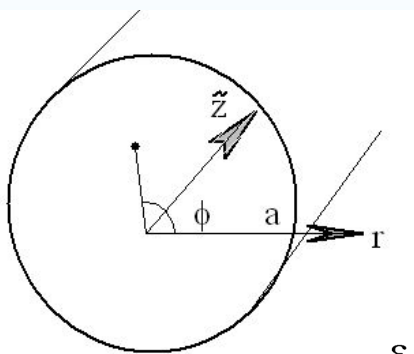


# Моделирование электромагнитных волн в цилиндрическом волноводе



$$\text{rot} \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\text{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{D} = \epsilon \epsilon_0 \vec{E},$$

$$\vec{B} = \mu \mu_0 \vec{H},$$

$$\vec{E}(r, t) = \vec{E}(r) e^{i\omega t},$$

$$\vec{H}(r, t) = \vec{H}(r) e^{i\omega t},$$

$$\epsilon_0 = \left(\frac{1}{36\pi}\right) \cdot 10^{-9} \left[\frac{F}{m}\right] \quad \mu_0 = 4\pi \cdot 10^{-7} \left[\frac{Gn}{m}\right], \quad \rho_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{4\pi \cdot 10^{-7} \cdot 36\pi \cdot 10^9} = 120\pi [Om],$$

$$\text{rot} \vec{H} = i\omega \epsilon_0 \epsilon \vec{E},$$

$$\text{rot} \vec{E} = -i\omega \mu_0 \mu \vec{H},$$

$$\vec{e} = i\vec{E},$$

$$\text{rot} \vec{H} = \omega \epsilon_0 \epsilon \vec{e},$$

$$\text{rot} \vec{e} = \omega \mu_0 \mu \vec{H},$$

$$\text{rot} \vec{h} = \omega \epsilon_0 \epsilon \sqrt{\frac{\mu_0}{\epsilon_0}} \vec{e},$$

$$\text{rot} \vec{h} = \omega \sqrt{\epsilon_0 \mu_0} \epsilon \vec{e},$$

$$\vec{h} = \rho_0 \vec{H},$$

$$\text{rot} \vec{h} = \omega \epsilon_0 \epsilon \rho_0 \vec{e},$$

$$\text{rot} \rho_0 \vec{e} = \omega \mu_0 \mu \vec{h},$$

$$\text{rot} \vec{e} = \omega \mu_0 \mu \sqrt{\frac{\epsilon_0}{\mu_0}} \vec{h},$$

$$\text{rot} \vec{e} = \omega \sqrt{\epsilon_0 \mu_0} \mu \vec{h},$$

$$\text{rot} \vec{h} = k_0 \epsilon \vec{e},$$

$$\text{rot} \vec{e} = k_0 \mu \vec{h},$$

$$k_0 = \omega \sqrt{\varepsilon_0 \mu_0}, \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}, \quad k_0 = \frac{\omega}{c}, \quad c = \lambda_0 F, \quad \omega = 2\pi F, \quad k_0 = \frac{2\pi}{\lambda_0}.$$

$$\begin{aligned} \text{rot } \vec{h} &= k_0 \varepsilon \vec{e}, \\ \text{rot } \vec{e} &= k_0 \mu \vec{h}, \end{aligned}$$

$r, \phi, z.$

$$\text{rot } A = \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \vec{i}_r + \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) \vec{i}_\phi + \left( \frac{1}{r} \frac{\partial (r A_\phi)}{\partial r} - \frac{1}{r} \frac{\partial A_r}{\partial \phi} \right) \vec{i}_z,$$

$$\nabla^2 A = \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right) A,$$

$$\frac{1}{r} \frac{\partial h_z}{\partial \phi} - \frac{\partial h_\phi}{\partial z} = k_0 \varepsilon e_r,$$

$$\frac{1}{r} \frac{\partial e_z}{\partial \phi} - \frac{\partial e_\phi}{\partial z} = k_0 \mu h_r,$$

$$\frac{\partial h_r}{\partial z} - \frac{\partial h_z}{\partial r} = k_0 \varepsilon e_\phi,$$

