to the market for sales



A firm sells pens for \$2 each. Hence, the revenue function is

$$R(Q) = 2Q$$

A firm has \$150 dollar *fixed cost*\* and \$1.5 *variable costs*\*.  
Hence, the cost function is

$$C(Q) = \$1.5Q + 150$$

A firm produces books. Books supply and demand depend upon the price. Therefore, the supply and demand function are:

$$Q_D(P) = 100 - 0.5P$$

$$Q_S(P) = 80 + 0.5P$$

# Rule of Differentiation (1)

## 1. The constant rule.

**If  $f(x) = Cg(x)$  ( $C$  is constant),**

$$f'(x) = (Cg(x))' = Cg'(x)$$

*Ex:* For  $f(x) = 3x^2$ ,

$$f'(x) = (3x^2)' = 3(x^2)' = 3(2x) = 6x$$

# Rule of Differentiation (2)

2. The sum and difference rule.

$$\text{If } f(x) = g(x) \pm h(x),$$

$$\begin{aligned} f'(x) &= (g(x) \pm h(x))' = \\ &= g'(x) \pm h'(x) \end{aligned}$$

*Ex:* For  $f(x) = 3x^3 + x^2,$

$$f'(x) = (3x^3 + x^2)' =$$

$$= (3x^3)' + (x^2)' = 9x^2 + 2x$$

# Rule of Differentiation (3)

## 3. The product rule.

$$\begin{aligned}\text{If } f(x) &= g(x)h(x), \\ f'(x) &= (g(x)h(x))' = \\ &= g'(x)h(x) + g(x)h'(x)\end{aligned}$$

*Ex:* If  $f(x) = xe^x$ , then

$$\begin{aligned}f'(x) &= [(x)(e^x)]' = \\ &= (x)'(e^x) + (x)(e^x)' = \\ &= e^x + 2xe^x.\end{aligned}$$

# Rule of Differentiation (4)

## 4. The quotient rule.

$$\text{If } f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{h^2(x)}$$

$$\text{Ex: If } f(x) = \frac{x-1}{x^2},$$

$$\begin{aligned} \text{then } f'(x) &= \left( \frac{x-1}{x^2} \right)' = \\ &= \frac{(x-1)'(x^2) - (x-1)(x^2)'}{(x^2)^2} = \\ &= \frac{x^2 - (x-1)2x}{x^4} = \frac{-x^2 + 2x}{x^4} \end{aligned}$$

# Rule of Differentiation (5)

## 5. The chain rule

$$\text{if } f(x) = g(h(x)),$$

$$f'(x) = g'(h(x)) = g'(h(x)) * h'(x)$$

**Example:**

$$f(x) = g(h(x)) = (2e^x - 1)^2$$

$$\text{Where, } g(x) = x^2 \text{ and } h(x) = (2e^x - 1)$$

$$\begin{aligned} f'(x) &= [(2e^x - 1)^2]' = \\ &= 2(2e^x - 1) * (2e^x - 1)' = \\ &= 2(2e^x - 1) * 2e^x = \\ &= 8e^{2x} - 4e^x \end{aligned}$$

## Today, you learnt

- Definition of derivatives
- Limits and derivation of derivatives
- Derivatives and their usage in real world



1. Jon Curwin..., “Quantitative methods...”, Ch 23
2. Glyn Burton..., “Quantitative methods...”, Ch 12
4. Mik Wisniewski..., “Foundation Quantitative...”, Ch 10-11
5. Clare Morris, “Quantitative Approaches...”, Ch 1
6. Louise Swift “Quantitative methods...”, Ch EM3, MM2.