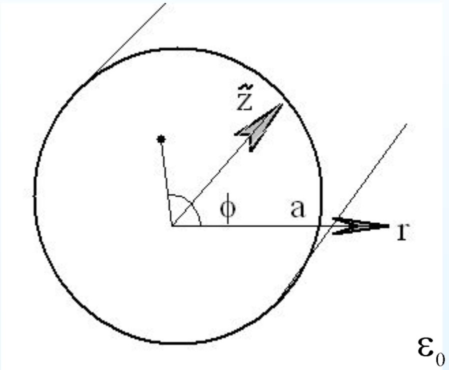
Моделирование электромагнитных волн в цилиндрическом волноводе



$$rot\vec{H} = \frac{\partial \vec{D}}{\partial t}, \qquad rot\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{D} = \varepsilon\varepsilon_0\vec{E}, \qquad \vec{B} = \mu\mu_0\vec{H},$$

$$\vec{E}(r,t) = \vec{E}(r)e^{i\omega\cdot t}, \qquad \vec{H}(r,t) = \vec{H}(r)e^{i\omega\cdot t},$$

$$\varepsilon_0 = \left(\frac{1}{36\pi}\right)\cdot 10^{-9}\left[\frac{F}{m}\right] \quad \mu_0 = 4\pi\cdot 10^{-7}\left[\frac{Gn}{m}\right], \quad \rho_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{4\pi\cdot 10^{-7}\cdot 36\pi\cdot 10^{9}} = 120\pi\left[Om\right],$$

$$rot\,\vec{H} = i\omega\varepsilon_0\varepsilon\,\vec{E}, \qquad rot\,\vec{E} = -i\omega\mu_0\mu\,\vec{H},$$

$$\vec{e} = i\vec{E},$$

$$rot\,\vec{H} = \omega\varepsilon_0\varepsilon\vec{e}, \qquad rot\,\vec{e} = \omega\mu_0\mu\,\vec{H},$$

$$rot\,\vec{h} = \omega\varepsilon_0\varepsilon\sqrt{\frac{\mu_0}{\varepsilon_0}}\vec{e}, \qquad rot\,\vec{h} = \omega\sqrt{\varepsilon_0\mu_0}\varepsilon\vec{e},$$

$$\vec{h} = \rho_0\vec{H},$$

$$rot\,\vec{e} = \omega\mu_0\mu\sqrt{\frac{\varepsilon_0}{\mu_0}}\vec{h}, \qquad rot\,\vec{e} = \omega\sqrt{\varepsilon_0\mu_0}\mu\vec{h},$$

$$rot\,\vec{h} = \omega\varepsilon_0\varepsilon\rho_0\vec{e}, \qquad rot\rho_0\vec{e} = \omega\mu_0\mu\vec{h},$$

$$rot\,\vec{h} = k_0\varepsilon\vec{e}, \qquad rot\,\vec{e} = k_0\mu\vec{h},$$

$$k_0 = \omega\sqrt{\varepsilon_0\mu_0}, \quad c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}, \quad k_0 = \frac{\omega}{c}, \quad c = \lambda_0 F, \quad \omega = 2\pi F, \quad k_0 = \frac{2\pi}{\lambda_0}.$$

$$rot\,\vec{h} = k_0\varepsilon\vec{e},$$
$$rot\,\vec{e} = k_0\mu\vec{h},$$

$r, \varphi, z.$

$$rot\,\vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\vec{i}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{i}_\varphi + \left(\frac{1}{r}\frac{\partial\left(rA_\varphi\right)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial\varphi}\right)\vec{i}_z,$$

$$\nabla^2\vec{A} = \left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}\right)\vec{A},$$

$$\frac{1}{r}\frac{\partial h_z}{\partial\varphi} - \frac{\partial h_\varphi}{\partial z} = k_0\varepsilon e_r, \qquad \frac{1}{r}\frac{\partial e_z}{\partial\varphi} - \frac{\partial e_\varphi}{\partial z} = k_0\mu h_r,$$

