

COMP290-084  
Clockless Logic

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# Acknowledgment

Michael Theobald and Steven Nowick,  
for providing slides for this lecture.

# An Implicit Method for Hazard-Free Two-Level Logic Minimization

Michael Theobald and Steven M. Nowick

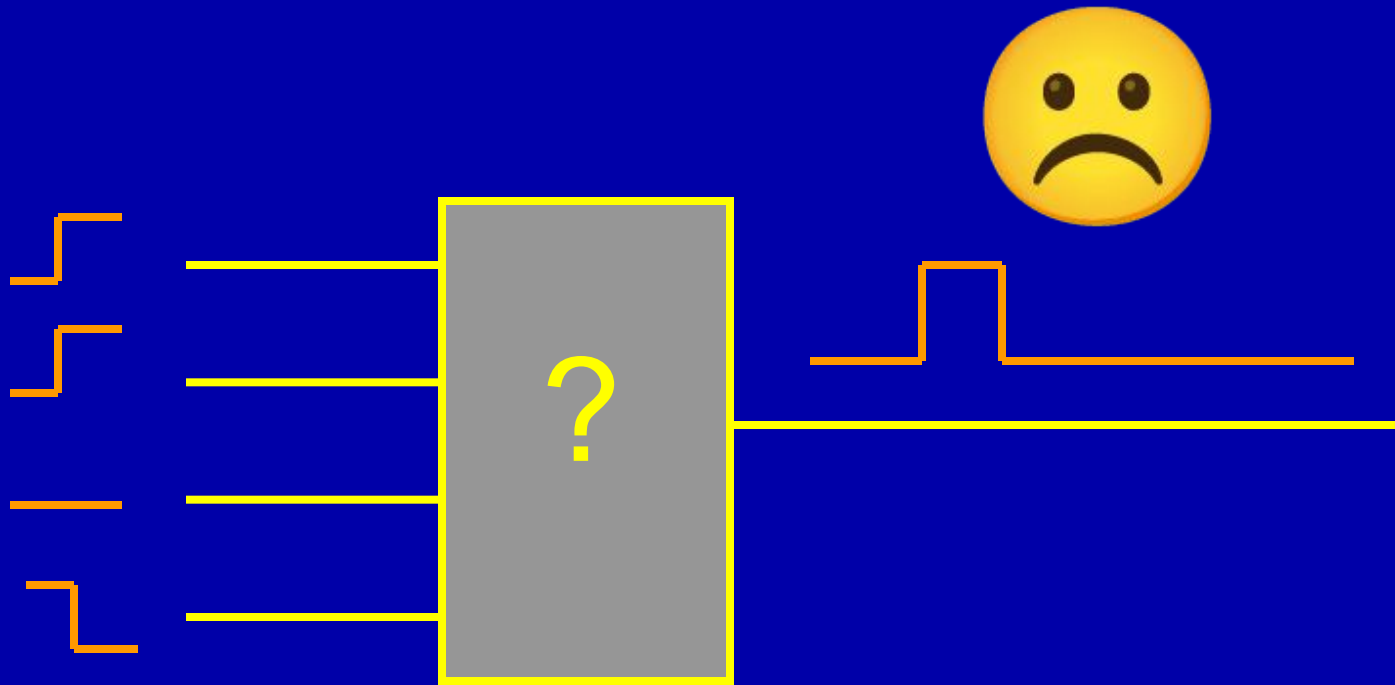
Columbia University, New York, NY

Paper appeared in Async-98

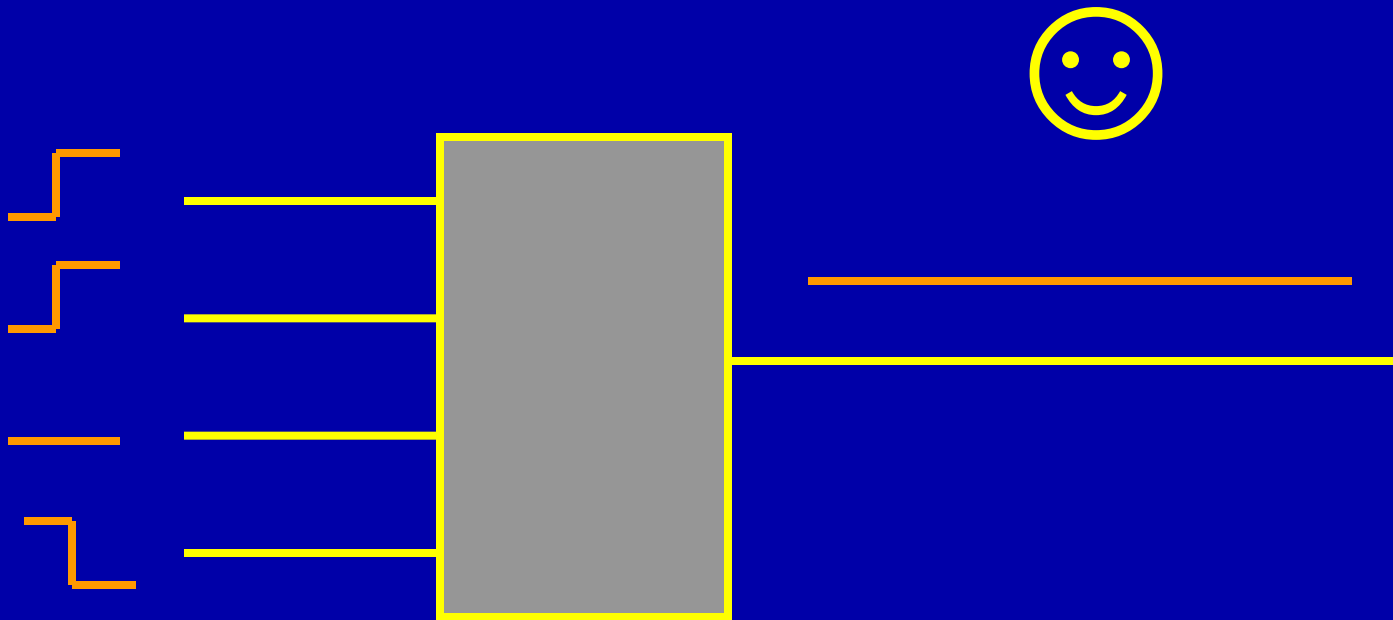
(Best Paper Finalist)

# Hazard-Free Logic Minimization

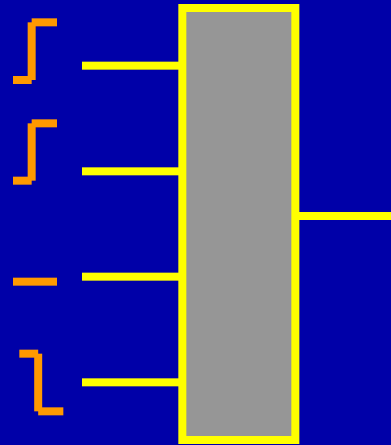
Given: Boolean function and multi-input change



