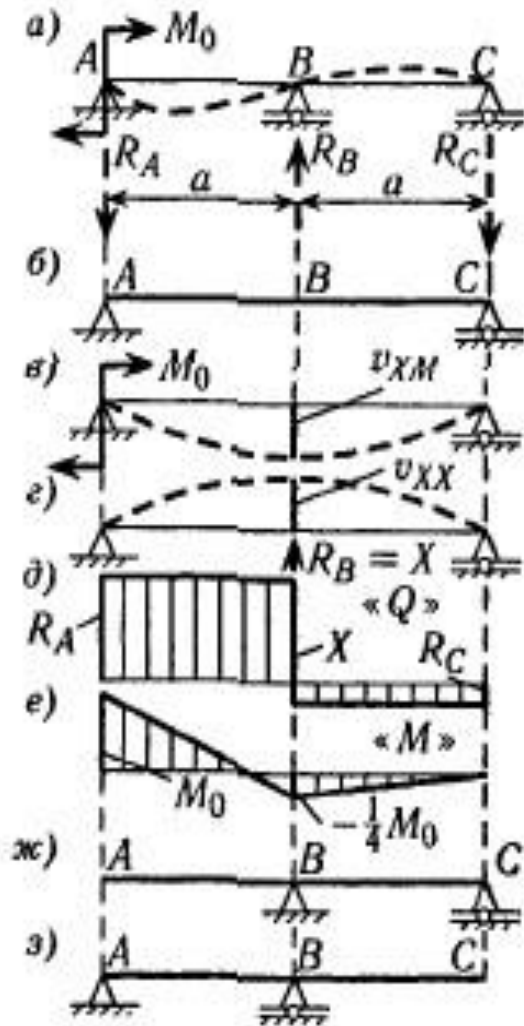


**ВЫЧИСЛЕНИЕ РЕАКЦИЙ И  
ПЕРЕМЕЩЕНИЙ В  
СТАТИЧЕСКИ  
НЕОПРЕДЕЛИМЫХ СИСТЕМАХ**

# 1. ОСНОВНЫЕ СИСТЕМЫ И ЛИШНИЕ НЕИЗВЕСТНЫЕ



$$\sum F_y = -R_A + R_B - R_C = 0, \quad (1)$$

$$\sum m_{z(B)} = R_A a - R_C a - M_0 = 0. \quad (2)$$

РИС. 6 «ОСНОВНАЯ СИСТЕМА»

$R_B = X$ . «ЛИШНЯЯ НЕИЗВЕСТНАЯ»

$$v_{XM} + v_{XX} = 0.$$

см. табл.

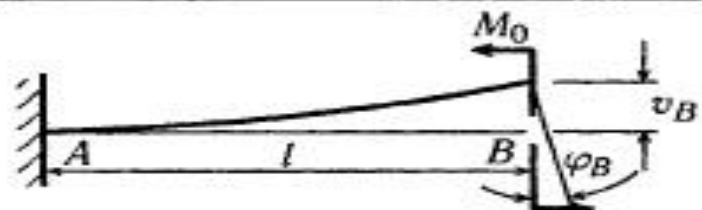
$$v_{XM} = -\frac{M_0(2a)^2}{16EI_z}, \quad v_{XX} = \frac{X(2a)^3}{48EI_z},$$

$$-\frac{M_0(2a)^2}{16EI_z} + \frac{X(2a)^3}{48EI_z} = 0.$$

$$X = \frac{3M_0}{2a}.$$

$$R_A = \frac{5M_0}{4a}, \quad R_C = \frac{M_0}{4a}.$$

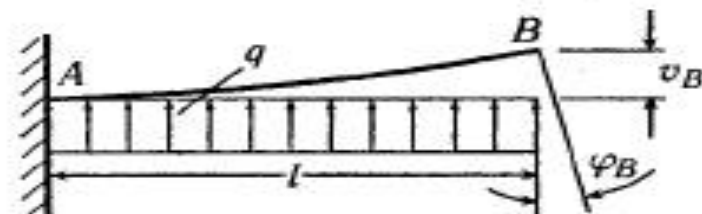
Прогибы и углы поворота в некоторых сечениях простейших балок



