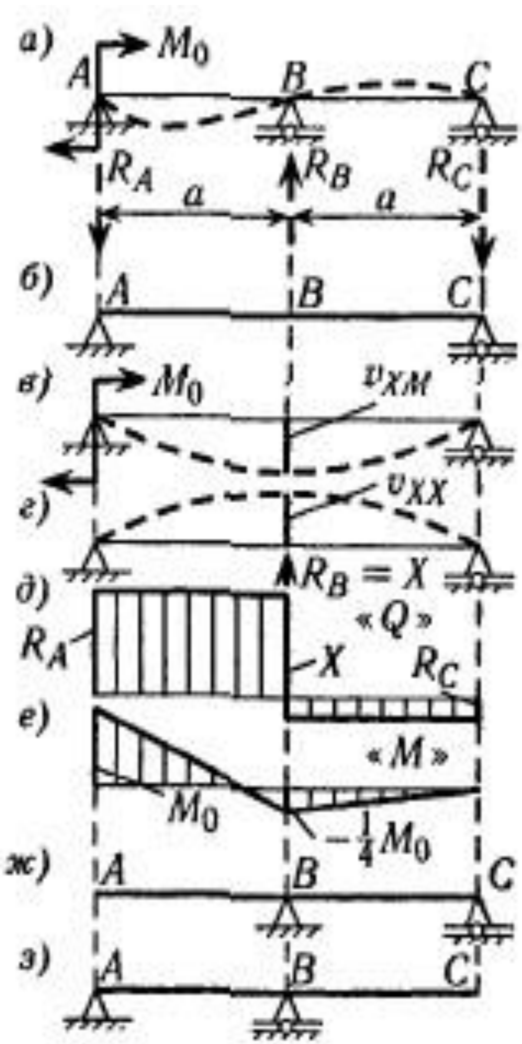
ВЫЧИСЛЕНИЕ РЕАКЦИЙ И ПЕРЕМЕЩЕНИЙ В СТАТИЧЕСКИ НЕОПРЕДЕЛИМЫХ СИСТЕМАХ

1. ОСНОВНЫЕ СИСТЕМЫ И ЛИШНИЕ НЕИЗВЕСТНЫЕ



$$\sum F_y = -R_A + R_B - R_C = 0, \quad (1)$$

$$\sum m_{z(B)} = R_A a - R_C a - M_0 = 0. \quad (2)$$

РИС. б «ОСНОВНАЯ СИСТЕМА»

$R_B = X.$ **«лишняя неизвестная»**

$$v_{XM} + v_{XX} = 0.$$

см. табл.

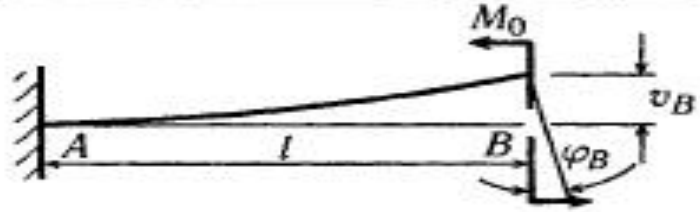
$$v_{XM} = -\frac{M_0(2a)^2}{16EI_z}, \qquad v_{XX} = \frac{X(2a)^3}{48EI_z},$$