$$\frac{\partial e_r}{\partial z} - \frac{\partial e_z}{\partial r} = k_0 \mu h_\phi,$$

$$\frac{1}{r} \frac{\partial (r h_\phi)}{\partial r} - \frac{1}{r} \frac{\partial h_r}{\partial \phi} = k_0 \varepsilon e_z,$$

$$\frac{1}{r} \frac{\partial (r e_\phi)}{\partial r} - \frac{1}{r} \frac{\partial e_r}{\partial \phi} = k_0 \mu h_z,$$

$$\rho = k_0 r, \quad z = k_0 z. \quad \text{Координаты безразмерные.}$$

$$\frac{1}{\rho} \frac{\partial h_z}{\partial \varphi} - \frac{\partial h_\varphi}{\partial z} = \varepsilon e_\rho,$$

$$\frac{1}{\rho} \frac{\partial e_z}{\partial \varphi} - \frac{\partial e_\varphi}{\partial z} = \mu h_\rho,$$

$$\frac{\partial h_\rho}{\partial z} - \frac{\partial h_z}{\partial \rho} = \varepsilon e_\varphi,$$

$$\frac{\partial e_\rho}{\partial z} - \frac{\partial e_z}{\partial \rho} = \mu h_\varphi,$$

$$\begin{pmatrix} h \\ e \end{pmatrix} \boxtimes e^{i\omega t}$$

$$\begin{pmatrix} h \\ e \end{pmatrix} \boxtimes e^{i\tilde{\gamma}z} e^{i\omega t}$$

$$\frac{1}{\rho} h_\varphi + \frac{\partial h_\varphi}{\partial \rho} - \frac{1}{\rho} \frac{\partial h_\rho}{\partial \varphi} = \varepsilon e_z,$$

$$\frac{1}{\rho} e_\varphi + \frac{\partial e_\varphi}{\partial \rho} - \frac{1}{\rho} \frac{\partial e_\rho}{\partial \varphi} = \mu h_z,$$

$$\gamma = \frac{\omega}{v_\phi},$$

$$v_\phi = \frac{1}{\sqrt{\varepsilon \varepsilon_0 \mu \mu_0}},$$

$$\tilde{z} = \frac{z}{k_0}, \quad \tilde{\gamma}z = \Gamma z, \quad \Gamma = \frac{\gamma}{k_0}, \quad \begin{pmatrix} h \\ e \end{pmatrix} \boxtimes e^{i\Gamma z}$$

$$v_\phi = \lambda F \quad \Gamma = \frac{2\pi}{\Lambda}. \quad \Lambda = \frac{\lambda}{\lambda_0}.$$

$$(1) \quad \frac{1}{\rho} \frac{\partial h_z}{\partial \varphi} - i\Gamma h_\varphi = \varepsilon e_\rho,$$

$$(4) \quad \frac{1}{\rho} \frac{\partial e_z}{\partial \varphi} - i\Gamma e_\varphi = \mu h_\rho,$$

$$(2) \quad i\Gamma h_\rho - \frac{\partial h_z}{\partial \rho} = \varepsilon e_\varphi,$$

$$(5) \quad i\Gamma e_\rho - \frac{\partial e_z}{\partial \rho} = \mu h_\varphi,$$

$$(3) \quad \frac{1}{\rho} h_\varphi + \frac{\partial h_\varphi}{\partial \rho} - \frac{1}{\rho} \frac{\partial h_\rho}{\partial \varphi} = \varepsilon e_z,$$

$$(6) \quad \frac{1}{\rho} e_\varphi + \frac{\partial e_\varphi}{\partial \rho} - \frac{1}{\rho} \frac{\partial e_\rho}{\partial \varphi} = \mu h_z,$$

Выразим  $e_\rho, h_\rho, e_\varphi, h_\varphi$  через  $e_z, h_z$ .

Выразим  $e_\rho, h_\rho, e_\varphi, h_\varphi$  через  $e_z, h_z$ .

