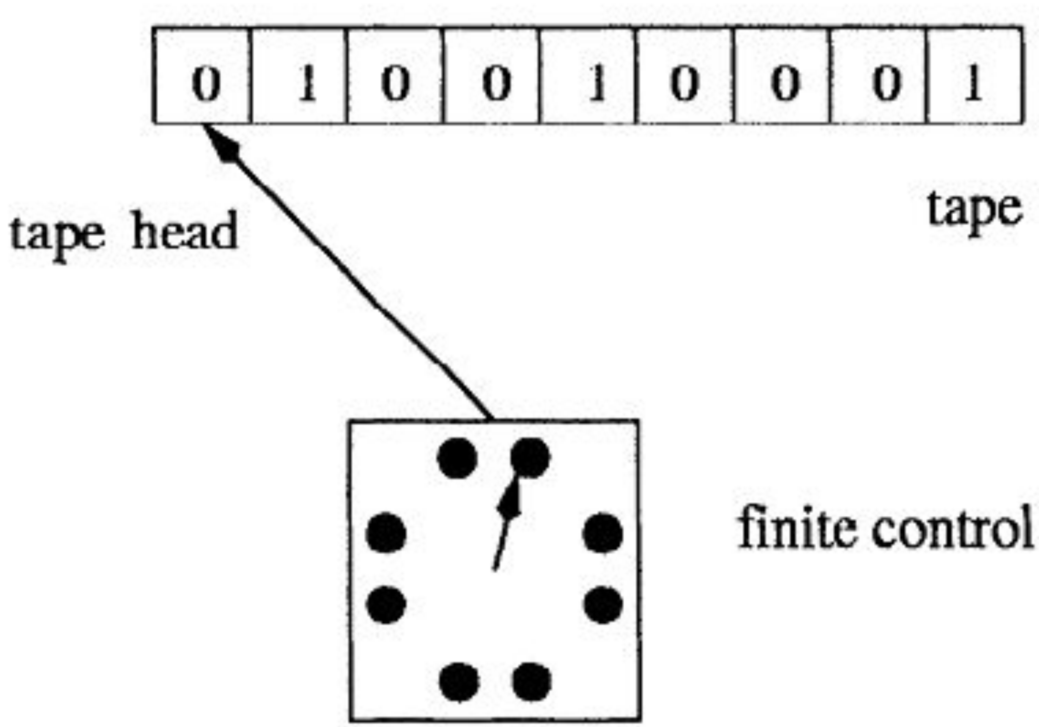
Finite automata

Irina Prosvirnina

- Deterministic finite automata
- Nondeterministic finite automata

Deterministic finite automata

Deterministic finite automaton (DFA) consists of three parts: a tape, a tape head (or, simply, head), and a finite control.

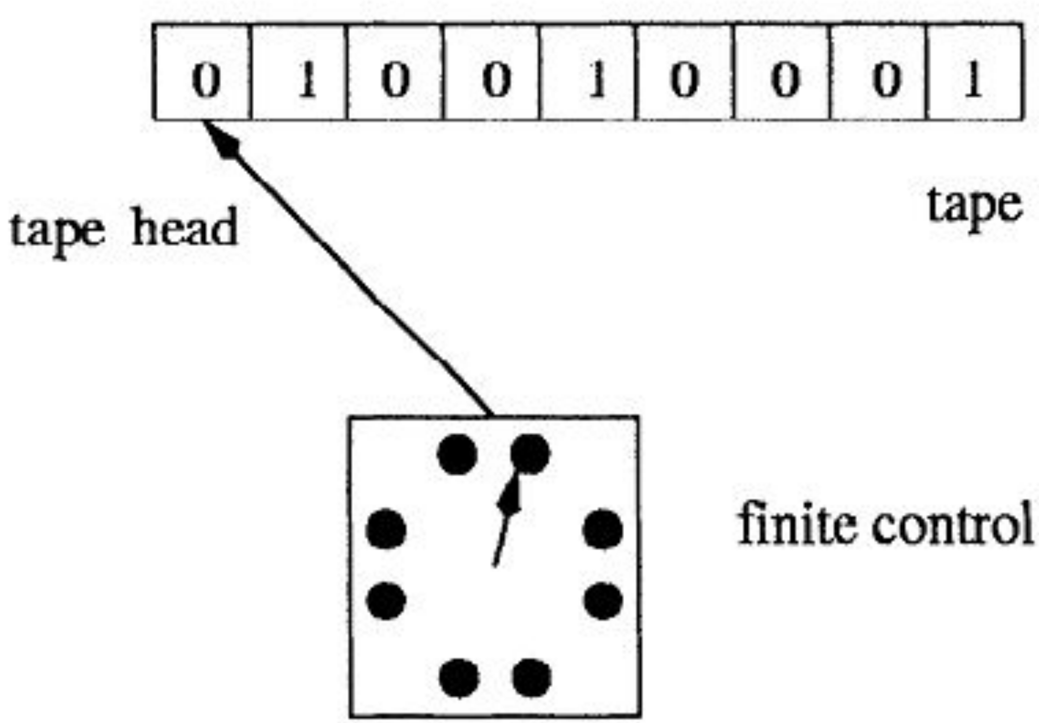


Deterministic finite automata

- In a formal theory, it is necessary to fix the set of symbols used to form strings.

Such a finite set of symbols is called an **alphabet**.

For exam



A string c **y**
string.

