ВЫЧИСЛЕНИЕ РЕАКЦИЙ И ПЕРЕМЕЩЕНИЙ В СТАТИЧЕСКИ НЕОПРЕДЕЛИМЫХ СИСТЕМАХ
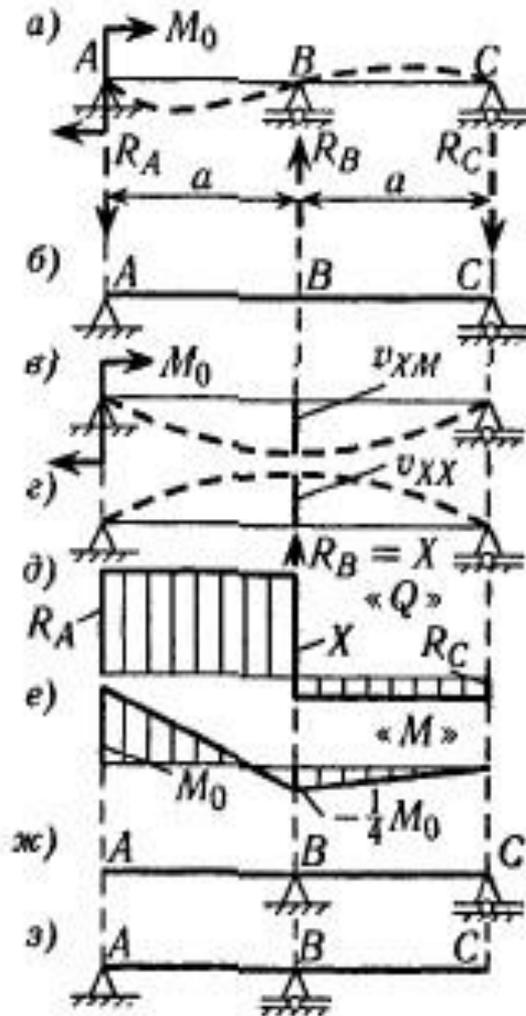
1. ОСНОВНЫЕ СИСТЕМЫ И ЛИШНИЕ НЕИЗВЕСТНЫЕ

$$\sum F_y = -R_A + R_B - R_C = 0, \quad (1)$$

$$\sum m_{z(B)} = R_A a - R_C a - M_0 = 0. \quad (2)$$

РИС. б «ОСНОВНАЯ СИСТЕМА»

$R_B = X.$ **«лишняя неизвестная»**
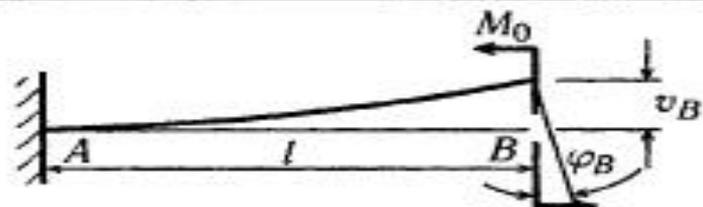
$$v_{XM} + v_{XX} = 0.$$

см. табл.

$$v_{XM} = -\frac{M_0(2a)^2}{16EI_z}, \qquad v_{XX} = \frac{X(2a)^3}{48EI_z},$$

$$-\frac{M_0(2a)^2}{16EI_z} + \frac{X(2a)^3}{48EI_z} = 0.$$

$$X = \frac{3M_0}{2a}.$$

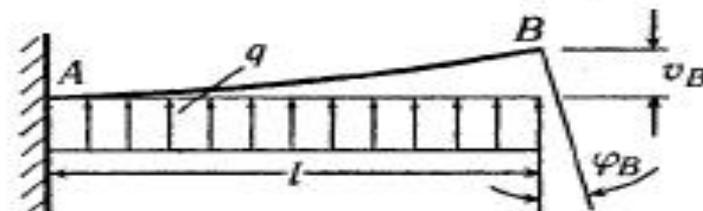$$R_A = \frac{5M_0}{4a}, \qquad R_C = \frac{M_0}{4a}.$$