$$\frac{\partial h_r}{\partial z} - \frac{\partial h_z}{\partial r} = k_0\varepsilon e_\varphi, \qquad \frac{\partial e_r}{\partial z} - \frac{\partial e_z}{\partial r} = k_0\mu h_\varphi,$$

$$\frac{1}{r}\frac{\partial\left(rh_\varphi\right)}{\partial r} - \frac{1}{r}\frac{\partial h_r}{\partial\varphi} = k_0\varepsilon e_z, \qquad \frac{1}{r}\frac{\partial\left(re_\varphi\right)}{\partial r} - \frac{1}{r}\frac{\partial e_r}{\partial\varphi} = k_0\mu h_z,$$

$\rho = k_0 r, \quad z = k_0 z.$ Координаты безразмерные.

$$\frac{1}{\rho}\frac{\partial h_z}{\partial \varphi} - \frac{\partial h_\varphi}{\partial z} = \varepsilon e_\rho, \qquad \frac{1}{\rho}\frac{\partial e_z}{\partial \varphi} - \frac{\partial e_\varphi}{\partial z} = \mu h_\rho,$$

$$\frac{\partial h_\rho}{\partial z} - \frac{\partial h_z}{\partial \rho} = \varepsilon e_\varphi, \qquad \frac{\partial e_\rho}{\partial z} - \frac{\partial e_z}{\partial \rho} = \mu h_\varphi, \qquad \begin{pmatrix} h \\ e \end{pmatrix} \sim e^{i\omega t} \qquad \begin{pmatrix} h \\ e \end{pmatrix} \sim e^{i\tilde{\gamma} z} e^{i\omega t}$$

$$\frac{1}{\rho} h_\varphi + \frac{\partial h_\varphi}{\partial \rho} - \frac{1}{\rho}\frac{\partial h_\rho}{\partial \varphi} = \varepsilon e_z, \qquad \frac{1}{\rho} e_\varphi + \frac{\partial e_\varphi}{\partial \rho} - \frac{1}{\rho}\frac{\partial e_\rho}{\partial \varphi} = \mu h_z,$$

$$\gamma = \frac{\omega}{v_\phi}, \qquad v_\phi = \frac{1}{\sqrt{\varepsilon\varepsilon_0\mu\mu_0}},$$

$$\breve{z} = \frac{z}{k_0}, \qquad \tilde{\gamma} z = \Gamma z, \qquad \Gamma = \frac{\gamma}{k_0}, \qquad \begin{pmatrix} h \\ e \end{pmatrix} \sim e^{i\Gamma z}$$

$$v_\phi = \lambda F \qquad \Gamma = \frac{2\pi}{\Lambda}. \quad \Lambda = \frac{\lambda}{\lambda_0}.$$

$$(1) \quad \frac{1}{\rho}\frac{\partial h_z}{\partial \varphi} - i\Gamma h_\varphi = \varepsilon e_\rho, \qquad (4) \quad \frac{1}{\rho}\frac{\partial e_z}{\partial \varphi} - i\Gamma e_\varphi = \mu h_\rho,$$

$$(2) \quad i\Gamma h_\rho - \frac{\partial h_z}{\partial \rho} = \varepsilon e_\varphi, \qquad (5) \quad i\Gamma e_\rho - \frac{\partial e_z}{\partial \rho} = \mu h_\varphi,$$

$$(3) \quad \frac{1}{\rho} h_\varphi + \frac{\partial h_\varphi}{\partial \rho} - \frac{1}{\rho}\frac{\partial h_\rho}{\partial \varphi} = \varepsilon e_z, \qquad (6) \quad \frac{1}{\rho} e_\varphi + \frac{\partial e_\varphi}{\partial \rho} - \frac{1}{\rho}\frac{\partial e_\rho}{\partial \varphi} = \mu h_z,$$

Выразим $e_\rho, h_\rho, e_\varphi, h_\varphi$ через e_z, h_z..

Выразим $e_\rho, h_\rho, e_\varphi, h_\varphi$ через e_z, h_z..