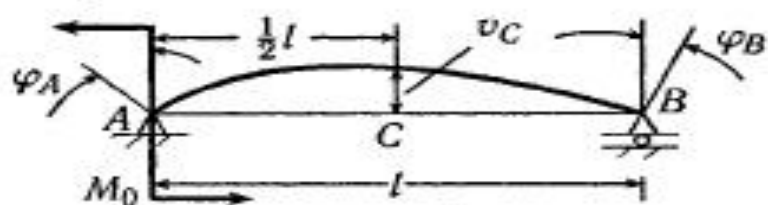
$$\varphi_B = \frac{M_0 l}{EI}, v_B = \frac{M_0 l^2}{2EI}$$



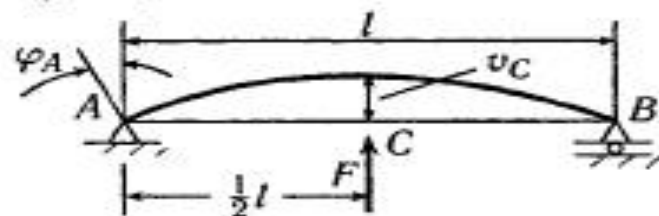
$$\varphi_B = \frac{Fl^2}{2EI}, v_B = \frac{Fl^3}{3EI}$$



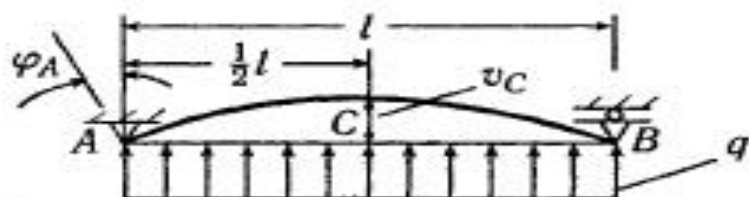
$$\varphi_B = \frac{ql^3}{6EI}, v_B = \frac{ql^4}{8EI}$$



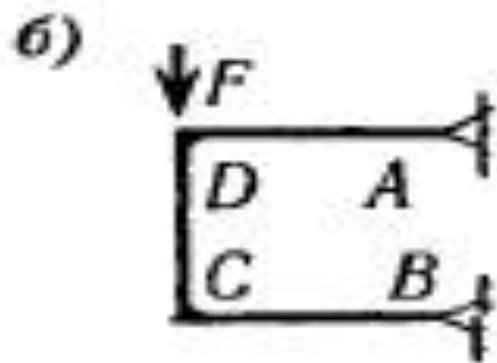
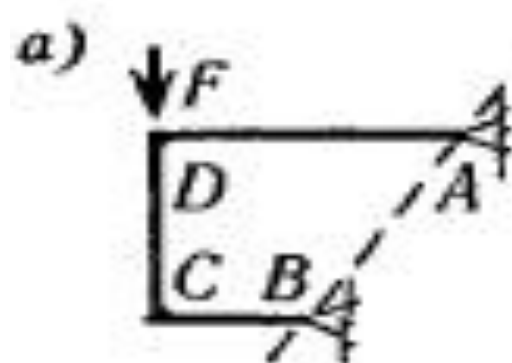
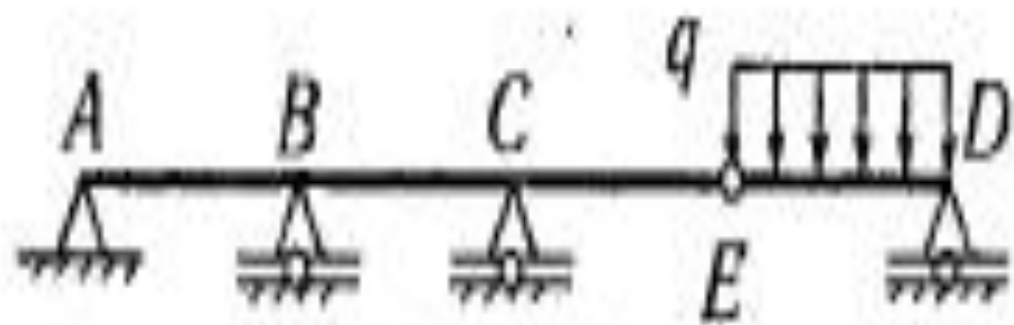
$$\varphi_A = \frac{M_0 l}{3EI}, \varphi_B = -\frac{M_0 l}{6EI}, v_C = \frac{M_0 l^2}{16EI} *$$