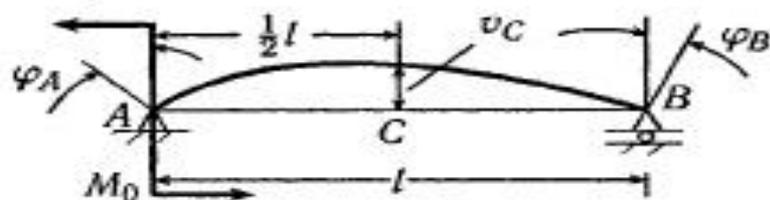## Прогибы и углы поворота в некоторых сечениях простейших балок

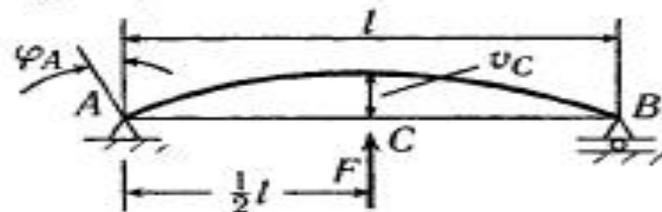$$\varphi_B = \frac{M_0 l}{EI}, \ v_B = \frac{M_0 l^2}{2EI}$$

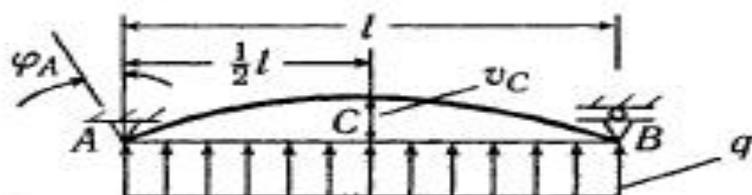$$\varphi_B = \frac{F l^2}{2EI}, \ v_B = \frac{F l^3}{3EI}$$

$$\varphi_B = \frac{q l^3}{6EI}, \ v_B = \frac{q l^4}{8EI}$$
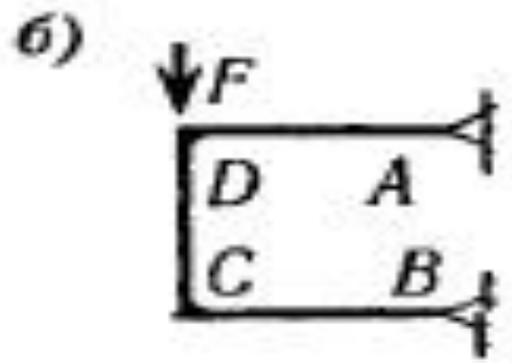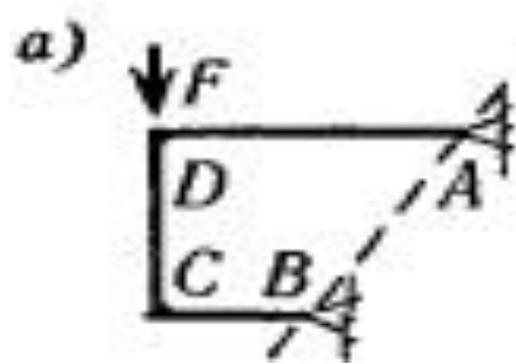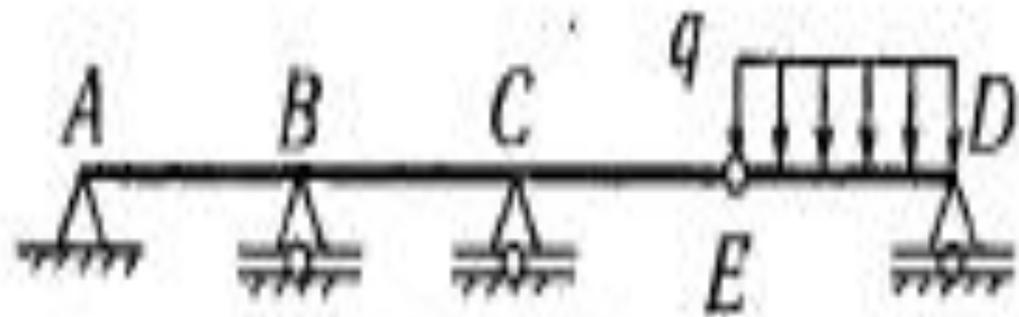
$$\varphi_A = \frac{M_0 l}{3EI}, \ \varphi_B = -\frac{M_0 l}{6EI},$$
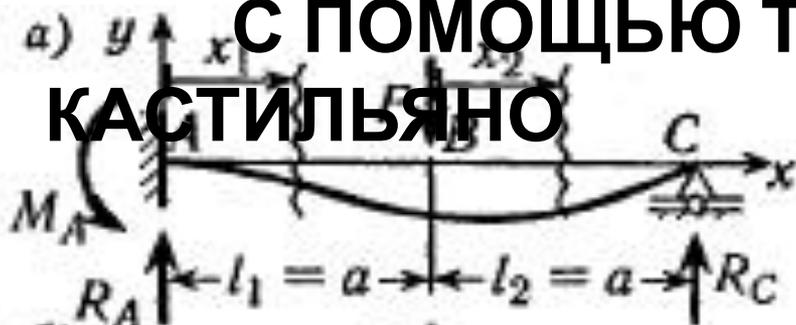$$v_C = \frac{M_0 l^2}{16EI} \quad *$$