# Hazard-Free Logic Minimization

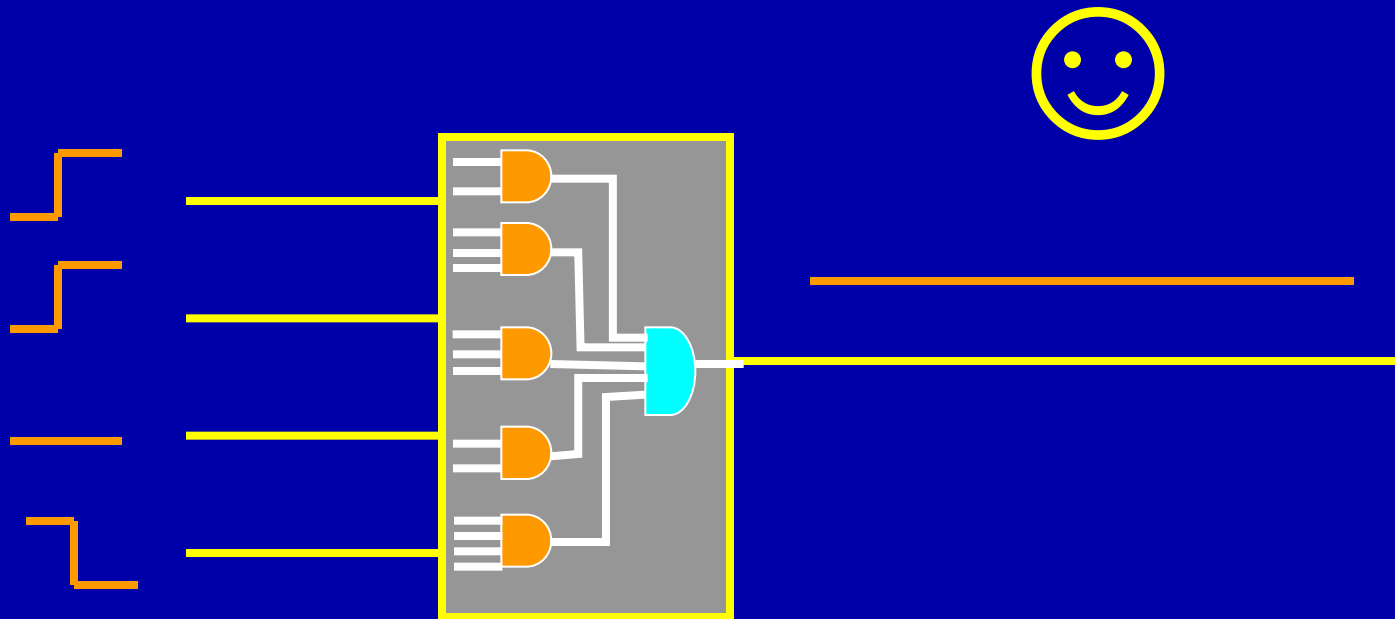


# Hazard-Free Logic Minimization



$f(A)$ □ $f(B)$	😊	😞
<b>0</b> □ <b>0</b>		
<b>0</b> □ <b>1</b>		
<b>1</b> □ <b>1</b>		
<b>1</b> □ <b>0</b>		

# Hazard-Free 2-Level Logic Minimization



# Classic 2-Level Logic Minimization

## Quine-McCluskey Algorithm

### Step 1. Generate Prime Implicants

1's: "Minterms"

Ovals: "Prime Implicants"

Karnaugh-Map:

0	0
0	1
1	1
1	0

### Step 2. Select Minimum # of Primes ..to cover all Minterms

	Prime implicants
Minterms	



# 2-level Logic Minimization: Classic vs. Hazard-Free

- Classic (Quine-McCluskey):

<On-set minterms, Prime implicants>

- Hazard-Free:

<Required cubes, DHF-Prime implicants>

- Given: Boolean function & set of “multi-input” changes
- Find: *min-cost* 2-level implementation guaranteed to be glitch-free
- Required cubes = sets of minterms
- DHF-Prime implicants =  
maximal implicants that do not intersect privileged cubes illegally

# Hazard-Free Logic Minimization

## Multi-Input Changes:

### ■ Non-monotonic

- *function hazard*
- no implementation hazard-free

### ■ Monotonic

- *function-hazard-free*

0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0



Restriction to **monotonic** changes

# Hazard-Freedom Conditions: 1 -> 1 transition

0	0	0	0
0	1	0	0
0	1	0	0
0	1	-	0



0	0	0	0
0	1	0	0
0	1	0	0
0	1	-	0



Required Cube  
must be covered

# Hazard-Freedom Conditions: 1 -> 0 transition

0	0	0	0
0	1	1	0
0	1	0	0
0	1	0	0

The diagram shows a 4x4 grid representing a Karnaugh map. The cells contain binary values (0 or 1). The values are as follows:

- Row 1: 0, 0, 0, 0
- Row 2: 0, 1, 1, 0
- Row 3: 0, 1, 0, 0
- Row 4: 0, 1, 0, 0

Yellow circles highlight the 1s in the second column (rows 2, 3, 4) and the 1s in the second row (columns 2, 3). A red arrow points from the 1 in the second row, first column to the 0 in the second row, third column, indicating a transition.

# Hazard-Freedom Conditions: 1 -> 0 transition

0	0	0	0
0	1	1	0
0	1	0	0
0	1	0	0

# Hazard-Freedom Conditions: 1 -> 0 transition

0	0	0	0
0	1	1	0
0	1	0	0
0	1	0	0

# Hazard-Freedom Conditions: 1 -> 0 transition

0	0	0	0
0	1	1	0
0	1	0	0
0	1	0	0

The image shows a 4x4 Karnaugh map with the following values:

0	0	0	0
0	1	1	0
0	1	0	0
0	1	0	0

Yellow circles highlight the 1s in the second column (rows 2, 3, 4) and the 1s in the second and third rows (columns 1, 2). A red arrow points from the 1 in the second row, second column to the 0 in the second row, third column, illustrating a 1-to-0 transition.