Из (1,5): 
$$e_\rho = \frac{1}{\varepsilon} \left( \frac{1}{\rho} \frac{\partial h_z}{\partial \varphi} - i\Gamma h_\varphi \right), \quad h_\varphi = \frac{1}{\mu} \left( i\Gamma e_\rho - \frac{\partial e_z}{\partial \rho} \right),$$

$$e_\rho = \frac{1}{\mu\varepsilon - \Gamma^2} \left( \frac{\mu}{\rho} \frac{\partial h_z}{\partial \varphi} + i\Gamma \frac{\partial e_z}{\partial \rho} \right),$$

$$h_\varphi = \frac{1}{\mu\varepsilon - \Gamma^2} \left( \frac{i\Gamma}{\rho} \frac{\partial h_z}{\partial \varphi} - \varepsilon \frac{\partial e_z}{\partial \rho} \right),$$

Из (2,4):

$$h_\rho = \frac{1}{\mu\varepsilon - \Gamma^2} \left( \frac{\varepsilon}{\rho} \frac{\partial e_z}{\partial \varphi} + i\Gamma \frac{\partial h_z}{\partial \rho} \right),$$

$$e_\varphi = \frac{1}{\mu\varepsilon - \Gamma^2} \left( \frac{i\Gamma}{\rho} \frac{\partial e_z}{\partial \varphi} - \mu \frac{\partial h_z}{\partial \rho} \right),$$

Подставим в (3) поперечные компоненты:

$$\frac{1}{\rho} h_\varphi + \frac{\partial h_\varphi}{\partial \rho} - \frac{1}{\rho} \frac{\partial h_\rho}{\partial \varphi} = \varepsilon e_z, \quad (3)$$

$$\rho^2 \frac{\partial^2 e_z}{\partial \rho^2} + \rho \frac{\partial e_z}{\partial \rho} + \rho^2 (\mu\varepsilon - \Gamma^2) e_z + \frac{\partial^2 e_z}{\partial \varphi^2} = 0$$

Подставим в (6) поперечные компоненты:

$$\frac{1}{\rho} e_\varphi + \frac{\partial e_\varphi}{\partial \rho} - \frac{1}{\rho} \frac{\partial e_\rho}{\partial \varphi} = \mu h_z, \quad (6)$$

$$\rho^2 \frac{\partial^2 h_z}{\partial \rho^2} + \rho \frac{\partial h_z}{\partial \rho} + \rho^2 (\mu\varepsilon - \Gamma^2) h_z + \frac{\partial^2 h_z}{\partial \varphi^2} = 0.$$

$$\rho^2 \frac{\partial^2 e_z}{\partial \rho^2} + \rho \frac{\partial e_z}{\partial \rho} + \rho^2 (\mu \varepsilon - \Gamma^2) e_z + \frac{\partial^2 e_z}{\partial \varphi^2} = 0,$$

$$e_\rho = \frac{1}{\mu \varepsilon - \Gamma^2} \left( \frac{\mu}{\rho} \frac{\partial h_z}{\partial \varphi} + i \Gamma \frac{\partial e_z}{\partial \rho} \right),$$

$$h_\varphi = \frac{1}{\mu \varepsilon - \Gamma^2} \left( \frac{i \Gamma}{\rho} \frac{\partial h_z}{\partial \varphi} - \varepsilon \frac{\partial e_z}{\partial \rho} \right),$$

$$h_\rho = \frac{1}{\mu \varepsilon - \Gamma^2} \left( \frac{\varepsilon}{\rho} \frac{\partial e_z}{\partial \varphi} + i \Gamma \frac{\partial h_z}{\partial \rho} \right),$$

$$e_\varphi = \frac{1}{\mu \varepsilon - \Gamma^2} \left( \frac{i \Gamma}{\rho} \frac{\partial e_z}{\partial \varphi} - \mu \frac{\partial h_z}{\partial \rho} \right),$$

$$\rho^2 \frac{\partial^2 h_z}{\partial \rho^2} + \rho \frac{\partial h_z}{\partial \rho} + \rho^2 (\mu \varepsilon - \Gamma^2) h_z + \frac{\partial^2 h_z}{\partial \varphi^2} = 0$$

