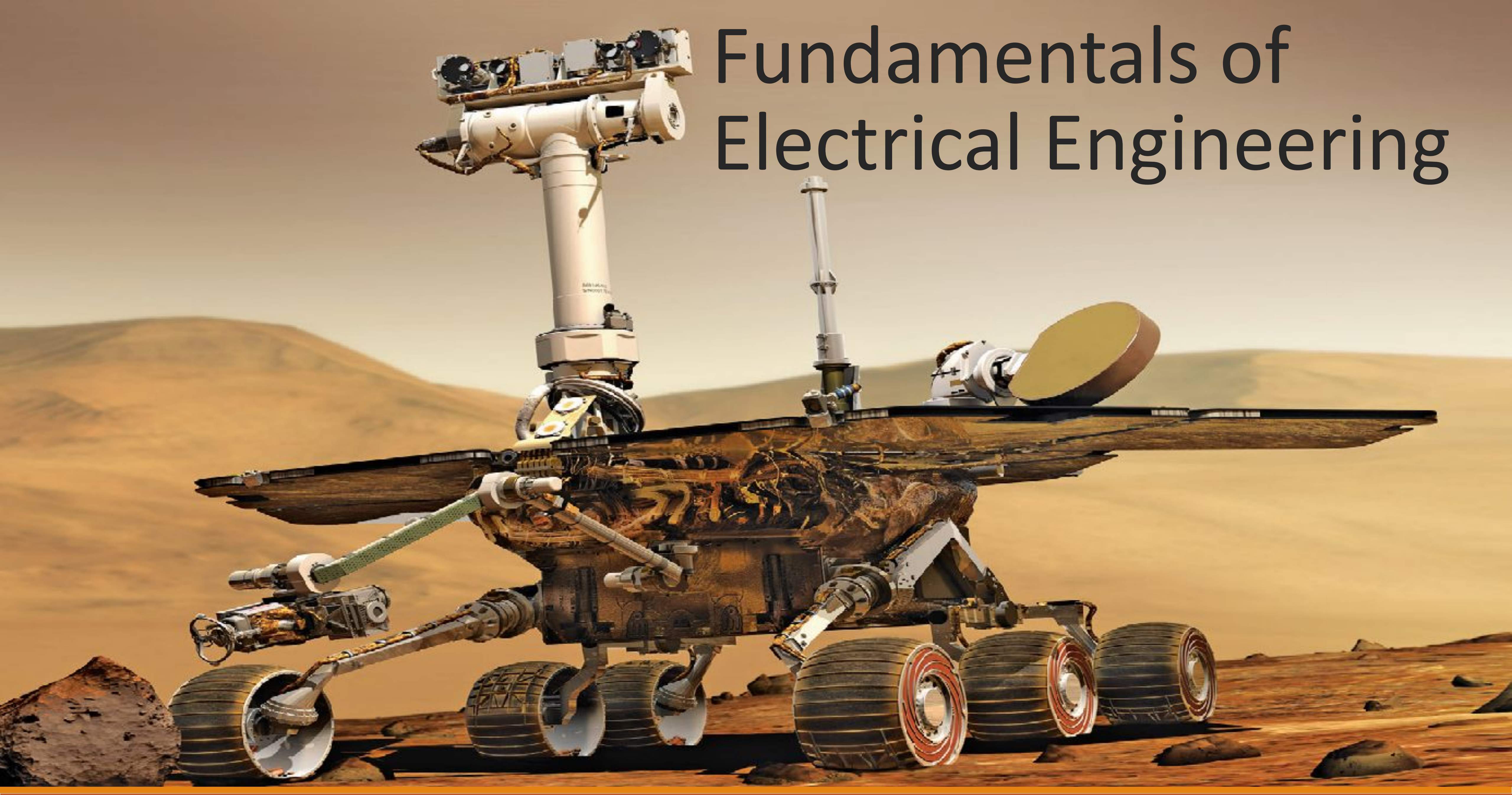
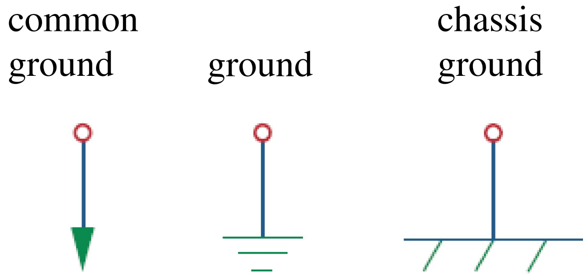
Fundamentals of Electrical Engineering



COMPILED BY RAKHIM AIBAT

INTRODUCTION

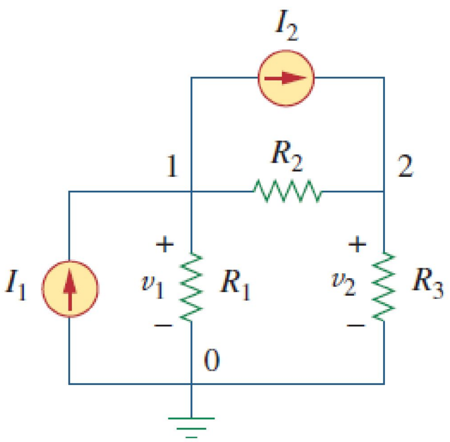
- So far covered: Ohm's law and Kirchhoff's laws
- This lecture covers powerful techniques for circuit analysis
  - nodal analysis - based on a systematic application of KCL
  - mesh analysis - based on a systematic application of KVL
- With these techniques we can analyze any linear circuit by obtaining a set of simultaneous equations that are then solved to obtain the required values of current or voltage
  - substitution method
  - elimination method
  - Cramer's rule
  - matrix inversion

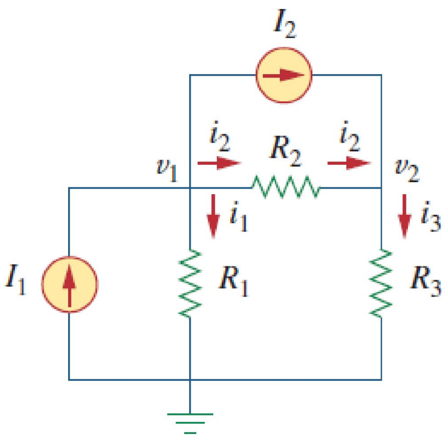


Nodal Analysis

- In nodal analysis, we are interested in finding the node voltages by applying KCL
  1. Select a node as the reference node, assign voltages V1, V2, …, Vn-1 to the remaining nodes, the voltages are referenced with respect to the reference node
  2. Apply KCL to each of the nonreference nodes, use Ohm's law to express the branch currents in terms of node voltages.
  3. Solve the resulting simultaneous equations to obtain the unknown node voltages