Из (1,5):
$$e_\rho = \frac{1}{\varepsilon}\left(\frac{1}{\rho}\frac{\partial h_z}{\partial \varphi} - i\Gamma h_\varphi\right), \qquad h_\varphi = \frac{1}{\mu}\left(i\Gamma e_\rho - \frac{\partial e_z}{\partial \rho}\right),$$

$$e_\rho = \frac{1}{\mu\varepsilon - \Gamma^2}\left(\frac{\mu}{\rho}\frac{\partial h_z}{\partial \varphi} + i\Gamma\frac{\partial e_z}{\partial \rho}\right),$$

$$h_\varphi = \frac{1}{\mu\varepsilon - \Gamma^2}\left(\frac{i\Gamma}{\rho}\frac{\partial h_z}{\partial \varphi} - \varepsilon\frac{\partial e_z}{\partial \rho}\right),$$

Из (2,4):
$$h_\rho = \frac{1}{\mu\varepsilon - \Gamma^2}\left(\frac{\varepsilon}{\rho}\frac{\partial e_z}{\partial \varphi} + i\Gamma\frac{\partial h_z}{\partial \rho}\right),$$

$$e_\varphi = \frac{1}{\mu\varepsilon - \Gamma^2}\left(\frac{i\Gamma}{\rho}\frac{\partial e_z}{\partial \varphi} - \mu\frac{\partial h_z}{\partial \rho}\right),$$

Подставим в (3) поперечные компоненты:
$$\frac{1}{\rho}h_\varphi + \frac{\partial h_\varphi}{\partial \rho} - \frac{1}{\rho}\frac{\partial h_\rho}{\partial \varphi} = \varepsilon e_z, \quad (3)$$

$$\rho^2\frac{\partial^2 e_z}{\partial \rho^2} + \rho\frac{\partial e_z}{\partial \rho} + \rho^2\left(\mu\varepsilon - \Gamma^2\right)e_z + \frac{\partial^2 e_z}{\partial \varphi^2} = 0$$

Подставим в (6) поперечные компоненты:
$$\frac{1}{\rho}e_\varphi + \frac{\partial e_\varphi}{\partial \rho} - \frac{1}{\rho}\frac{\partial e_\rho}{\partial \varphi} = \mu h_z, \quad (6)$$

$$\rho^2\frac{\partial^2 h_z}{\partial \rho^2} + \rho\frac{\partial h_z}{\partial \rho} + \rho^2\left(\mu\varepsilon - \Gamma^2\right)h_z + \frac{\partial^2 h_z}{\partial \varphi^2} = 0.$$

$$\rho^2\frac{\partial^2 e_z}{\partial\rho^2}+\rho\frac{\partial e_z}{\partial\rho}+\rho^2\left(\mu\varepsilon-\Gamma^2\right)e_z+\frac{\partial^2 e_z}{\partial\varphi^2}=0,\qquad \rho^2\frac{\partial^2 h_z}{\partial\rho^2}+\rho\frac{\partial h_z}{\partial\rho}+\rho^2\left(\mu\varepsilon-\Gamma^2\right)h_z+\frac{\partial^2 h_z}{\partial\varphi^2}=0$$

$$e_\rho=\frac{1}{\mu\varepsilon-\Gamma^2}\left(\frac{\mu}{\rho}\frac{\partial h_z}{\partial\varphi}+i\Gamma\frac{\partial e_z}{\partial\rho}\right),\qquad e_\rho=\frac{1}{\mu\varepsilon-\Gamma^2}\left(\frac{\mu}{\rho}\frac{\partial h_z}{\partial\varphi}+i\Gamma\frac{\partial e_z}{\partial\rho}\right),$$

$$h_\varphi=\frac{1}{\mu\varepsilon-\Gamma^2}\left(\frac{i\Gamma}{\rho}\frac{\partial h_z}{\partial\varphi}-\varepsilon\frac{\partial e_z}{\partial\rho}\right),\qquad h_\rho=\frac{1}{\mu\varepsilon-\Gamma^2}\left(\frac{\varepsilon}{\rho}\frac{\partial e_z}{\partial\varphi}+i\Gamma\frac{\partial h_z}{\partial\rho}\right),$$