$$\varphi_A = \frac{F l^2}{16EI}, \ v_C = \frac{F l^3}{48EI} \quad *$$

$$\varphi_A = \frac{q l^3}{24EI}, \ v_C = \frac{5q l^4}{384EI}$$

A B C q D

E

$а)$ F $б)$ F

D A D A

C B C B

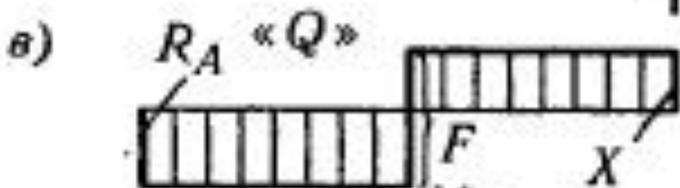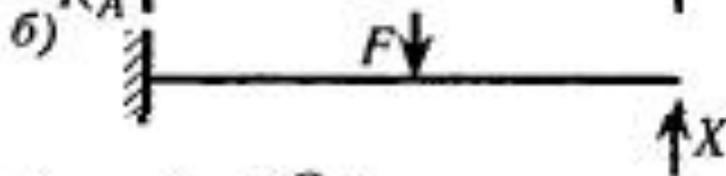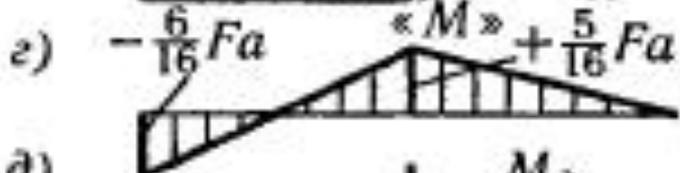# ВЫЧИСЛЕНИЕ РЕАКЦИЙ И ПЕРЕМЕЩЕНИЙ С ПОМОЩЬЮ ТЕОРЕМЫ КАСТИЛЬЯНО

$$\begin{cases} \sum F_y = R_A + X - F = 0, \\ \sum m_{z(A)} = M_A - Fa + 2aX = 0. \end{cases}$$

$$X = R_C.$$

$$v_X = \frac{\partial U}{\partial X} = \frac{1}{EI_2}\left(\int\limits_{l_1} M_1 \frac{\partial M_1}{\partial X}\, dx + \int\limits_{l_2} M_2 \frac{\partial M_2}{\partial X}\, dx \right) = 0.$$

$$R_A = F - X, \quad M_A = Fa - 2aX.$$
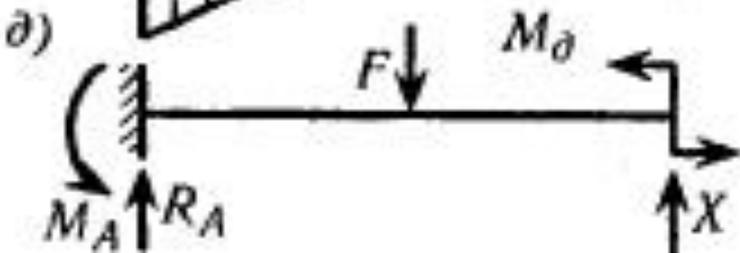
$$M_1 = -M_A + R_A x = -Fa + 2aX + (F - X)x,$$
$$M_2 = -M_A + R_A(x + a) - Fx =$$
$$= -Fa + 2aX + (F - X)(x + a) - Fx = X(a - x),$$
$$\frac{\partial M_1}{\partial X} = 2a - x,$$
$$\frac{\partial M_2}{\partial X} = a - x.$$

