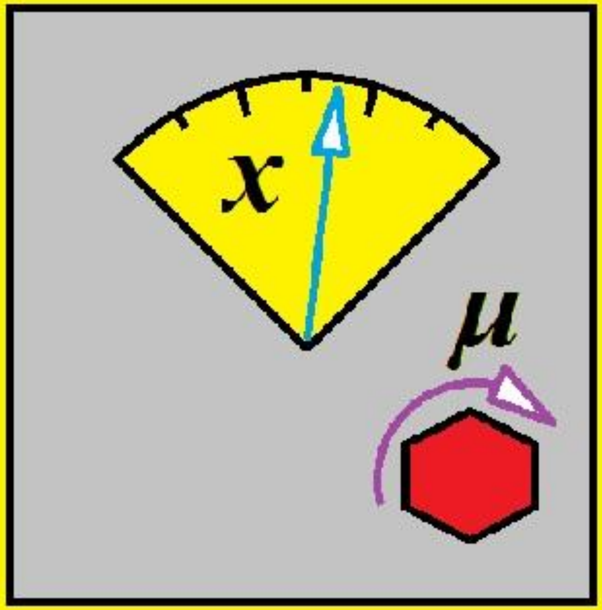
Climate tipping as a noisy bifurcation: a predictive technique

- J Michael T Thompson (DAMTP, Cambridge)
- Jan Sieber (Maths, Portsmouth)

- Part I (JMTT) Bifurcations and their precursors

- Part II (JS) Normal form estimates

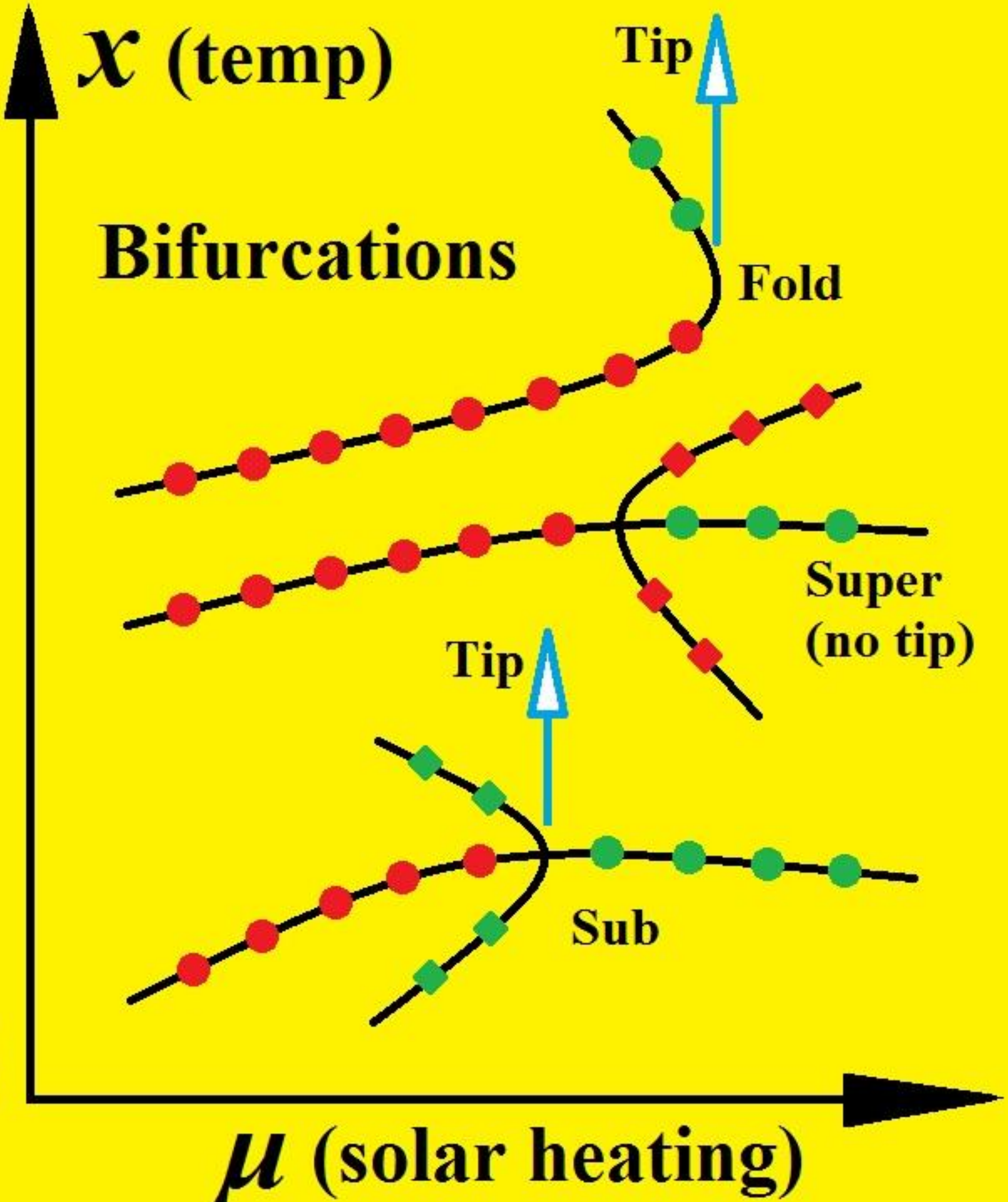
Dynamical System



Control μ

Response x

- Stable (type 1 response)
- Stable (type 2 response)
- Unstable (type 1 response)
- Unstable (type 2 response)