$$h_\rho=\frac{1}{\mu\varepsilon-\Gamma^2}\left(\frac{\varepsilon}{\rho}\frac{\partial e_z}{\partial\varphi}+i\Gamma\frac{\partial h_z}{\partial\rho}\right),\qquad h_\varphi=\frac{1}{\mu\varepsilon-\Gamma^2}\left(\frac{i\Gamma}{\rho}\frac{\partial h_z}{\partial\varphi}-\varepsilon\frac{\partial e_z}{\partial\rho}\right),$$

$$e_\varphi=\frac{1}{\mu\varepsilon-\Gamma^2}\left(\frac{i\Gamma}{\rho}\frac{\partial e_z}{\partial\varphi}-\mu\frac{\partial h_z}{\partial\rho}\right),\qquad e_\varphi=\frac{1}{\mu\varepsilon-\Gamma^2}\left(\frac{i\Gamma}{\rho}\frac{\partial e_z}{\partial\varphi}-\mu\frac{\partial h_z}{\partial\rho}\right),$$

Вариант 2 (в векторной форме).

$$rot\,\vec h=k_0\varepsilon\vec e,\qquad rot\,h=k_0\varepsilon\;e,\qquad rot\,rot\,e=grad\,div\,e-\nabla^2 e,\quad div\,e=0.$$

$$rot\,\vec e=k_0\mu\vec h,\qquad rot\;rot\;e=k_0\mu\;rot\,h,\qquad rot\;rot\;e=k_0^2\varepsilon\,\mu\,e,\qquad rot\;rot\;e=-\nabla^2 e$$

$$\left(\nabla^2+k_0^2\varepsilon\mu\right)\vec e=0,\qquad r,\phi,z,\qquad \nabla^2\vec A=\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}\right)\vec A,$$

$$\left(\nabla_\perp^2+\frac{\partial^2}{\partial z^2}+\varepsilon\mu\right)\vec e=0,\qquad e(\rho,\varphi,z)=e(\rho,\varphi)e^{i\Gamma z},\qquad \nabla_\perp^2\vec e=\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right)\vec e$$

$$\rho^2\frac{\partial^2\vec e}{\partial\rho^2}+\rho\frac{\partial\vec e}{\partial\rho}+\rho^2\left(\varepsilon\mu-\Gamma^2\right)\vec e+\frac{\partial^2\vec e}{\partial\varphi^2}=0,$$

Метод разделения переменных

$$\rho^2 \frac{\partial^2 e_z}{\partial \rho^2} + \rho \frac{\partial e_z}{\partial \rho} + \rho^2 \chi^2 e_z + \frac{\partial^2 e_z}{\partial \varphi^2} = 0, \qquad e(\rho, \varphi, z) = e(\rho, \varphi) e^{i\Gamma z},$$

$$\chi^2 = \left(\mu\varepsilon - \Gamma^2\right), \quad \Gamma = \sqrt{\mu\varepsilon - \chi^2}. \qquad e_z(\rho, \varphi) = R(\rho)\Phi(\varphi),$$

$$\rho^2 \Phi(\varphi) \frac{\partial^2 R(\rho)}{\partial \rho^2} + \rho \Phi(\varphi) \frac{\partial R(\rho)}{\partial \rho} + \rho^2 \chi^2 R(\rho)\Phi(\varphi) + R(\rho) \frac{\partial^2 \Phi(\varphi)}{\partial \varphi^2} = 0,$$

$$\frac{\rho^2}{R(\rho)} \frac{\partial^2 R(\rho)}{\partial \rho^2} + \frac{\rho}{R(\rho)} \frac{\partial R(\rho)}{\partial \rho} + \rho^2 \chi^2 = -\frac{1}{\Phi(\varphi)} \frac{\partial^2 \Phi(\varphi)}{\partial \varphi^2},$$

$$\frac{\rho^2}{R} \frac{d^2 R}{d\rho^2} + \frac{\rho}{R} \frac{dR}{d\rho} + \rho^2 \chi^2 = n^2, \qquad -\frac{1}{\Phi} \frac{d^2 \Phi}{d\varphi^2} = n^2,$$