$$e_\rho = \frac{1}{\mu \varepsilon - \Gamma^2} \left( \frac{\mu}{\rho} \frac{\partial h_z}{\partial \varphi} + i \Gamma \frac{\partial e_z}{\partial \rho} \right),$$

$$h_\rho = \frac{1}{\mu \varepsilon - \Gamma^2} \left( \frac{\varepsilon}{\rho} \frac{\partial e_z}{\partial \varphi} + i \Gamma \frac{\partial h_z}{\partial \rho} \right),$$

$$h_\varphi = \frac{1}{\mu \varepsilon - \Gamma^2} \left( \frac{i \Gamma}{\rho} \frac{\partial h_z}{\partial \varphi} - \varepsilon \frac{\partial e_z}{\partial \rho} \right),$$

$$e_\varphi = \frac{1}{\mu \varepsilon - \Gamma^2} \left( \frac{i \Gamma}{\rho} \frac{\partial e_z}{\partial \varphi} - \mu \frac{\partial h_z}{\partial \rho} \right),$$

**Вариант 2** (в векторной форме).

$$\text{rot } h = k_0 \varepsilon e, \quad \text{rot } h = k_0 \varepsilon e, \quad \text{rot rot } e = \text{grad } \text{dive } e - \nabla^2 e, \quad \text{dive } e = 0.$$

$$\text{rot } e = k_0 \mu h, \quad \text{rot rot } e = k_0 \mu \text{rot } h, \quad \text{rot rot } e = k_0^2 \varepsilon \mu e, \quad \text{rot rot } e = -\nabla^2 e$$

$$\left( \nabla^2 + k_0^2 \varepsilon \mu \right) e = 0, \quad r, \phi, z, \quad \nabla^2 A = \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2} \right) A,$$

$$\left( \nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + \varepsilon \mu \right) e = 0, \quad e(\rho, \varphi, z) = e(\rho, \varphi) e^{i \Gamma z}, \quad \nabla_\perp^2 e = \left( \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} \right) e$$

$$\rho^2 \frac{\partial^2 e}{\partial \rho^2} + \rho \frac{\partial e}{\partial \rho} + \rho^2 (\varepsilon \mu - \Gamma^2) e + \frac{\partial^2 e}{\partial \varphi^2} = 0,$$

# Метод разделения переменных

$$\rho^2 \frac{\partial^2 e_z}{\partial \rho^2} + \rho \frac{\partial e_z}{\partial \rho} + \rho^2 \chi^2 e_z + \frac{\partial^2 e_z}{\partial \varphi^2} = 0,$$

$$e(\rho, \varphi, z) = e(\rho, \varphi) e^{i\Gamma z},$$

$$\chi^2 = (\mu\varepsilon - \Gamma^2), \quad \Gamma = \sqrt{\mu\varepsilon - \chi^2}.$$

$$e_z(\rho, \varphi) = R(\rho)\Phi(\varphi),$$

$$\rho^2 \Phi(\varphi) \frac{\partial^2 R(\rho)}{\partial \rho^2} + \rho \Phi(\varphi) \frac{\partial R(\rho)}{\partial \rho} + \rho^2 \chi^2 R(\rho) \Phi(\varphi) + R(\rho) \frac{\partial^2 \Phi(\varphi)}{\partial \varphi^2} = 0,$$

$$\frac{\rho^2}{R(\rho)} \frac{\partial^2 R(\rho)}{\partial \rho^2} + \frac{\rho}{R(\rho)} \frac{\partial R(\rho)}{\partial \rho} + \rho^2 \chi^2 = -\frac{1}{\Phi(\varphi)} \frac{\partial^2 \Phi(\varphi)}{\partial \varphi^2},$$

$$\frac{\rho^2}{R} \frac{d^2 R}{d\rho^2} + \frac{\rho}{R} \frac{dR}{d\rho} + \rho^2 \chi^2 = n^2, \quad -\frac{1}{\Phi} \frac{d^2 \Phi}{d\varphi^2} = n^2,$$