$$\varphi_A = \frac{Fl^2}{16EI}, v_C = \frac{Fl^3}{48EI} *$$



$$\varphi_A = \frac{ql^3}{24EI}, v_C = \frac{5ql^4}{384EI}$$

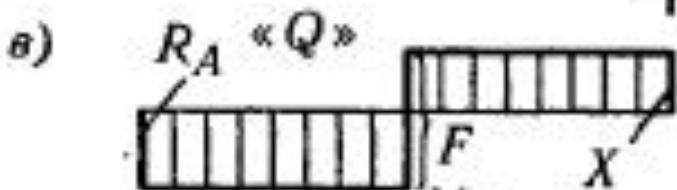
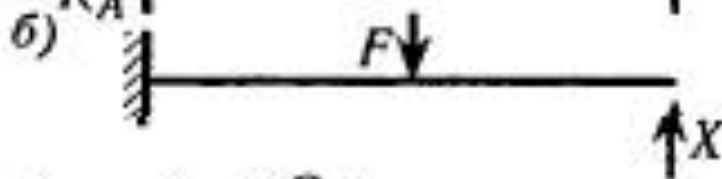


# ВЫЧИСЛЕНИЕ РЕАКЦИЙ И ПЕРЕМЕЩЕНИЙ С ПОМОЩЬЮ ТЕОРЕМЫ КАСТИЛЬЯНО



$$\begin{cases} \sum F_y = R_A + X - F = 0, \\ \sum m_{z(A)} = M_A - Fa + 2aX = 0. \end{cases}$$

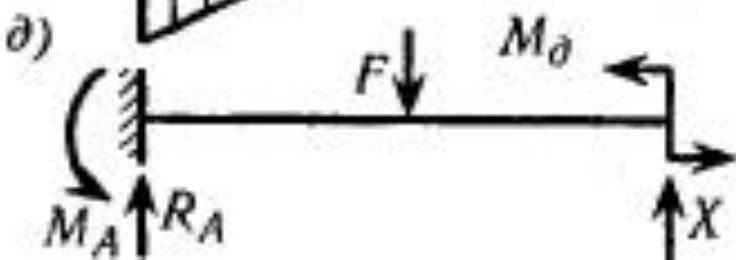
$$X = R_C.$$



$$v_X = \frac{\partial U}{\partial X} = \frac{1}{EI_2} \left( \int_{l_1} M_1 \frac{\partial M_1}{\partial X} dx + \int_{l_2} M_2 \frac{\partial M_2}{\partial X} dx \right) = 0.$$



$$R_A = F - X, \quad M_A = Fa - 2aX.$$



$$M_1 = -M_A + R_A x = -Fa + 2aX + (F - X)x,$$

$$M_2 = -M_A + R_A(x + a) - Fx =$$

$$= -Fa + 2aX + (F - X)(x + a) - Fx = X(a - x),$$

$$\frac{\partial M_1}{\partial X} = 2a - x,$$

$$\frac{\partial M_2}{\partial X} = a - x.$$

$$\int_0^a (-Fa + 2aX + (F - X)x)(2a - x) dx + \int_0^a (X(a - x))(a - x) dx = 0,$$

$$X = \frac{5}{16}F.$$

$$R_A = \frac{11}{16}F, \quad M_A = \frac{3}{8}Fa,$$

$$\begin{cases} \sum F_y = R_A - F + X = 0, \\ \sum m_{z(A)} = M_A - Fa + 2aX + M_D = 0. \end{cases}$$

$$R_A = \frac{11}{16}F, \quad M_A = \frac{3}{8}Fa - M_\theta.$$

$$\varphi_C = \frac{\partial U}{\partial M_\theta} = \frac{1}{EI_z} \left( \int_{l_1} M_1 \frac{\partial M_1}{\partial M_\theta} dx + \int_{l_2} M_2 \frac{\partial M_2}{\partial M_\theta} dx \right).$$

