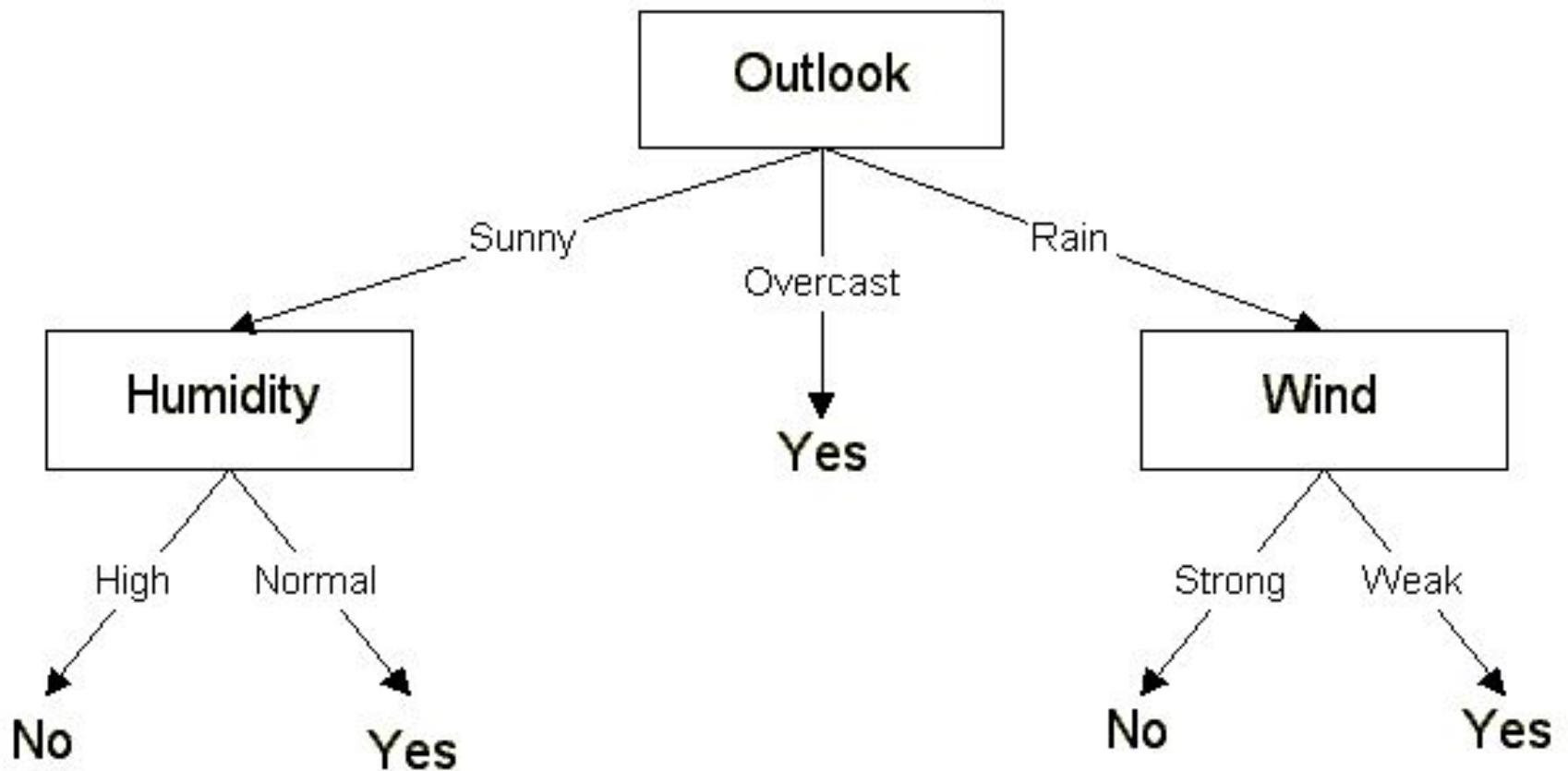


# **Arbori de decizie.**

## **Algoritmul IDE3**



$$H(X) = - \sum_{i=1}^n p(x_i) \log_b p(x_i),$$

$$\text{entropy}(p_1, p_2, \dots, p_n) = -p_1 \log p_1 - p_2 \log p_2 \dots - p_n \log p_n$$



Ludwig Boltzmann  
1844 - 1906 Viena  
*Austrian physicist and philosopher*

**The entropy law is  
sometimes referred to  
as the second law of  
thermodynamics**

This second law states that for any irreversible process, entropy always increases.

