A RELATION

Consider the relationship between the weight of five students and their ages as shown below.

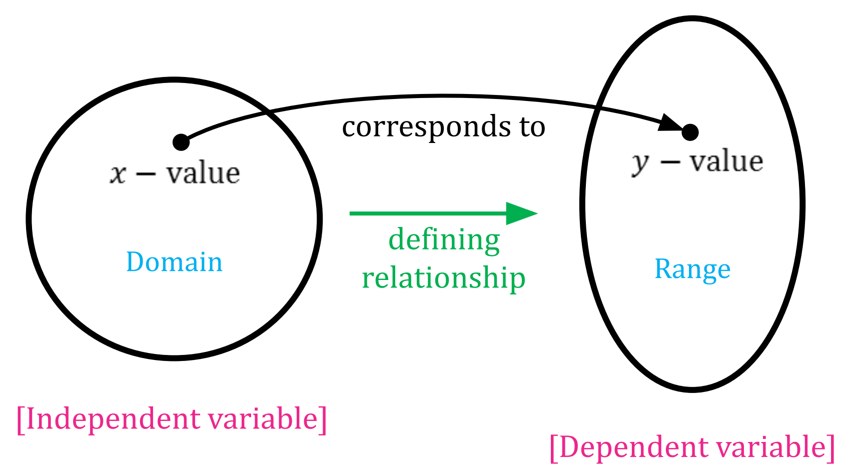
We can represent this information as a ***set of ordered pairs***.
An age of 10 years would correspond to a weight of 31 kg. An age of 16 years would correspond to a weight of 53 kg and so on.
This type of information represents a relation between two sets of data. This in formation could then be represented as a set of ordered pairs.

Age (years)	Weight (kg)
10	31
12	36
14	48
16	53
18	65

$$\{(10, 31), (12, 36), (14, 48), (16, 53), (18, 65)\}$$

The ***set of all first elements*** of the ordered pair is called the ***domain*** of the relation and is referred to as the ***independent variable***. The ***set of all second elements*** is called the ***range*** and is referred to as the dependent variable.
For the above example,
the domain $= \{10, 12, 14, 16, 18\}$
the range $= \{31, 36, 48, 53, 65\}$

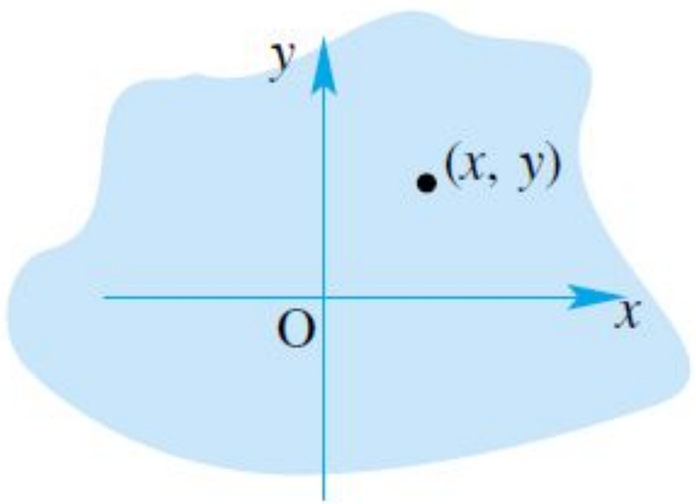
Summary



A RELATION

Relation
A **relation** is a correspondence between a first set, called the **domain**, and a second set, called the **range**, such that each member of the domain corresponds to *at least one* member of the range.

A relation is any subset of the Cartesian plane and can be represented by a set of ordered pairs $\{(x, y)\}$.