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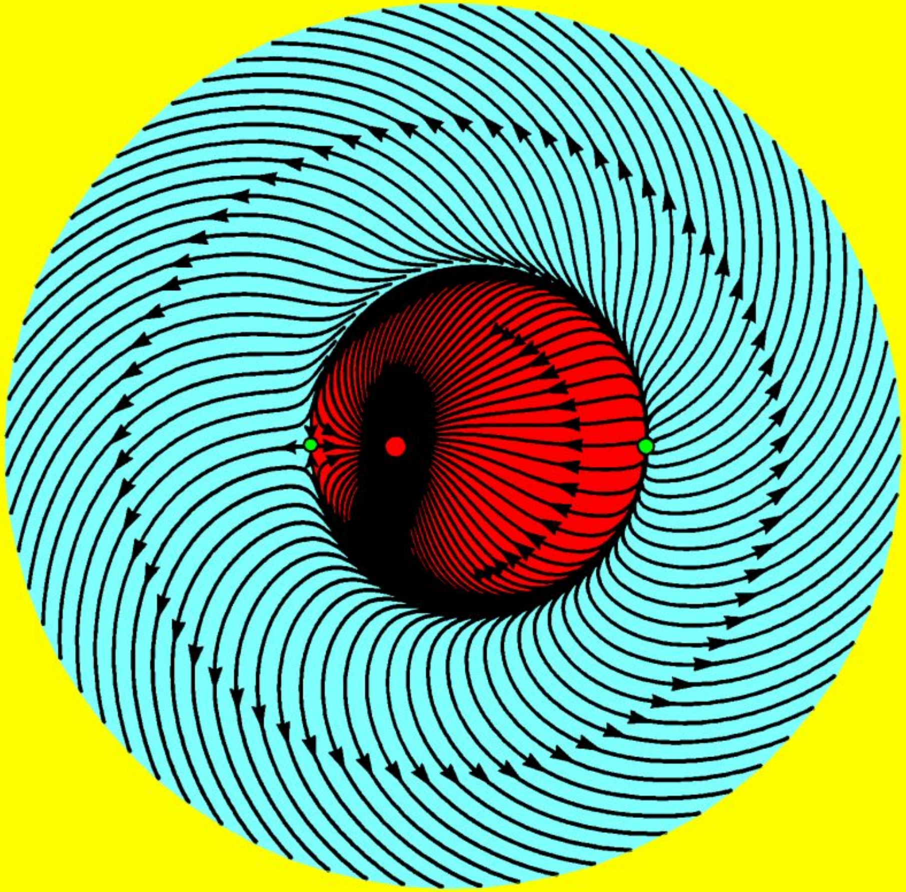
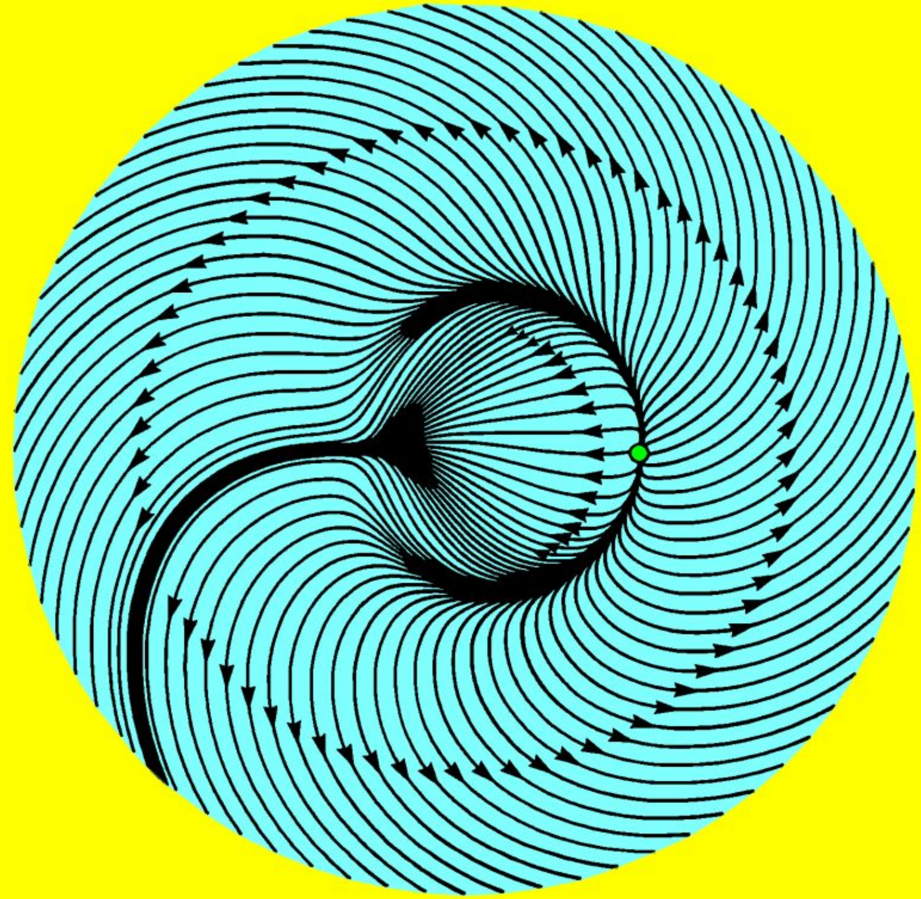
Instantaneous Basin loss at a Fold