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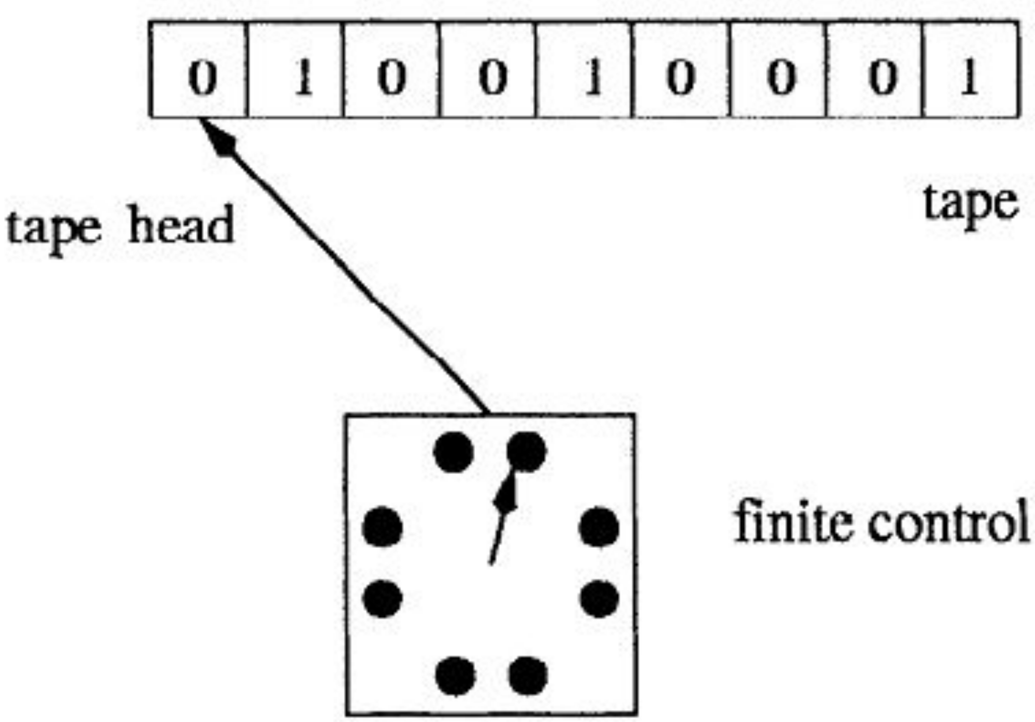
The tape head scans the tape, reads symbols from the tape, and passes the information to the finite control.



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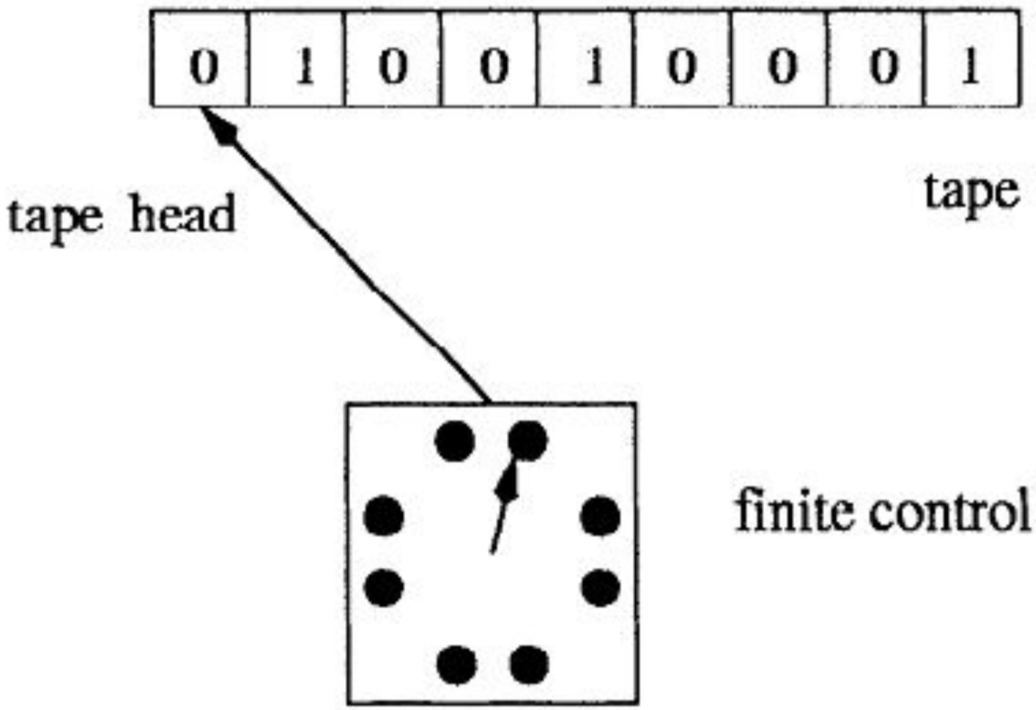
At each move of the DFA, the head scans one cell of the tape and reads the symbol in the cell, and then moves to the next cell to the right.



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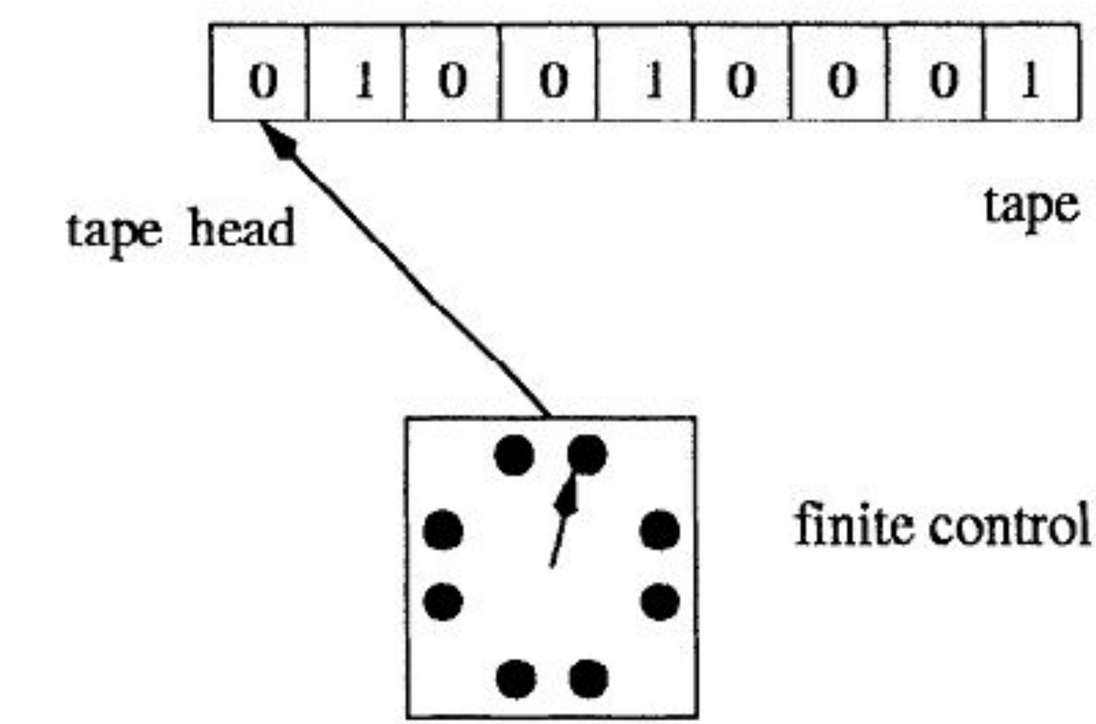
The **length** of a string x, denoted by $|x|$, is the number of symbols contained in the string.