$$-\frac{M_0(2a)^2}{16EI_z} + \frac{X(2a)^3}{48EI_z} = 0.$$

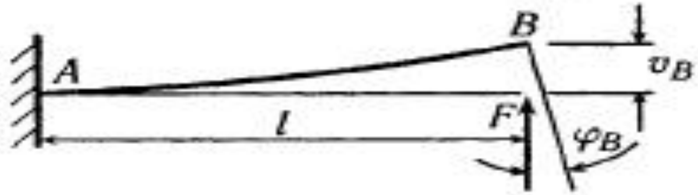
$$X = \frac{3M_0}{2a}.$$

$$R_A = \frac{5M_0}{4a}, \qquad R_C = \frac{M_0}{4a}.$$

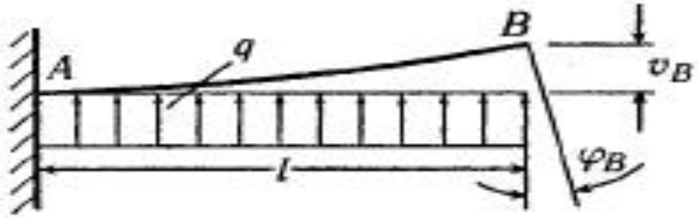
Прогибы и углы поворота в некоторых сечениях простейших балок



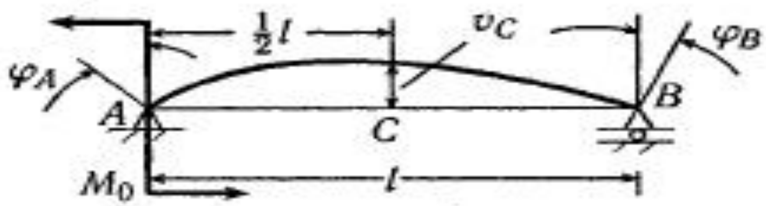
$\varphi_B = \frac{M_0 l}{EI}, v_B = \frac{M_0 l^2}{2EI}$



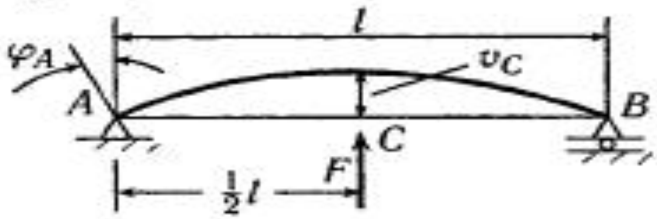
$\varphi_B = \frac{Fl^2}{2EI}, v_B = \frac{Fl^3}{3EI}$



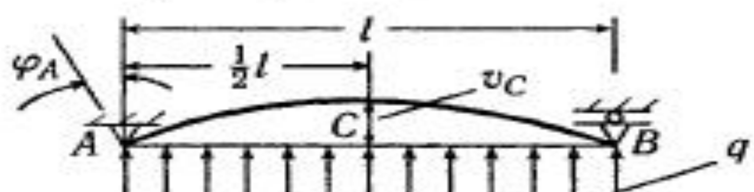
$\varphi_B = \frac{ql^3}{6EI}, v_B = \frac{ql^4}{8EI}$



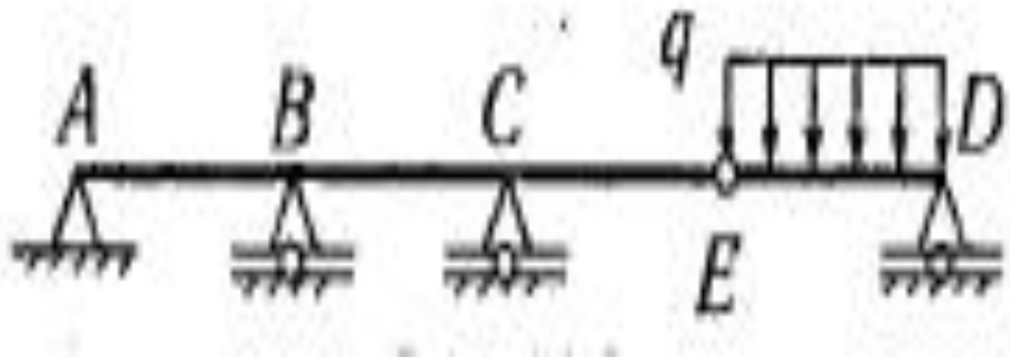
$\varphi_A = \frac{M_0 l}{3EI}, \varphi_B = -\frac{M_0 l}{6EI},$
$v_C = \frac{M_0 l^2}{16EI}$ *