# Hazard-Freedom Conditions: 1 -> 0 transition

0	0	0	0
0	1	1	0
0	1	0	0
0	1	0	0

The diagram shows a 4x4 Karnaugh map with the following values:

- Row 1: 0, 0, 0, 0
- Row 2: 0, 1, 1, 0
- Row 3: 0, 1, 0, 0
- Row 4: 0, 1, 0, 0

Annotations:

- A yellow oval groups the 1s in the first column of rows 2 and 3.
- A red arrow points from the 1 in row 2, column 1 to the 0 in row 3, column 2.



# Hazard-Freedom Conditions: 1 -> 0 transition

0	0	0	0
0	1	1	0
0	1	0	0
0	1	0	0



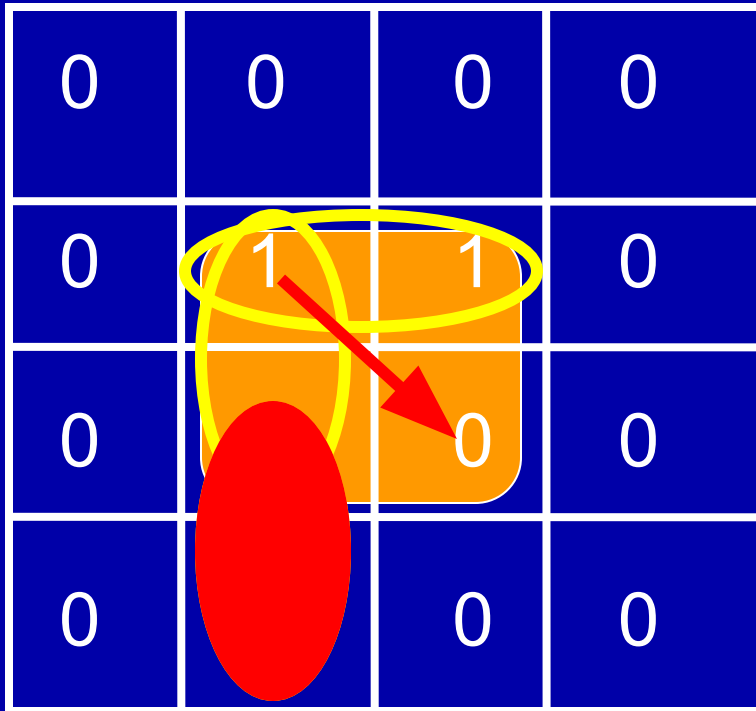
# Hazard-Freedom Conditions: 1 -> 0 transition

0	0	0	0
0	1	1	0
0	0	0	0
0	0	0	0

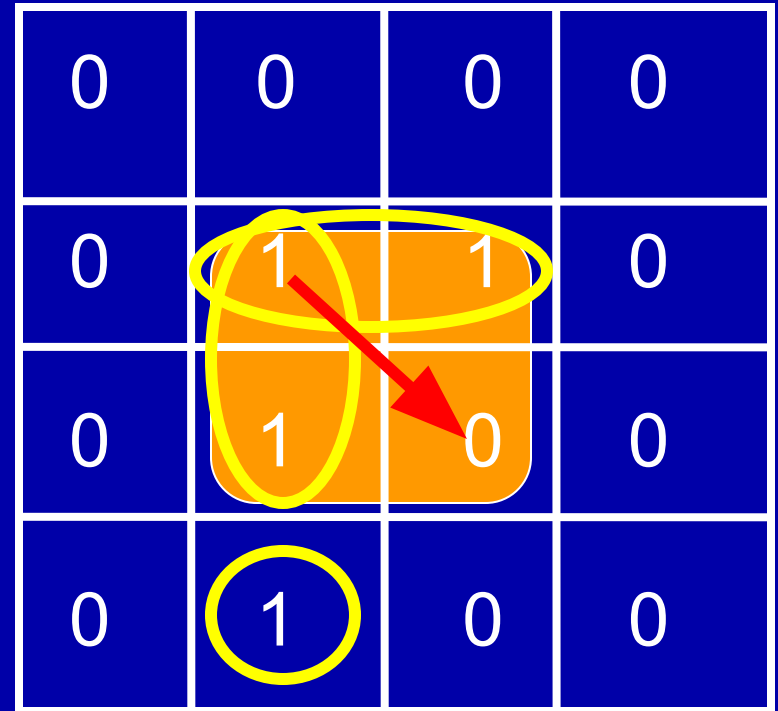


illegal intersection

# Hazard-Freedom Conditions: 1 -> 0 transition



illegal intersection



No illegal  
intersection  
of privileged cube

# Dynamic-Hazard-Free Prime Implicants

0	0
0	1
1	1
1	0

**Prime**

0	0
0	0
1	1
1	0

**NO DHF-Prime  
illegal  
intersection**

0	0
0	1
1	1
1	0

**DHF-Prime**

# 2-level Logic Minimization: Classic vs. Hazard-Free

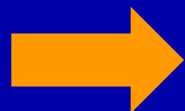
- Classic (Quine-McCluskey):