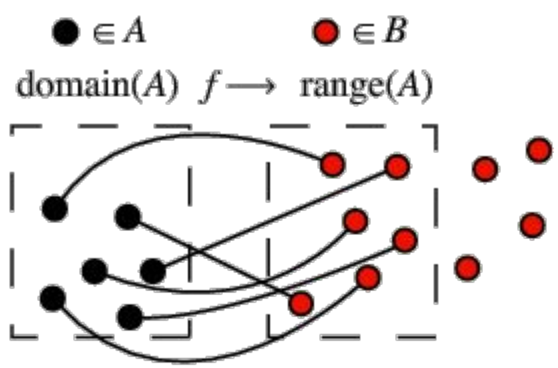


B

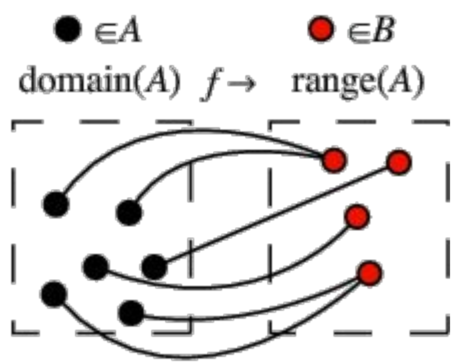
FUNCTIONS

- Functions are special relations.
- Every set of ordered pairs is a relation, but every relation is not a function
- Functions make up a subset of all relations.
- A function is defined as a relation that is either one to one or many to one., i.e. no ordered pairs have the same first element.

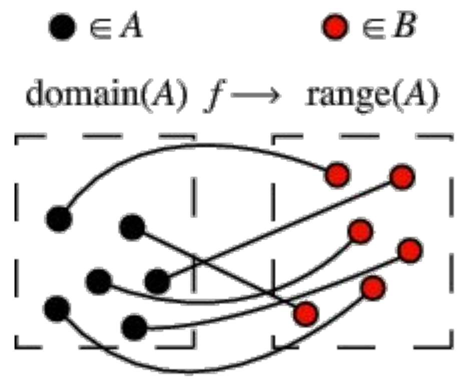
B TYPES OF FUNCTIONS



one-to-one and not onto
(injection but not surjection)



onto and not one-to-one
(surjection but not injection)



one-to-one and onto
(bijection)

Surjective

A surjective function is a function whose image is equal to its codomain. Equivalently, a function f with domain A and codomain B is surjective if for every b in B there exists at least one a in A with $f(a) = b$.

If $f: A \to B$, then f is said to be surjective if:

$\forall b \in B \;\; \exists \; a \in A$, such that $f(a) = b$.

Injective

A function f is injective if and only if for all a and b in A, if $f(a) = f(b)$, then $a = b$; that is, $f(a) = f(b)$ implies $a = b$. Equivalently, if $a \neq b$, then $f(a) \neq f(b)$.

If $f: A \to B$, then f is said to be injective if:

$\forall a, b \in A \;\; if \;\; f(a) = f(b) \Rightarrow \; a = b$.

Bijective

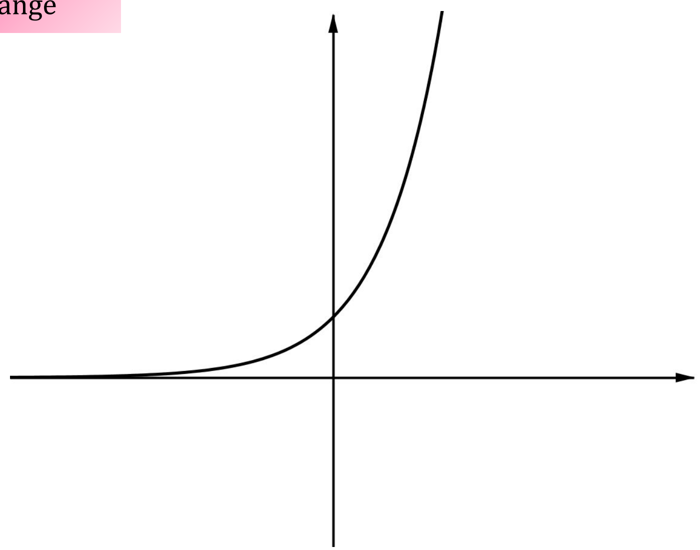
A function is **bijective** (**one-to-one and onto** or **one-to-one correspondence**) if every element of the codomain is mapped to by *exactly* one element of the domain. (That is, the function is *both* injective and surjective.)