For example,

|stri

|cs5

|10(



The **emp** no symbol.

Clearly, |

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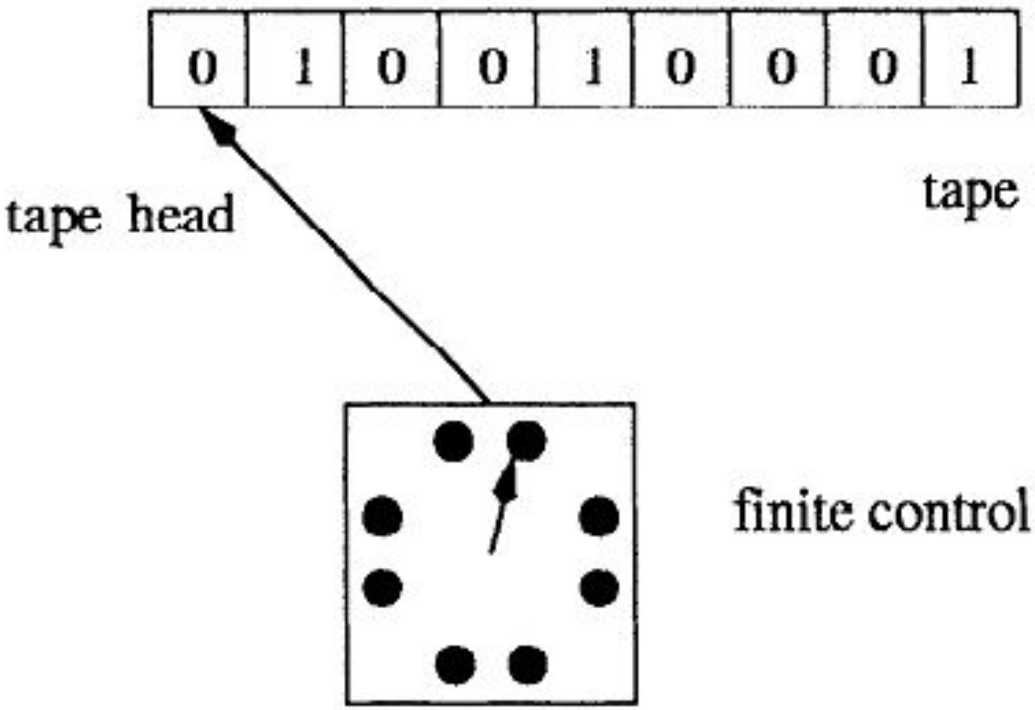
Then, it determines, from the current state and the symbol read by the tape, how the state is changed to a new state.



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Example 1

How many strings over the alphabet $A = \{a_1, a_2, \ldots, a_k\}$ are there which are of length n, where n is a nonnegative integer?

Solution. There are n positions in such a string, and each position can hold one of k possible symbols. Therefore, there are k^n strings of length exactly n.

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- For a string x over alphabet Σ the **reversal** of x, denoted by x^R, is defined by

$$x^R = \begin{cases} \varepsilon, \text{if } x = \varepsilon, \\ x_n \ldots x_2 x_1, \text{if } x = x_1 x_2 \ldots x_n. \end{cases}$$

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Example 2

For strings x and y

$$(xy)^R = y^R x^R.$$

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When the DFA halts, it accepts the input string if it halts in one of the final states.

Otherwise, the input string is rejected.

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A **language** is a set of strings.

Let Σ be an alphabet. We write Σ^* to denote the set of all strings over Σ.

Thus, a language L over Σ is just a subset of Σ^*.

For any finite language A we write $|A|$ to denote the size of A (i.e. the number of strings in A).

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The transition diagram of a DFA is an alternative way to represent the DFA.

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- **Union**.

If A and B are two languages, then

$$A \cup B = \{x |\ x \in A \text{ or } x \in B\}.$$

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- **Intersection**.

If A and B are two languages, then

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

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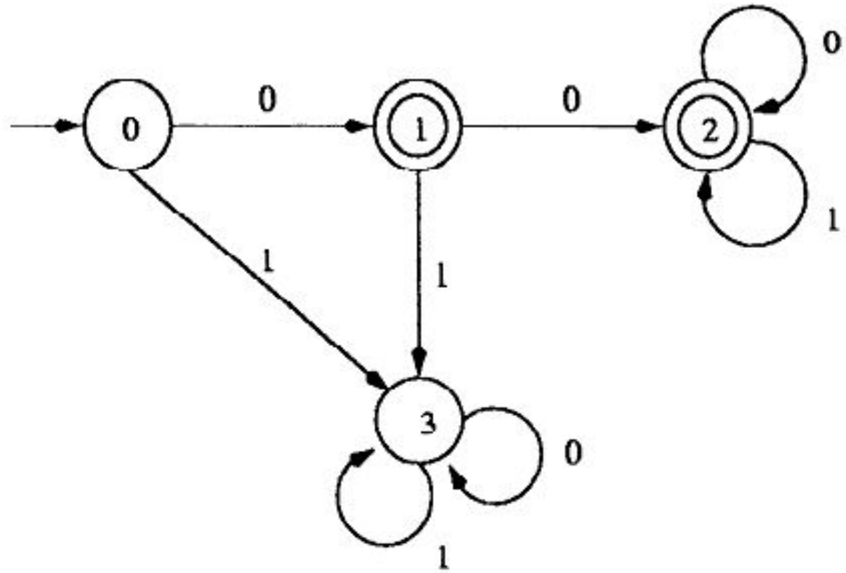
- **Complementation.**

If A is a language over the alphabet Σ, then

$$\bar{A} = \Sigma^* - A.$$

- **Concatenation.**

If A and B are two languages, then their concatenation is



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- Since the transition function $\delta: Q \times \Sigma \rightarrow Q$ is well-defined on $Q \times \Sigma$, the transition diagram of the DFA has the property that for every vertex (state) q and every symbol a, there exists exactly one edge with label a leaving q.