<On-set minterms, Prime implicants>

- Hazard-Free:

<Required cubes, DHF-Prime implicants>

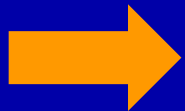
- Given: Boolean function & set of “multi-input” changes
- Find: *min-cost* 2-level implementation guaranteed to be glitch-free
- Required cubes = sets of minterms
- DHF-Prime implicants =  
maximal implicants that do not intersect privileged cubes illegally



Main challenge: Computing DHF-prime implicants

# Hazard-Free 2-level Logic Minimization: Previous Work

- Early work (1950s-1970s):
  - Eichelberger, Unger, Beister, McCluskey
- Initial solution: Nowick/Dill [ICCAD 1992]
- Improved approaches:
  - HFMIN: Fuhrer/Nowick [ICCAD 1995]
  - Rutten et al. [Async 1999]
  - Myers/Jacobson [Async 2001]



No approach can solve large examples

# IMPYMIN: an exact 2-level minimizer

- Two main ideas:
  - novel reformulation of hazard-freedom constraints
    - used for dhf-prime generation
    - recasts an **asynchronous** problem as a **synchronous** one
  - uses an “implicit” method
    - represents & manipulates large # of objects **simultaneously**
    - avoids explicit enumeration
    - makes use of BDDs, ZBDDs
- Outperforms existing tools by orders of magnitude

# Review: Primes vs. DHF-Primes

Classic (Quine-McCluskey):

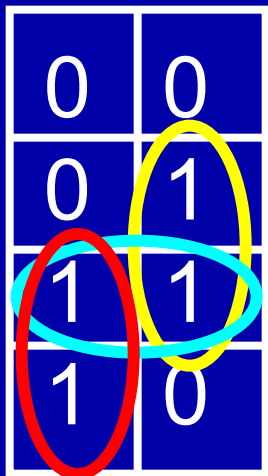
<On-set minterms, **Prime implicants**>

Hazard-Free:

<Required cubes, **DHF-Prime implicants**>

**DHF-Prime Implicants** = maximal implicants that do not intersect  
"privileged cubes" illegally

0	0
0	1
1	1
1	0

A 2x2 Karnaugh map with cells containing 0, 0, 1, 1, 1, 0. Three overlapping circles represent prime implicants: a red circle around the bottom-left cell (1,1), a yellow circle around the top-right cell (0,1), and a cyan circle around the two middle cells (1,1) and (0,1).

**Primes**

0	0
0	1
1	1
1	0

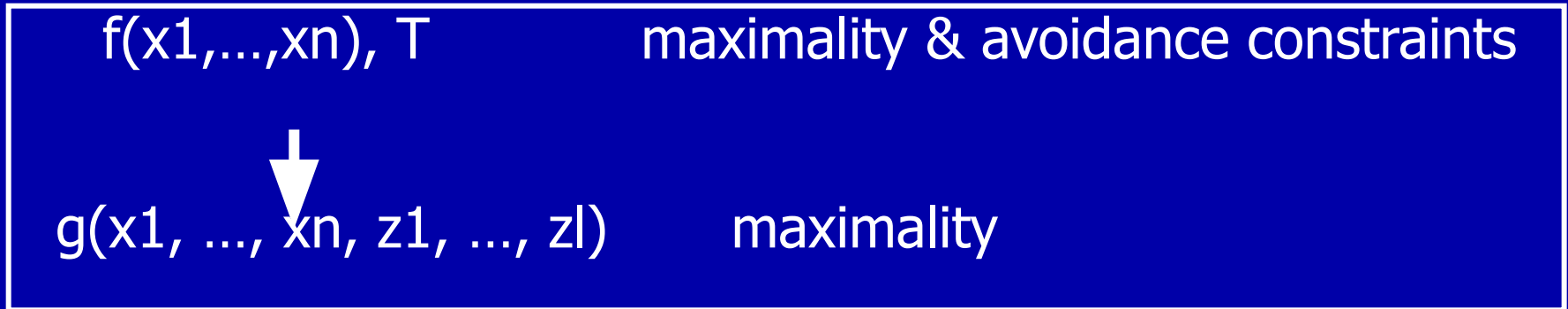
A 2x2 Karnaugh map with cells containing 0, 0, 1, 1, 1, 0. Three overlapping circles represent DHF-Prime implicants: a red circle around the bottom-left cell (1,1), a yellow circle around the top-right cell (0,1), and a cyan circle around the two middle cells (1,1) and (0,1). The two middle cells are shaded orange, and white arrows point from the red and yellow circles towards the orange shaded area.