- The reference node is commonly called the ground since it is assumed to have zero potential





$$I_1 = I_2 + i_1 + i_2 \qquad I_2 + i_2 = i_3$$

$$i_1 = \frac{v_1 - 0}{R_1} \quad \text{or} \quad i_1 = G_1 v_1$$

$$i_2 = \frac{v_1 - v_2}{R_2} \quad \text{or} \quad i_2 = G_2(v_1 - v_2)$$

$$i_3 = \frac{v_2 - 0}{R_3} \quad \text{or} \quad i_3 = G_3 v_2$$

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$$

$$I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$$

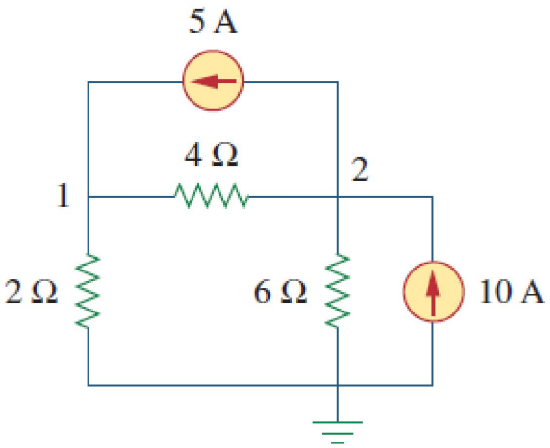
$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2)$$

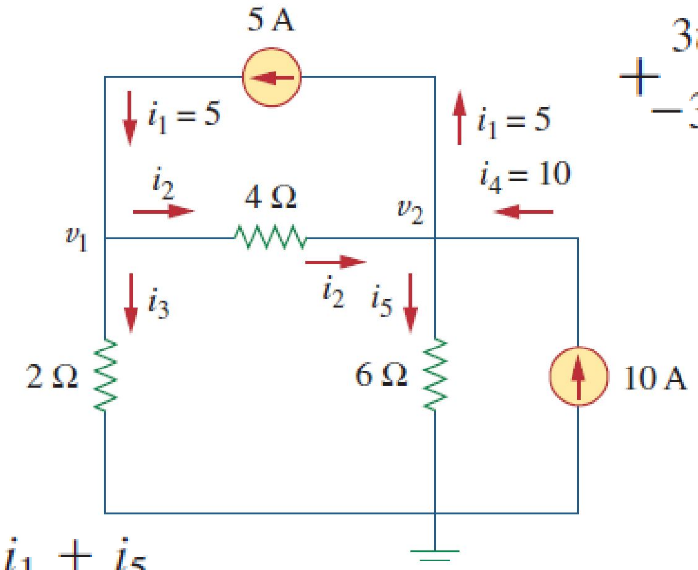
$$I_2 + G_2(v_1 - v_2) = G_3 v_2$$

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Example

Calculate the node voltages in the circuit shown





$$i_1 = i_2 + i_3$$

$$5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

$$20 = v_1 - v_2 + 2v_1$$

$$3v_1 - v_2 = 20$$

$$i_2 + i_4 = i_1 + i_5$$

$$\frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

$$3v_1 - 3v_2 + 120 = 60 + 2v_2$$

$$-3v_1 + 5v_2 = 60$$

METHOD 1 Using the elimination technique

$$\begin{array}{r} 3v_1 - v_2 = 20 \\ + \quad -3v_1 + 5v_2 = 60 \end{array} \Rightarrow \quad 4v_2 = 80 \qquad v_2 = 20 \text{ V}$$

$$v_1 = \frac{40}{3} = 13.333 \text{ V}$$

$$i_1 = 5 \text{ A}$$

$$i_2 = \frac{v_1 - v_2}{4} = -1.6668 \text{ A}$$

$$i_3 = \frac{v_1}{2} = 6.666 \text{ A}$$

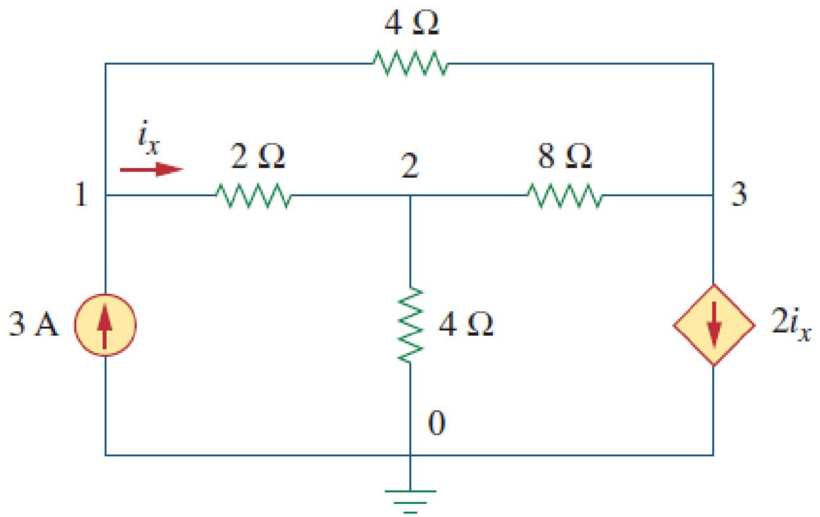
$$i_4 = 10 \text{ A}$$

$$i_5 = \frac{v_2}{6} = 3.333 \text{ A}$$

The fact that i2 is negative shows that the current flows in the direction opposite to the one assumed