This implies that for each string x, there exists exactly one path starting from q_0 whose labels form the string x.

This path is called the computation path of the DFA on x. We note that a string x is accepted by M if and only if its computation path ends at one of the final states.

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Example 4

The language $\{0,10\}^*$ is the set of all binary strings having no substring 11 and ending with 0.

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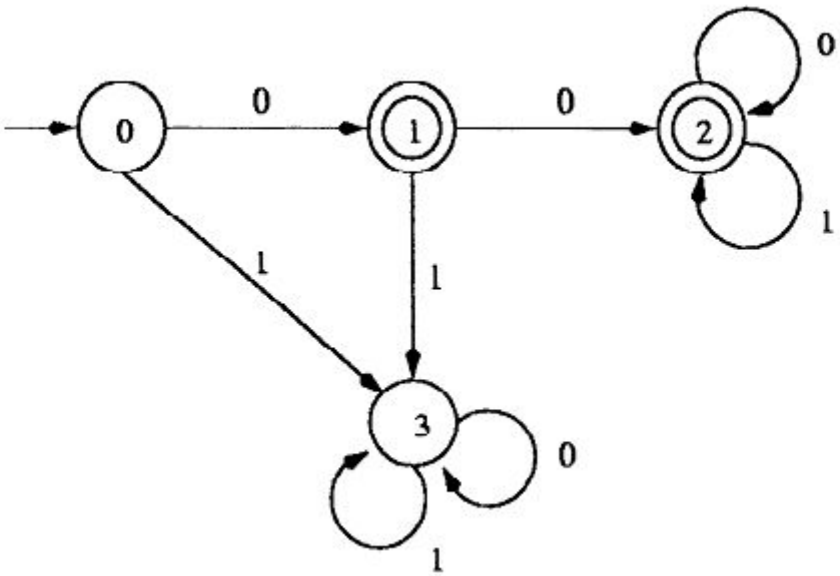
- The concept of **regular languages** (or, regular sets) over an alphabet Σ is defined recursively as follows:

1) The empty set $\emptyset$ is a regular language.
2) For every symbol $a \in \Sigma$, $\{a\}$ is a regular language.
3) If A and B are regular languages, then $A \cup B$, AB, A^* are all regular languages.
4) Nothing else is a regular language.

- Example 5

The set $\{\varepsilon\}$ is a regular language, because $\{\varepsilon\} = \emptyset^*$.



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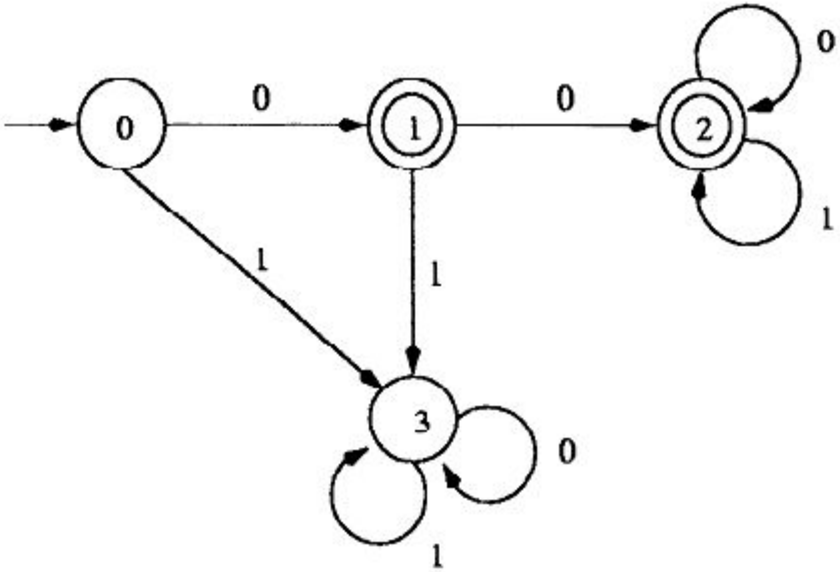
Example 6

The set $\{001, 110\}$ is a regular language over the binary alphabet:

$$\{001, 110\} = (\{0\}\{0\}\{1\}) \cup (\{1\}\{1\}\{0\}).$$

Example 5

The set $\{\varepsilon\}$ is a regular language, because $\{\varepsilon\} = \emptyset^*$.



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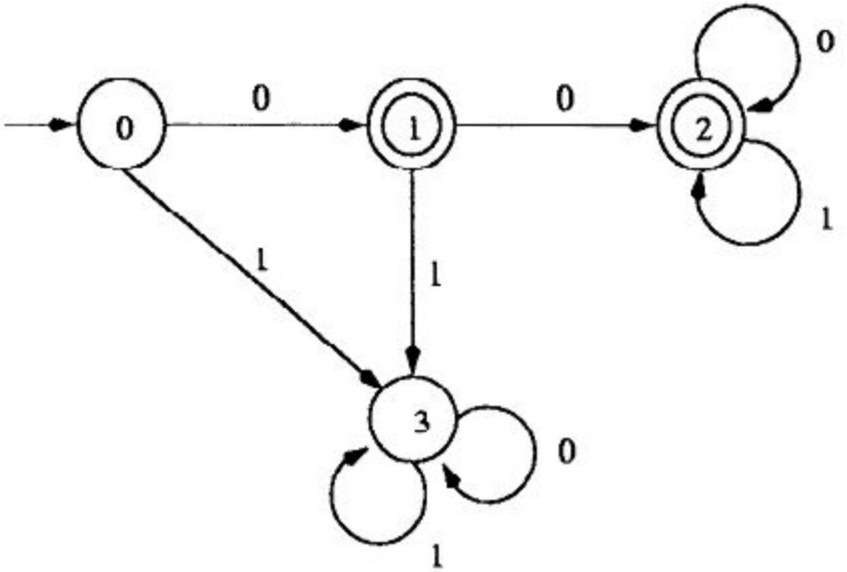
- To simplify the representations for regular languages, we define the notion of regular expressions over alphabet Σ.