**DHF-Primes**



# DHF-Prime Generation

- **Challenge:** Two types of constraints
  - **maximality** constraints: “we want maximally large implicants”
  - **avoidance** constraints: “we must avoid illegal intersections”
- **New Approach:** Unify constraints by “lifting” the problem into a higher-dimensional space:



# Auxiliary Synchronous Function $g$

$f$

0	0
0	1
1	1
1	0



Add one new dimension per privileged cube

$g$

0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0

$z=0$        $z=1$

0-half-space:  $g$  is defined as  $f$

1-half-space:  $g$  is defined as  $f$   
BUT priv-cube is  
filled with 0's

# Prime Implicants of g

f

0	0
0	1
1	1
1	0



g

0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0

Expansion in z-dimension  
guarantees avoidance  
of priv-cube in original domain

# Prime Implicants of g

f

0	0
0	1
1	1
1	0



g

0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0

Expansion in x-dimension corresponds to enlarging cube in original domain.

# Summary: Auxiliary Synchronous Function $g$

The definition of auxiliary function  $g$  **exactly** ensures :

- Expansion in a z-dimension corresponds to **avoiding the privileged cube** in the original domain.
- Expansion in a x-dimension corresponds to **enlarging the cube** in the original domain.

# New approach: DHF-Prime Generation

Goal: Efficient new method for DHF-Prime generation

Approach:

- translate original function  $f$  into **synchronous** function  $g$
- generate  $\text{Primes}(g)$
- after filtering step, retrieve  $\text{dhf-primes}(f)$

# Prime Generation of g

f

0	0
0	1
1	1
1	0



g

0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0

Prime implicants of g

# Filtering Primes of g

f

0	0
0	1
1	1
1	0

Lifting

g

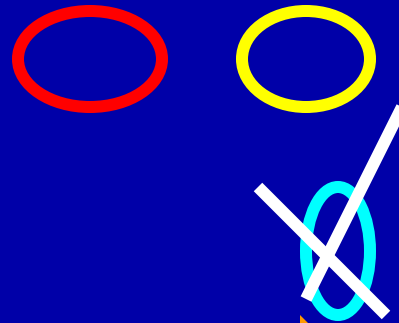
0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0

Prime implicants of g

Transforming Prime(g) into DHF-Prime(f,T):

3 classes of primes of synchronous fct g:

- 1. do not intersect priv-cube (in original domain)
- 2. intersect legally
- 3. intersect illegally



Filter

0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0



# Projection

f

0	0
0	1
1	1
1	0

0	0
0	1
1	1
1	0

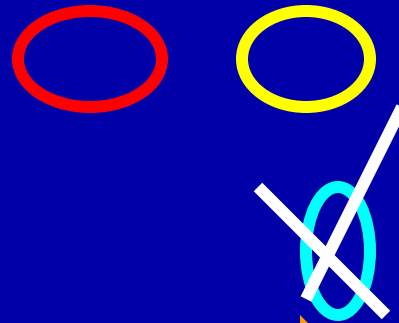
DHF-Prime(f,T)

