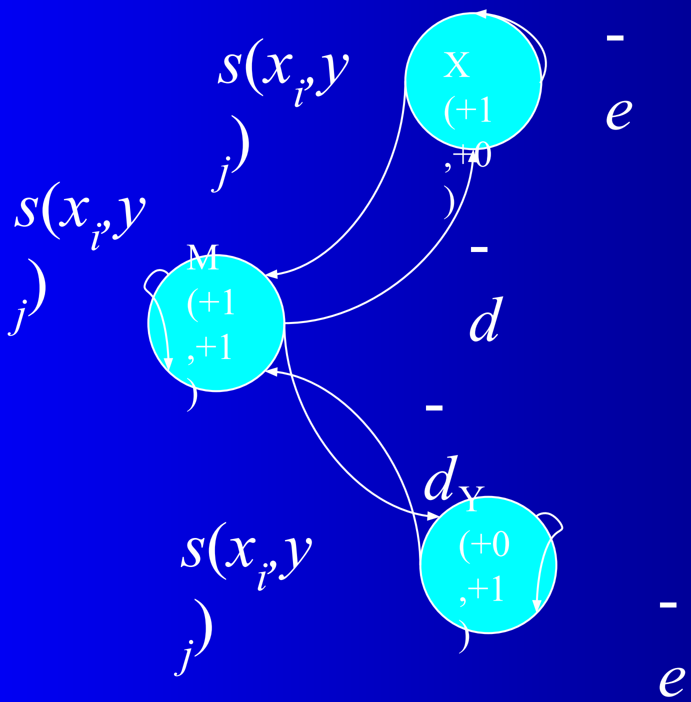
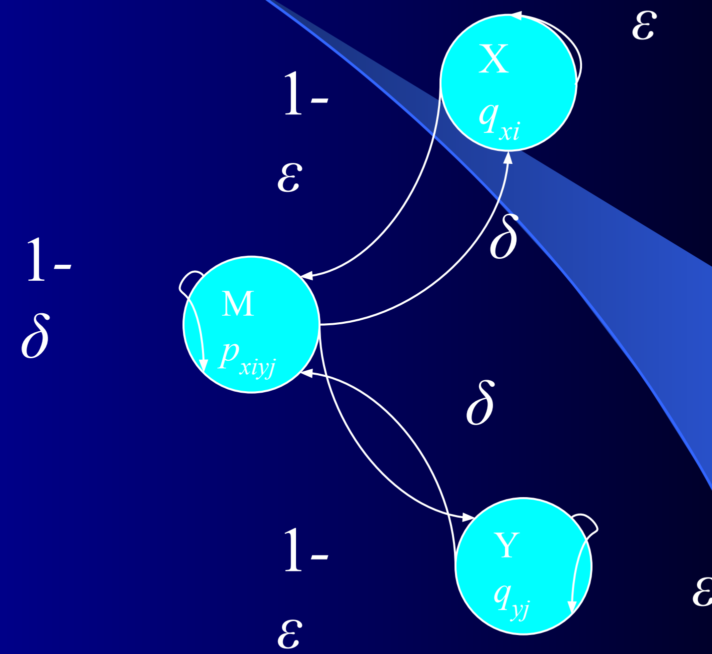
HMM выравнивание