Before



After

Introduction

- Focus on the Earth, or a relevant sub-system (Lenton).
- Regard it as a nonlinear dissipative dynamical system.
- Ignore discontinuities and memory effects.
- We have a large but finite set of ODEs and phase space.
- This large complex system has activity at many scales.

Effective Noise

Small fast action is noise to the overall dynamics (OD)

Models of the OD might need added random noise

Bifurcations of the OD may underlie climate tipping

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Control Parameters

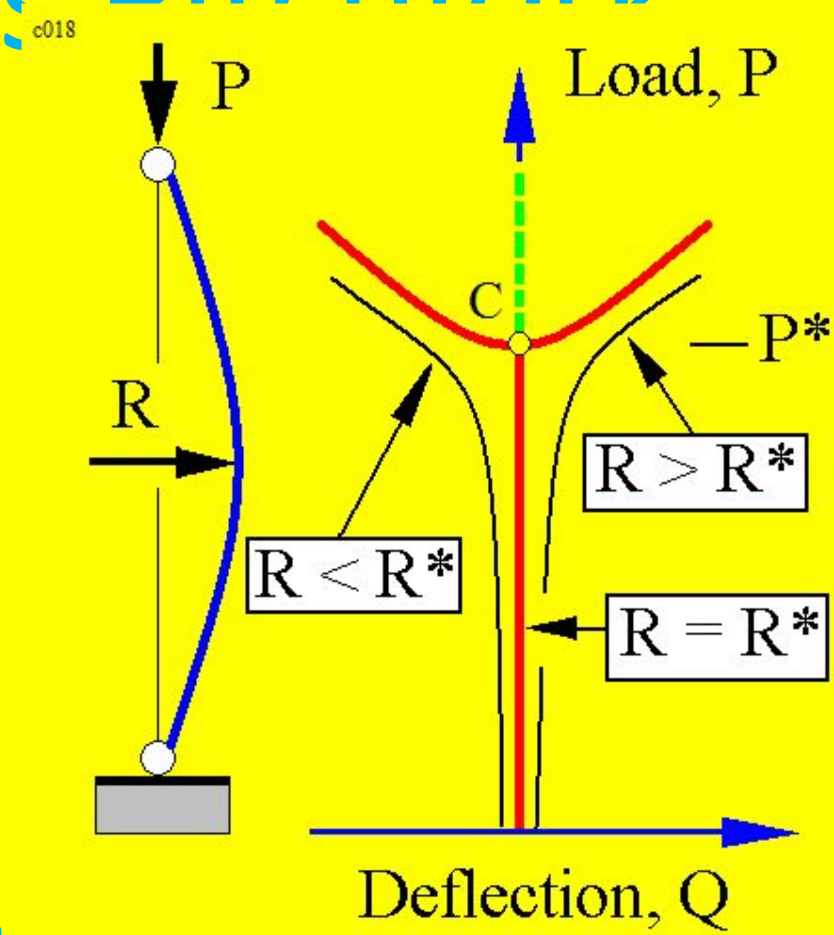
We may have many slowly-varying control parameters, μ_i

But they can subsumed into a single μ (eg. slow time)

This limits the relevant bifurcations to those with co-dimension (*CD*) = 1

We now explain the co-dimension concept, before moving on to classify the *CD* = 1 bifurcations

Unfolding Euler's Pitchfork



A real column has imperfections.
With P it does not reach pitchfork, C.

Catastrophe Theory shows that only one extra control is needed to hit C.

One such control is the side load, R.
R = R* cancels out the imperfections.

Needing 2 controls to be observable we say a pitchfork has co-dimension 2.

A climate tip from a single slow evolution must be co-dimension 1.

Co-Dimension 1 Bifurcations (we shall be listing all 18)

- Bifurcations can be classified as:

- (a) Safe Bifurcations
- (b) Explosive Bifurcations
- (c) Dangerous Bifurcations