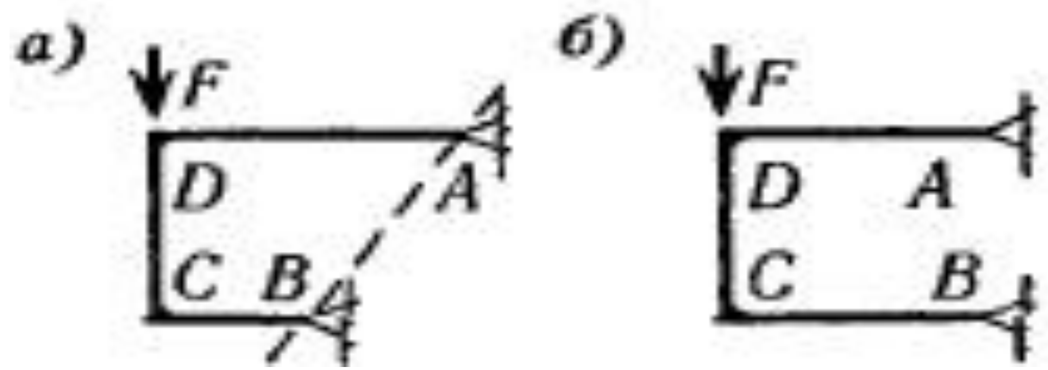


$\varphi_A = \frac{Fl^2}{16EI}, v_C = \frac{Fl^3}{48EI}$ *

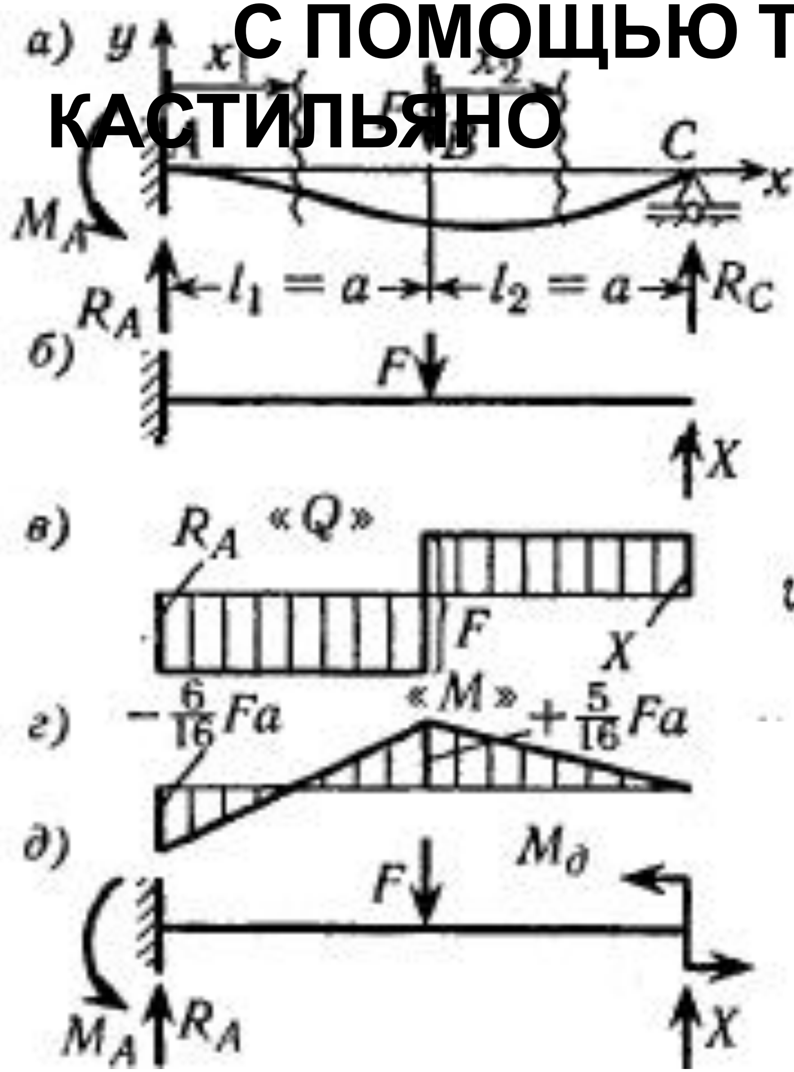


$\varphi_A = \frac{ql^3}{24EI}, v_C = \frac{5ql^4}{384EI}$


q
A
B
C
D
E


а)
F
D
A
C
B
б)
F
D
A
C
B

ВЫЧИСЛЕНИЕ РЕАКЦИЙ И ПЕРЕМЕЩЕНИЙ С ПОМОЩЬЮ ТЕОРЕМЫ КАСТИЛЬЯНО



$$\begin{cases} \sum F_y = R_A + X - F = 0, \\ \sum m_{z(A)} = M_A - Fa + 2aX = 0. \end{cases}$$

$$X = R_C.$$

$$v_X = \frac{\partial U}{\partial X} = \frac{1}{EI_z}\left(\int_{l_1} M_1 \frac{\partial M_1}{\partial X}\,dx + \int_{l_2} M_2 \frac{\partial M_2}{\partial X}\,dx\right) = 0.$$

$$R_A = F - X, \quad M_A = Fa - 2aX.$$

$$M_1 = -M_A + R_A x = -Fa + 2aX + (F - X)x,$$

$$M_2 = -M_A + R_A(x + a) - Fx =$$

$$= -Fa + 2aX + (F - X)(x + a) - Fx = X(a - x),$$

$$\frac{\partial M_1}{\partial X} = 2a - x,$$

$$\frac{\partial M_2}{\partial X} = a - x.$$

$$\int_0^a (-Fa + 2aX + (F - X)x)(2a - x)\,dx + \int_0^a (X(a - x))(a - x)\,dx = 0,$$

$$X = \frac{5}{16}F.$$

$$R_A = \frac{11}{16}F, \qquad M_A = \frac{3}{8}Fa,$$

$$\begin{cases} \sum F_y = R_A - F + X = 0, \\ \sum m_{z(A)} = M_A - Fa + 2aX + M_\partial = 0. \end{cases}$$

$$R_A = \frac{11}{16}F, \qquad M_A = \frac{3}{8}Fa - M_\partial.$$

$$\varphi_C = \frac{\partial U}{\partial M_\partial} = \frac{1}{EI_z}\left(\int_{l_1} M_1 \frac{\partial M_1}{\partial M_\partial}\,dx + \int_{l_2} M_2 \frac{\partial M_2}{\partial M_\partial}\,dx\right).$$