Entropy is a measure of disorder.

*!!! Since virtually all natural processes are irreversible, the entropy law implies that the universe is "running down"*

$$S = k \cdot \log W$$



LUDWIG  
BOLTZMANN  
1844-1906

HENRIETTE  
BOLTZMANN  
FEB. EDLE VON AIGENTLER  
1854-1938

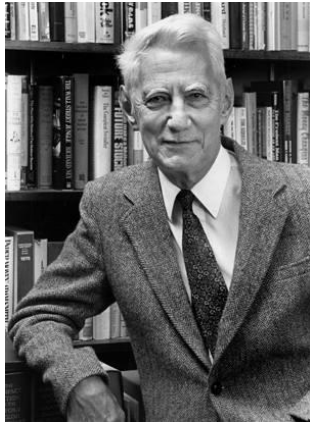
DR. PHIL. PAULA  
BOLTZMANN  
GEB. CHLARI  
1891-1977  
ARTHUR  
BOLTZMANN  
DIPL. ING. DR. PHIL. HOFRA  
1881-1952  
LUDWIG  
BOLTZMANN  
1923-1943  
ÄLTSTER MÄNNLICHER NACHKOMME  
GEFALLEN BEI SMOLENSK

**Entropy** can be seen as a measure of the quality of energy:

Low entropy sources of energy are of high quality. Such energy sources have a high energy density.

High entropy sources of energy are closer to randomness and are therefore less available for use

# Entropy in Information Theory



Claude Shannon

1916 –2001

*American mathematician, electrical engineer, and cryptographer known as "the father of information theory".*

- Claude Shannon transferred some of these ideas to the world of information processing. Information is associated by him with low entropy.
- Contrasted with information is "noise", randomness, high entropy.

- At the extreme of no information are random number.
- Of course, data may only look random.
  - But there may be hidden patterns, information in the data. The whole point of ML is to dig out the patterns. The discovered patterns are usually presented as rules or decision trees. Shannon's information theory can be used to construct decision trees.



A collection of random numbers has maximum entropy





# Entropy in Information Theory

- Shannon defined the entropy  $H$  (Greek capital letter eta) of a discrete variable  $X$  with possible values  $\{x_1, \dots, x_n\}$  and probability  $p$  as:

$$H(X) = - \sum_{i=1}^n p(x_i) \log_b p(x_i),$$

- where

measure is the *bit*.

# Example

• Presupunem evenimentul aruncării unui zar cu 6 fețe. Valorile variabilei  $X$  sunt  $\{1,2,3,4,5,6\}$  iar probabilitățile obținerii oricărei valori sunt egale.

-În acest caz entropia este:

$$H(X) = - \sum_{i=1}^6 (1/6) \log_2(1/6) = -6 * (1/6) \log_2(1/6) = -\log_2(1/6) = 2.58$$

**bits.**

## Which is the best attribute?

- ❖ Aim: to get the smallest tree
- ❖ Heuristic
  - choose the attribute that produces the “purest” nodes
  - i.e. the greatest information gain
- ❖ Information theory: measure information in bits