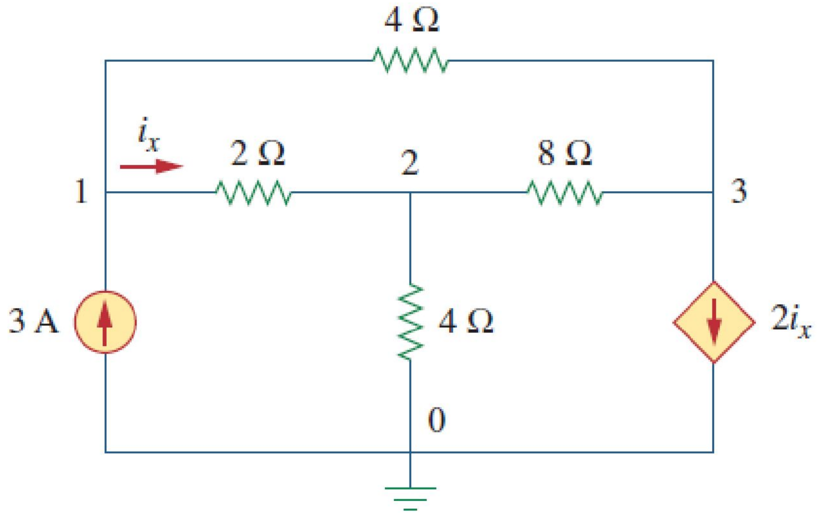
Problems

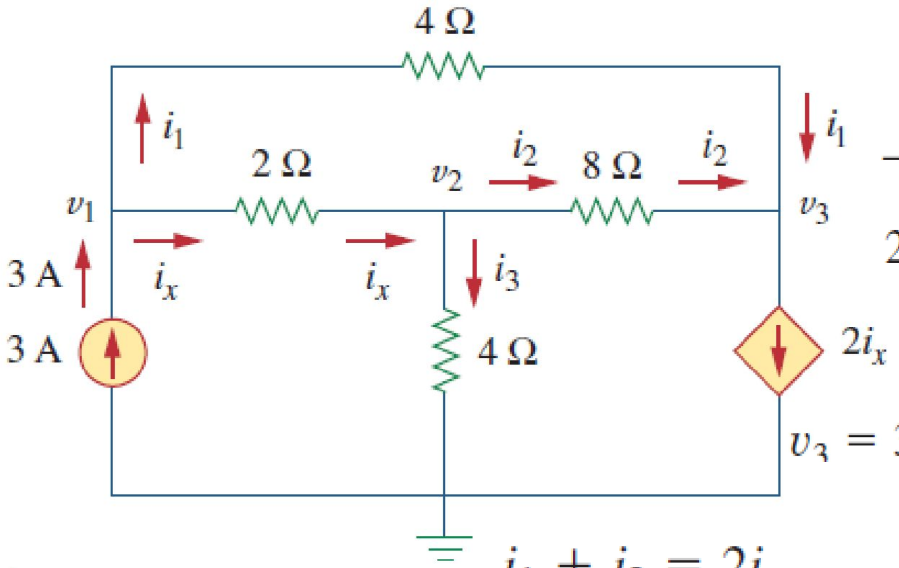
Determine the voltages at the nodes



Problems

Determine the voltages at the nodes





$$5v_1 - 5v_2 = 12$$

$$v_1 - v_2 = \frac{12}{5} = 2.4$$

$$-2v_1 + 4v_2 = 0 \qquad v_1 = 2v_2$$

$$2v_2 - v_2 = 2.4 \qquad v_2 = 2.4$$

$$v_1 = 2v_2 = 4.8\text{ V}$$

$$v_3 = 3v_2 - 2v_1 = 3v_2 - 4v_2 = -v_2 = -2.4\text{ V}$$

$$3 = i_1 + i_x$$

$$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

$$3v_1 - 2v_2 - v_3 = 12$$

$$i_x = i_2 + i_3$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$-4v_1 + 7v_2 - v_3 = 0$$

$$i_1 + i_2 = 2i_x$$

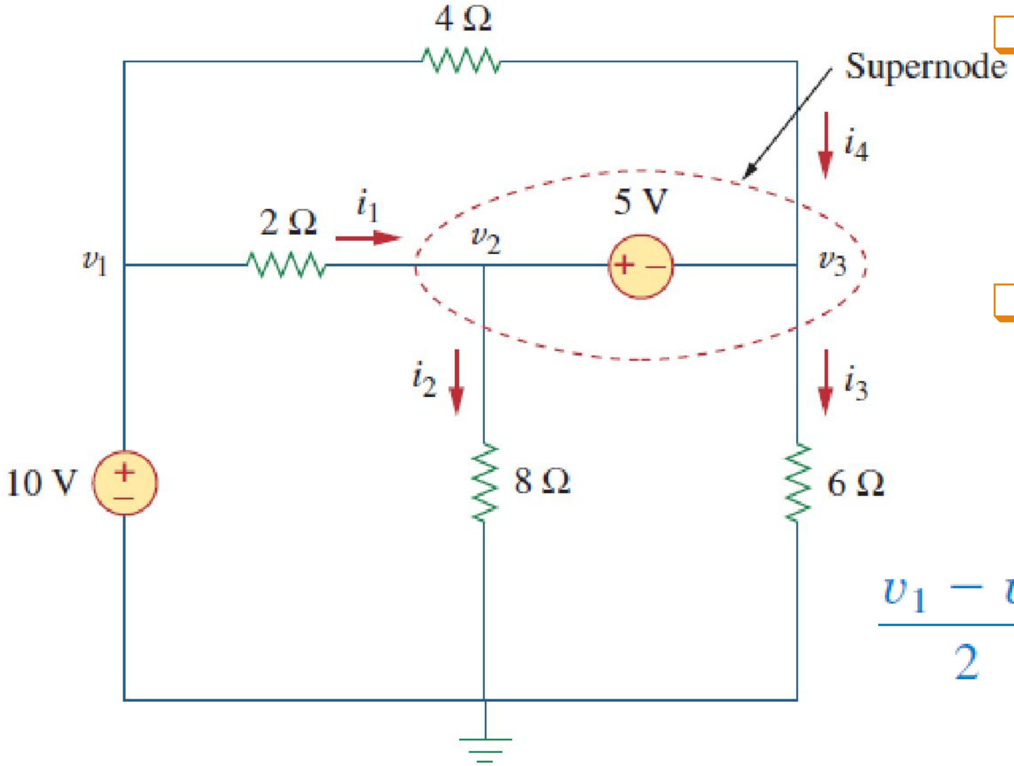
$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

$$2v_1 - 3v_2 + v_3 = 0$$

Nodal Analysis with Voltage Sources

- CASE 1: If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source

$$v_1 = 10 \text{ V}$$



- CASE 2: If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode; we apply both KCL and KVL to determine the node voltages