$$M_1 = -M_A + R_A x = -\frac{3}{8}Fa + M_\partial + \frac{11}{16}Fx,$$

$$M_2 = -M_A + R_A(x+a) - Fx = -\frac{3}{8}Fa + M_\partial +$$

$$+\frac{11}{16}F(x+a) - Fx = \frac{5}{16}Fa - \frac{5}{16}Fx + M_\partial,$$

$$\frac{\partial M_1}{\partial M_\partial} = +1,$$

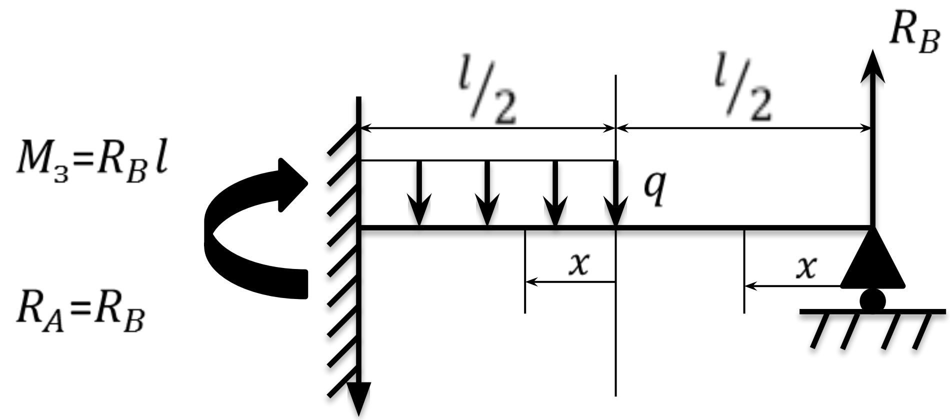
$$\frac{\partial M_2}{\partial M_\partial} = +1.$$

$$\varphi_c = \frac{1}{EI_z}\left(\int_0^a \left(-\frac{3}{8}Fa + \frac{11}{16}Fx + M_\partial\right)\Big|_{M_\partial=0} (+1)\,dx + \right.$$

$$\left. + \int_0^a \left(\frac{5}{16}Fa - \frac{5}{16}Fx + M_\partial\right)\Big|_{M_\partial=0} (+1)\,dx\right) = \frac{Fa^2}{8EI_z}.$$

$$P = R_B$$

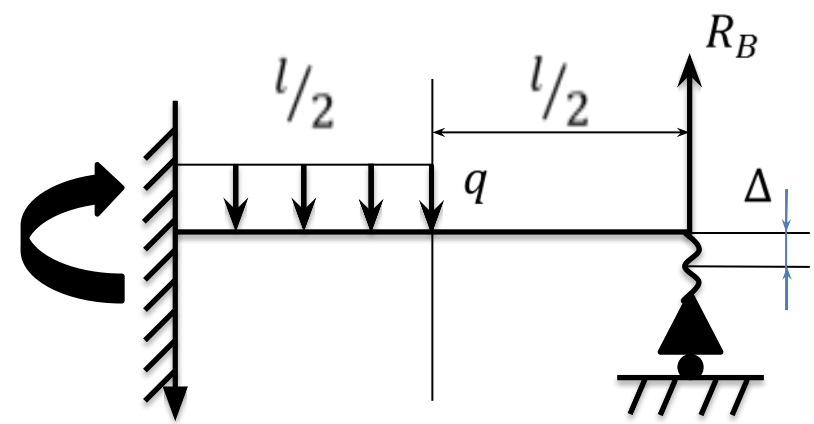
$$\frac{\partial U}{\partial R_B} = 0$$



- $$\int_0^{l/2} R_B\, x x dx + \int_0^{\frac{l}{2}} \left[R_B \left(x + \frac{l}{2} \right) - q \frac{x^2}{2} \right] \left(x + \frac{l}{2} \right) dx = 0 \to R_B$$