$$M_1 = -M_A + R_A x = -\frac{3}{8}Fa + M_\theta + \frac{11}{16}Fx,$$

$$M_2 = -M_A + R_A(x+a) - Fx = -\frac{3}{8}Fa + M_\theta +$$

$$+ \frac{11}{16}F(x+a) - Fx = \frac{5}{16}Fa - \frac{5}{16}Fx + M_\theta,$$

$$\frac{\partial M_1}{\partial M_\theta} = +1,$$

$$\frac{\partial M_2}{\partial M_\theta} = +1.$$



$$\begin{aligned}
\varphi_c = & \frac{1}{EI_z} \left( \int_0^a \left( -\frac{3}{8}Fa + \frac{11}{16}Fx + M_\partial \right) \Big|_{M_\partial=0} (+1) dx + \right. \\
& \left. + \int_0^a \left( \frac{5}{16}Fa - \frac{5}{16}Fx + M_\partial \right) \Big|_{M_\partial=0} (+1) dx \right) = \frac{Fa^2}{8EI_z}.
\end{aligned}$$

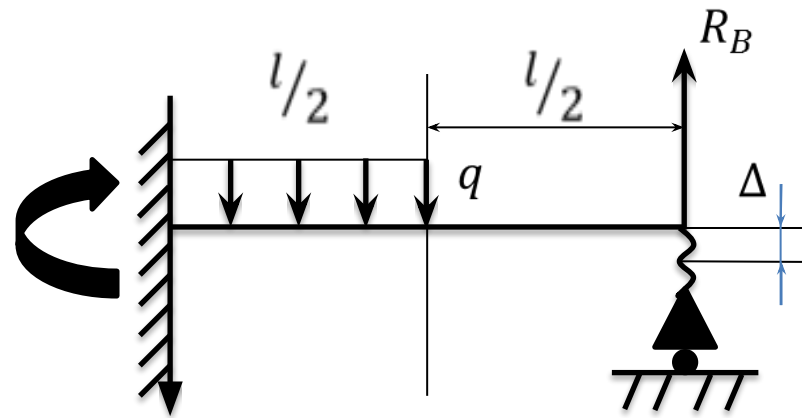
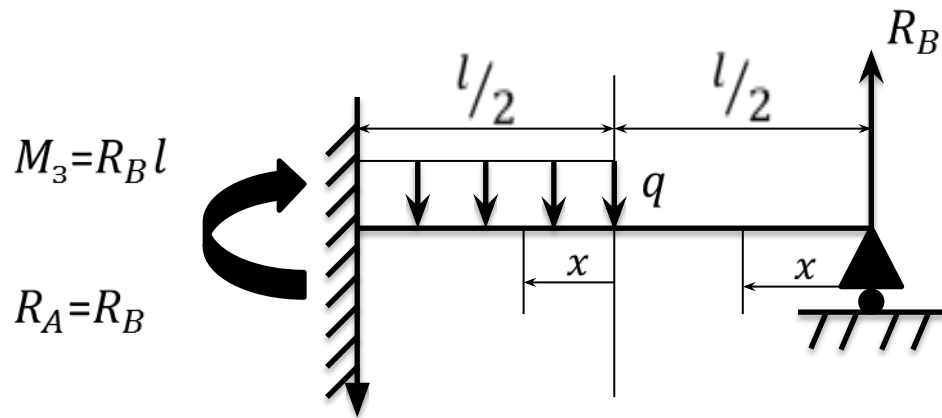
$$P = R_B$$

$$\frac{\partial U}{\partial R_B} = 0$$

$$\int_0^{l/2} R_B x dx + \int_0^{l/2} \left[ R_B \left( x + \frac{l}{2} \right) - q \frac{x^2}{2} \right] \left( x + \frac{l}{2} \right) dx = 0 \rightarrow R_B$$

$$\frac{\partial U}{\partial R_B} = -\Delta = -\frac{R_B}{C}$$

C - жѐсткость



# **Вычисление внутренних усилий и перемещений в замкнутых рамках**

$$H_A = R_A = H_B = R_B = 0$$

$$\varphi = 0, \quad u = 0, \quad v = 0 \quad (1)$$

Для выполнения условия совместности перемещений (1) нужно подобрать условия

$$X_1 = M, \quad X_2 = N, \quad X_3 = Q \quad (2)$$

$$\begin{cases} \varphi = \frac{\partial U}{\partial M} = 0, \\ u = \frac{\partial U}{\partial N} = 0, \\ v = \frac{\partial U}{\partial Q} = 0. \end{cases} \quad (3)$$