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + (\chi^2 \rho^2 - n^2) R = 0,$$

$$\frac{d^2 \Phi}{d\varphi^2} + n^2 \Phi = 0.$$

$$\frac{\partial^2 \Phi}{\partial \varphi^2} + n^2 \Phi = 0,$$

$$\Phi = D e^{in\varphi},$$

$$\Phi = D e^{\pm in\varphi}, \quad n = 1, 2, 3, \dots$$

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + (\chi^2 \rho^2 - n^2) R = 0,$$

$$r = \chi \rho,$$

$$r^2 \frac{d^2 R}{dr^2} + r \frac{dR}{dr} + (r^2 - n^2) R = 0, \quad r = 0, \quad r = \infty,$$

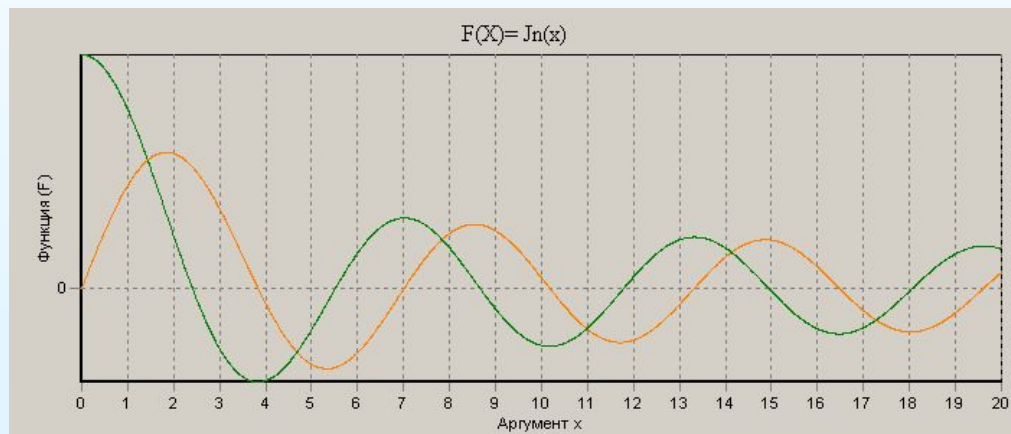
$$R_{1,2} = r^\lambda \sum_{k=0}^{\infty} C_k r^k,$$

$$\frac{dR}{dr} = r^{\lambda-1} \sum_{k=0}^{\infty} C_k (\lambda + k) r^k, \quad \frac{d^2R}{dr^2} = r^{\lambda-2} \sum_{k=0}^{\infty} C_k (\lambda + k)(\lambda + k - 1) r^k,$$

$$\sum_{k=0}^{\infty} C_k (\lambda + k)(\lambda + k - 1) r^k + \sum_{k=0}^{\infty} C_k (\lambda + k) r^k + (r^2 - n^2) \sum_{k=0}^{\infty} C_k r^k = 0$$

$$(\lambda^2 - n^2) = 0, \quad \lambda_{1,2} = \pm n, \quad R_{1,2} = r^{\pm n} \sum_{k=0}^{\infty} C_k r^k,$$

$$R_1(r) = r^n \sum_{k=0}^{\infty} \frac{(-1)^k r^{2k}}{k!(n+k)!} = J_n(r)$$



Функции Бесселя  $J_0(x)$ - зеленая кривая,  $J_1(x)$ - оранжевая кривая

Корни функции  $\xi_{n,m}$

$n \setminus m$	1	2	3	...
0	2.41	5.52...	8.65...	...
1	3.83	7.02	10.17	...
2	5.14	8.42	11.62	...
...	...	...	...	...