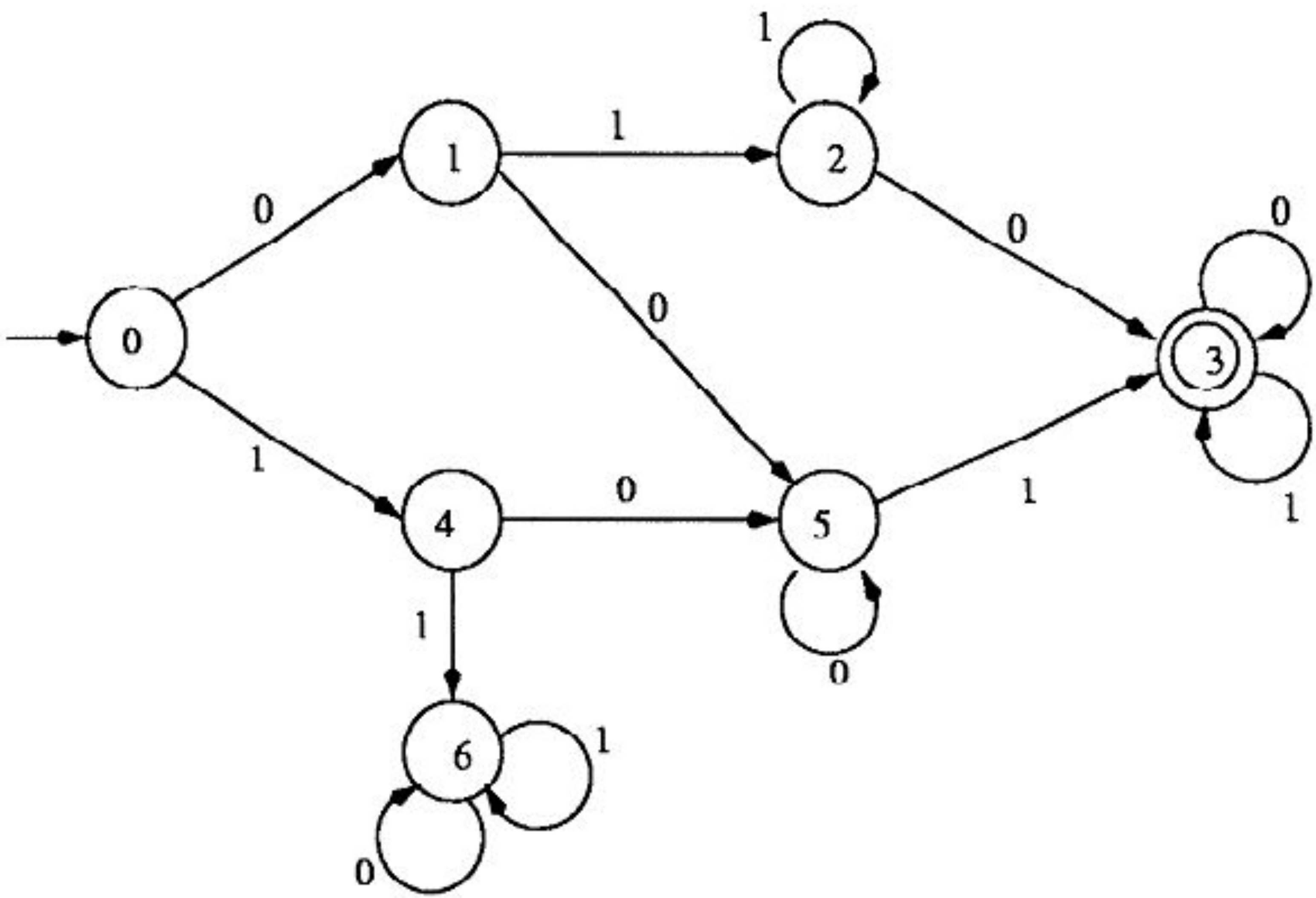
- The concept of regular expressions over an alphabet Σ is defined recursively as follows:
1) Ø is a regular expression which represents the



5) Nothing else is a regular expression over Σ.