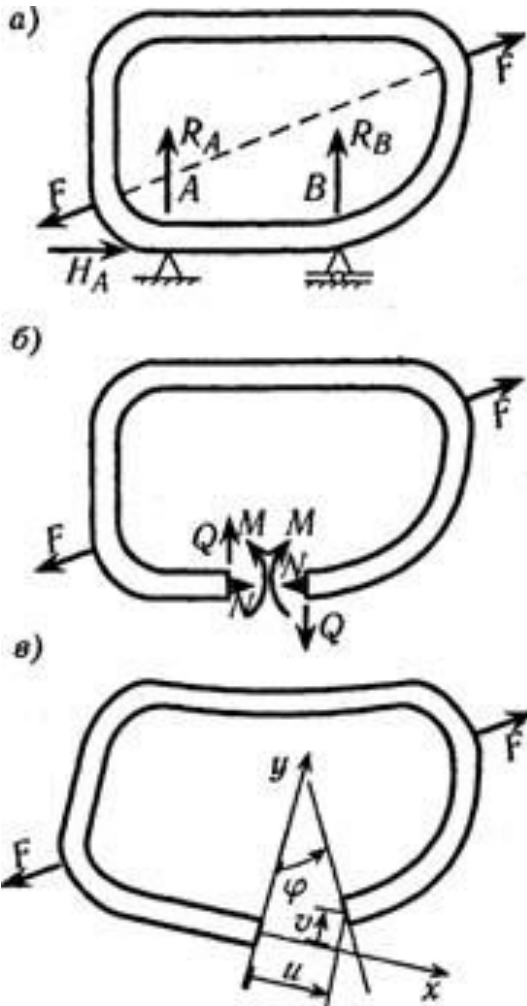


Рис.1

Две оси симметрии  $mm$  и  $nn$ :