$$\frac{\partial U}{\partial R_B} = -\Delta = -\frac{R_B}{C}$$

C - жёсткость



Вычисление внутренних усилий и перемещений в замкнутых рамках

$$H_A = R_A = H_B = R_B = 0$$

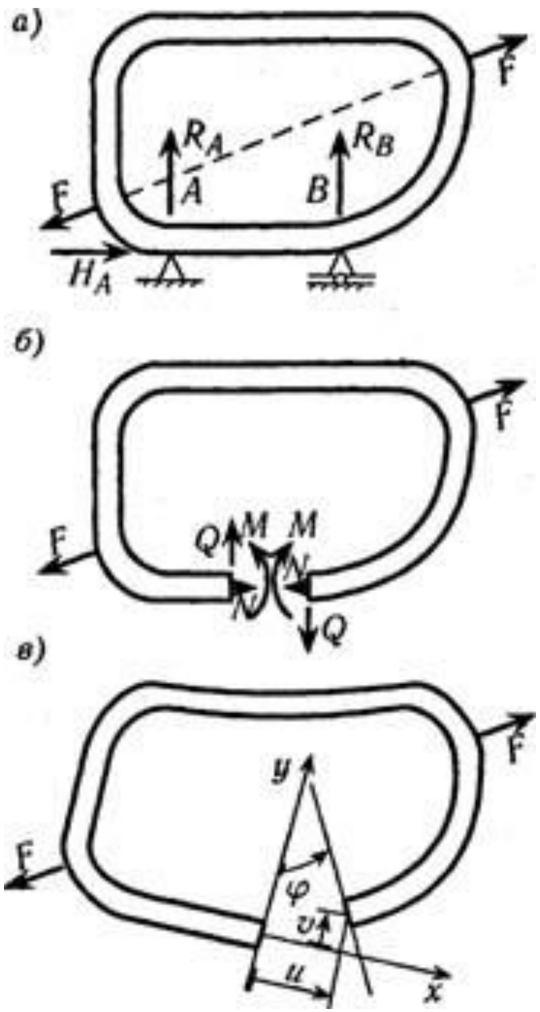


Рис.1

$$\varphi = 0, \qquad u = 0, \qquad v = 0 \tag{1}$$

Для выполнения условия совместности перемещений (1) нужно подобрать условия

$$X_1 = M, \quad X_2 = N, \quad X_3 = Q \tag{2}$$

$$\begin{cases} \varphi = \dfrac{\partial U}{\partial M} = 0, \\ u = \dfrac{\partial U}{\partial N} = 0, \\ v = \dfrac{\partial U}{\partial Q} = 0. \end{cases} \tag{3}$$

Две оси симметрии mm и nn:

