

Семинар 8

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Занятие 8

Смешанные задачи на тему
» Прямая и плоскость в пространстве «

1011 $M_1(-6; 6; -5)$, $M_2(12; -6; 1)$, $M_1, M_2 \in \ell$

$\ell \cap xOy = N_1 = ?$, $\ell \cap xOz = N_2 = ?$, $\ell \cap yOz = N_3 = ?$

Решение: $6 \cdot \vec{\tau} = \overrightarrow{M_1 M_2} = \{18; -12; 6\}$

$\Rightarrow \vec{\tau} = \{3; -2; 1\} \Rightarrow \ell: \begin{cases} x = -6 + 3t \\ y = 6 - 2t \\ z = -5 + t \end{cases}$

$N_1: xOy: z=0 \Rightarrow -5+t=0 \Rightarrow t=5$

$\Rightarrow \boxed{N_1(9; -4; 0)}$

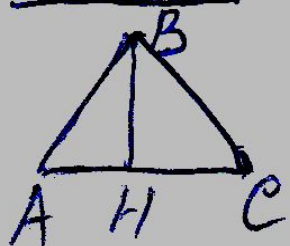
$N_2: xOz: y=0 \Rightarrow 6-2t=0 \Rightarrow t=3$

$\Rightarrow \boxed{N_2(3; 0; -2)}$

$N_3: yOz: x=0 \Rightarrow -6+3t=0 \Rightarrow t=2$

$\Rightarrow \boxed{N_3(0; 2; -3)}$

1015 | $\Delta ABC: A(1; -2; -4), B(3; 1; -3), C(5; 1; -7)$



BH - ?

Решение: $\vec{r} = \lambda \vec{a} = \vec{AC} \times (\vec{AB} \times \vec{AC})$

$$\vec{b} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 4 & 3 & -3 \end{vmatrix} = -12\vec{i} + 10\vec{j} - 6\vec{k}$$

$$\vec{a} = \vec{AC} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & -3 \\ -12 & 10 & -6 \end{vmatrix} = 12\vec{i} + 60\vec{j} + 76\vec{k}$$

$$\vec{r} = \{3; 15; 19\}$$

$$\Rightarrow \ell: \begin{cases} x = 3 + 3t \\ y = 1 + 15t \\ z = -3 + 19t \end{cases}$$

1018] l_1 - ? $l_1 \in M_1(2; 3; -5)$, $l_1 \parallel \ell: \begin{cases} 3x - y + 2z - 7 = 0 \\ x + 3y - 2z + 3 = 0 \end{cases}$

Решение $\vec{\tau}_1 = \lambda \vec{\tau} = \vec{n}_1 \times \vec{n}_2$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 2 \\ 1 & 3 & -2 \end{vmatrix} = (2-6)\vec{i} - (-6-2)\vec{j} + (9+1)\vec{k} = \{-4; 8; 10\}$$

$$\Rightarrow \vec{\tau}_1 = \{-2; 4; 5\}$$

$$\boxed{l_1: \frac{x-2}{-2} = \frac{y-3}{4} = \frac{z+5}{5}}$$

1025] $l_1: \begin{cases} x - y - 4z - 5 = 0 \\ 2x + y - 2z - 4 = 0 \end{cases}$, $l_2: \begin{cases} x - 6y - 6z + 2 = 0 \\ 2x + 2y + 9z - 1 = 0 \end{cases}$

$\hat{(l_1, l_2)} = ?$

Решение $\vec{\tau}_1 = \vec{n}_{11} \times \vec{n}_{12} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -4 \\ 2 & 1 & -2 \end{vmatrix} = \{6; -6; 3\}$

$$\vec{\tau}_2 = \vec{n}_{21} \times \vec{n}_{22} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -6 & -6 \\ 2 & 2 & 9 \end{vmatrix} = \{42; -21; 14\}$$