$$\int_0^a (-Fa + 2aX + (F - X)x)(2a - x)dx + \int_0^a (X(a - x))(a - x)dx = 0,$$

$$X = \frac{5}{16}F.$$

$$R_A = \frac{11}{16}F, \qquad M_A = \frac{3}{8}Fa,$$

$$\begin{cases} \sum F_y = R_A - F + X = 0, \\ \sum m_{z(A)} = M_A - Fa + 2aX + M_\partial = 0. \end{cases}$$

$$R_A = \frac{11}{16}F, \qquad M_A = \frac{3}{8}Fa - M_\partial.$$

$$\varphi_C = \frac{\partial U}{\partial M_\partial} = \frac{1}{EI_z}\left(\int_{l_1} M_1 \frac{\partial M_1}{\partial M_\partial}\, dx + \int_{l_2} M_2 \frac{\partial M_2}{\partial M_\partial}\, dx\right).$$

$$M_1 = -M_A + R_A x = -\frac{3}{8}Fa + M_\partial + \frac{11}{16}Fx,$$

$$M_2 = -M_A + R_A(x+a) - Fx = -\frac{3}{8}Fa + M_\partial +$$

$$+ \frac{11}{16}F(x+a) - Fx = \frac{5}{16}Fa - \frac{5}{16}Fx + M_\partial,$$

$$\frac{\partial M_1}{\partial M_\partial} = +1,$$

$$\frac{\partial M_2}{\partial M_\partial} = +1.$$

$$\varphi_c = \frac{1}{EI_z}\left(\int\limits_0^a \left(-\frac{3}{8}Fa + \frac{11}{16}Fx + M_\partial\right)\Big|_{M_\partial=0}(+1)\,dx + \right.$$

$$\left. + \int\limits_0^a \left(\frac{5}{16}Fa - \frac{5}{16}Fx + M_\partial\right)\Big|_{M_\partial=0}(+1)\,dx\right) = \frac{Fa^2}{8EI_z}.$$
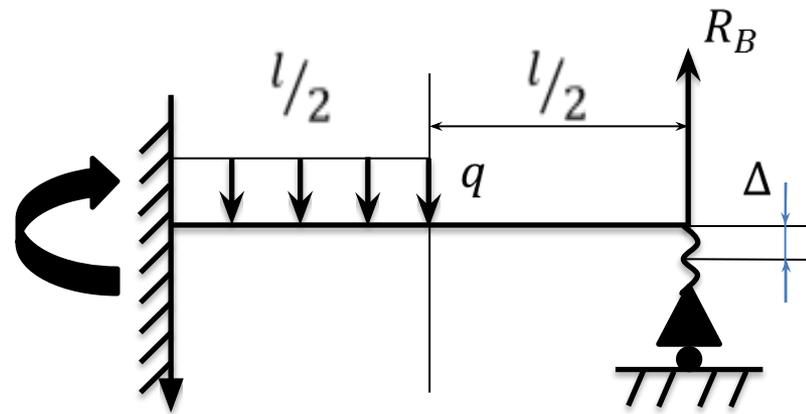
$$P = R_B$$

$$\frac{\partial U}{\partial R_B} = 0$$

$$M_з = R_B l$$

$$R_A = R_B$$

$$\int_0^{l/2} R_B \, x x dx + \int_0^{\frac{l}{2}} \left[R_B \left(x + \frac{l}{2} \right) - q \frac{x^2}{2} \right] \left(x + \frac{l}{2} \right) dx = 0 \rightarrow R_B$$

$$\frac{\partial U}{\partial R_B} = -\Delta = -\frac{R_B}{C}$$

C - жёсткость
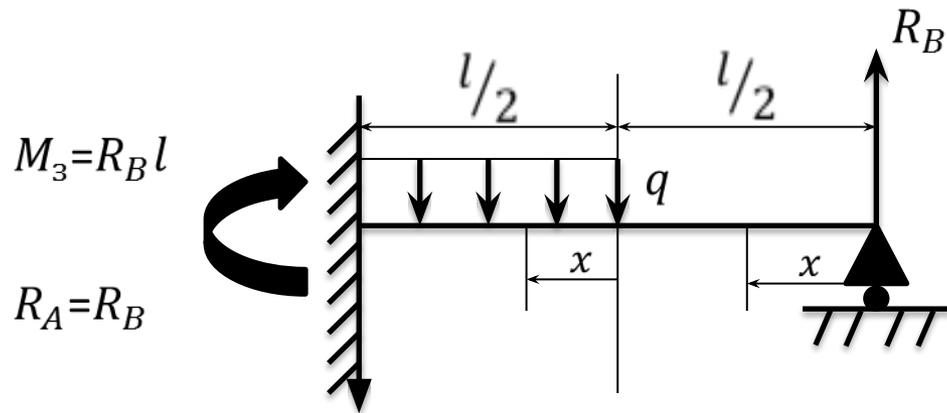
Вычисление внутренних усилий и перемещений в замкнутых рамках