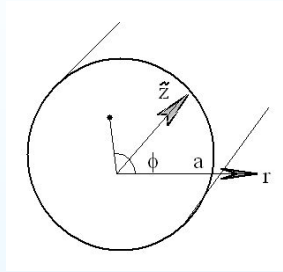
$$R_2(r) = \frac{dR_1}{d\lambda} \Big|_{\lambda=\lambda_2} = Y_n(r),$$

$$R(\rho) = AJ_n(\chi\rho) + BY_n(\chi\rho),$$

$$e_z(\rho, \varphi, z, t) = (AJ_n(\chi\rho) + BY_n(\chi\rho))e^{in\varphi}e^{i\Gamma z}e^{i\omega t}$$

$$h_z(\rho, \varphi, z)$$

## Краевые задачи



Задача Дирихле.  $a|_{L_1} = 0, \quad \{a\}_{n=1}^{\infty}, \quad \{\chi_n\}_{n=1}^{\infty}.$

$$e_z|_{L_1} = 0, \quad \{e_z\}_{n=1}^{\infty}, \quad \{\chi_n\}_{n=1}^{\infty}.$$

Задача Неймана.  $\left. \frac{\partial a}{\partial n} \right|_{L_1} = 0, \quad \{a\}_{n=1}^{\infty}, \quad \{\chi_n\}_{n=1}^{\infty},$

$$\left. \frac{\partial h_z}{\partial n} \right|_{L_1} = 0, \quad \{h_z\}_{n=1}^{\infty}, \quad \{\chi_n\}_{n=1}^{\infty},$$

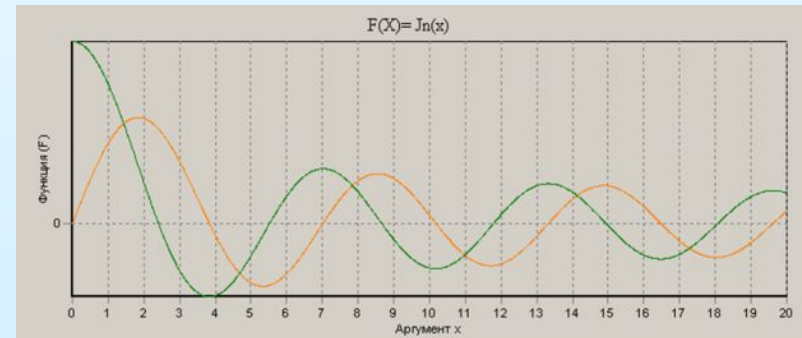
## Задача Дирихле. E – волны.

$$e_z(\rho) = AJ_n(\chi\rho) + BY_n(\chi\rho), \quad e_z(\rho)|_{\rho=a} = 0$$

$$B = 0, \quad e_z = AJ_n(\chi\rho),$$

$$e_z|_a = AJ_n(\chi a) = 0,$$

$$J_n(\chi a) = 0, \quad \chi a = \xi_{n,m}, \quad \chi = \frac{\xi_{n,m}}{a}, \quad \Gamma = \sqrt{\varepsilon\mu - \chi^2} = \sqrt{\varepsilon\mu - \left(\frac{\xi_{n,m}}{a}\right)^2},$$





$$\Gamma = \sqrt{\varepsilon\mu - \chi^2} = \sqrt{\varepsilon\mu - \left(\frac{\xi_{n,m}}{a}\right)^2},$$

$$m_{мет} = 0.03, \quad \varepsilon = 1, \quad \mu = 1, \quad \gamma = 10$$

$$k_0 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} \approx \frac{6.3 \cdot 10 \cdot 10^9}{3 \cdot 10^8} \approx 210 \quad \text{м}^{-1}$$