- A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

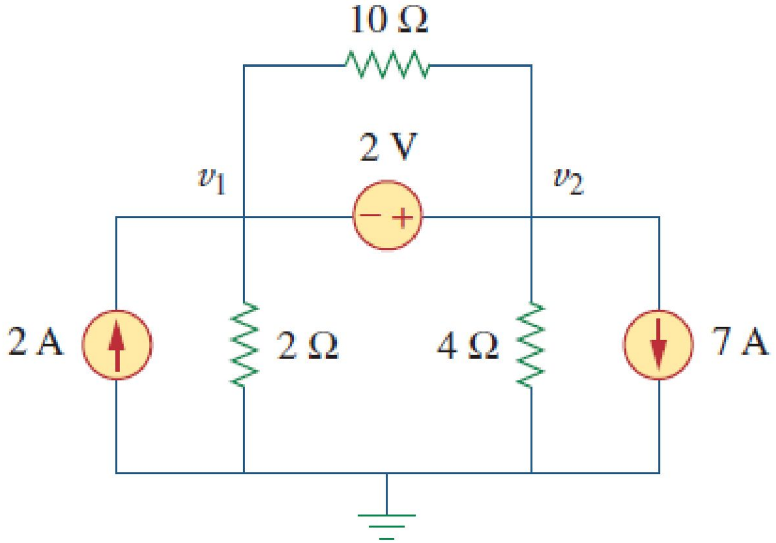
$$i_1 + i_4 = i_2 + i_3$$

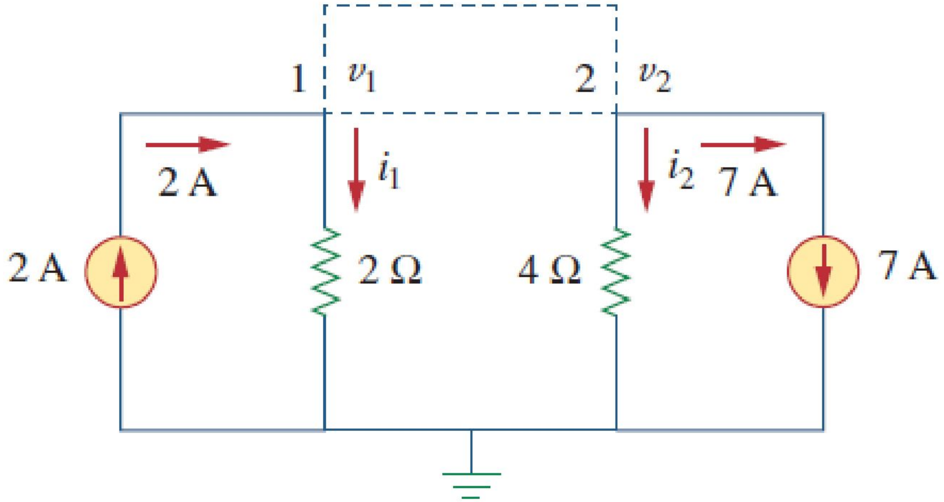
$$v_2 - v_3 = 5$$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

Example

For the circuit shown find the node voltages.





$$2 = i_1 + i_2 + 7$$

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7$$

$$8 = 2v_1 + v_2 + 28$$

$$v_2 = -20 - 2v_1$$

$$2 = v_2 - v_1$$

$$v_2 = v_1 + 2$$

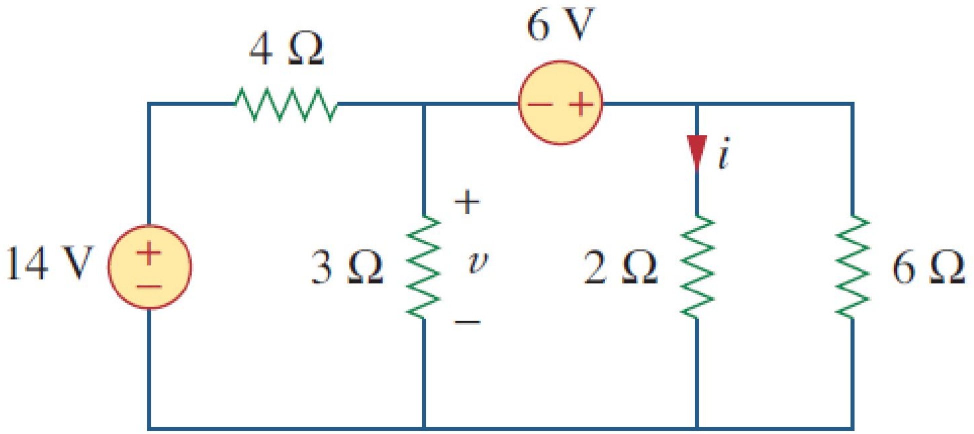
$$v_2 = v_1 + 2 = -20 - 2v_1$$

$$3v_1 = -22 \qquad v_1 = -7.333 \text{ V}$$

$$v_2 = v_1 + 2 = -5.333 \text{ V.}$$

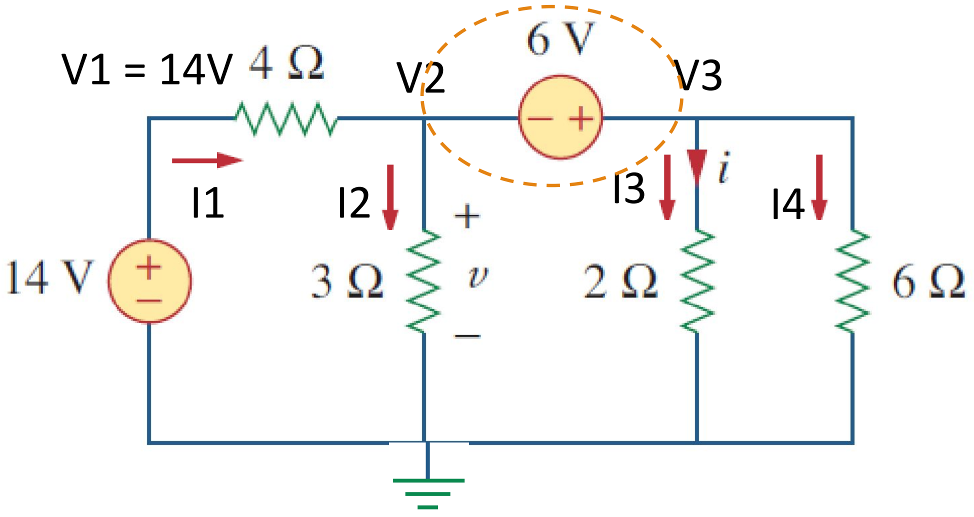
Problems

Find v and i in the circuit



Problems

Find v and i in the circuit



V1 = 14 V

6 = V3 - V2

V3 = 6 + V2

I1 = I2 + I3 + I4

(14 – V2)/4 = V2/3 + V3/2 + V3/6

42 – 3*V2 = 4*V2 + 6*V3 + 2*V3

42 = 7*V2 + 8*V3

42 = 7*V2 + 8*(6 + V2)