Lifting

Projection

g

0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0



Filter

0	0	0	0
0	1	1	0
1	1	0	0
1	0	0	0

Prime implicants of g

# Formal Characterization of DHF-Prime( $f, T$ )

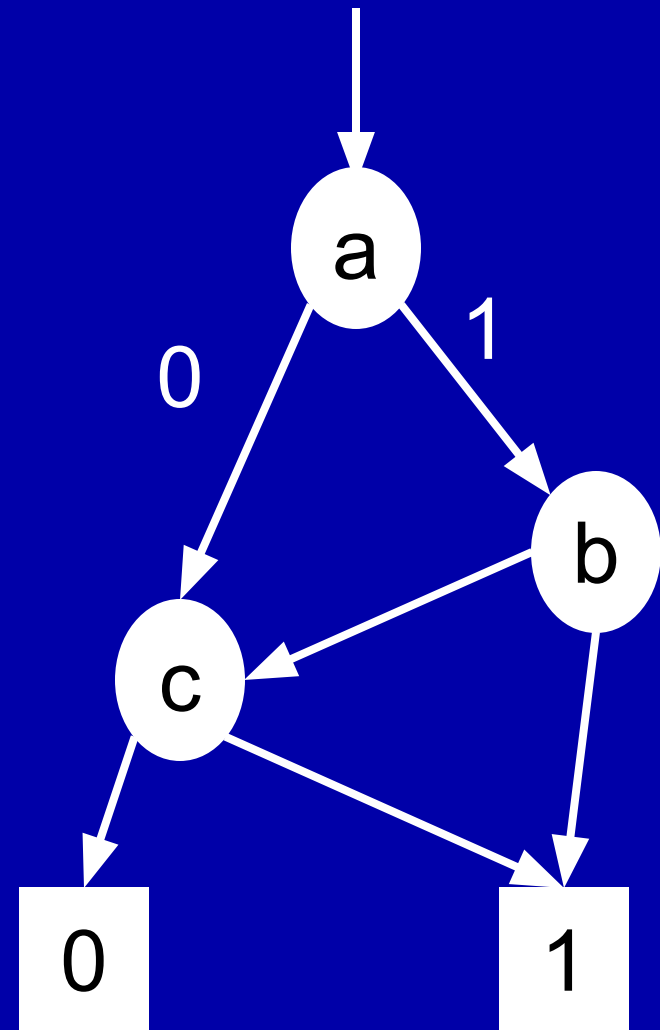
$$g(x_1, \mathbb{X}, x_n, z_1, \mathbb{X}, z_l) = f \bullet \prod_{1 \leq i \leq l} (\overline{z_i} + \overline{p_i})$$

# IMPYMIN

- CAD tool for Hazard-Free 2-Level Logic
- Two main ideas:
  - Computes DHF-Primes in higher-dimension space
  - ➔ – **Implicit Method**: makes use of BDDs, ZBDDs