$$a = k_0 r = 0.03 \cdot 210 = 6.3,$$

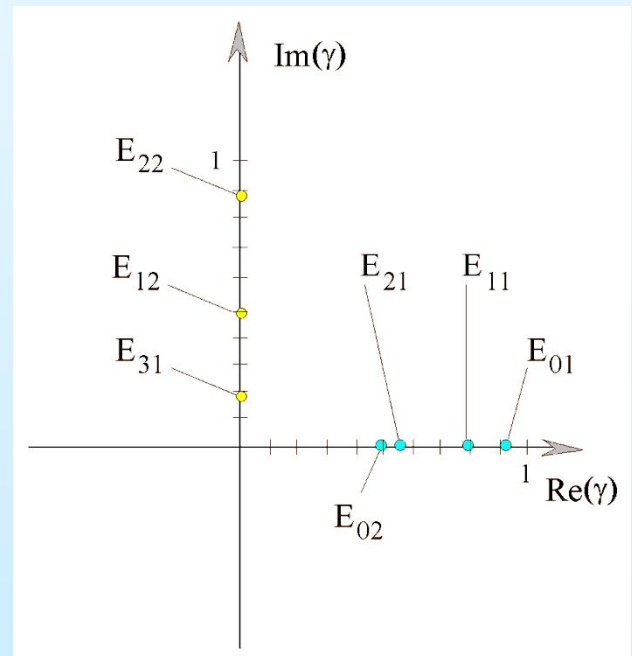
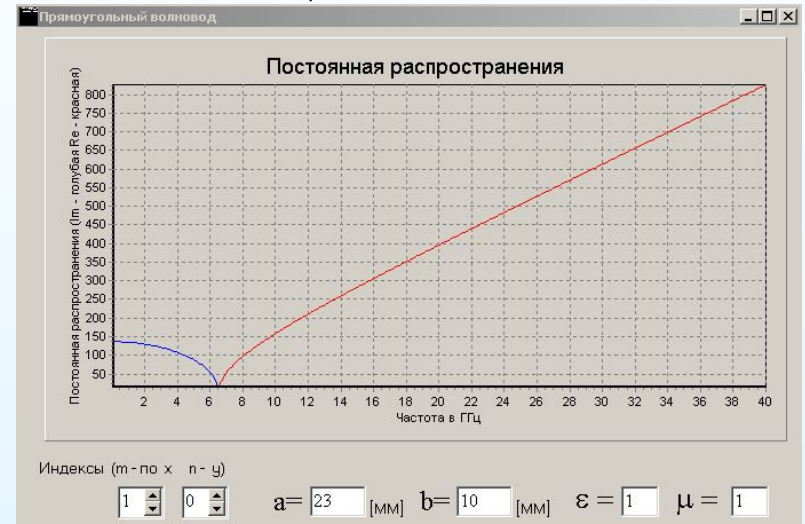
$$\Gamma = \sqrt{\varepsilon\mu - \left(\frac{\xi_{n,m}}{a}\right)^2} \approx \sqrt{1 - \left(\frac{\xi_{n,m}}{6.3}\right)^2},$$

Корни функции Бесселя

n \ m	1	2	3	...
0	2.41	5.52...	8.65...	...
1	3.83	7.02	10.17	...
2	5.14	8.42	11.62	...
...	...	...	...	...

параметр распространения $\Gamma$				
n \ m	1	2	3	...
0	≈ 0.92	≈ 0.48	≈ i 0.95	...
1	≈ 0.79	≈ i 0.50	≈ i 1.27	...
2	≈ 0.56	≈ i 0.89	≈ i 1.56	...
3	≈ i 0.18	≈ i 1.19	≈ i 1.82	...
...	...	...	...	...

$$\Gamma = \sqrt{\varepsilon\mu - \chi^2} = \sqrt{\varepsilon\mu - \left[ \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right]}.$$



## Структура поля.

$$\begin{aligned}
 e_z &= E_0 J_n(\chi\rho), & h_z &= 0, \\
 e_\rho &= i\Gamma \frac{E_0}{\chi^2} \left[ \frac{n}{\chi\rho} J_n(\chi\rho) - J_{n+1}(\chi\rho) \right], & h_\rho &= -\frac{i\varepsilon}{\Gamma} e_\varphi, \\
 e_\varphi &= -\frac{\Gamma n}{\chi^2 \rho} e_z, & h_\varphi &= \frac{i\varepsilon}{\Gamma} e_\rho.
 \end{aligned}
 \quad \chi^2 = \varepsilon\mu - \Gamma^2 = \left( \frac{\xi_{n,m}}{a} \right)^2,$$

Критический размер волновода.