Example 8

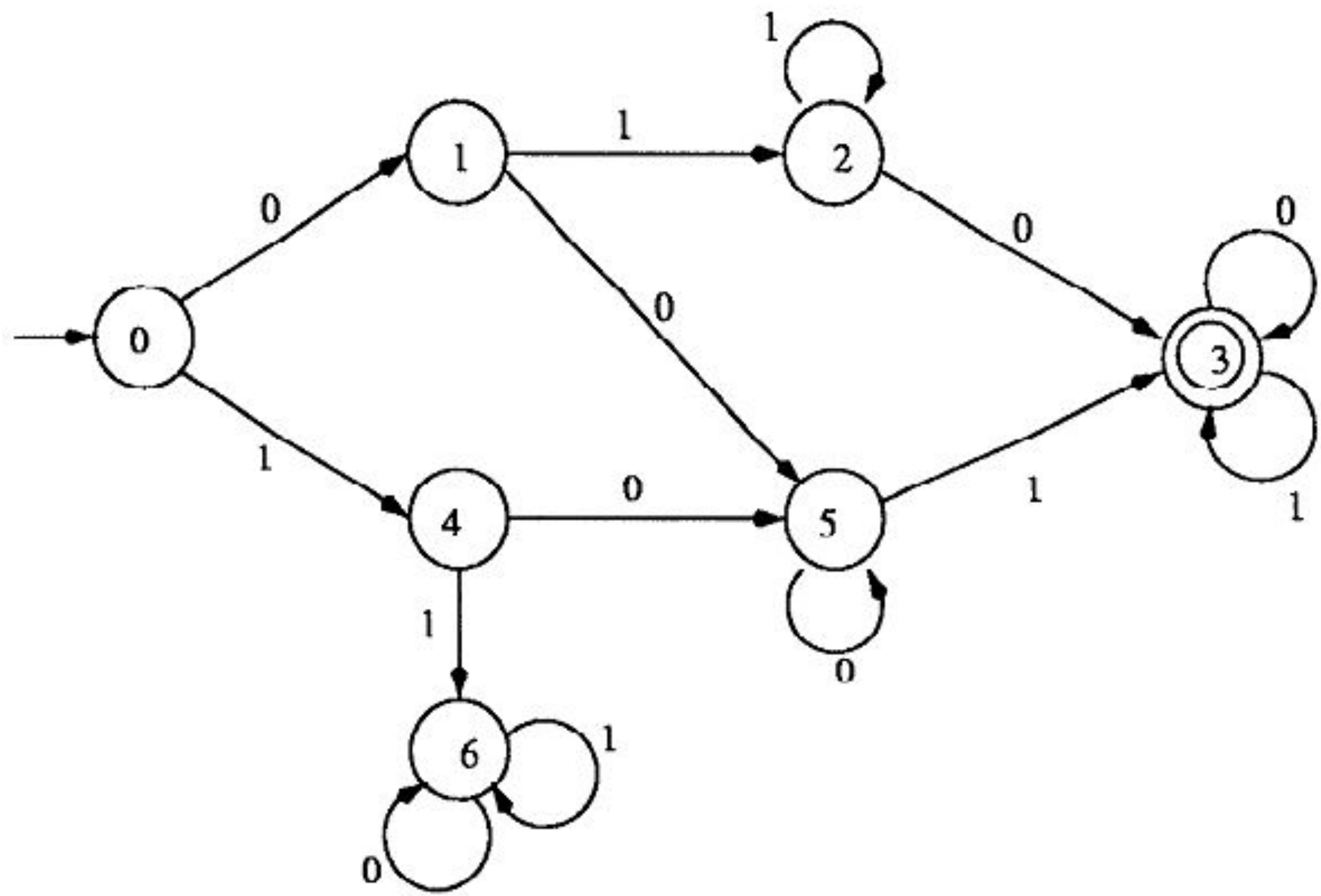
Language $A = \{0\}^*$ has a regular expression $r_A = (\mathbf{0})^*$.
Language $B = \{00\}^* \cup \{0\}$ has a regular expression $r_B = (((\mathbf{0})(\mathbf{0}))^*) + (\mathbf{0})$.



We can simplify the regular expression $r_A = (\mathbf{0})^*$ to $\mathbf{0}^*$.

We can simplify the regular expression
$r_B = (((\mathbf{0})(\mathbf{0}))^*) + (\mathbf{0})$
to $(\mathbf{00})^* + \mathbf{0}$.

We can simplify the regular expression
$(\{0\}^* \cup (\{1\}\{0\}\{0\}^*))\ \{1\}\{0\}^*((\{0\}\{1\}^*) \cup \{1\}^*)$
to $(\mathbf{0}^* + \mathbf{100}^*)\mathbf{10}^*(\mathbf{01}^* + \mathbf{1}^*)$.



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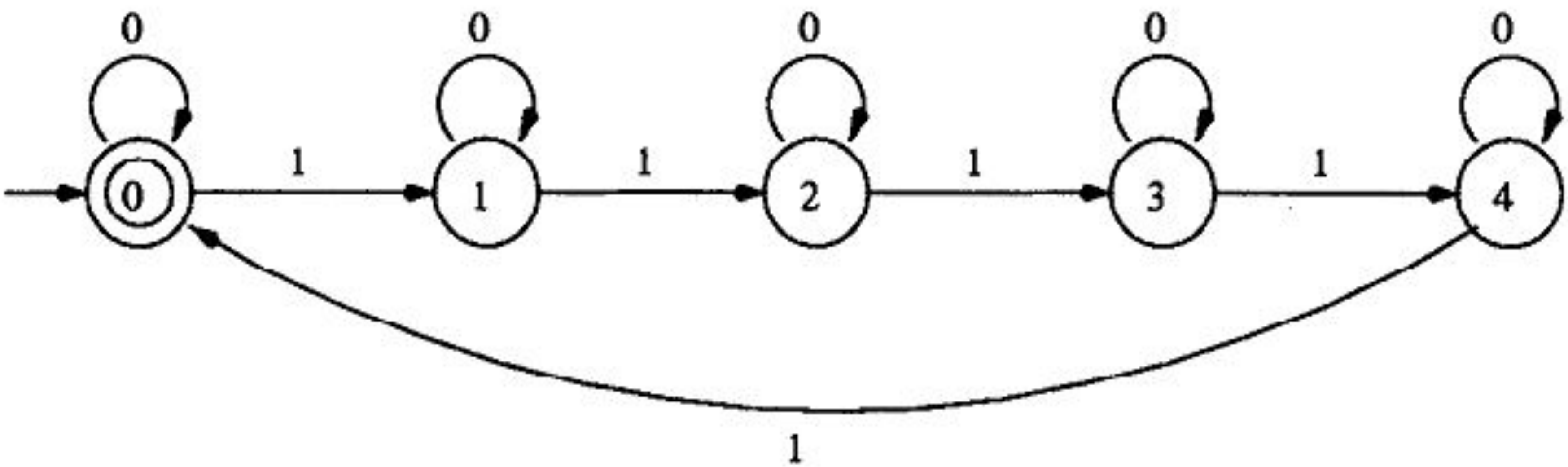
- The operations $+$ and $\cdot$ in a regular expression satisfy the **distributive law**: for any regular r, s and t,