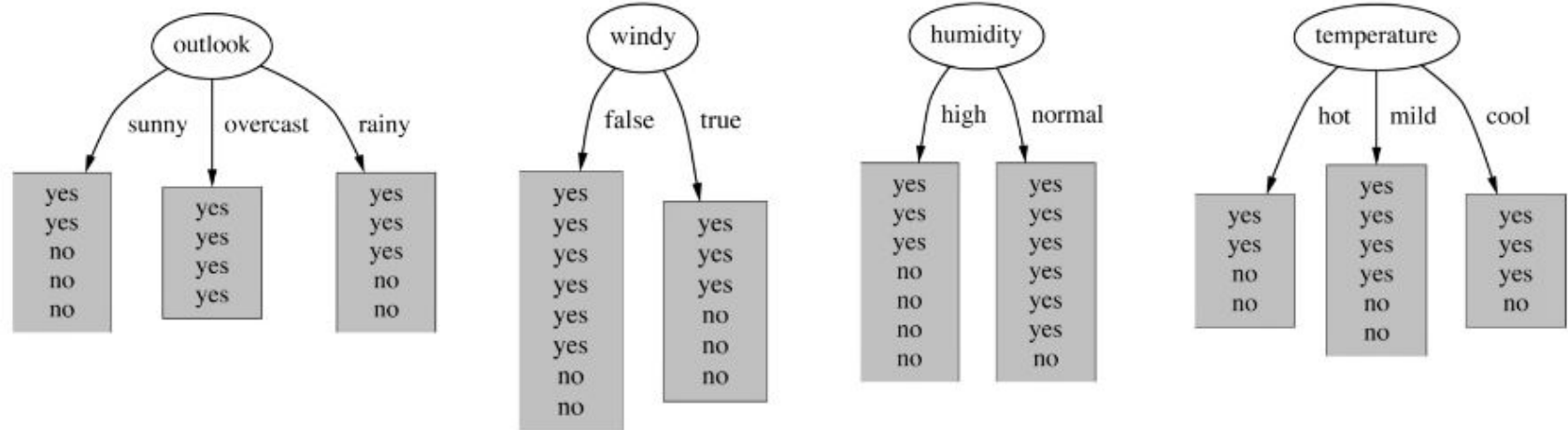
$$\text{entropy}(p_1, p_2, \dots, p_n) = -p_1 \log p_1 - p_2 \log p_2 \dots - p_n \log p_n$$

### *Information gain*

- Amount of information gained by knowing the value of the attribute
- (Entropy of distribution before the split) – (entropy of distribution after it)

	outlook text	temperature text	humidity text	windy text	play text
1	sunny	hot	high	FALSE	no
2	sunny	hot	high	TRUE	no
3	overcast	hot	high	FALSE	yes
4	rainy	mild	high	FALSE	yes
5	rainy	cool	normal	FALSE	yes
6	rainy	cool	normal	TRUE	no
7	overcast	cool	normal	TRUE	yes
8	sunny	mild	high	FALSE	no
9	sunny	cool	normal	FALSE	yes
10	rainy	mild	normal	FALSE	yes
11	sunny	mild	normal	TRUE	yes
12	overcast	mild	high	TRUE	yes
13	overcast	hot	normal	FALSE	yes
14	rainy	mild	high	TRUE	no