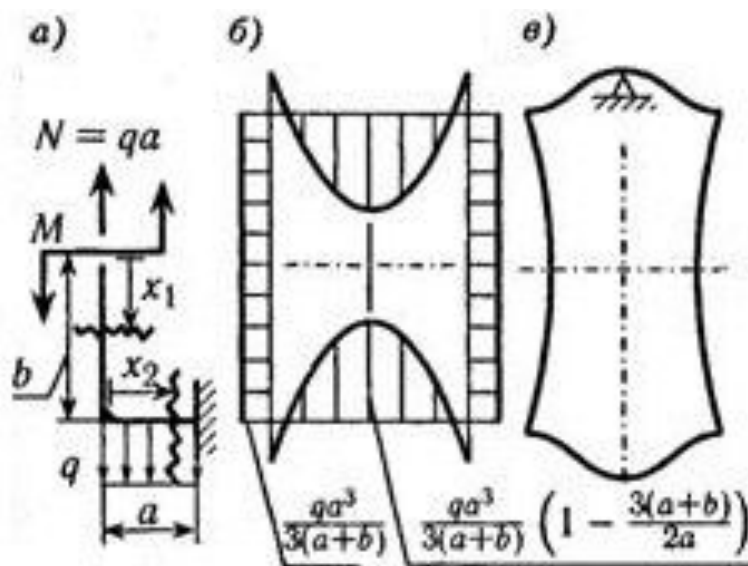
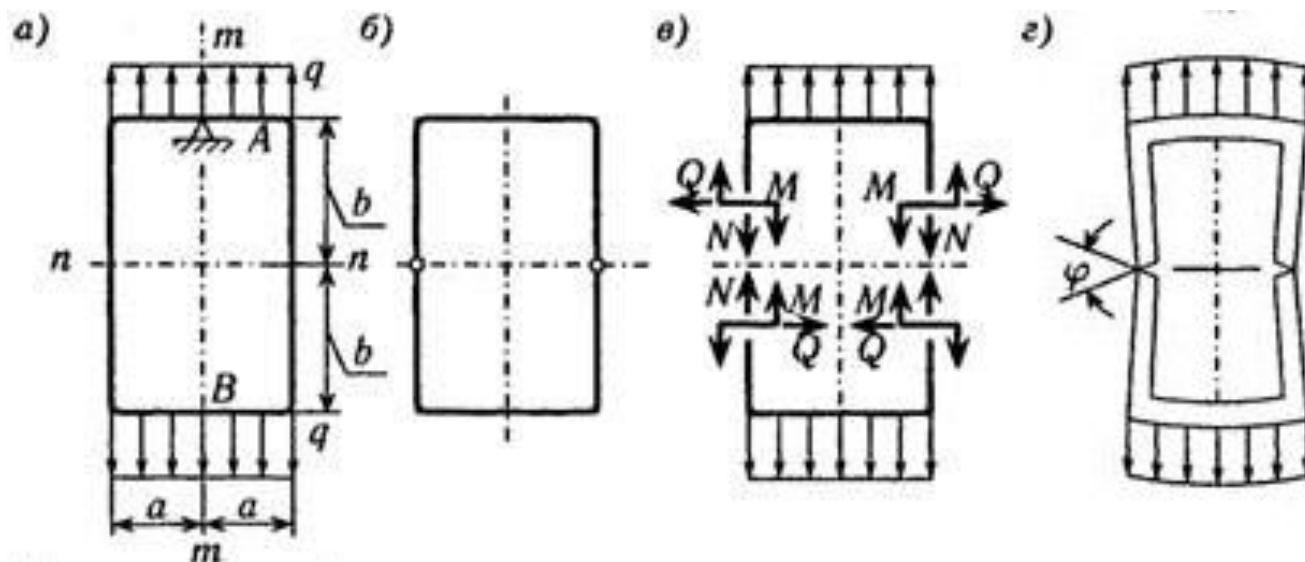


Рис.2

**Из симметрии относительно  
оси nn**

$$\Rightarrow Q = 0$$

(4)

$$N = qa.$$

(5)

$$\varphi = \frac{\partial U}{\partial M} = \sum_{i=1}^n \frac{1}{EI} \int_{l_i} M_i \frac{\partial M_i}{\partial M} dx = 0$$

(6)

$$\varphi = \frac{\partial U}{\partial M} = \frac{4}{EI} \left( \int_0^b M_1 \frac{\partial M_1}{\partial M} dx + \int_0^a M_2 \frac{\partial M_2}{\partial M} dx \right) = 0$$

(7)

$$M_1 = M \quad \frac{\partial M_1}{\partial M} = 1 \quad (0 \leq x \leq b)$$

(8)

$$M_2 = M + \frac{1}{2}qx^2 - qax \quad \frac{\partial M_2}{\partial M} = 1 \quad (0 \leq x \leq a)$$

**Из (8)  $\Rightarrow$**

$$\left( \int_0^b M \cdot 1 \cdot dx + \int_0^a \left( M + \frac{1}{2}qx^2 - qax \right) \cdot 1 \cdot dx \right) = 0$$

(9)

$$M = \frac{qa^3}{3(a+b)}$$

(10)