Конечный автомат FSA



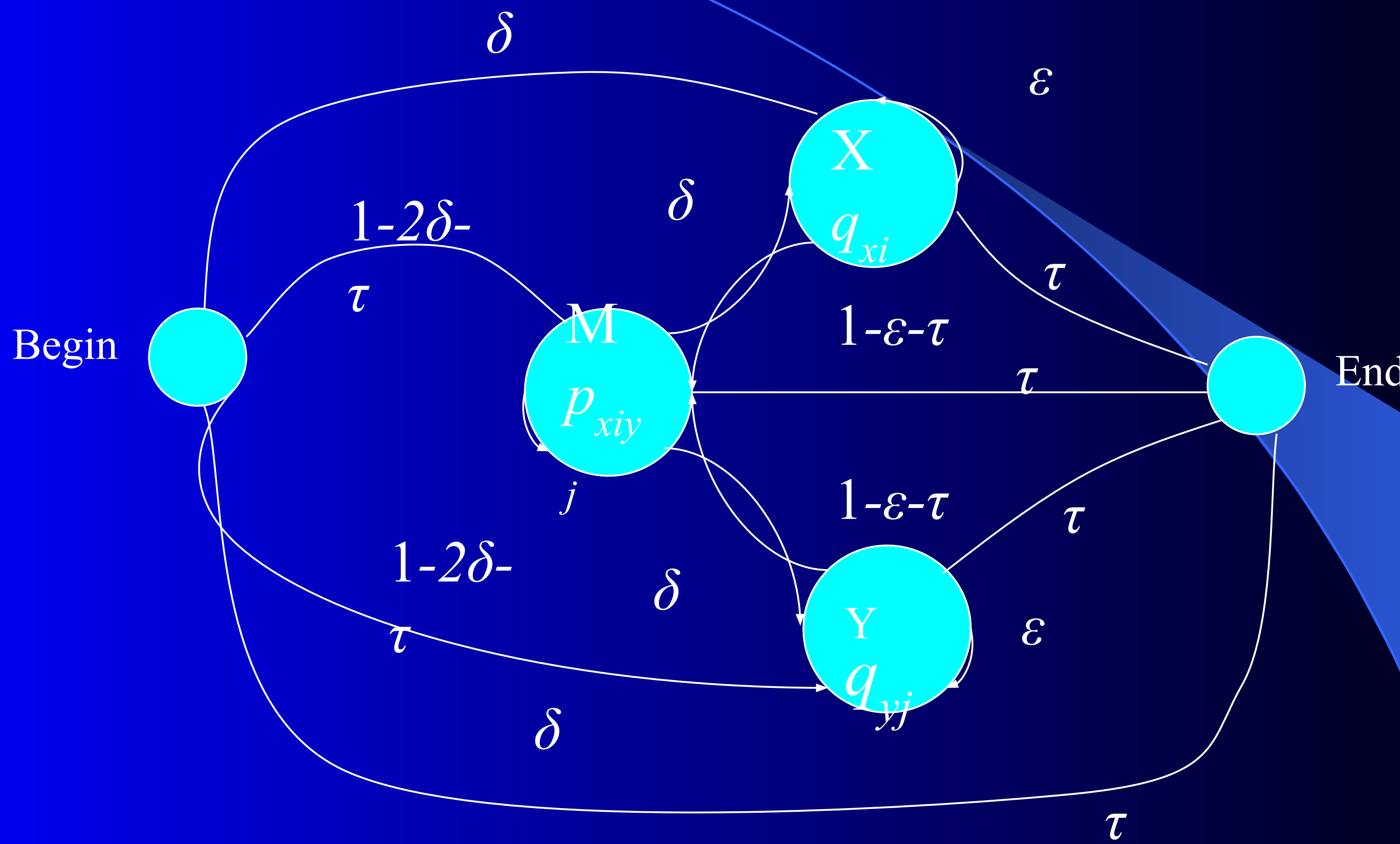
HMM

Рекурсия FSA

$$V^{\mathrm{M}}(i,j) = s(x_i, y_i) + \max\begin{cases} V^{\mathrm{M}}(i-1, j-1) \\ V^{\mathrm{X}}(i-1, j-1) \\ V^{\mathrm{Y}}(i-1, j-1) \end{cases}$$

$$V^{\mathrm{X}}(i,j) = \max\begin{cases} V^{\mathrm{M}}(i-1, j) - d \\ V^{\mathrm{X}}(i-1, j) - e \end{cases}$$

$$V^{\mathrm{Y}}(i,j) = \max\begin{cases} V^{\mathrm{M}}(i, j-1) - d \\ V^{\mathrm{Y}}(i, j-1) - e \end{cases}$$



δ
ε
X
q_{xi}
δ
1-2δ-
τ
τ
Begin
M
p_{xiy}
1-ε-τ
τ
End
j
1-ε-τ
τ
1-2δ-
δ
τ
Y
q_{yj}
ε
δ
τ

Алгоритм Витерби

– Начало:
  - $v^{M}(0, 0) = 1$. $v^{X}(0, 0) = v^{Y}(0, 0) = 0$ $v^{*}(-1, j) = v^{*}(i, -1) = 0$.