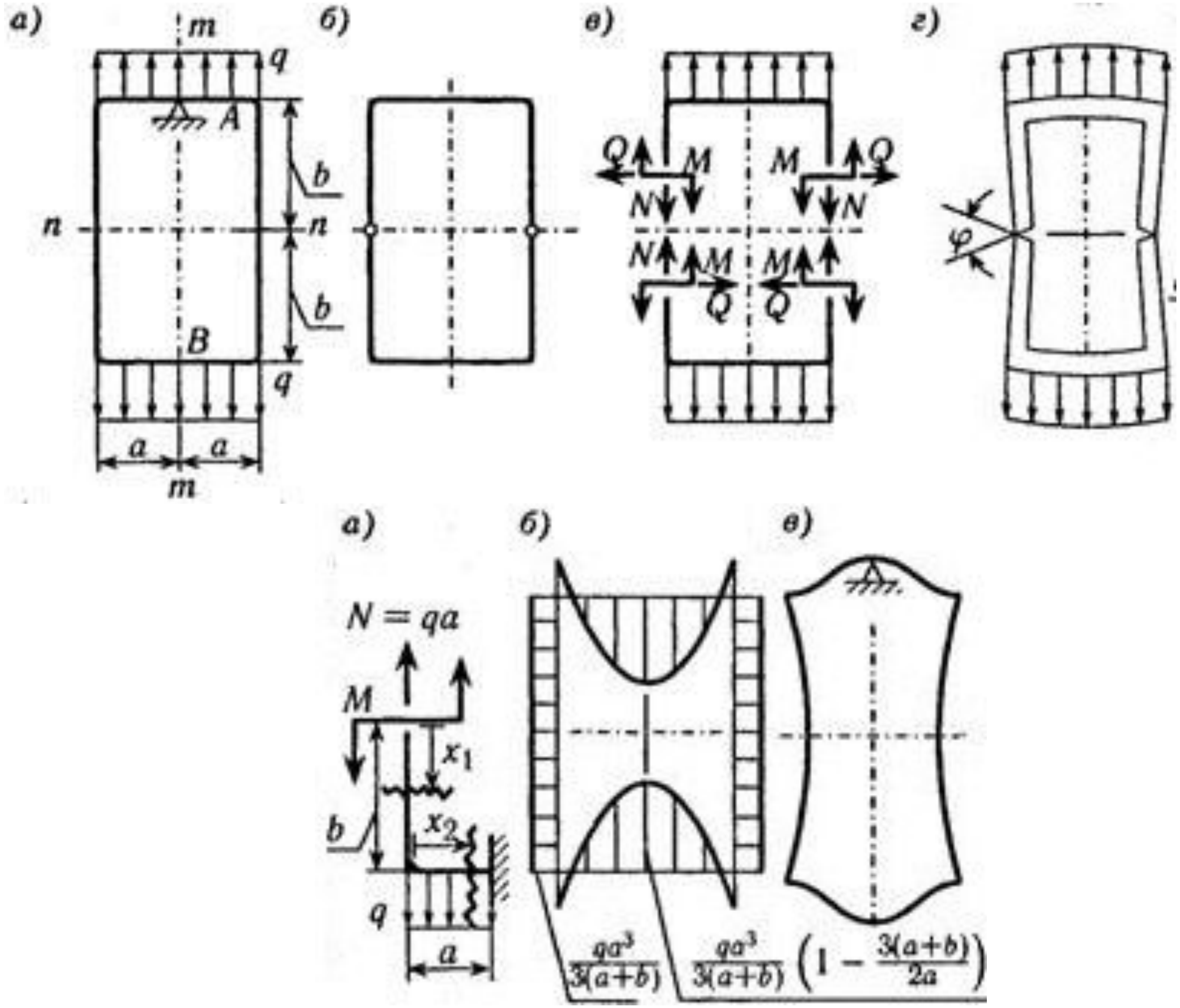


Рис.2

Из симметрии относительно оси nn $\Longrightarrow Q = 0$ (4)

N =qa. (5)

$$\varphi = \frac{\partial U}{\partial M} = \sum_{i=1}^{n} \frac{1}{EI} \int_{l_i} M_i \frac{\partial M_i}{\partial M} dx = 0 \qquad (6)$$

$$\varphi = \frac{\partial U}{\partial M} = \frac{4}{EI}\left(\int_0^b M_1 \frac{\partial M_1}{\partial M} dx + \int_0^a M_2 \frac{\partial M_2}{\partial M} dx \right) = 0 \qquad (7)$$

$$M_1 = M \qquad \frac{\partial M_1}{\partial M} = 1 \quad (0 \le x \le b) \qquad (8)$$