# What is a BDD ?

- Compact representation for  
**Boolean function**

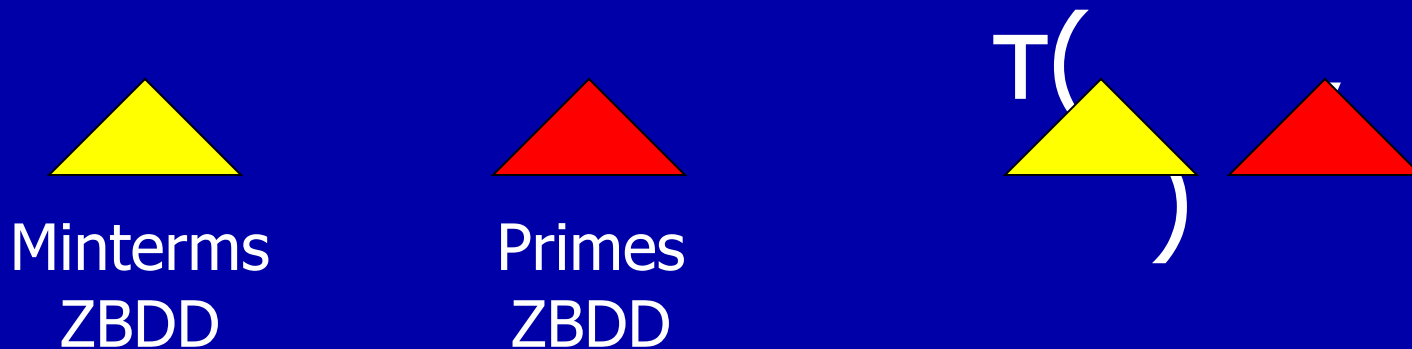


# What is implicit logic minimization?

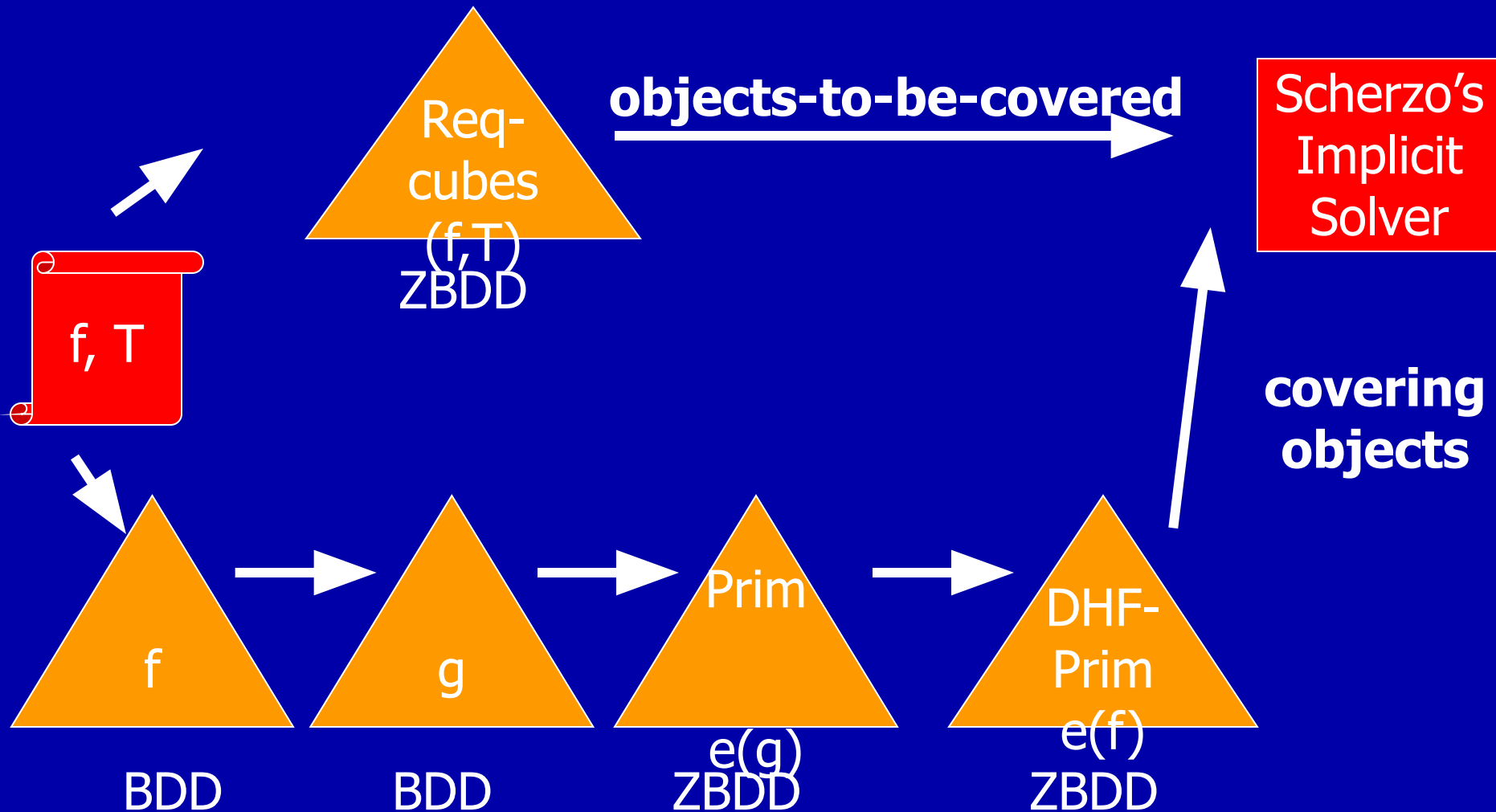
- Classic Quine-McCluskey:

	Prime implicants
Minterms	

- Scherzo [Coudert] (implicit logic minimization):



# IMPYMIN Overview: Implicit Hazard-free 2-Level Minimizer



# Impymin vs. HFMIN: Results

added variables



	I/O	#C	HFMIN	IMPYMIN	#z
cache	20/23	97	impossible	301	<b>39</b>
pscsi	16/11	77	1656	105	<b>23</b>
sd	18/22	34	172	52	<b>0</b>
Stetson1	32/33	60	>72000	813	<b>9</b>
Stetson2	18/22	37	151	49	<b>0</b>

# IMPYMIN: Conclusions

- New idea: incorporate hazard-freedom constraints
  - transformed **asynchronous** problem into **synchronous** problem
- Presented implicit minimizer IMPYMIN:
  - significantly outperforms existing minimizers
- Idea may be applicable to other problems, e.g. testing