– Рекурсия: $i = 0,\ldots,n$, $j = 0,\ldots,m$, except for(0,0);

$$v^{M}(i,j) = p_{x_i y_j} \max\begin{cases}(1-2\delta-\tau)v^{M}(i-1,j-1)\\(1-\varepsilon-\tau)v^{X}(i-1,j-1)\\(1-\varepsilon-\tau)v^{Y}(i-1,j-1)\end{cases}$$

$$v^{X}(i,j) = q_{x_i} \max\begin{cases}\delta v^{M}(i-1,j)\\\varepsilon v^{X}(i-1,j)\end{cases}$$

$$v^{Y}(i,j) = q_{y_j} \max\begin{cases}\delta v^{M}(i,j-1)\\\varepsilon v^{X}(i,j-1)\end{cases}$$

– Вывод: $v^{E} = \tau \max(v^{M}(n,m), v^{X}(n,m), v^{Y}(n,m))$

Полная вероятность выравниваний

- Алгоритм: Forward для парных HMMs
  - Начало:
    - $f^{M}(0, 0) = 1, f^{X}(0,0) = f^{Y}(0,0) = 0.$
    - All $f^{\bullet}(i,-1), f^{\bullet}(-1, j)$ are set to 0.
  - Рекурсия: $i = 0,\ldots,n, j = 0,\ldots,m$ except (0,0);

$$f^{M}(i,j) = p_{x_i y_j}[(1-2\delta-\tau)f^{M}(i-1,j-1) +$$

$$(1-\varepsilon-\tau)(f^{X}(i-1,j-1) + f^{Y}(i-1,j-1))];$$

$$f^{X}(i,j) = q_{x_i}[\delta f^{M}(i-1,j) + \varepsilon f^{X}(i-1,j)];$$

$$f^{Y}(i,j) = q_{y_j}[\delta v^{M}(i,j-1) + \varepsilon v^{X}(i,j-1)].$$

  - Вывод:

$$f^{E}(n,m) = \tau[f^{M}(n,m) + f^{X}(n,m) + f^{Y}(n,m)];$$

Вероятность выровненных x_i и y_j

$$P(x_i \Diamond y_j \mid x, y) = \frac{P(x, y, x_i \Diamond y_j)}{P(x, y)}$$

Forward algorithm

$$P(x, y, x_i \Diamond y_j) = P(x_{1\ldots i}, y_{1\ldots j}, x_i \Diamond y_j) P(x_{i+1\ldots n}, y_{j+1\ldots m} \mid x_{1\ldots i}, y_{1\ldots j}, x_i \Diamond y_j)$$

$$= P(x_{1\ldots i}, y_{1\ldots j}, x_i \Diamond y_j) P(x_{i+1\ldots n}, y_{j+1\ldots m} \mid x_i \Diamond y_j)$$

Forward algorithm

Backward algorithm

Backward Algorithm

- Алгоритм: Backward для парных HMMs
  - Начало:
    - $b^M(n, m) = b^X(n, m) = b^Y(n,m) = \tau$.
    - All $b^{\bullet}(i, m+1)$, $b^{\bullet}(n+1, j)$ are set to 0.
  - Рекурсия: $i = 1,\ldots,n$, $j = 1,\ldots,m$ except (n, m);

$$b^M(i,j) = (1-2\delta-\tau)p_{x_{i+1}y_{j+1}}b^M(i+1,j+1)$$
$$+\delta[q_{x_i+1}b^X(i+1,j)+q_{y_j+1}b^Y(i,j+1)];$$
$$b^X(i,j) = (1-\varepsilon-\tau)p_{x_{i+1}y_{j+1}}b^M(i+1,j+1)+\varepsilon q_{x_{i+1}}b^X(i+1,j)];$$
$$b^Y(i,j) = (1-\varepsilon-\tau)p_{x_{i+1}y_{j+1}}b^M(i+1,j+1)+\varepsilon q_{y_{j+1}}b^Y(i,j+1)].$$