PART 3-1 (THE DETERMINANT OF A MATRIX)

Exercises

Definition of the Determinant of a 2×2 Matrix

The **determinant** of the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

is given by

$$\det(A) = |A| = a_{11}a_{22} - a_{21}a_{12}.$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

Note

The determinant of a matrix of order 1 is defined simply as the entry of the matrix. For instance, if $A = [-2]$, then

$$\det(A) = -2.$$

find The determinant of The matrix:-

① $[1]$

$\therefore$ det $= 1$

③ $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

det $= 2 * 4 - 3 * 1$

$= 8 - 3 = 5$

(5) $\begin{bmatrix} 5 & 2 \\ -6 & 3 \end{bmatrix}$

$$\det = 5 * 3 - (-6) * 2$$
$$= 15 + 12 = 27$$

(11) $\begin{bmatrix} \lambda-3 & 2 \\ 4 & \lambda-1 \end{bmatrix}$

$$\det = \left[(\lambda-3)(\lambda-1)\right] - 4 * 2$$
$$= \lambda^2 - 4\lambda + 3 - 8$$
$$= \lambda^2 - 4\lambda - 5$$

Definitions of Minors and Cofactors of a Matrix

If A is a square matrix, then the **minor** M_{ij} of the element a_{ij} is the determinant of the matrix obtained by deleting the ith row and jth column of A. The **cofactor** C_{ij} is given by

$$C_{ij} = (-1)^{i+j} M_{ij}.$$

For example, if A is a 3×3 matrix, then the minors and cofactors of a_{21} and a_{22} are as shown in the diagram below.

Minor of a_{21}

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

Delete row 2 and column 1.

Cofactor of a_{21}

$$C_{21} = (-1)^{2+1} M_{21}$$
$$= -M_{21}$$

Minor of a_{22}

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}$$

Delete row 2 and column 2.

Cofactor of a_{22}

$$C_{22} = (-1)^{2+2} M_{22}$$
$$= M_{22}$$

(13) find The minors and Cofactors of

The matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$m_{11} = \det[4] = 4$ & $C_{11} = (-1)^{1+1} * 4 = 4$

$m_{12} = \det[3] = 3$ & $C_{12} = (-1)^{1+2} * 3 = -3$

$m_{21} = \det[2] = 2$ & $C_{21} = (-1)^{2+1} * 2 = -2$

$m_{22} = \det[1]$ & $C_{22} = (-1)^{2+2} * 1 = 1$

(15)

$$\begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{bmatrix}$$

$$m_{11} = \det \begin{bmatrix} 5 & 6 \\ -3 & 1 \end{bmatrix} = 5 * 1 - (-3) * 6$$
$$= 5 + 18 = 23$$

$$C_{11} = (-1)^{1+1} m_{11} = 23$$

$$m_{12} = \det \begin{bmatrix} 4 & 6 \\ 2 & 1 \end{bmatrix} = -8 \ \& \ C_{12} = (-1)^{1+2} * -8 = 8$$

$$m_{13} = \det \begin{bmatrix} 4 & 5 \\ 2 & -3 \end{bmatrix} = -22 \ \& \ C_{13} = (-1)^{1+3} * -22 = -22$$