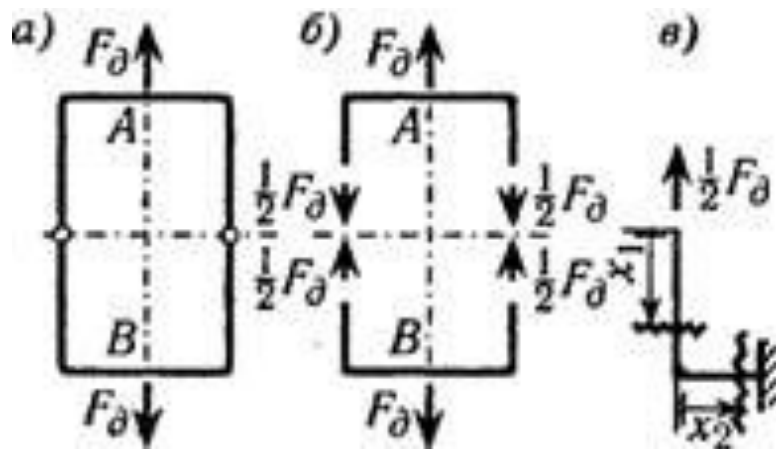


Рис.3

$$M_1^* = 0 \quad M_2^* = -\frac{1}{2}F_\partial x \quad (11)$$

$$\frac{\partial M_1^*}{\partial F_\partial} = 0 \quad \frac{\partial M_2^*}{\partial F_\partial} = -\frac{1}{2}x \quad (12)$$

$$v_{AB} = \frac{\partial U}{\partial F_\partial} = \frac{4}{EI} \left( \int_0^b M_1 \frac{\partial M_1^*}{\partial F_\partial} dx + \int_0^a M_2 \frac{\partial M_2^*}{\partial F_\partial} dx \right) \quad (13)$$

$$v_{AB} = \frac{4}{EI} \left( \int_0^b M \cdot 0 \cdot dx + \int_0^a \left( M + \frac{1}{2}qx^2 - qax \right) \left( -\frac{1}{2}x \right) dx \right) \quad (14)$$

$$v_{AB} = \frac{qa^4}{48EI} \cdot \frac{a+5b}{a+b} \quad (15)$$

$$R_F = R_B = R = F \quad (16)$$

$$Q = 0 \quad (17)$$



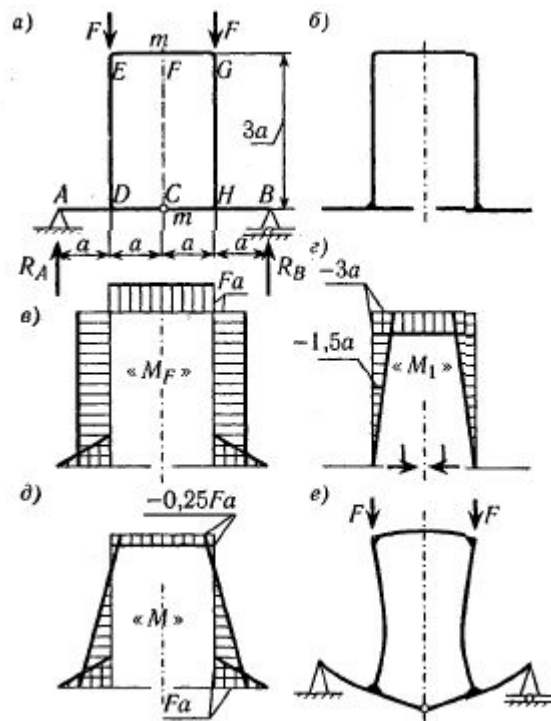


Рис.4

Из симметрии, а так же в шарнире  $M=0$

$$X=0$$

(18)

$$\sum_1 = 2 \left( \frac{3a}{6EI_z} (Fa \cdot 0 + 4 \cdot Fa \cdot (-1,5a) + Fa \cdot (-3a)) + \frac{a}{6EI_z} (Fa \cdot (-3a) + 4 \cdot Fa \cdot (-3a) + Fa \cdot (-3a)) \right) = -\frac{15Fa^3}{EI_z}, \quad (19)$$

$$\sum_2 = 2 \left( \frac{3a}{6EI_z} (0 + 4 \cdot (-1,5a)^2 + (-3a)^2) + \frac{a}{6EI_z} ((-3a)^2 + 4 \cdot (-3a)^2 + (-3a)^2) \right) = \frac{36a^3}{EI_z}. \quad (20)$$

$$\chi = N = -\frac{\sum_1}{\sum_2} = -\left(-\frac{15Fa^3}{EI_z}\right) \cdot \left(\frac{EI_z}{36a^3}\right) = \frac{5}{12}F = 0,417F. \quad (21)$$

**БЛАГОДАРЮ  
ЗА**

**ВНИМАНИЕ**