Safe and dangerous forms of the Hopf bifurcation

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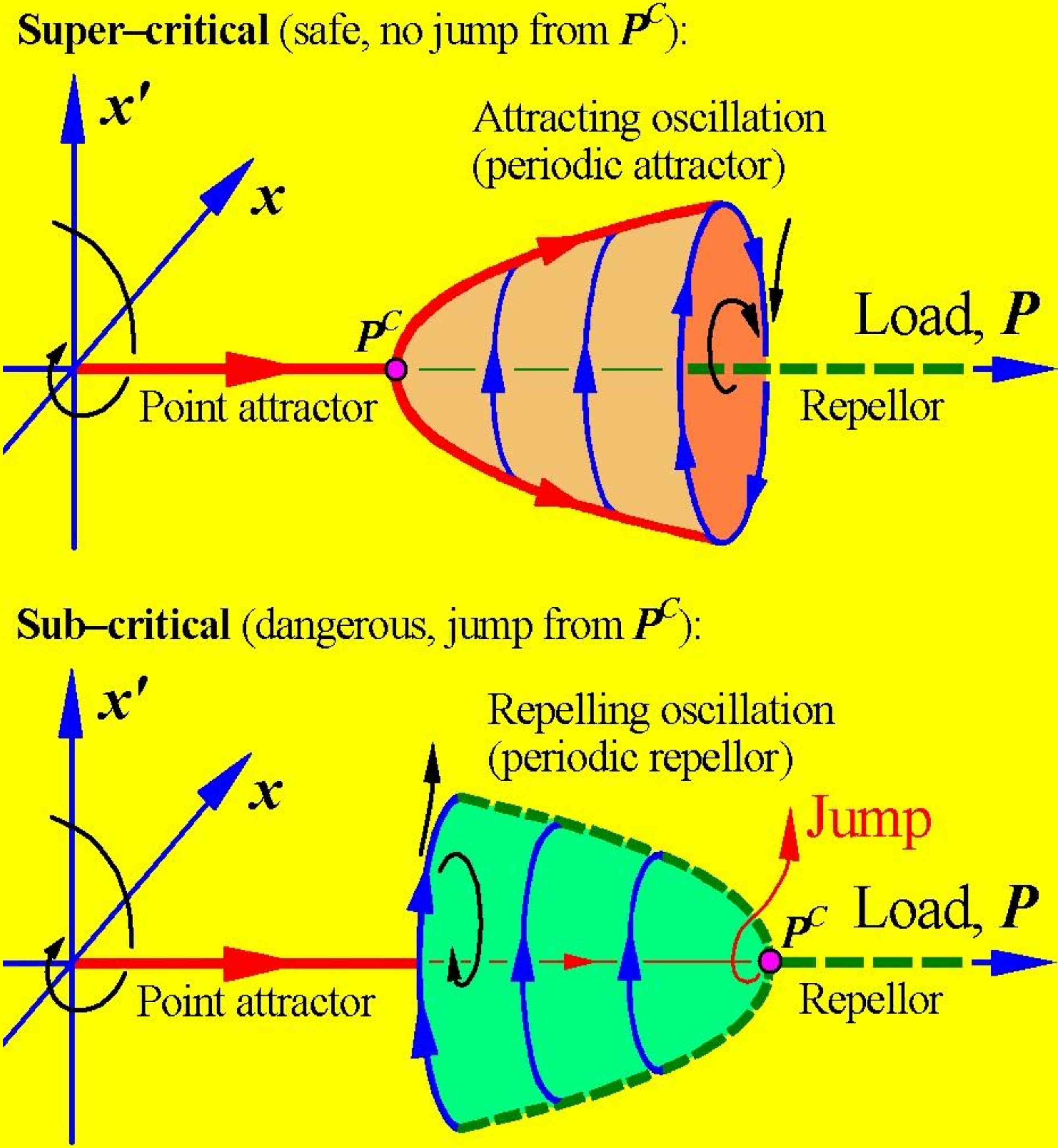
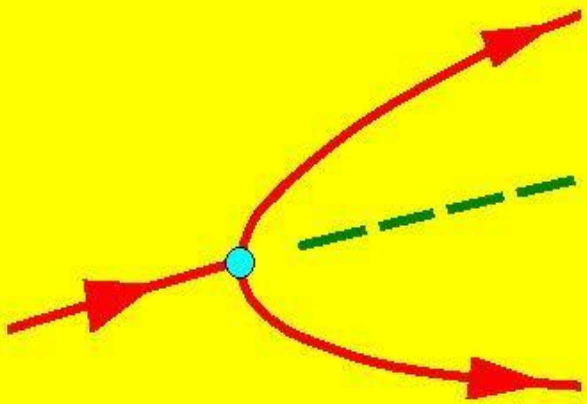


(a) Safe Bifurcations

(a.1) Local Supercritical Bifurcations

1. Supercritical Hopf — Point to cycle
2. Supercritical Neimark — Cycle to torus
3. Supercritical Flip — Cycle to cycle



(a.2) Global Bifurcations

4. Band Merging — Chaos to chaos

SUBTLE: continuous supercritical growth of new attractor path

SAFE: no fast jump or enlargement of the attracting set

DETERMINATE: single outcome even with small noise

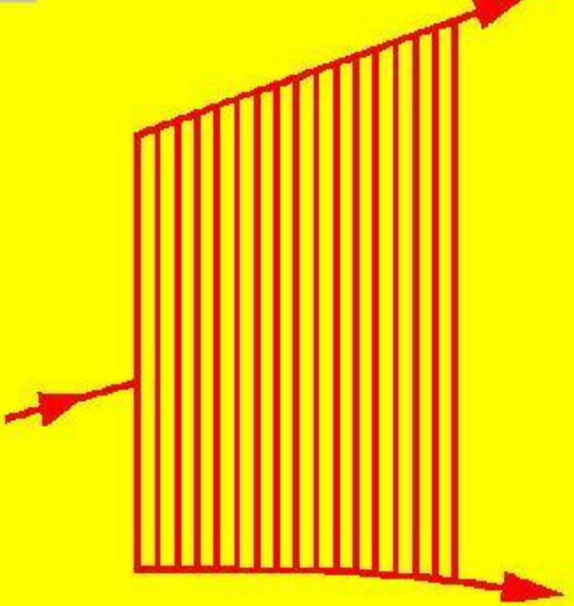
NO HYSTERESIS: path retraced on reversal of control sweep