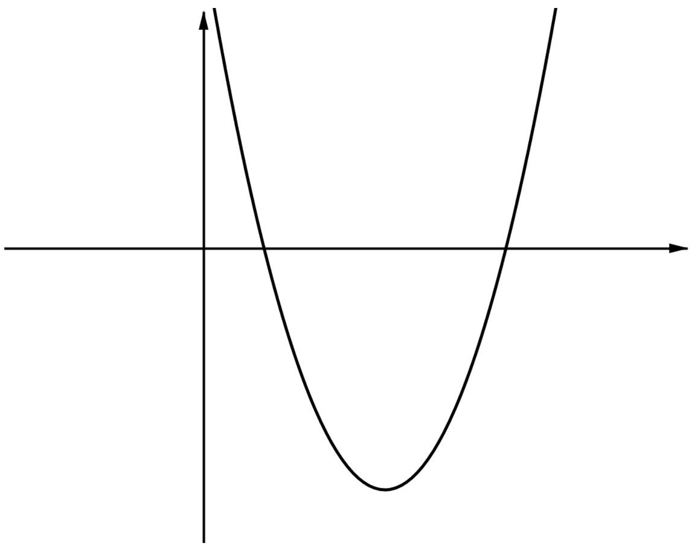
C NATURAL DOMAIN

Every relation has a domain, the set of (input) values over which it is defined. If the domain is not stated, by convention we take the domain to be the largest set of (real) numbers for which the expression defining the function can be evaluated.
We call this the "natural domain" of the function.

Domain

Range

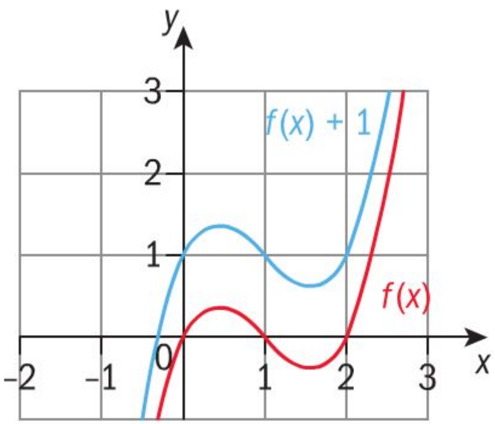




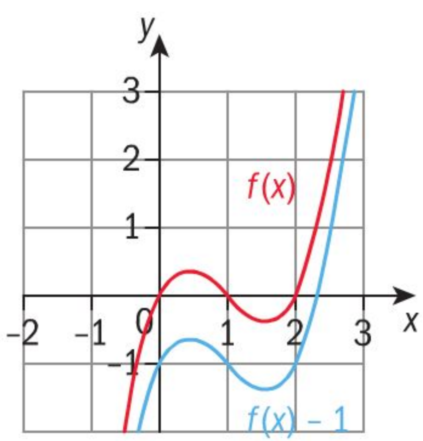
D TRANSLATIONS

Shift upward of downward

$f(x) + k$ translates $f(x)$ vertically a distance of k units **upward**.

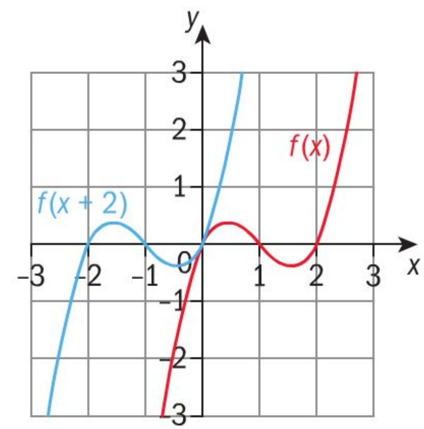


$f(x) - k$ translates $f(x)$ vertically a distance of k units **downward**.

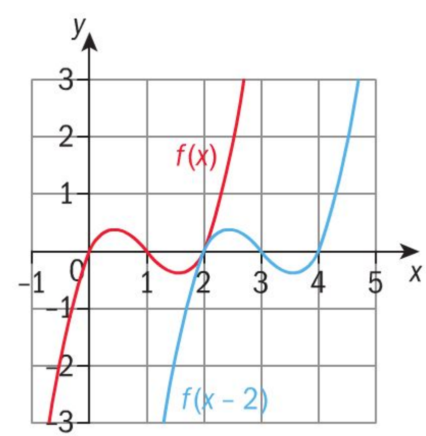


Shift to the right or left

$f(x + k)$ translates $f(x)$ horizontally a distance of k units to the **left**, when $k > 0$.