## Which attribute to select?



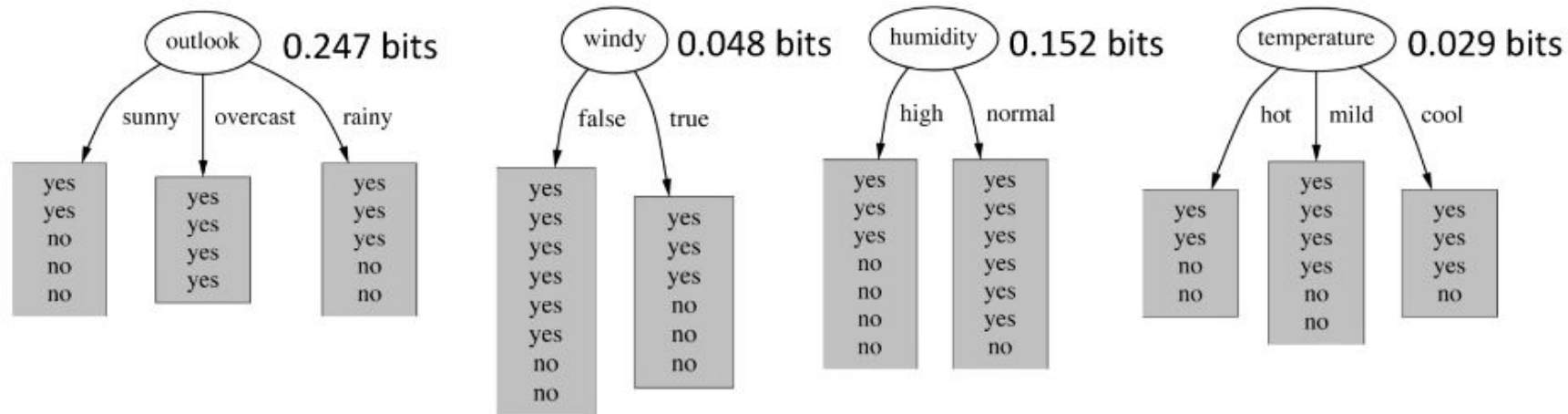
There are 14 instances: 9 yes, 5 no

TotalEntropy([9,5]) = 0.940 bits.

Entropy<sub>outlook</sub>([2,3], [4,0],[3,2]) =  $(5/14) * 0.971 + (4/14) * 0.0 + (5/14) * 0.971 = \mathbf{0.693}$ bits

InfoGain =  $0.940 - 0.693 = 0.247$  bits of information

## Which attribute to select?



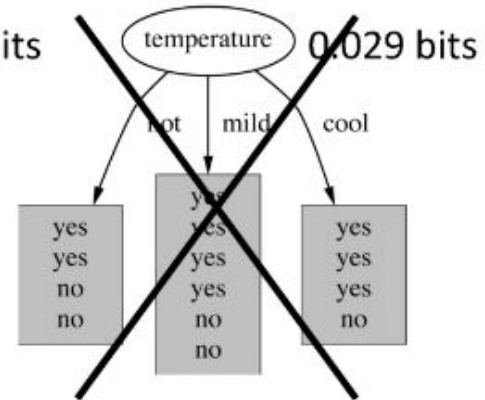
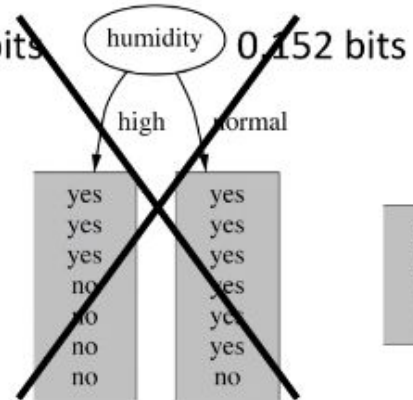
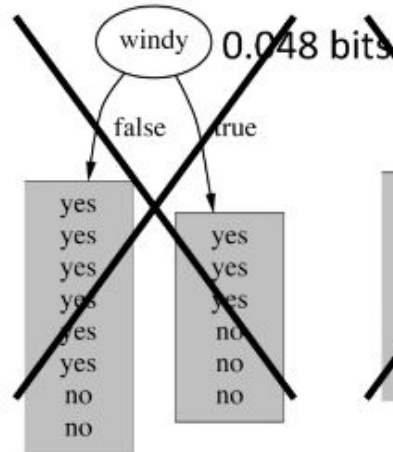
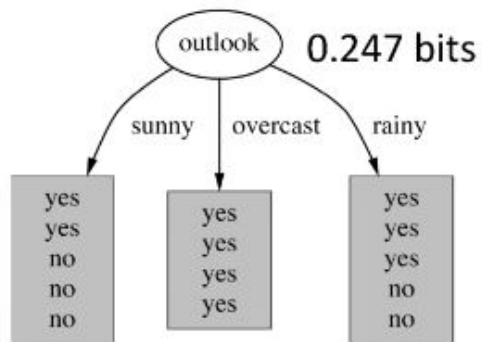
$\text{gain}(\text{outlook}) = 0.940 - 0.693 = 0.247$  bits of information.