42 = 7*V2 + 48 + 8*V2

15*V2 = -6

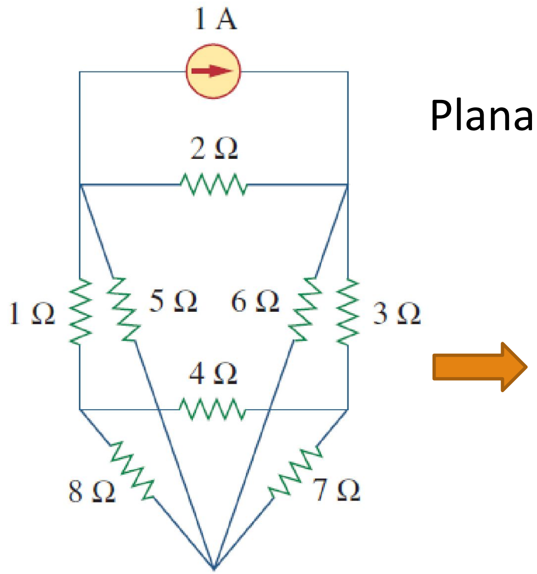
V2 = -6/15 = -0.4 V

V3 = 6 + (-0.4) = 5.6 V

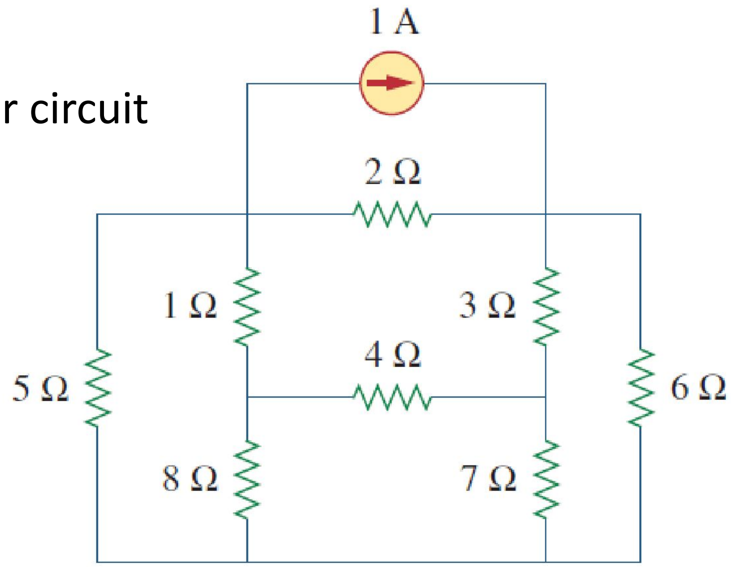
I = V3/2 = 5.6 / 2 = 2.8 A

Mesh Analysis (Planar / Nonplanar)

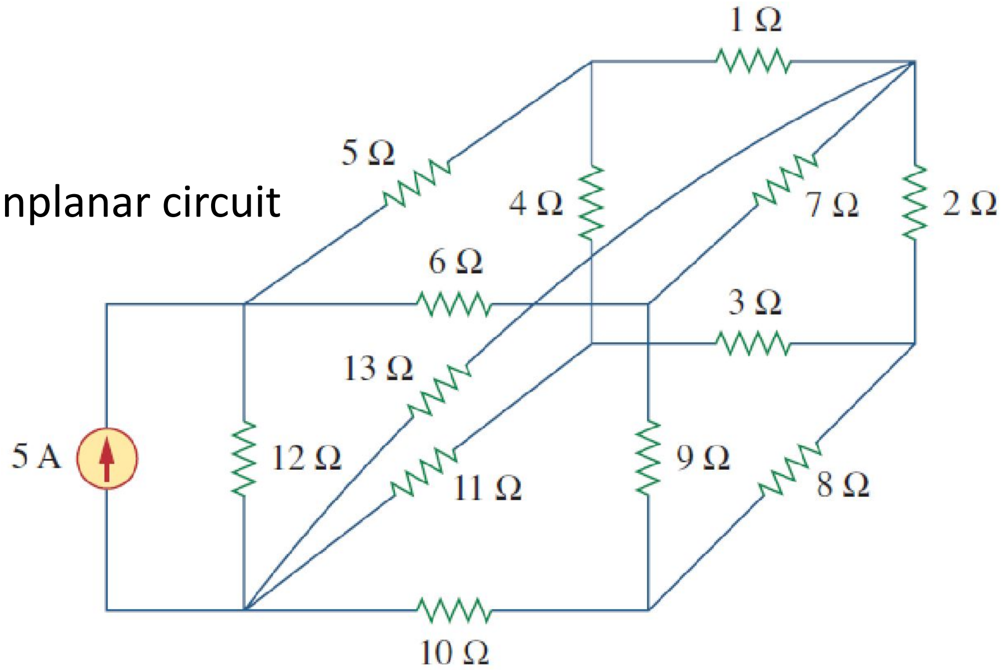
- mesh analysis applies KVL to find mesh currents
- Mesh analysis is not quite general because it is only applicable to a circuit that is planar
- planar circuit is one that can be drawn in a plane with no branches crossing one another
- Nonplanar circuits can be handled using nodal analysis



Planar circuit



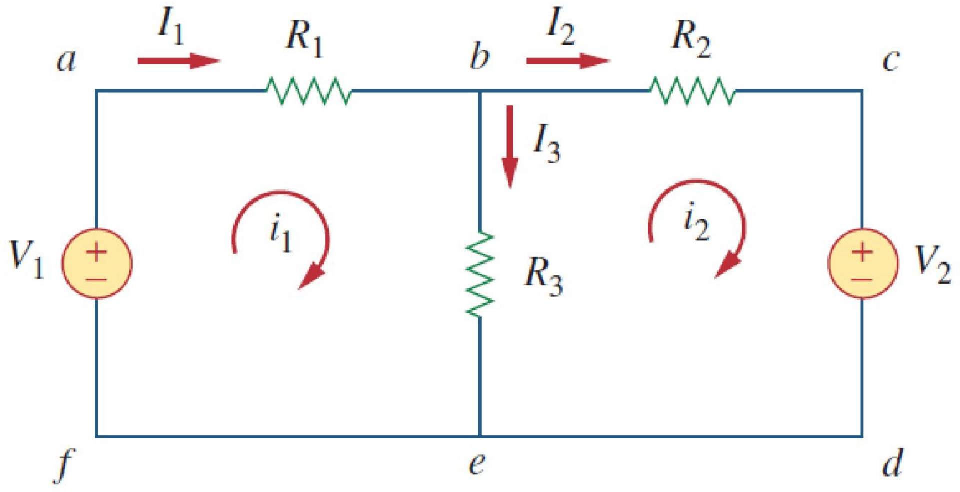
Nonplanar circuit



Mesh Analysis

- **A mesh (independent loop) is a loop which does not contain any other loops within it** $l = b - n + 1$

1. Assign mesh currents I1, I2, …, In to the n meshes
2. Apply KVL to each of the n meshes. Use Ohm’s law to express the voltages in terms of the mesh currents
3. Solve the resulting n simultaneous equations to get the mesh currents.