Рис.1

$$H_A = R_A = H_B = R_B = 0$$

$$\varphi = 0, \qquad u = 0, \qquad v = 0 \qquad (1)$$

Для выполнения условия совместности перемещений (1) нужно подобрать условия

$$X_1 = M, \;\; X_2 = N, \;\; X_3 = Q \qquad (2)$$

$$\begin{cases} \varphi = \dfrac{\partial U}{\partial M} = 0, \\[2mm] u = \dfrac{\partial U}{\partial N} = 0, \\[2mm] v = \dfrac{\partial U}{\partial Q} = 0. \end{cases} \qquad (3)$$
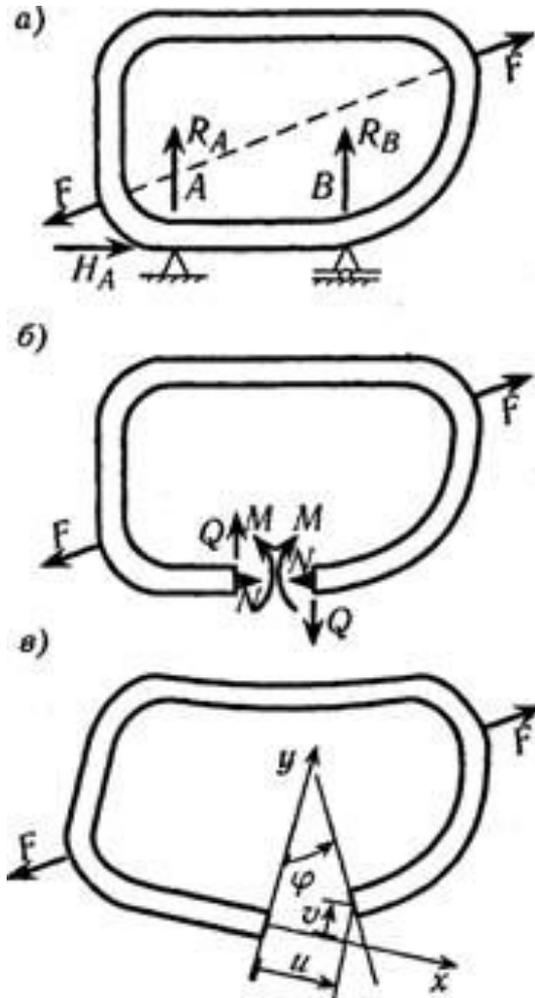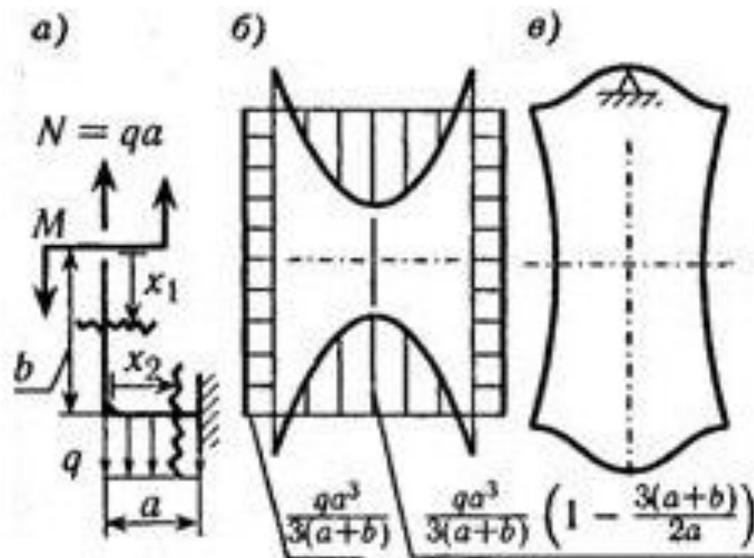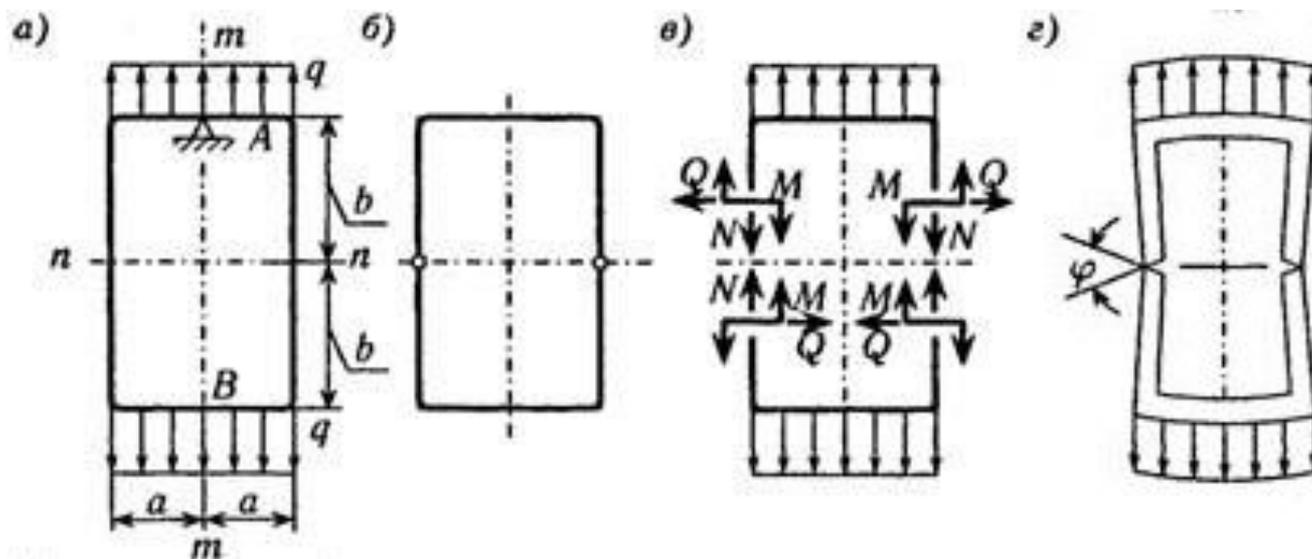
Две оси симметрии mm и nn:

Рис.2

Из симметрии относительно оси nn

$$\Rightarrow Q = 0 \tag{4}$$

$$N = qa. \tag{5}$$

$$\varphi = \frac{\partial U}{\partial M} = \sum_{i=1}^{n} \frac{1}{EI} \int_{l_i} M_i \frac{\partial M_i}{\partial M} dx = 0 \tag{6}$$

$$\varphi = \frac{\partial U}{\partial M} = \frac{4}{EI} \left(\int_0^b M_1 \frac{\partial M_1}{\partial M} dx + \int_0^a M_2 \frac{\partial M_2}{\partial M} dx \right) = 0 \tag{7}$$

$$M_1 = M \qquad \frac{\partial M_1}{\partial M} = 1 \quad (0 \leq x \leq b) \tag{8}$$

$$M_2 = M + \frac{1}{2} qx^2 - qax \qquad \frac{\partial M_2}{\partial M} = 1 \qquad (0 \leq x \leq a)$$

Из (8) $\Rightarrow$

$$\left(\int_0^b M \cdot 1 \cdot dx + \int_0^a (M + \frac{1}{2} qx^2 - qax) \cdot 1 \cdot dx \right) = 0 \tag{9}$$
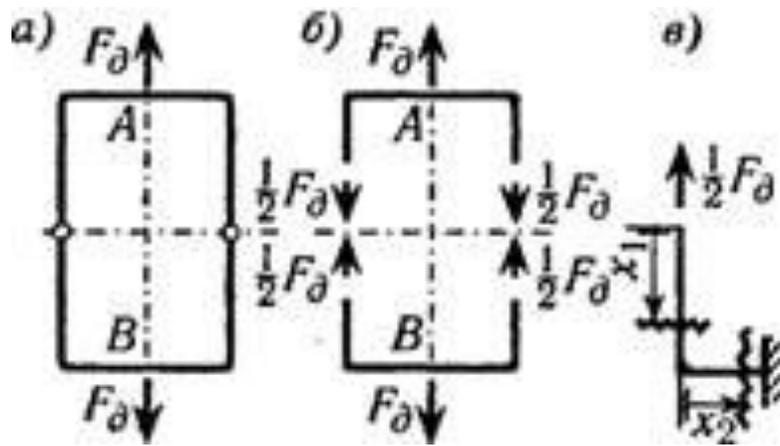
$$M = \frac{qa^3}{3(a+b)} \tag{10}$$