$\text{gain}(\text{temperature}) = 0.029$  bits

$\text{gain}(\text{humidity}) = 0.152$  bits

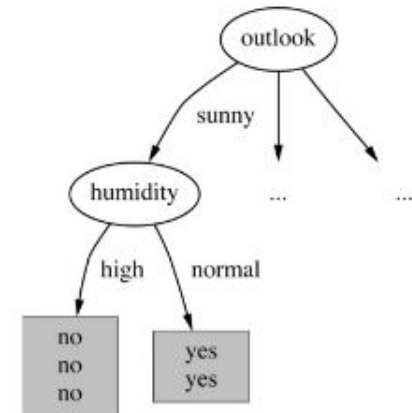
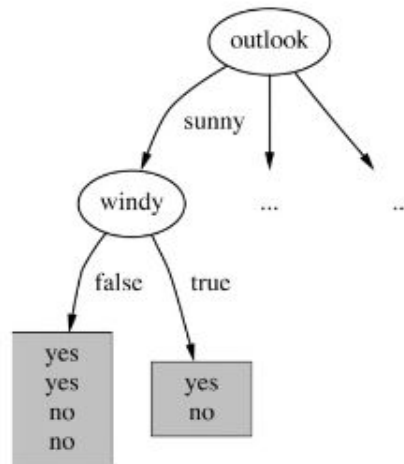
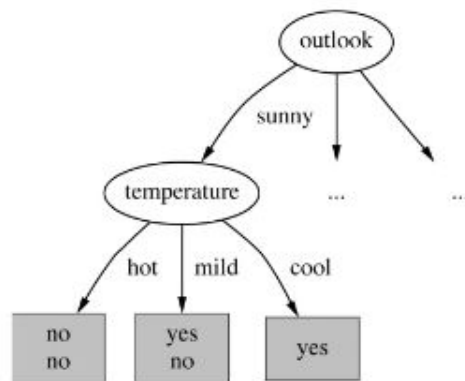
$\text{gain}(\text{windy}) = 0.048$  bits

Which attribute to select?





## Continue to split ...

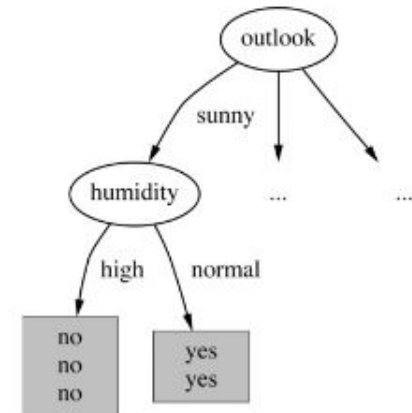
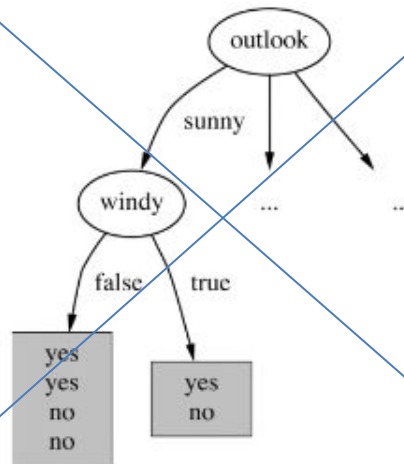
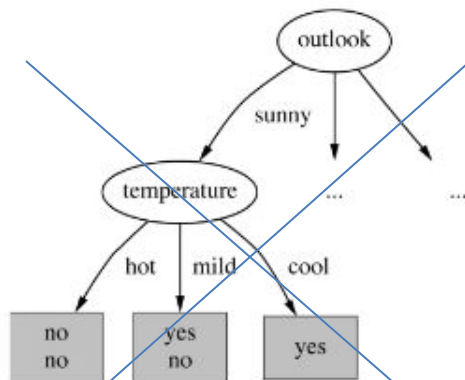