$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

$$(R_1 + R_3) i_1 - R_3 i_2 = V_1$$

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

$$-R_3 i_1 + (R_2 + R_3) i_2 = -V_2$$

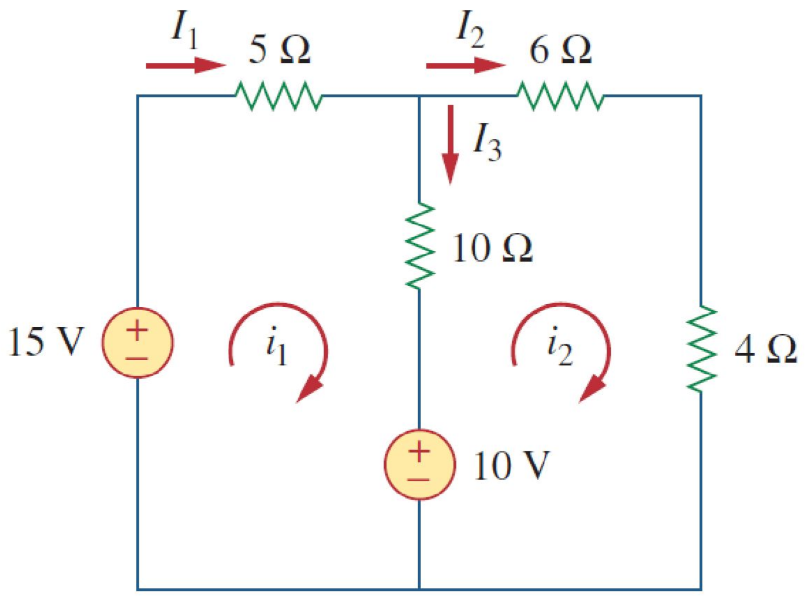
$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}$$

$$I_1 = i_1, \qquad I_2 = i_2,$$

$$I_3 = i_1 - i_2$$

Example

For the circuit, find the branch currents using mesh analysis



$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

$$3i_1 - 2i_2 = 1$$

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

$$i_1 = 2i_2 - 1$$

$$6i_2 - 3 - 2i_2 = 1$$

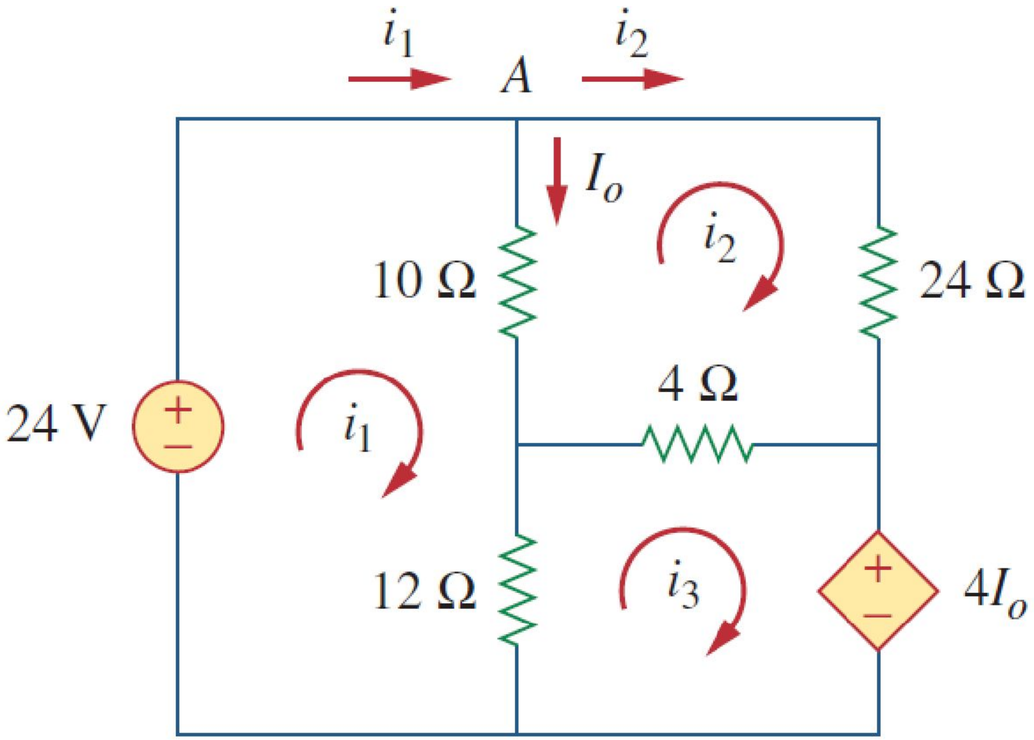
$$i_2 = 1 \text{ A}$$

$$i_1 = 2i_2 - 1 = 2 - 1 = 1 \text{ A}$$

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

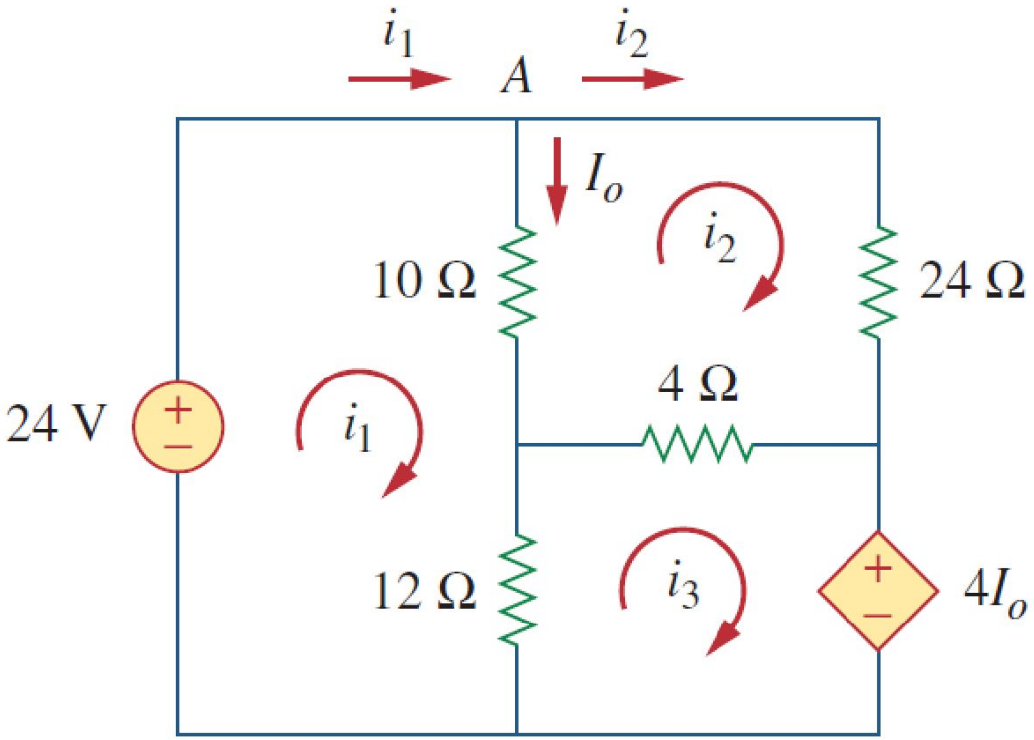
Problem

Use mesh analysis to find the current I0 in the circuit



Problem

Use mesh analysis to find the current I0 in the circuit



$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0 \rightarrow 11i_1 - 5i_2 - 6i_3 = 12$$