NO BASIN CHANGE: basin boundary remote from attractors

NO INTERMITTENCY: in the responses of the attractors

(b) Explosive Bifurcations

5. Flow Explosion	Point to cycle
6. Map Explosion	Cycle to torus
7. Intermittency Explosion: Flow	Point to chaos
8. Intermittency Explosion: Map	Cycle to chaos
9. Regular-Saddle Explosion	Chaos to chaos
10. Chaotic-Saddle Explosion	Chaos to chaos



CATASTROPHIC: global events, abrupt enlargement of attracting set

EXPLOSIVE: enlargement, but no jump to remote attractor

DETERMINATE: with single outcome even with small noise

NO HYSTERESIS: paths retraced on reversal of control sweep

NO BASIN CHANGE: basin boundary remote from attractors

INTERMITTENCY: lingering in old domain, flashes through the new

Example of an Explosive Event

Flow-explosion transforms point attractor to a cycle

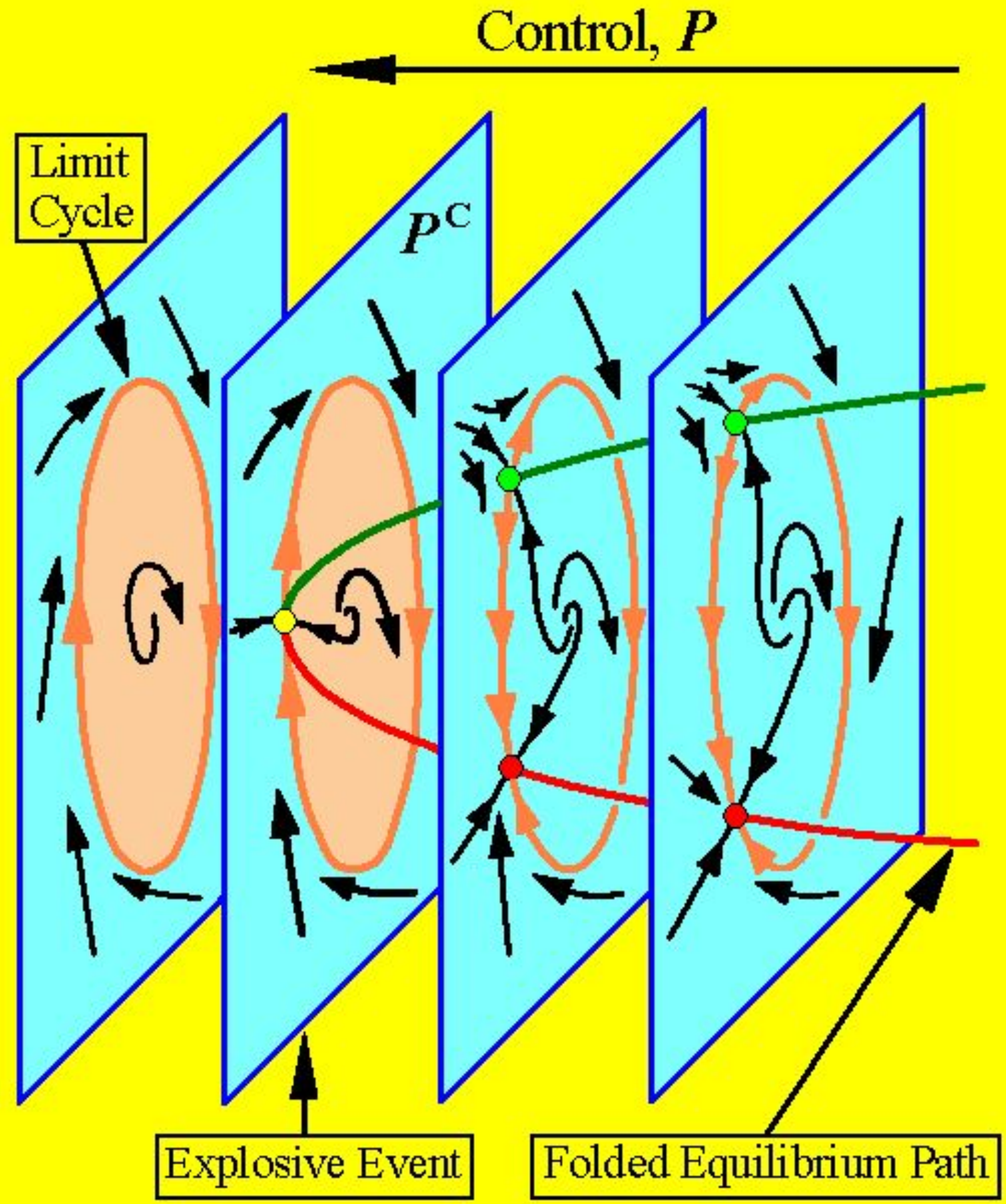
Equilibrium path has a regular saddle-node fold.