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + \left(\chi^2 \rho^2 - n^2\right) R = 0, \qquad \frac{d^2 \Phi}{d\varphi^2} + n^2 \Phi = 0. \qquad \frac{\partial^2 \Phi}{\partial \varphi^2} + n^2 \Phi = 0, \qquad \Phi = D e^{in\varphi},$$

$$\Phi = D e^{\pm in\varphi}, \quad n = 1, 2, 3, \ldots$$

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + \left(\chi^2 \rho^2 - n^2\right) R = 0, \qquad r = \chi\rho,$$

$$R_{1,2} = r^\lambda \sum_{k=0}^{\infty} C_k r^k,$$

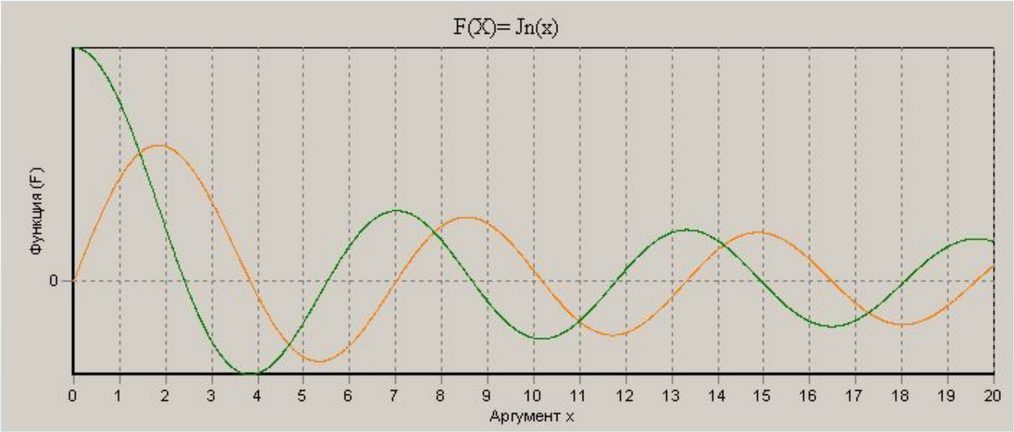
$$r^2 \frac{d^2 R}{dr^2} + r \frac{dR}{dr} + \left(r^2 - n^2\right) R = 0, \qquad r = 0, \quad r = \infty,$$

$$\frac{dR}{dr} = r^{\lambda-1}\sum_{k=0}^{\infty} C_k(\lambda+k)r^k, \qquad \frac{d^2R}{dr^2} = r^{\lambda-2}\sum_{k=0}^{\infty} C_k(\lambda+k)(\lambda+k-1)r^k,$$

$$\sum_{k=0}^{\infty} C_k(\lambda+k)(\lambda+k-1)r^k + \sum_{k=0}^{\infty} C_k(\lambda+k)r^k + \left(r^2-n^2\right)\sum_{k=0}^{\infty} C_k r^k = 0$$

$$\left(\lambda^2-n^2\right)=0, \quad \lambda_{1,2}=\pm n, \quad R_{1,2} = r^{\pm n}\sum_{k=0}^{\infty} C_k r^k,$$

$$R_1(r) = r^n \sum_{k=0}^{\infty}\frac{(-1)^k r^{2k}}{k!(n+k)!} = J_n(r)$$



Функции Бесселя $J_0(x)$ - зеленая кривая, $J_1(x)$ - оранжевая кривая

Корни функции $\xi_{n,m}$

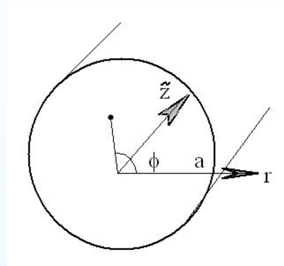
n\m	1	2	3	…
0	2.41	5.52…	8.65…	…
1	3.83	7.02	10.17	…
2	5.14	8.42	11.62	…
…	…	…	…	…

$$R_2(r) = \left.\frac{dR_1}{d\lambda}\right|_{\lambda=\lambda_2} = Y_n(r),$$

$$R(\rho) = AJ_n(\chi\rho) + BY_n(\chi\rho),$$

$$e_z(\rho,\varphi,z,t) = \left(AJ_n(\chi\rho) + BY_n(\chi\rho)\right)e^{in\varphi}e^{i\Gamma z}e^{i\omega t} \qquad h_z(\rho,\varphi,z)$$