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0 \rightarrow -5i_1 + 19i_2 - 2i_3 = 0$$

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$I_o = i_1 - i_2 \rightarrow 4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

$$-i_1 - i_2 + 2i_3 = 0$$

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{vmatrix}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix} = 24 + 120 = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix} = 60 + 228 = 288$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}$$

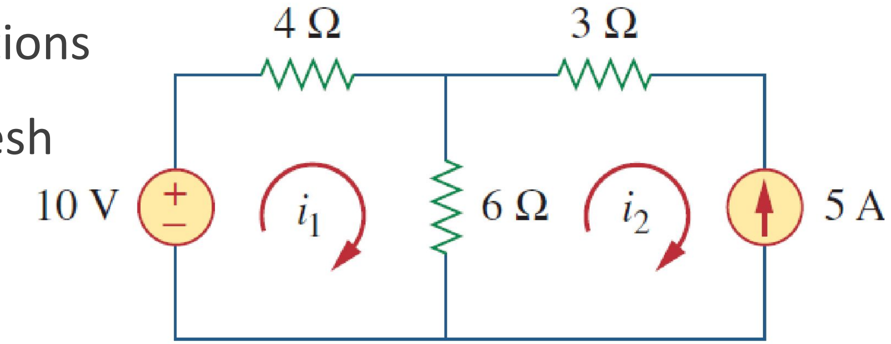
$$i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A}$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

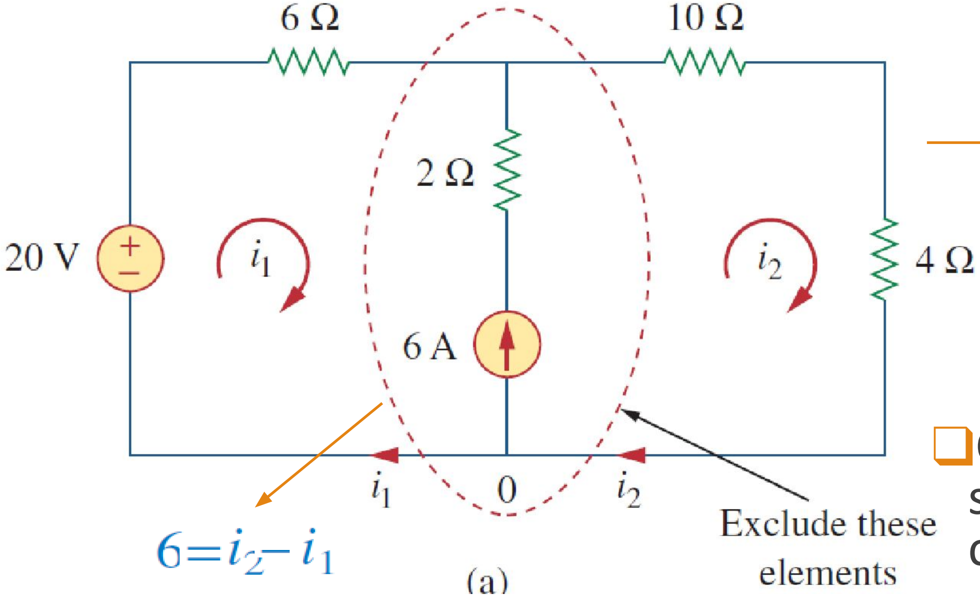
$$I_o = i_1 - i_2 = 1.5 \text{ A}$$

Mesh Analysis with Current Sources

- the presence of the current sources reduces the number of equations
- CASE 1: When a current source exists only in one mesh we set mesh current equal to algebraic sum of current sources in that mesh

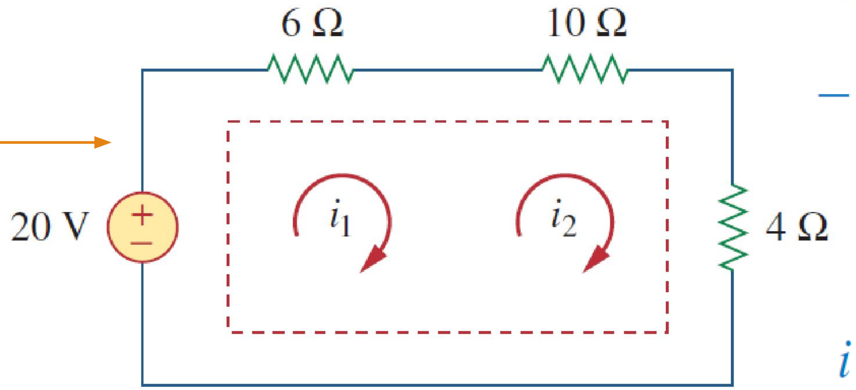


$i_2 = -5$ A $\quad -10 + 4i_1 + 6(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 = -2$ A



$6 = i_2 - i_1$

(a)



$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20$$

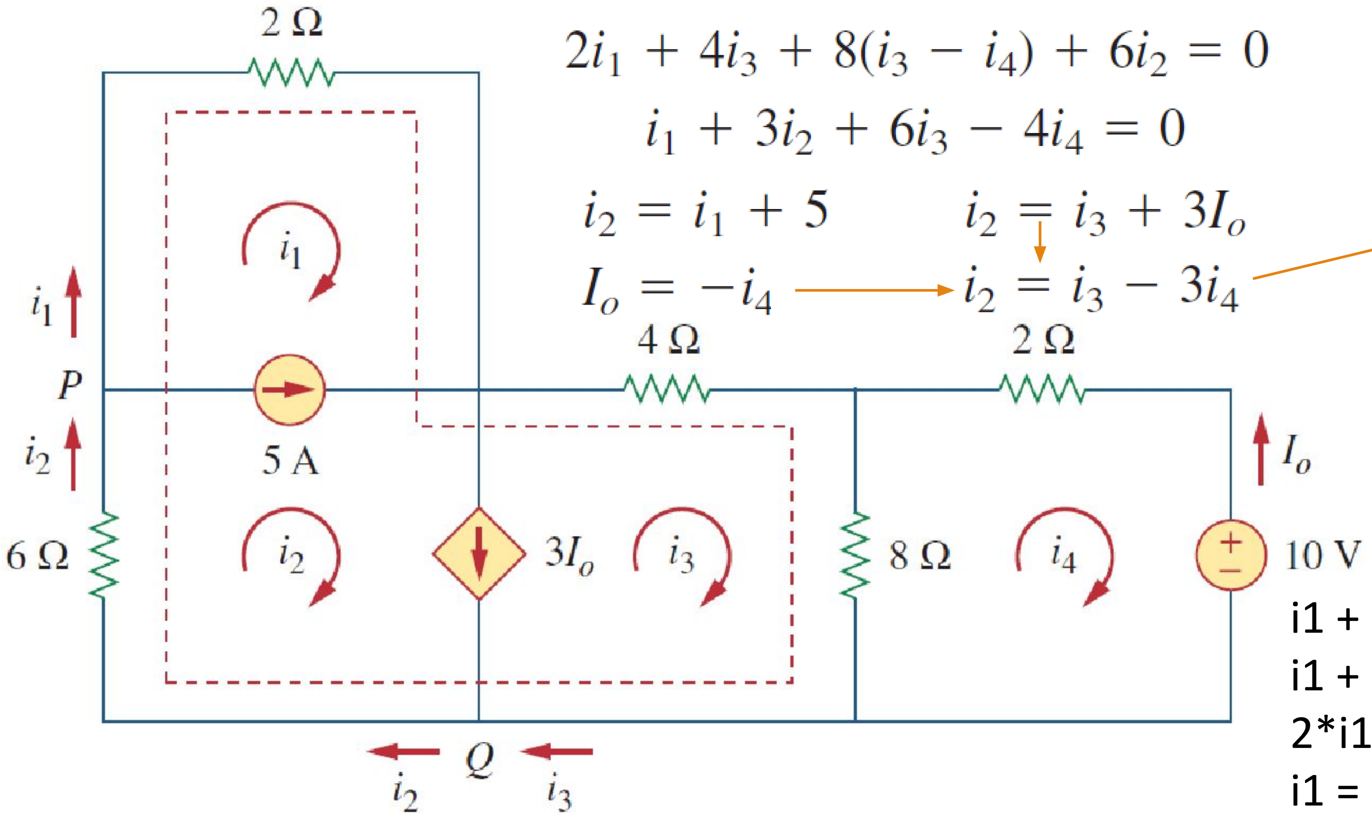
$$6 = i_2 - i_1 \longrightarrow i_2 = i_1 + 6$$