Saddle outset flows around a closed loop to the node.

A stable cycle is created.

Initial period is infinite (critical slowing).

Precursor: same as static fold.



(c) Dangerous Bifurcations

(c.1) Local Saddle-Node Bifurcations

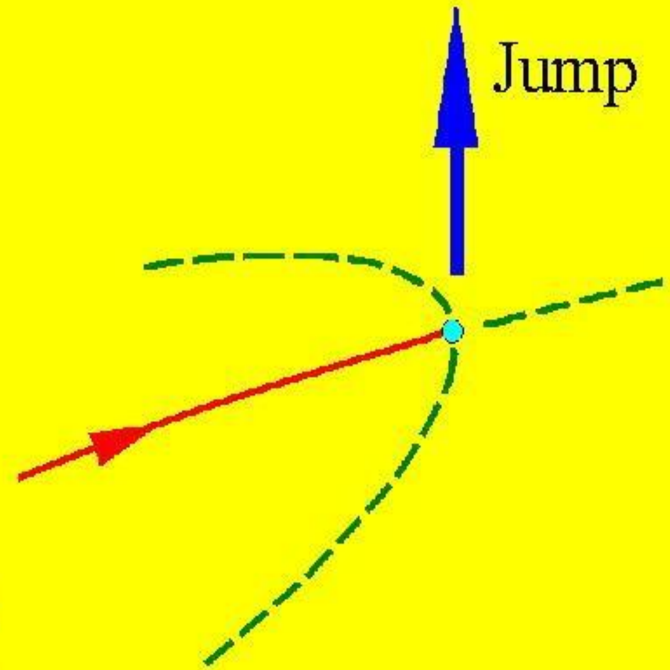
11. Static Fold	from Point
12. Cyclic Fold	from Cycle

(c.2) Local Subcritical Bifurcations

13. Subcritical Hopf	from Point
14. Subcritical Neimark	from Cycle
15. Subcritical Flip	from Cycle

(c.3) Global Bifurcations

16. Saddle Connection	from Cycle
17. Regular-Saddle Catastrophe	from Chaos
18. Chaotic-Saddle Catastrophe	from Chaos



CATASTROPHIC: blue-sky disappearance of attractor

DANGEROUS: sudden jump to new attractor (of any type)

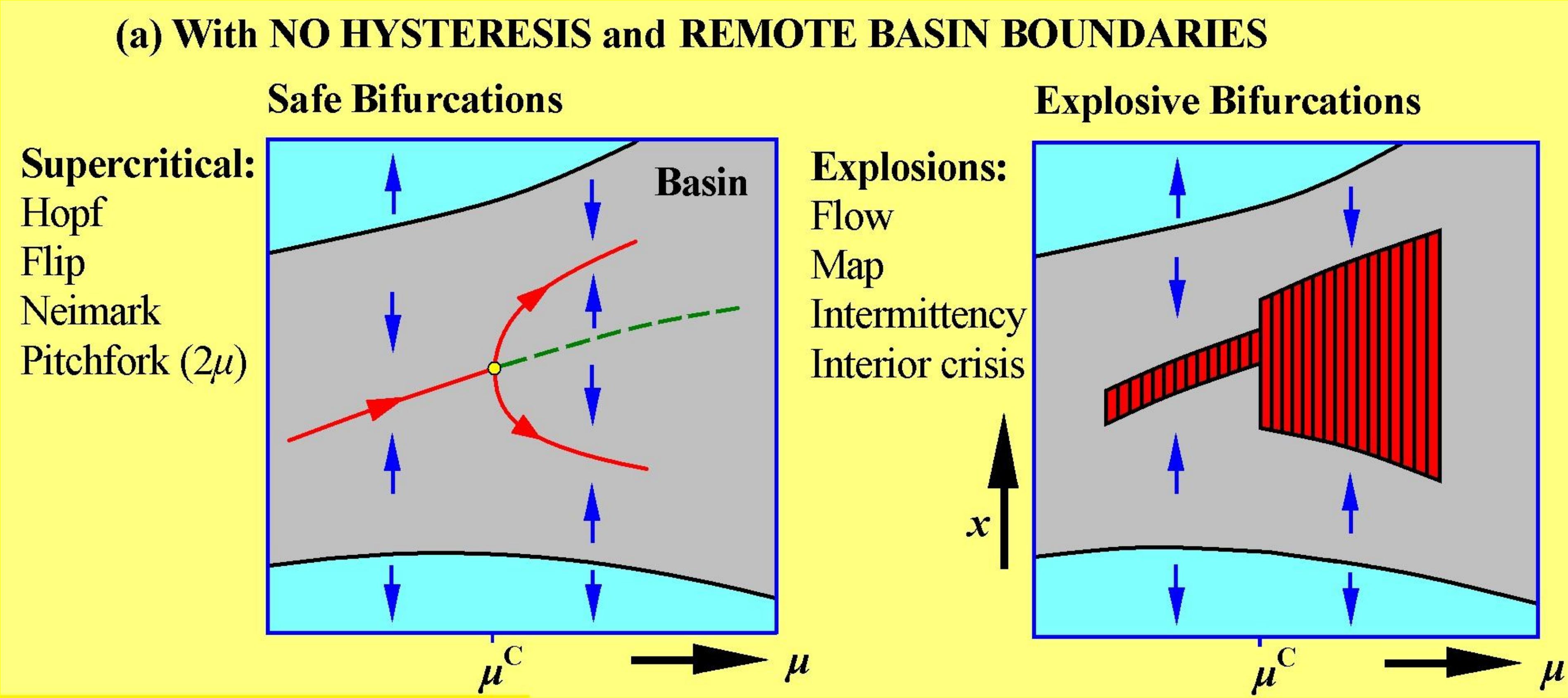
INDETERMINACY: outcome can depend on global topology

