

*PART 3-1*  
*(THE DETERMINANT*  
*OF A MATRIX)*

*Exercises*

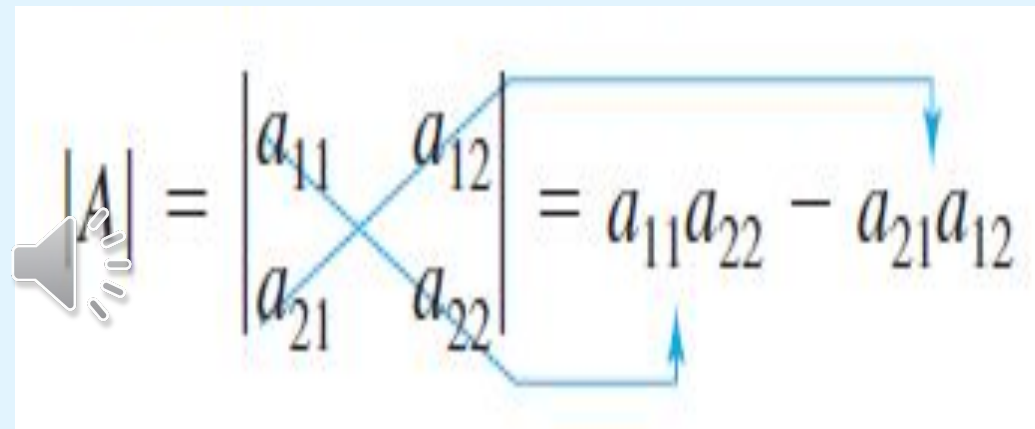
## Definition of the Determinant of a $2 \times 2$ Matrix

The **determinant** of the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

is given by

$$\det(A) = |A| = a_{11}a_{22} - a_{21}a_{12}.$$


$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

## *Note*

The determinant of a matrix of order 1 is defined simply as the entry of the matrix. For instance, if  $A = [-2]$ , then

$$\det(A) = -2.$$



find The determinant of The matrix; -

$$\textcircled{1} [1]$$

$$\therefore \det = 1$$



$$\textcircled{3} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\begin{aligned} \det &= 2 \times 4 - 3 \times 1 \\ &= 8 - 3 = 5 \end{aligned}$$

$$\textcircled{5} \begin{bmatrix} 5 & 2 \\ -6 & 3 \end{bmatrix}$$

$$\begin{aligned} \det &= 5 \times 3 - (-6) \times 2 \\ &= 15 + 12 = 27 \end{aligned}$$

$$\textcircled{11} \begin{bmatrix} \lambda - 3 & 2 \\ 4 & \lambda - 1 \end{bmatrix}$$

$$\det = [(\lambda - 3)(\lambda - 1)] - 4 \times 2$$

$$= \lambda^2 - 4\lambda + 3 - 8$$

$$= \lambda^2 - 4\lambda - 5$$

## Definitions of Minors and Cofactors of a Matrix

If  $A$  is a square matrix, then the **minor**  $M_{ij}$  of the element  $a_{ij}$  is the determinant of the matrix obtained by deleting the  $i$ th row and  $j$ th column of  $A$ . The **cofactor**  $C_{ij}$  is given by

$$C_{ij} = (-1)^{i+j}M_{ij}.$$

For example, if  $A$  is a  $3 \times 3$  matrix, then the minors and cofactors of  $a_{21}$  and  $a_{22}$  are as shown in the diagram below.

*Minor of  $a_{21}$*

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \xrightarrow{M_{21}} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

Delete row 2 and column 1.

*Cofactor of  $a_{21}$*

$$\begin{aligned} C_{21} &= (-1)^{2+1}M_{21} \\ &= -M_{21} \end{aligned}$$

*Minor of  $a_{22}$*

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \xrightarrow{M_{22}} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

Delete row 2 and column 2.

*Cofactor of  $a_{22}$*

$$\begin{aligned} C_{22} &= (-1)^{2+2}M_{22} \\ &= M_{22} \end{aligned}$$

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⑬ find The minors and Cofactors of

The matrix  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$m_{11} = \det [4] = 4 \quad \& \quad C_{11} = (-1)^{1+1} * 4 = 4$$

$$m_{12} = \det [3] = 3 \quad \& \quad C_{12} = (-1)^{1+2} * 3 = -3$$

$$m_{21} = \det [2] = 2 \quad \& \quad C_{21} = (-1)^{2+1} * 2 = -2$$

$$m_{22} = \det [1] \quad \& \quad C_{22} = (-1)^{2+2} * 1 = 1$$

(15)

$$\begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{bmatrix}$$

$$m_{11} = \det \begin{bmatrix} 5 & 6 \\ -3 & 1 \end{bmatrix} = 5 \times 1 - (-3) \times 6 = 5 + 18 = 23$$

$$C_{11} = (-1)^{1+1} m_{11} = 23$$

$$m_{12} = \det \begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} = -8 \quad C_{12} = (-1)^{1+2} \times -8 = 8$$

$$m_{13} = \det \begin{bmatrix} 4 & 5 \\ 2 & -3 \end{bmatrix} = -22 \quad C_{13} = (-1)^{1+3} \times -22 = -22$$