$$\vec{\tau}_1' = \{2; -2; 1\}, \quad \vec{\tau}_2' = \{6; 3; -2\}$$

$$\cos(\hat{\tau}_1, \hat{\tau}_2) = \frac{\vec{\tau}_1' \cdot \vec{\tau}_2'}{|\vec{\tau}_1'| |\vec{\tau}_2'|} = \frac{12 - 6 - 2}{\sqrt{9} \cdot \sqrt{49}} = \frac{4}{7 \cdot 7} = \frac{4}{49}$$

$$\boxed{\cos(\hat{l}_1, \hat{l}_2) = \frac{4}{49}}$$

$$\underline{1031)} \quad l_1: \begin{cases} x = 3t - 7 \\ y = -2t + 4 \\ z = 3t + 4 \end{cases}, \quad l_2: \begin{cases} x = t + 1 \\ y = 2t - 8 \\ z = -t - 12 \end{cases}$$

$$l - ? \quad l \perp l_1, \quad l \perp l_2$$

$$\text{Решение } M_1(-7; 4; 4), \quad \vec{\tau}_1 = \{3; -2; 3\}$$

$$M_2(1; -8; -12), \quad \vec{\tau}_2 = \{1; 2; -1\}$$

$$\vec{\tau} = \vec{\tau}_1 \times \vec{\tau}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 3 \\ 1 & 2 & -1 \end{vmatrix} = -4\vec{i} + 6\vec{j} + 8\vec{k}$$

$$\vec{\tau} = \{-2; 3; 4\}$$

$$\Pi: \Pi \ni l_1, \quad \Pi \ni \vec{\tau}$$

$$\Pi: \begin{vmatrix} x+7 & y-4 & z-4 \\ 3 & -2 & 3 \\ -2 & 3 & 4 \end{vmatrix} = 0 \Rightarrow$$

$$(x+7)(-17) + (y-4)(-18) + (z-4) \cdot 5 = 0$$

$$\Pi: 17x + 18y - 5z + 67 = 0$$

$$M_0 = l_2 \cap \Pi: 17(t+1) + 18(2t-8) - 5(-t-12) + 67 = 0$$

$$\Rightarrow 17t + 17 + 36t - 144 + 5t + 60 + 67 = 0$$

$$58t = 0 \quad \Rightarrow t = 0$$

$$\Rightarrow M_0(1; -8; -12)$$

$$l: \begin{cases} x = 1 - 2t \\ y = -8 + 3t \\ z = -12 + 4t \end{cases}$$

$$1039) \ell: \begin{cases} 5x - 3y + 2z - 5 = 0 \\ 2x - y - z - 1 = 0 \end{cases} \quad ?$$

$$\Pi: 4x - 3y + 7z - 7 = 0 \quad \ell \in \Pi$$

Решение Пучок плоскостей

$$\alpha(5x - 3y + 2z - 5) + \beta(2x - y - z - 1) = 0$$

$$\Rightarrow (5\alpha + 2\beta)x + (-3\alpha - \beta)y + (2\alpha - \beta)z - 5\alpha - \beta = 0$$

$$\begin{cases} 5\alpha + 2\beta = 4 \\ -3\alpha - \beta = -3 \end{cases} \cdot 2^+ \Rightarrow -\alpha = -2 \Rightarrow \alpha = 2, \beta = -3$$

$$\Rightarrow 2\alpha - \beta = 7 \quad \text{и} \quad -5\alpha - \beta = -7$$

Итак, при $\alpha = 2, \beta = -3$ плоскость Π принадлежит пучку плоскостей с осью ℓ . $\Rightarrow \ell \in \Pi$.

$$1041) \ell - ? \quad \ell \ni M_0(2; -4; -1)$$

$$\ell_1: \begin{cases} 3x + 4y + 5z - 26 = 0 \\ 3x - 3y - 2z - 5 = 0 \end{cases}$$

$$\Pi_1: 5x + 3y - 4z + 11 = 0; \quad \Pi_2: 5x + 3y - 4z - 41 = 0$$

Решение. $M_1 = \ell_1 \cap \Pi_1$, $M_2 = \ell_1 \cap \Pi_2$

$$M_1: \begin{cases} 3x + 4y + 5z - 26 = 0 \\ 3x - 3y - 2z - 5 = 0 \\ 5x + 3y - 4z + 11 = 0 \end{cases} \Rightarrow \begin{cases} 21x + 7z - 98 = 0 \\ 8x - 6z + 6 = 0 \end{cases} \begin{matrix} 6^+ \\ 7^+ \end{matrix}$$