HYSTERESIS: path not reinstated on control reversal

BASIN: tends to zero (c.2), attractor hits edge of residual basin (c.1, c.3)

NO INTERMITTENCY: but critical slowing in global events

BASINS (1)

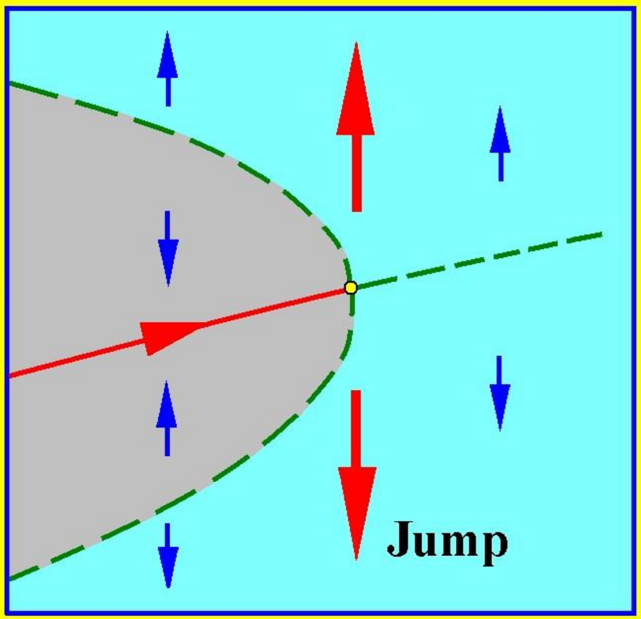

(a) With NO HYSTERESIS and REMOTE BASIN BOUNDARIES
Safe Bifurcations
Supercritical:
Hopf
Flip
Neimark
Pitchfork (2μ)
Basin
μC
μ
Explosive Bifurcations
Explosions:
Flow
Map
Intermittency
Interior crisis
x
μC
μ

BASINS (2)

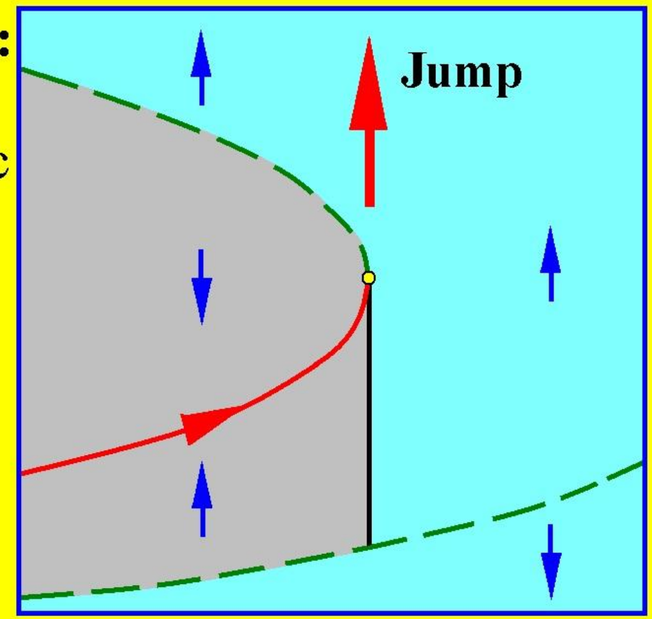
(b) With HYSTERESIS, blue-sky JUMPS and SHRINKING BASINS

Dangerous Bifurcations

Subcritical:
Hopf
Flip
Neimark
Pitchfork (2μ)