$$i_1 = -3.2 \text{ A} \qquad i_2 = 2.8 \text{ A}$$

- CASE 2: When a current source exists between two meshes we create a supermesh by excluding the current source and any elements connected in series with it, and apply KVL around supermesh

Example

For the circuit, find i1 to i4 using mesh analysis



$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$
$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$
$$i_2 = i_1 + 5 \qquad i_2 = i_3 + 3I_o$$
$$I_o = -i_4 \qquad i_2 = i_3 - 3i_4$$

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$
$$5i_4 - 4i_3 = -5$$

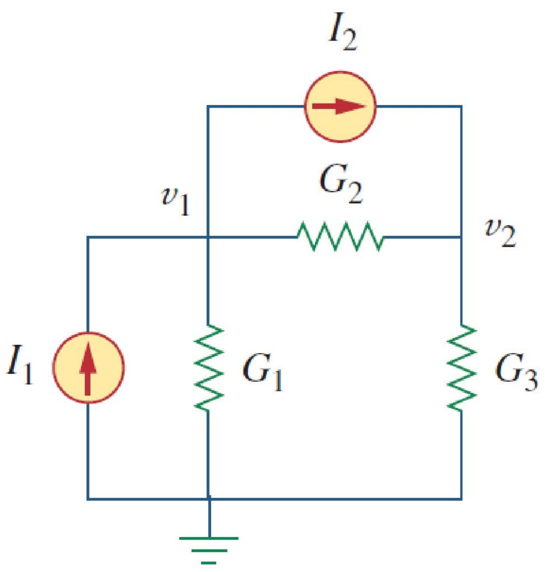
i3 = i2 + 3*i4

i4 = 0.8*i3 - 1

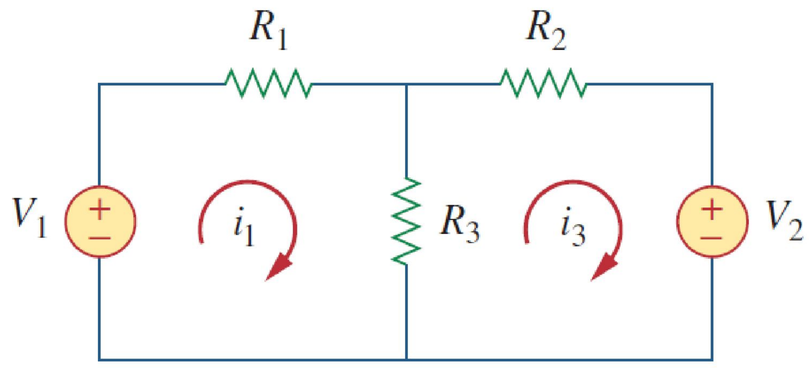
i3 = i2 + 3*(0.8*i3 - 1)
i3 = i2 + 2.4*i3 – 3
i3 = (3 – i2)/1.4
I3 = (– i1 – 2)/1.4
I4 = 0.8*(– i1 – 2)/1.4 - 1

i1 + 3*(i1 + 5) + 6*(– i1 – 2)/1.4 – 4*(0.8*(– i1 – 2)/1.4 – 1) = 0
i1 + 3*i1 + 15 – 4.3*i1 – 8.6 + 4 + 2.3*i1 + 4.6 = 0
2*i1 +15 = 0
i1 = -7.5A   i2 = -2.5A   i3 = 3.93A   i4 = 2.143A

Nodal and Mesh Analyses by Inspection



Voltage of the selected node multiplied by sum of conductances connected to that node minus all neighboring nodes multiplied with conductance that makes that node neighbor to the selected one equals to algebraic sum of current sources at the selected node



(R1+R3)*i1 – R3*i2 = v1
(R2+R3)*i2 – R3*i1 = -v2

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

(G1+G2)*v1 – G2*v2 = I1 – I2
(G2+G3)*v2 – G2*v1 = I2

Current of the selected mesh multiplied by sum of resistanses around that mesh minus all neighboring meshes multiplied with resistance that makes that mesh neighbor to the selected one equals to algebraic sum of voltage sources around the selected mesh

Nodal Versus Mesh Analysis

- first factor is the nature of the particular network
  - Networks that contain many series-connected elements, voltage sources, or supermeshes are more suitable for mesh analysis
  - whereas networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis
  - circuit with fewer nodes than meshes is better analyzed using nodal analysis
  - circuit with fewer meshes than nodes is better analyzed using mesh analysis
- second factor is the information required
  - If node voltages are required, it may be better to apply nodal analysis
  - If branch or mesh currents are required, it may be better to use mesh analysis

Home Work

From book Fundamentals of Electric Circuits (FIFTH EDITION) by Charles K. Alexander and Matthew N. O. Sadiku solve:

Chapter 3, Problems section (pages 114-124):

Problems – 3.1, 3.2, 3.10, 3.11, 3.15, 3.16, 3.33, 3.36, 3.41, 3.44, 3.45, 3.49, 3.51