$$\Rightarrow 182x = 546 \Rightarrow x = 3, z = 5, y = -2$$

$$\Rightarrow M_1(3; -2; 5)$$

$$M_2: \begin{cases} 3x + 4y + 5z - 26 = 0 \\ 3x - 3y - 2z - 5 = 0 \\ 5x + 3y - 4z - 41 = 0 \end{cases} \begin{cases} \cdot 3 \\ \cdot 4 \\ + \frac{1}{2} \end{cases} \begin{cases} 21x + 7z - 98 = 0 \\ 4x - 3z - 23 = 0 \end{cases} \begin{cases} | 3 \\ | 7 \end{cases}$$

$$\Rightarrow 91x = 455 \Rightarrow x = 5, z = -1, y = 4$$

$$M_2(5; 4; -1)$$

$$\Rightarrow M_c\left(\frac{3+5}{2}; \frac{-2+4}{2}; \frac{5-1}{2}\right) = M_c(4; 1; 2)$$

$$\vec{M_0 M_c} = \{2; 5; 3\}$$

$$\boxed{l: \frac{x-2}{2} = \frac{y+4}{5} = \frac{z+1}{3}}$$

$$1045) \quad l: \frac{x+1}{3} = \frac{y-2}{m} = \frac{z+3}{-2}; \quad \Pi: x - 3y + 6z + 7 = 0$$

$l \parallel \Pi$ $m = ?$
Решение $l \parallel \Pi \Rightarrow \vec{\tau} \perp \vec{n} \Rightarrow \vec{\tau} \cdot \vec{n} = 0$

$$\vec{\tau} = \{3; m; -2\}, \quad \vec{n} = \{1; -3; 6\}$$

$$\Rightarrow \vec{\tau} \cdot \vec{n} = 3 - 3m - 12 = 0 \Rightarrow \boxed{m = -3}$$

$$1051) \quad P(4; 1; 6), \quad l: \begin{cases} x - y - 4z + 12 = 0 \\ 2x + y - 2z + 3 = 0 \end{cases}$$

? Q - точка симметричная к точке P относительно прямой l.

Решение Построим плоскость $\Pi \perp l$, $\Pi \ni P$

$$\vec{n}' = \vec{\tau}_l = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -4 \\ 2 & 1 & -2 \end{vmatrix} = 6\vec{i} - 6\vec{j} + 3\vec{k}$$

$$\Rightarrow \vec{n} = \{2; -2; 1\}$$

$$\Pi: 2(x-4) - 2(y-1) + (z-6) = 0$$

$$\Pi: 2x - 2y + z - 12 = 0$$

$$l \cap \Pi = 0 \Rightarrow \begin{cases} x - y - 4z + 12 = 0 \\ 2x + y - 2z + 3 = 0 \\ 2x - 2y + z - 12 = 0 \end{cases} \begin{array}{l} + 3x - 6z + 15 = 0 \\ + 6x - 3z - 6 = 0 \end{array}$$

$$\Rightarrow \begin{cases} x - 2z + 5 = 0 \\ 2x - z - 2 = 0 \end{cases} \begin{array}{l} | -2 \\ + \end{array} \Rightarrow -3x + 9 = 0 \Rightarrow x = 3$$

$$\Rightarrow z = 4 \Rightarrow y = -1 \Rightarrow O(3; -1; 4)$$

$$x_Q = 2x_0 - x_P = 6 - 4 = 2$$

$$y_Q = 2y_0 - y_P = -2 - 1 = -3 \Rightarrow \boxed{Q(2; -3; 2)}$$

$$z_Q = 2z_0 - z_P = 8 - 6 = 2$$

$$1062) P(1; -1; -2); l: \frac{x+3}{3} = \frac{y+2}{2} = \frac{z-8}{-2}$$

$$r(P, l) = ?$$

Решение $M_0(-3; -2; 8), \vec{r} = \{3; 2; -2\}$

$$r(P, l) = h = \frac{|\vec{M}_0 P \times \vec{r}|}{|\vec{r}|}$$

$$\frac{r}{M_0} \begin{array}{c} \vec{h} \\ \vec{r} \end{array} \quad \vec{M}_0 P \times \vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 1 & -10 \\ 3 & 2 & -2 \end{vmatrix} = \{18; -22; 5\}$$