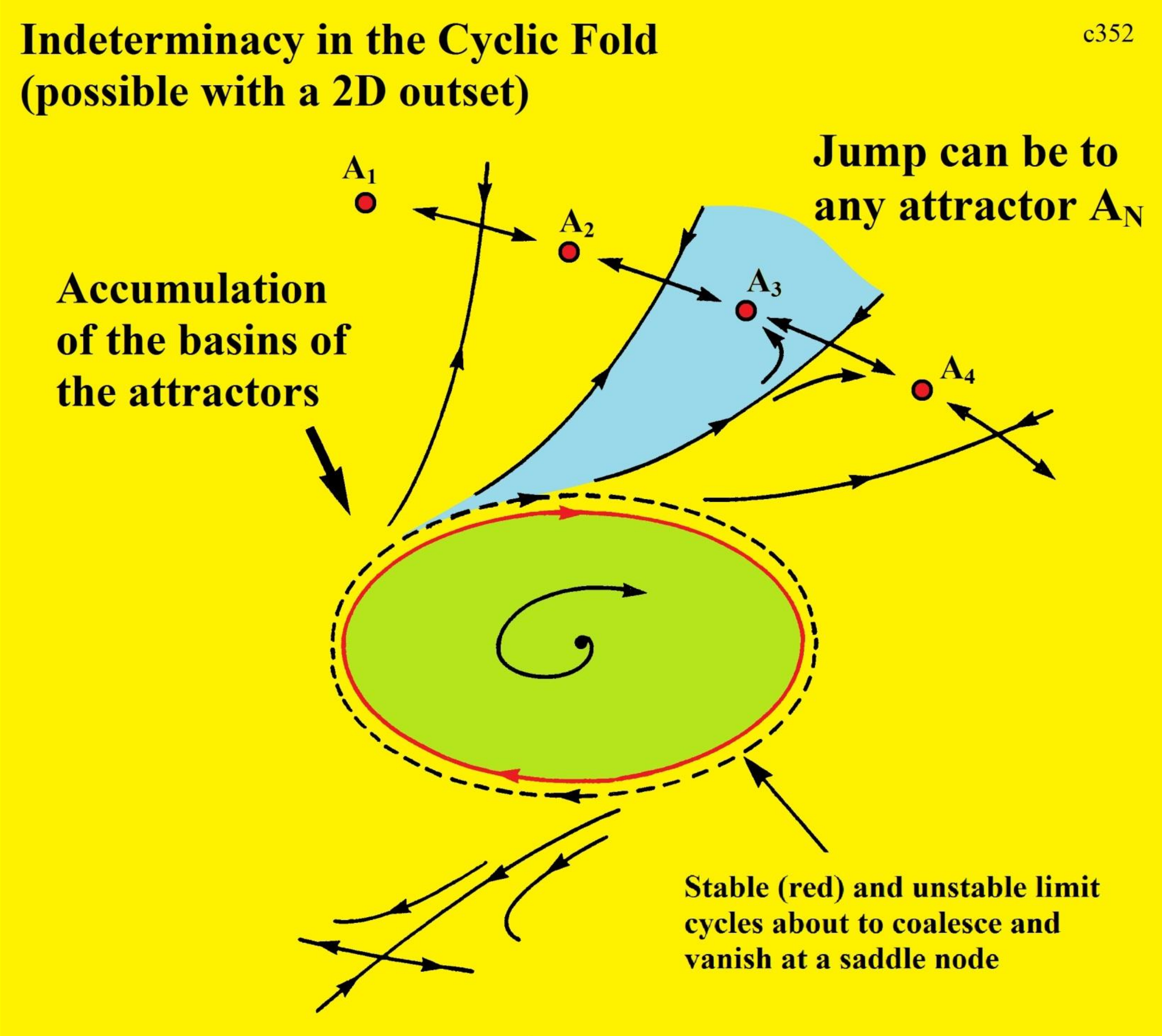
Folds:
Static
Cyclic



Precursors of our 18 bifurcations

Precursors of codimension–one bifurcations (● local decay rate of transients → 0)		
Supercritical Hopf	S: point to cycle	● linearly with control c276a
Supercritical Neimark	S: cycle to torus	● linearly with control
Supercritical flip	S: cycle to cycle	● linearly with control
Band merging	S: chaos to chaos	lingers near impinging boundary
Flow explosion	E: point to cycle	● linearly along folding path
Map explosion	E: cycle to torus	● linearly along folding path
Intermittency expl: flow	E: point to chaos	● linearly with control
Intermittency expl: map	E: cycle to chaos	● as for trigger (fold, flip, Neimark)
Regular interior crisis	E: chaos to chaos	lingers near impinging saddle
Chaotic interior crisis	E: chaos to chaos	lingers near impinging saddle
Static fold	D: from point	● linearly along folding path
Cyclic fold	D: from cycle	● linearly along folding path
Subcritical Hopf	D: from point	● linearly with control
Subcritical Neimark	D: from cycle	● linearly with control
Subcritical flip	D: from cycle	● linearly with control
Saddle connection	D: from cycle	period of cycle tends to infinity
Regular exterior crisis	D: from chaos	lingers near impinging saddle
Chaotic exterior crisis	D: from chaos	lingers near impinging saddle

INDETERMINATE JUMP



Concluding Remarks

- Bifurcation concepts for climate studies:
- Co-dimension-one events in dissipative systems.
- Safe, explosive and dangerous forms.
- Hysteresis and basin boundary structure
- Slowing of transients prior to an instability.

Our recent publications

All can be found in Jan Sieber’s Homepage

http://userweb.port.ac.uk/~sieberj

- J.M.T. Thompson & J. Sieber, Predicting climate tipping points, in *Geo-Engineering Climate Change* (eds. Launder & Thompson) CUP 2010.
- J.M.T. Thompson & Jan Sieber, Climate tipping as a noisy bifurcation: a predictive technique, to appear in *IMA J. Appl. Maths*. http://arxiv.org/abs/1007.1376
- J.M.T. Thompson & Jan Sieber, Predicting climate tipping as a noisy bifurcation: a review, to appear in *Int. J. Bifurcation & Chaos* (this is an extended version of the top paper).