$$M_2 = M + \frac{1}{2} q x^2 - qax \qquad \frac{\partial M_2}{\partial M} = 1 \qquad (0 \le x \le a)$$

Из (8) $\Rightarrow$

$$\left(\int_0^b M \cdot 1 \cdot dx + \int_0^a (M + \frac{1}{2} q x^2 - qax) \cdot 1 \cdot dx \right) = 0 \qquad (9)$$

$$M = \frac{qa^3}{3(a+b)} \qquad (10)$$

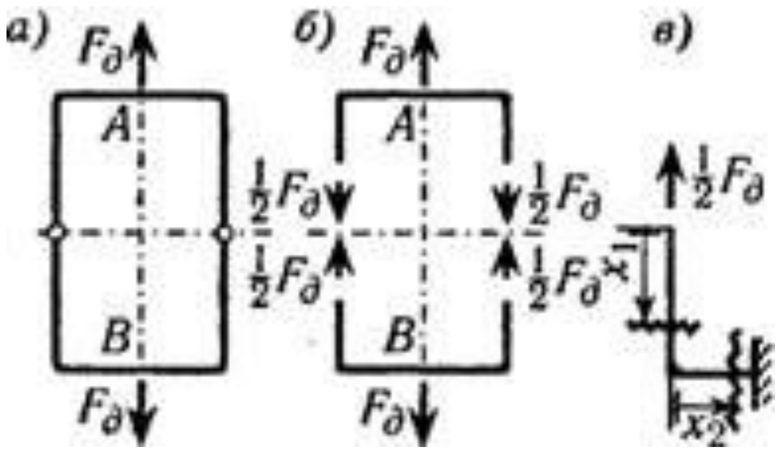


Рис.3

$$M_1^* = 0 \qquad M_2^* = -\frac{1}{2}F_\partial x \tag{11}$$

$$\frac{\partial M_1^*}{\partial F_\partial} = 0 \qquad \frac{\partial M_2^*}{\partial F_\partial} = -\frac{1}{2}x \tag{12}$$