$$r(P, l) = \frac{\sqrt{324 + 484 + 25}}{\sqrt{9 + 4 + 4}} = \frac{\sqrt{833}}{\sqrt{17}} = \sqrt{49} = \boxed{7}$$

$$1065] M_0(1; 2; -3) \quad \ell_1: \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-7}{3}$$

$$\ell_2: \frac{x+5}{3} = \frac{y-2}{-2} = \frac{z+3}{-1}$$

Π -? $\Pi \ni M_0$, $\Pi \parallel \ell_1$, $\Pi \parallel \ell_2$

Решение $\vec{\tau}_1 = \{2; -3; 3\}$, $\vec{\tau}_2 = \{3; -2; -1\}$

\forall точки $M(x, y, z) \in \Pi$ $\vec{M_0M}, \vec{\tau}_1, \vec{\tau}_2 \in \Pi$

$$\Rightarrow \vec{M_0M} \cdot \vec{\tau}_1 \cdot \vec{\tau}_2 = 0$$

$$\Rightarrow \Pi: \begin{vmatrix} x-1 & y-2 & z+3 \\ 2 & -3 & 3 \\ 3 & -2 & -1 \end{vmatrix} = 0 \Rightarrow (x-1)9 + (y-2)11 + (z+3)5 = 0$$

$$\boxed{\Pi: 9x + 11y + 5z - 16 = 0}$$

$$1079] \Pi-? \Pi \ni \ell: \frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-2}{2}$$

$$\Pi \perp \Pi_1: 3x + 2y - z - 5 = 0$$

Решение $\vec{n}_1 = \{3; 2; -1\} \in \Pi$

$$\Rightarrow \begin{vmatrix} x-1 & y+2 & z-2 \\ 2 & -3 & 2 \\ 3 & 2 & -1 \end{vmatrix} = 0$$

$$(x-1)(-1) + (y+2)8 + (z-2)13 = 0$$

$$\boxed{\Pi: x - 8y - 13z + 9 = 0}$$

$$1083) 2) l_1: \begin{cases} x=2t-4 \\ y=-t+4 \\ z=-2t-1 \end{cases}, l_2: \begin{cases} x=4t-5 \\ y=-3t+5 \\ z=-5t+5 \end{cases}$$

$\rho(l_1, l_2) - ?$

Решение $M_1(-4, 4, -1), \vec{\tau}_1 = \{2; -1; -2\}$

$M_2(-5; 5; 5), \vec{\tau}_2 = \{4; -3; -5\}$

$\vec{M}_1 \vec{M}_2 = \{-1; 1; 6\}$

$$\rho(l_1, l_2) = \frac{|\vec{M}_1 \vec{M}_2 \cdot \vec{\tau}_1 \cdot \vec{\tau}_2|}{|\vec{\tau}_1 \times \vec{\tau}_2|}$$

$$\vec{M}_1 \vec{M}_2 \cdot \vec{\tau}_1 \cdot \vec{\tau}_2 = \begin{vmatrix} -1 & 1 & 6 \\ 2 & -1 & -2 \\ 4 & -3 & -5 \end{vmatrix} = 1 + 2 - 12 = -9$$

$$\vec{\tau}_1 \times \vec{\tau}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & -2 \\ 4 & -3 & -5 \end{vmatrix} = -\vec{i} + 2\vec{j} - 2\vec{k}$$

$$\rho(l_1, l_2) = \frac{|-9|}{\sqrt{1+4+4}} = \frac{9}{3} = \boxed{3}$$

Дома: К. 1006, 1012, 1020, 1024, 1030, 1036,
1038, 1042, 1053, 1064, 1068, 1076,
1082, 1083 (ост.)