$$r(s+t) = rs + rt,$$



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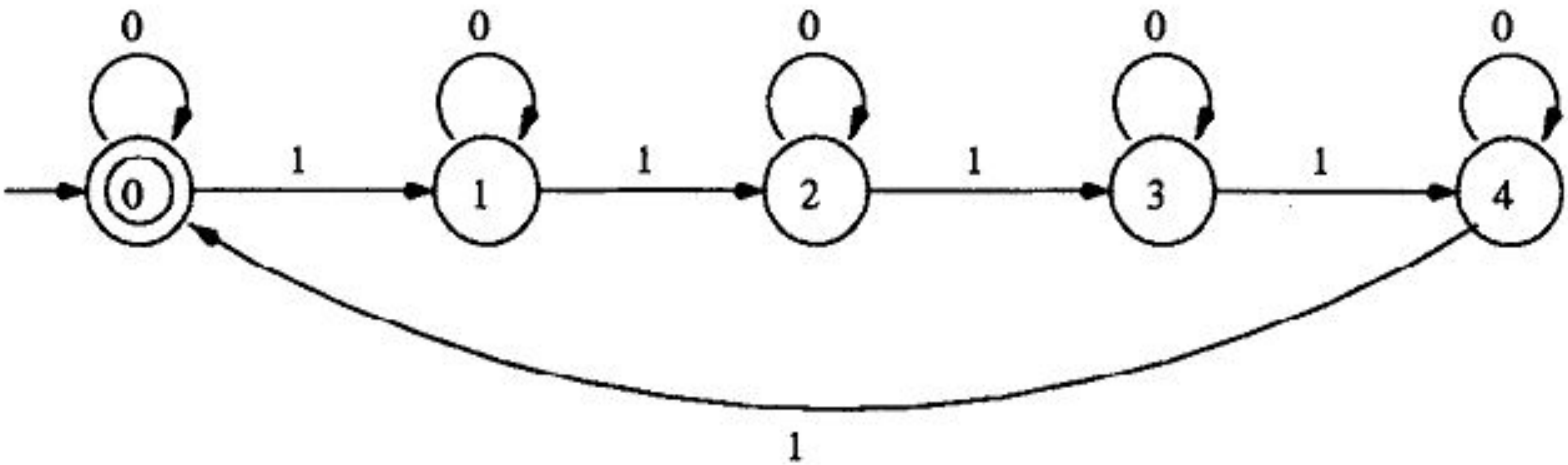
Example 10

Find a regular expression for the set of binary integers which are the expansions of 4.



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Nondeterministic finite automata

NFA’s, like DFA’s, can also be represented by transition diagrams.

In the transition diagram, we still use a vertex to represent a state and a labeled edge to represent a move, except that we allow multiple edges from one vertex to other vertices with the same label.

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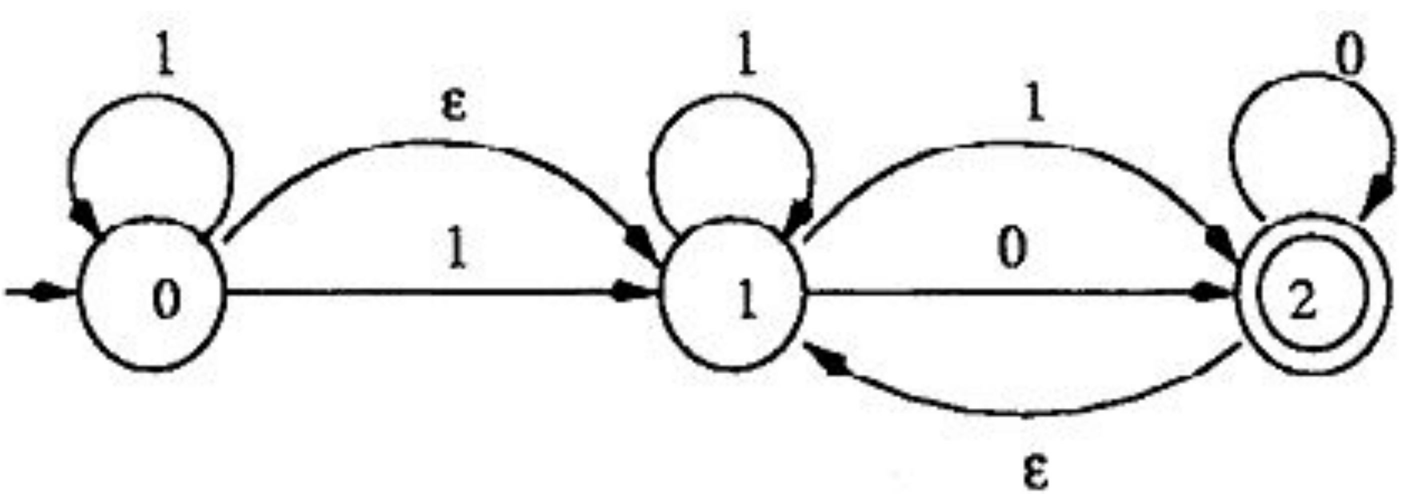

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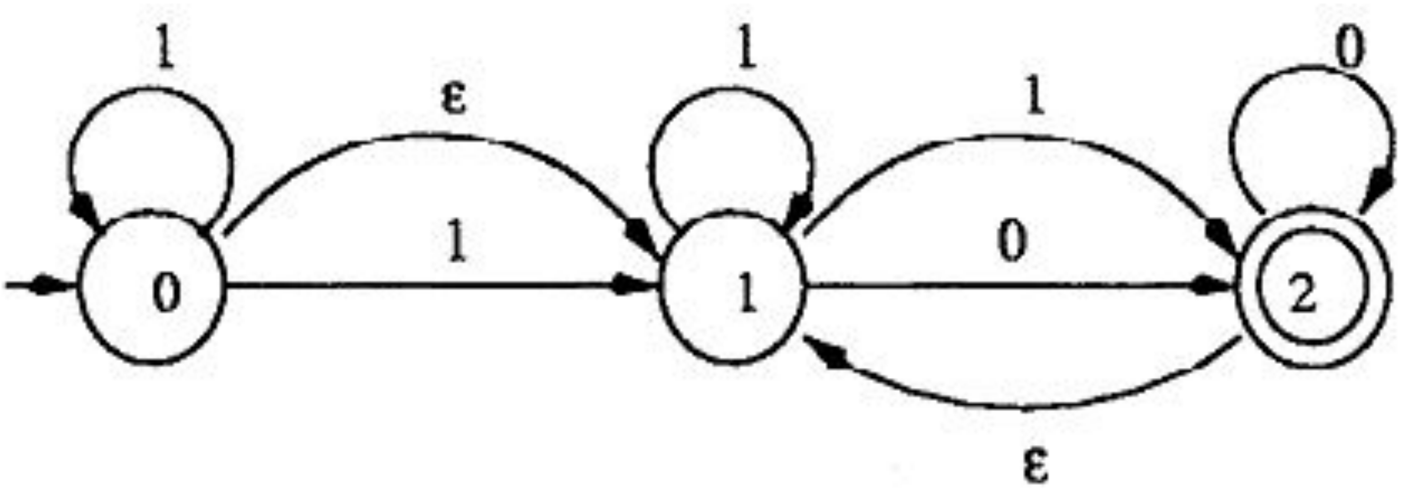