Краевые задачи



Задача Дирихле. $\left.a\right|_{L_1} = 0, \quad \{a\}_{n=1}^{\infty}, \quad \{\chi_n\}_{n=1}^{\infty}.$ $\quad \left.e_z\right|_{L_1} = 0, \quad \{e_z\}_{n=1}^{\infty}, \quad \{\chi_n\}_{n=1}^{\infty}.$

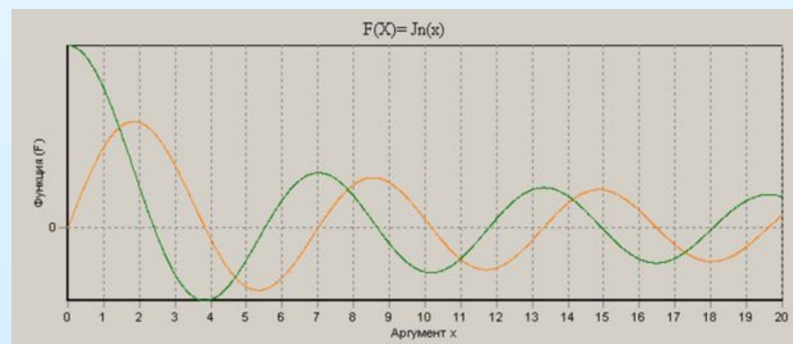
Задача Неймана. $\left.\dfrac{\partial a}{\partial n}\right|_{L_1} = 0, \quad \{a\}_{n=1}^{\infty}, \quad \{\chi_n\}_{n=1}^{\infty},$ $\quad \left.\dfrac{\partial h_z}{\partial n}\right|_{L_1} = 0, \quad \{h_z\}_{n=1}^{\infty}, \quad \{\chi_n\}_{n=1}^{\infty},$

Задача Дирихле. Е – волны.

$$e_z(\rho) = AJ_n(\chi\rho) + BY_n(\chi\rho), \qquad \left.e_z(\rho)\right|_{\rho=a} = 0$$

$$B = 0, \qquad e_z = AJ_n(\chi\rho),$$

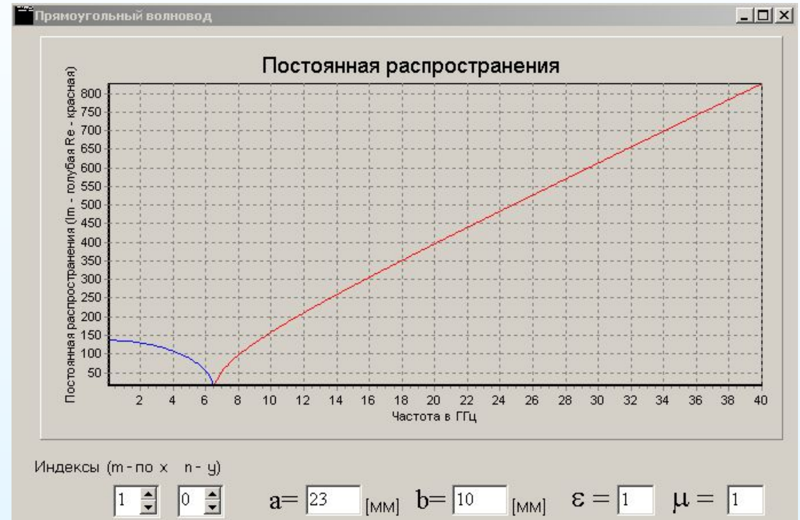
$$\left.e_z\right|_a = AJ_n(\chi a) = 0,$$