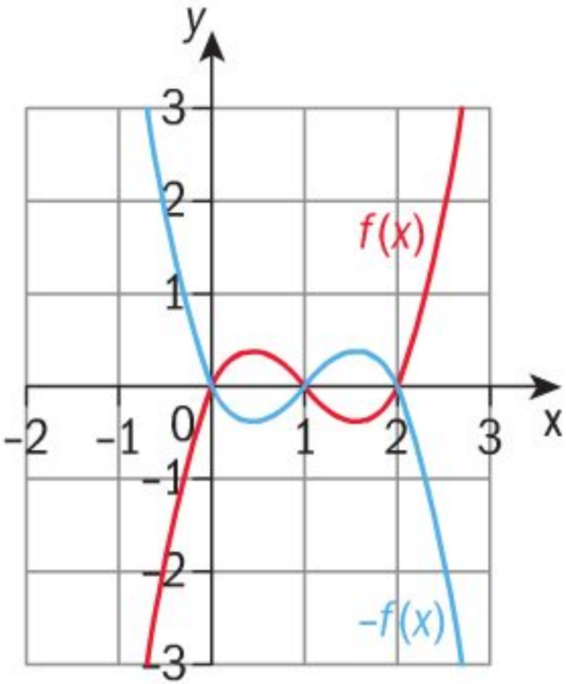
$f(x - k)$ translates $f(x)$ horizontally a distance of k units to the **right**, when $k > 0$.

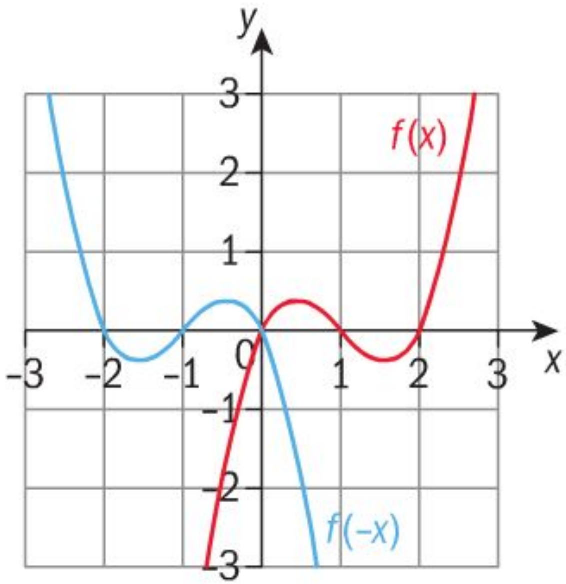


Reflections

$-f(x)$ reflects $f(x)$ in the $x-$axis.

$f(-x)$ reflects $f(x)$ in the $y-$axis.



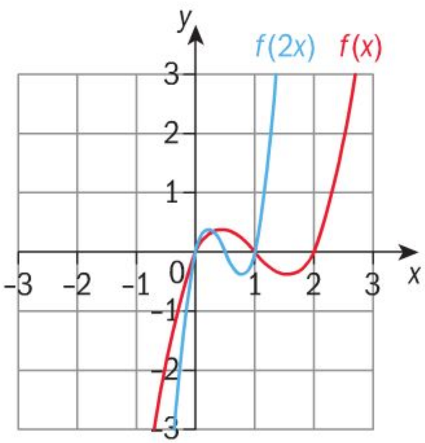


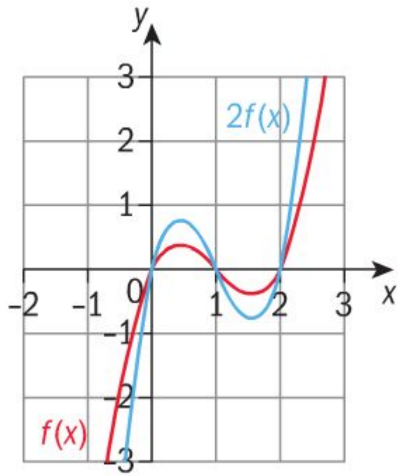
Stretches

$f(qx)$ stretches or compresses $f(x)$ horizontally with scale factor $\frac{1}{q}$.

$pf(x)$ stretches $f(x)$ vertically with a scale factor of p.

A stretch with a scale factor p where $0 < p < 1$ will actually compress the graph.





Students often make mistakes with stretches. It is important to remember the different effects of, for example, $2f(x)$ and $f(2x)$.

The transformation is a **horizontal stretch of scale factor** $\frac{1}{q}$.
When $q > 1$ the graph is compressed towards the $y-$axis.
When $0 < q < 1$ the graph is compressed away from the $y-$axis.

The transformation is a **vertical stretch of scale factor** p.
When $p > 1$ the graph stretches away from the $x-$axis.
When $0 < p < 1$ the graph is compressed towards the $x-$axis.