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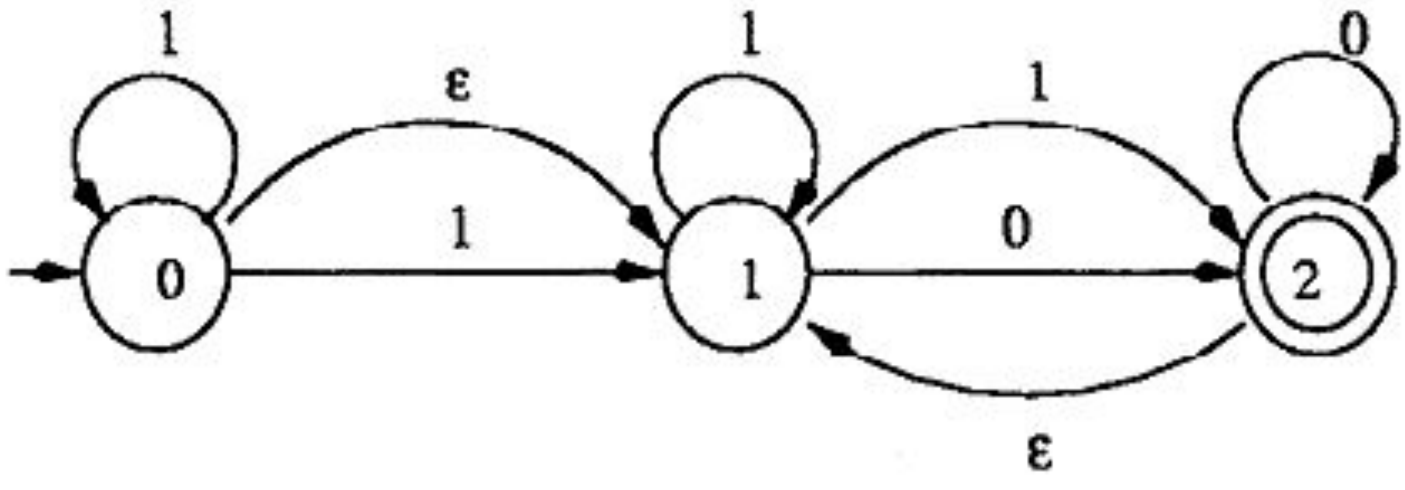
1
ε
1
1
0
1
0
0
1
2
0
ε

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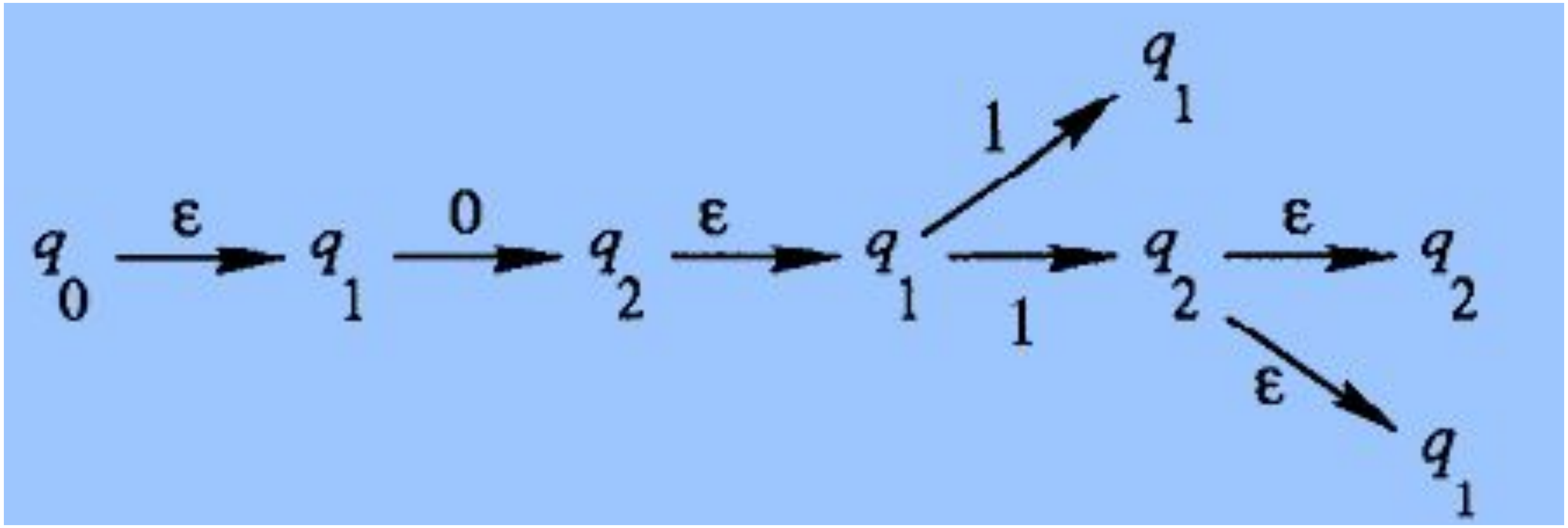

q_0
ε
q_1
0
q_2
ε
q_1
1
q_1
1
q_2
ε
q_2
ε
q_1

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Some of these computation paths lead to final states and some do not.

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