$$J_n(\chi a) = 0, \qquad \chi a = \xi_{n,m}, \qquad \chi = \frac{\xi_{n,m}}{a}, \qquad \Gamma = \sqrt{\varepsilon\mu - \chi^2} = \sqrt{\varepsilon\mu - \left(\frac{\xi_{n,m}}{a}\right)^2},$$

$$\Gamma = \sqrt{\varepsilon\mu - \chi^2} = \sqrt{\varepsilon\mu - \left(\frac{\xi_{n,m}}{a}\right)^2},$$

$$\Gamma = \sqrt{\varepsilon\mu - \chi^2} = \sqrt{\varepsilon\mu - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}.$$



$r_{мет} = 0.03$, $\varepsilon = 1$, $\mu = 1$, $f = 10$ ГГц,

$$k_0 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} \approx \frac{6.3 \cdot 10 \cdot 10^9}{3 \cdot 10^8} \approx 210\ {}^{-1}$$

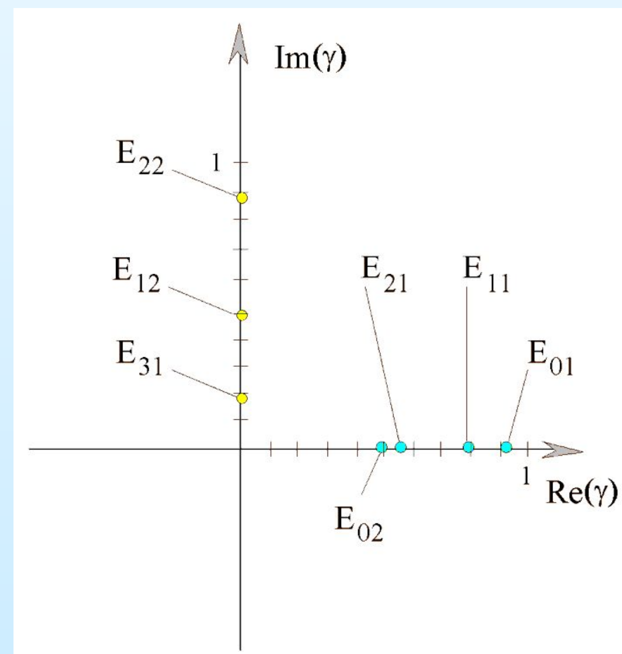
$$a = k_0 r = 0.03 \cdot 210 = 6.3,$$

$$\Gamma = \sqrt{\varepsilon\mu - \left(\frac{\xi_{n,m}}{a}\right)^2} \approx \sqrt{1 - \left(\frac{\xi_{n,m}}{6.3}\right)^2},$$

Корни функции Бесселя

n\m	1	2	3	…
0	2.41	5.52…	8.65…	…
1	3.83	7.02	10.17	…
2	5.14	8.42	11.62	…
…	…	…	…	…

параметр распространения Γ				
n\m	1	2	3	…
0	≈ 0.92	≈ 0.48	$\approx i\,0.95$	…
1	≈ 0.79	$\approx i\,0.50$	$\approx i\,1.27$	…
2	≈ 0.56	$\approx i\,0.89$	$\approx i\,1.56$	…
3	$\approx i\,0.18$	$\approx i\,1.19$	$\approx i\,1.82$	
…	…	…	…	…



Структура поля.

$$e_z = E_0 J_n(\chi\rho), \qquad h_z = 0,$$

$$e_\rho = i\Gamma \frac{E_0}{\chi^2}\left[\frac{n}{\chi\rho} J_n(\chi\rho) - J_{n+1}(\chi\rho)\right], \qquad h_\rho = -\frac{i\varepsilon}{\Gamma} e_\varphi, \qquad \chi^2 = \varepsilon\mu - \Gamma^2 = \left(\frac{\xi_{n,m}}{a}\right)^2,$$

$$e_\varphi = -\frac{\Gamma n}{\chi^2 \rho} e_z, \qquad h_\varphi = \frac{i\varepsilon}{\Gamma} e_\rho.$$