$$v_{AB} = \frac{\partial U}{\partial F_\partial} = \frac{4}{EI}\left(\int_0^b M_1 \frac{\partial M_1^*}{\partial F_\partial}dx + \int_0^a M_2 \frac{\partial M_2^*}{\partial F_\partial}dx\right) \tag{13}$$

$$v_{AB} = \frac{4}{EI}\left(\int_0^b M\cdot 0\cdot dx + \int_0^a (M + \frac{1}{2}qx^2 - qax)(-\frac{1}{2}x)dx\right) \tag{14}$$

$$v_{AB} = \frac{qa^4}{48EI} \cdot \frac{a+5b}{a+b} \tag{15}$$

$$R_F = R_B = R = F \tag{16}$$

$$Q = 0 \tag{17}$$

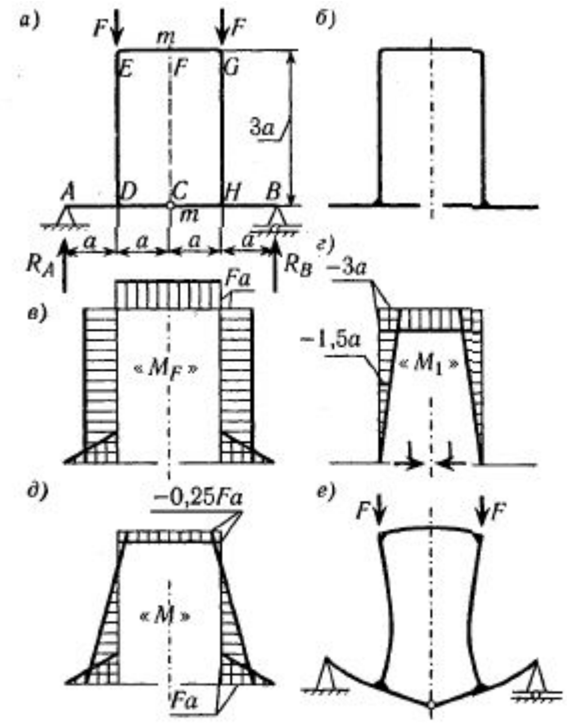


Рис.4

Из симметрии, а так же в шарнире M=0

$$X=0 \qquad (18)$$

$$\sum_1 = 2\left(\frac{3a}{6EI_z}(Fa \cdot 0 + 4 \cdot Fa \cdot (-1{,}5a) + Fa \cdot (-3a)) + \frac{a}{6EI_z}(Fa \cdot (-3a) + 4 \cdot Fa \cdot (-3a) + Fa \cdot (-3a))\right) = -\frac{15Fa^3}{EI_z}, \tag{19}$$

$$\sum_2 = 2\left(\frac{3a}{6EI_z}(0 + 4 \cdot (-1{,}5a)^2 + (-3a)^2) + \frac{a}{6EI_z}((-3a)^2 + 4 \cdot (-3a)^2 + (-3a)^2)\right) = \frac{36a^3}{EI_z}. \tag{20}$$

$$X = N = -\frac{\sum_1}{\sum_2} = -\left(-\frac{15Fa^3}{EI_z}\right) \cdot \left(\frac{EI_z}{36a^3}\right) = \frac{5}{12}F = 0{,}417F. \tag{21}$$

БЛАГОДАРЮ

ЗА

ВНИМАНИЕ