$\text{gain}(\text{temperature}) = 0.571$  bits

$\text{gain}(\text{windy}) = 0.020$  bits

$\text{gain}(\text{humidity}) = 0.971$  bits

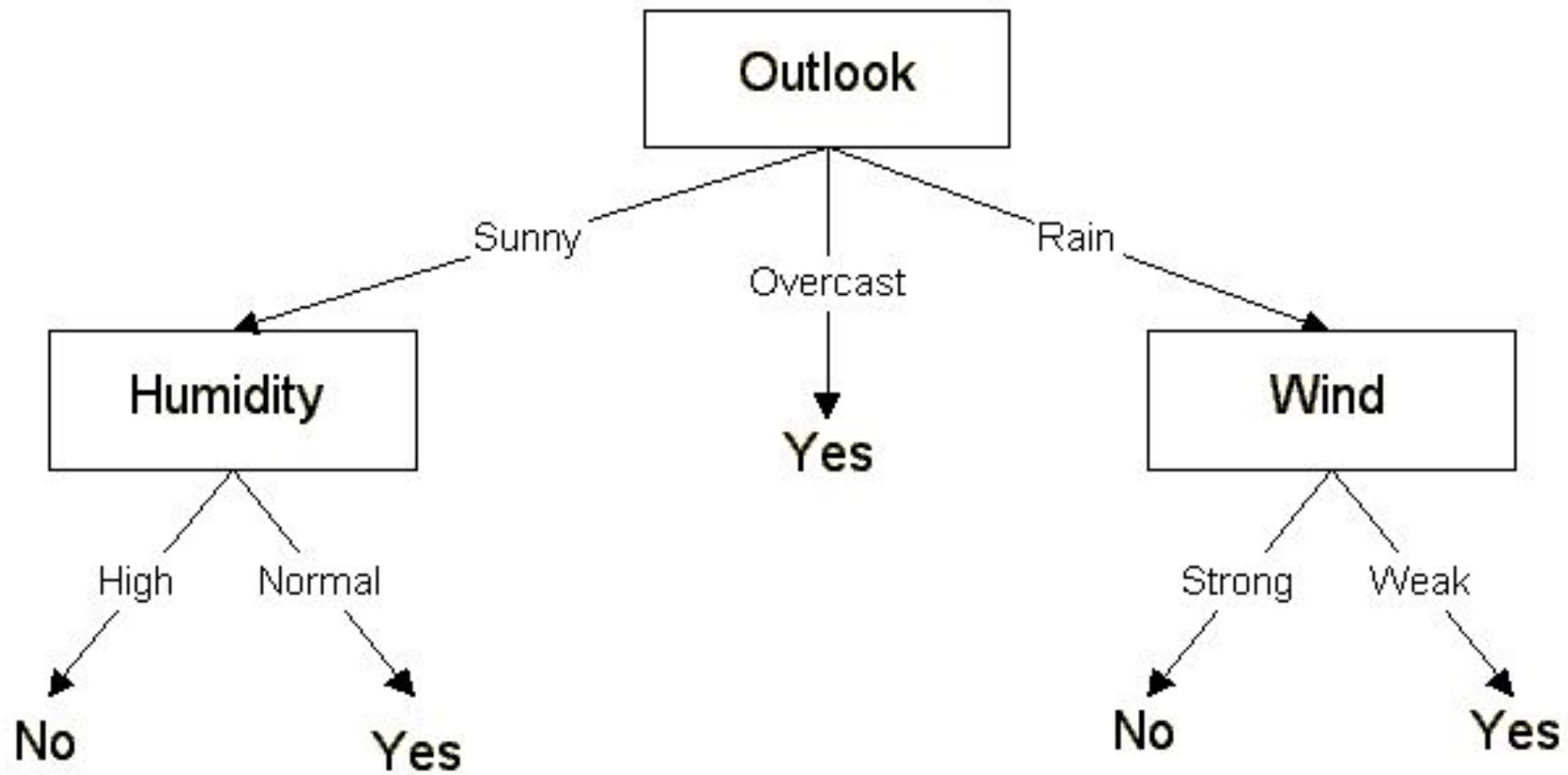
## Continue to split ...



$\text{gain}(\text{temperature}) = 0.571$  bits

$\text{gain}(\text{windy}) = 0.020$  bits

$\text{gain}(\text{humidity}) = 0.971$  bits



Thank you