Критический размер волновода.

$$\Gamma = \sqrt{\varepsilon\mu - \left(\frac{\xi_{n,m}}{a}\right)^2}, \qquad \Gamma = 0, \qquad \Rightarrow \qquad \varepsilon\mu = \left(\frac{\xi_{n,m}}{a}\right)^2, \quad a_{кр} = \frac{\xi_{n,m}}{\sqrt{\varepsilon\mu}}.$$

$$\gamma = \frac{2\pi}{\lambda_в}, \quad k_0 = \frac{2\pi}{\lambda_0}, \quad \Gamma = \frac{\gamma}{k_0} = \frac{\lambda_0}{\lambda_в}, \quad \Gamma = 0, \quad \lambda_в = \infty, \quad \lambda_в = \frac{\lambda_0}{\Gamma} = \frac{\lambda_0}{\sqrt{\varepsilon\mu - \left(\frac{\xi_{n,m}}{a}\right)^2}},$$

Критическая частота.

$$\Gamma = 0, \quad \lambda_в = \infty, \quad \sqrt{\varepsilon\mu} = \frac{\xi_{n,m}}{a}, \quad \times k_0, \quad k_0\sqrt{\varepsilon\mu} = k_0 \cdot \frac{\xi_{n,m}}{a}, \quad k_0\sqrt{\varepsilon\mu} = \frac{\xi_{n,m}}{R_в},$$

$$k_0\sqrt{\varepsilon\mu} = \frac{2\pi}{\lambda} = \frac{c}{f}, \qquad \frac{2\pi}{\lambda} = \frac{\xi_{n,m}}{R_в}, \qquad \lambda = \lambda_{кр}, \qquad \frac{2\pi}{\lambda_{кр}} = \frac{\xi_{n,m}}{R_в},$$

$$\lambda_{кр} = 2\pi \frac{R_в}{\mu_{n,m}}, \qquad f_{кр} = \frac{c \cdot \xi_{n,m}}{2\pi \cdot R},$$

Задача Неймана. Н – волны. $\left.\frac{\partial a}{\partial n}\right|_{L_1} = \left.\frac{\partial h}{\partial \rho}\right|_{L_1} = 0,$

$$h_{z(n)} = AJ_n(\chi\rho) + BY_n(\chi\rho), \qquad B = 0, \quad \left.\frac{\partial h_z}{\partial \rho}\right|_{\rho=a} = 0, \quad \left.\frac{\partial J_n(\chi\rho)}{\partial \rho}\right|_{\rho=a} = 0,$$

$$J_n'(\chi a) = 0, \quad \chi a = \eta_{n,m}, \quad \chi = \frac{\eta_{n,m}}{a}, \qquad \Gamma = \sqrt{\varepsilon\mu - \chi^2} = \sqrt{\varepsilon\mu - \left(\frac{\eta_{n,m}}{a}\right)^2},$$

Корни функции $\eta_{n,m}$

n\m	0	1	2	3	…
0	0	3.83	7.02	10.17	…
1	1.84	5.33	8.53	11.70	…
2	3.05	6.71	9.97	13.17	…
…		…	…	…	…

параметр распространения Γ

n\m	1	2	3	…
0	≈ 0.79	≈ 0.49	≈ i 1.26	…
1	≈ 0.96	≈ i 0.53	≈ i 1.94	…
2	≈ 0.87	≈ i 0.36	≈ i 1.23	…
3	≈ 0.75	≈ i 0.79	≈ i 1.50	
…	…	…	…	…

мм 30 , , $\varepsilon = 1$, $\mu = 1$

f = 10 ГГц , $k_0 = \frac{2\pi f}{c} \approx \frac{6.3\cdot 10^9}{3\cdot 10^8} \approx 210$ $^{-1}$

$a = k_0 r = 0.03\cdot 210 = 6.3,$

$$\Gamma = \sqrt{\varepsilon\mu - \left(\frac{\eta_{n,m}}{a}\right)^2} \approx \sqrt{1 - \left(\frac{\eta_{n,m}}{6.3}\right)^2},$$