$$\begin{aligned}
 \Gamma &= \sqrt{\varepsilon\mu - \left( \frac{\xi_{n,m}}{a} \right)^2}, & \Gamma &= 0, & \Rightarrow & \varepsilon\mu = \left( \frac{\xi_{n,m}}{a} \right)^2, & a_{кр} &= \frac{\xi_{n,m}}{\sqrt{\varepsilon\mu}}. \\
 \gamma &= \frac{2\pi}{\lambda_\varepsilon}, & k_0 &= \frac{2\pi}{\lambda_0}, & \Gamma &= \frac{\gamma}{k_0} = \frac{\lambda_0}{\lambda_\varepsilon}, & \Gamma &= 0, & \lambda_\varepsilon &= \infty, & \lambda_\varepsilon &= \frac{\lambda_0}{\Gamma} = \frac{\lambda_0}{\sqrt{\varepsilon\mu - \left( \frac{\xi_{n,m}}{a} \right)^2}},
 \end{aligned}$$

Критическая частота.

$$\Gamma = 0, \quad \lambda_\varepsilon = \infty, \quad \sqrt{\varepsilon\mu} = \frac{\xi_{n,m}}{a}, \quad \times k_0, \quad k_0 \sqrt{\varepsilon\mu} = k_0 \cdot \frac{\xi_{n,m}}{a}, \quad k_0 \sqrt{\varepsilon\mu} = \frac{\xi_{n,m}}{R_\varepsilon},$$

$$k_0 \sqrt{\varepsilon\mu} = \frac{2\pi}{\lambda} = \frac{c}{f}, \quad \frac{2\pi}{\lambda} = \frac{\xi_{n,m}}{R_\varepsilon}, \quad \lambda = \lambda_{кр}, \quad \frac{2\pi}{\lambda_{кр}} = \frac{\xi_{n,m}}{R_\varepsilon},$$

$$\lambda_{кр} = 2\pi \frac{R_\varepsilon}{\mu_{n,m}}, \quad f_{кр} = \frac{c \cdot \xi_{n,m}}{2\pi \cdot R},$$

# Задача Неймана. Н – волны.

$$\left. \frac{\partial a}{\partial n} \right|_{L_1} = \left. \frac{\partial h}{\partial \rho} \right|_{L_1} = 0,$$

$$h_{z(n)} = AJ_n(\chi\rho) + BY_n(\chi\rho),$$

$$B = 0, \quad \left. \frac{\partial h_z}{\partial \rho} \right|_{\rho=a} = 0, \quad \left. \frac{\partial J_n(\chi\rho)}{\partial \rho} \right|_{\rho=a} = 0,$$

$$J'_n(\chi a) = 0, \quad \chi a = \eta_{n,m}, \quad \chi = \frac{\eta_{n,m}}{a},$$

$$\Gamma = \sqrt{\varepsilon\mu - \chi^2} = \sqrt{\varepsilon\mu - \left(\frac{\eta_{n,m}}{a}\right)^2},$$

Корни функции  $\eta_{n,m}$

n\m	0	1	2	3	...
0	0	3.83	7.02	10.17	...
1	1.84	5.33	8.53	11.70	...
2	3.05	6.71	9.97	13.17	...
...	...	...	...	...	...

параметр распространения  $\Gamma$

n\m	1	2	3	...
0	$\approx 0.79$	$\approx 0.49$	$\approx i 1.26$	...
1	$\approx 0.96$	$\approx i 0.53$	$\approx i 1.94$	...
2	$\approx 0.87$	$\approx i 0.36$	$\approx i 1.23$	...
3	$\approx 0.75$	$\approx i 0.79$	$\approx i 1.50$	...
...	...	...	...	...

ММ 30 , ,  $\varepsilon = 1$  ,  $\mu = 1$

МФПД ,  $k_0 = \frac{2\pi f}{c} \approx \frac{6.3 \cdot 10^9}{3 \cdot 10^8} = 210$  -1

$a = k_0 r = 0.03 \cdot 210 = 6.3$ ,

$$\Gamma = \sqrt{\varepsilon\mu - \left(\frac{\eta_{n,m}}{a}\right)^2} \approx \sqrt{1 - \left(\frac{\eta_{n,m}}{6.3}\right)^2},$$