Рис.3

$$M_1^* = 0 \qquad M_2^* = -\frac{1}{2}F_\partial x \tag{11}$$

$$\frac{\partial M_1^*}{\partial F_\partial} = 0 \qquad \frac{\partial M_2^*}{\partial F_\partial} = -\frac{1}{2}x \tag{12}$$

$$v_{AB} = \frac{\partial U}{\partial F_\partial} = \frac{4}{EI}\left(\int_0^b M_1 \frac{\partial M_1^*}{\partial F_\partial}dx + \int_0^a M_2 \frac{\partial M_2^*}{\partial F_\partial}dx\right) \tag{13}$$

$$v_{AB} = \frac{4}{EI}\left(\int_0^b M \cdot 0 \cdot dx + \int_0^a (M + \frac{1}{2}qx^2 - qax)(-\frac{1}{2}x)dx\right) \tag{14}$$

$$v_{AB} = \frac{qa^4}{48EI} \cdot \frac{a+5b}{a+b} \qquad (15)$$
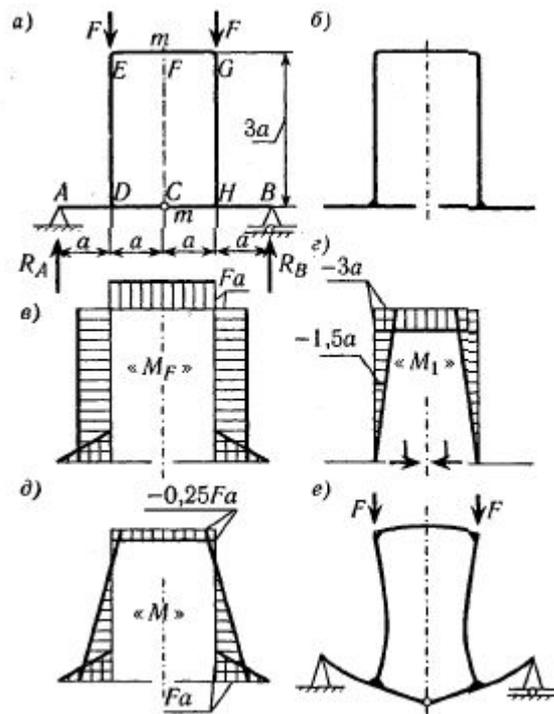
$$R_F = R_B = R = F \qquad (16)$$

$$Q = 0 \qquad (17)$$

Рис.4

Из симметрии, а так же в шарнире M=0

X=0

(18)

$$\sum_1 = 2\left(\frac{3a}{6EI_z}(Fa \cdot 0 + 4 \cdot Fa \cdot (-1{,}5a) + \right.$$
$$+ Fa \cdot (-3a)) + \frac{a}{6EI_z}(Fa \cdot (-3a) + 4 \cdot Fa \cdot (-3a) +$$
$$\left. + Fa \cdot (-3a))\right) = -\frac{15Fa^3}{EI_z},$$

(19)

$$\sum_2 = 2\left(\frac{3a}{6EI_z}(0 + 4 \cdot (-1{,}5a)^2 + (-3a)^2) + \right.$$
$$\left. + \frac{a}{6EI_z}((-3a)^2 + 4 \cdot (-3a)^2 + (-3a)^2)\right) = \frac{36a^3}{EI_z}.$$

(20)

$$X = N = -\frac{\sum_1}{\sum_2} = -\left(-\frac{15Fa^3}{EI_z}\right) \cdot \left(\frac{EI_z}{36a^3}\right) = \frac{5}{12}F = 0{,}417F.$$

(21)

БЛАГОДАРЮ ЗА

ВНИМАНИЕ