

Course: Corporate Finance.

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## *Session 2*

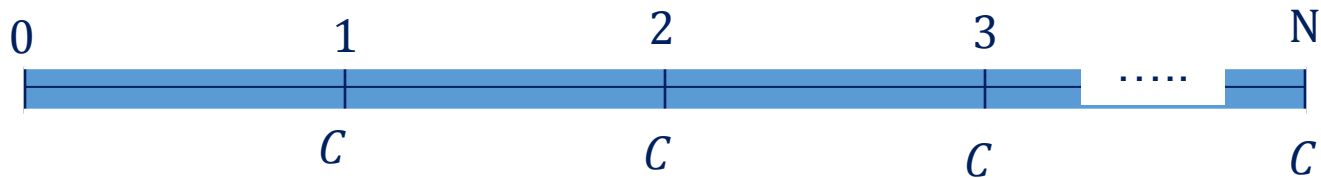
# Annuities, Perpetuities and Other Special Cases

- The formulas we have developed so far allow us to compute the present or future value of any cash flow stream.
- There are cash flows that follow a regular pattern □ We can use shortcuts to value them.
- In what follow, we consider two types of assets: *annuities* and *perpetuities*.

# 1. Annuities

An annuity is a constant cash flow that occurs at regular intervals for a fixed period of time.

Defining  $C$  to be the cash flow,



## 1.1. Present Value of an Annuity

1. The present value of an annuity can be calculated by taking each cash flow and discounting it back to the present, and adding up the present values.
2. Alternatively, *there is a shortcut* that can be used in the calculation:

$$PV \text{ of an annuity} = C \left[ \frac{1 - \frac{1}{(1+r)^n}}{r} \right] \quad (1)$$

$C$  is the cash flow.

## Exercise 1

1. What is the present value of an annuity of \$1,000 for the next five years, assuming a discount rate of 10%?

## Solution

1. The present value of an annuity of \$1,000 for the next five years, assuming a discount rate of 10% is:

$$PV = \$1000 * \left[ \frac{1 - \frac{1}{1.10^5}}{0.10} \right] = \$3,791$$

## 1.2. Annuity, Given the Present Value

When the present value is known and the annuity is to be estimated, then:

$$\text{Annuity given Present Value} = C = PV \left[ \frac{r}{1 - \frac{1}{(1+r)^n}} \right] \quad (2)$$

## 1.3. Future Value of an Annuity

1. The future value of an end-of-the-period annuity can also be calculated as follows:

$$FV \text{ of an annuity} = C \left[ \frac{(1+r)^n - 1}{r} \right] \quad (3)$$

## Exercise 2

What is the future value of \$1,000 each year for the next five years, at the end of the fifth year (assuming a 10% discount rate)?

## Solution

The future value of \$1,000 each year for the next five years, at the end of the fifth year (assuming a 10% discount rate) is

$$FV = \$1000 \left[ \frac{1.10^5 - 1}{0.10} \right] = \$6,105$$



## 1.4. Annuity, given Future Value

If we know the future value and we are looking for the annuity:

$$\text{Annuity given Future Value} = C = FV \left[ \frac{r}{(1+r)^n - 1} \right] \quad (4)$$

### Exercise 3: Saving for College Tuition

1. Assume that you want to send your newborn child to a private college (when he gets to be 18 years old). The tuition costs are \$ 16000/year now and that these costs are expected to rise 5% a year for the next 18 years. Assume that you can invest, after taxes, at 8%.
  - a. Calculate the expected tuition cost/year 18 years from now.
  - b. Calculate the PV of four years of tuition costs at \$38,506/year at the end of the 18th year (We assume that the tuition costs stop rising by 5% beyond the end of the 18<sup>th</sup> year).
2. If you need to set aside a lump sum now, what is the amount you would need to set aside?
3. If you need to set aside an annuity each year, starting one year from now, what is the amount of the annuity?

## Solution for Exercise 3: Saving for College Tuition

1. a. Expected tuition cost/year 18 years from now =  $16000 \times (1.05)^{18} = \$38,506$

b. PV of four years of tuition costs at \$38,506/year: We use formula (IV.1):

$$PV = \$38,506 \left[ \frac{1 - \frac{1}{(1.08)^4}}{0.08} \right] = \underline{\$127,537}$$

2. If you need to set aside a lump sum now, the amount you would need to set aside would be:  $\frac{\$127,357}{1.08^{18}} = \underline{\$31,916}$

3. We use formula (IV.3):  $C = \$127,537 \left[ \frac{0.08}{(1.08)^{18} - 1} \right] = \underline{\$3,405}$

## Exercise 4: How much is an MBA worth?

Assume that you were earning \$40,000/year before entering MBA program and that tuition costs are \$16000/year. If you decide to enter the MBA program, you will resign from working and focus on studying. Expected salary is \$ 54,000/year after graduation. You can invest money at 8%.

1. *For simplicity, assume that the first payment of \$16,000 has to be made at the start of the program and the second payment one year later.*

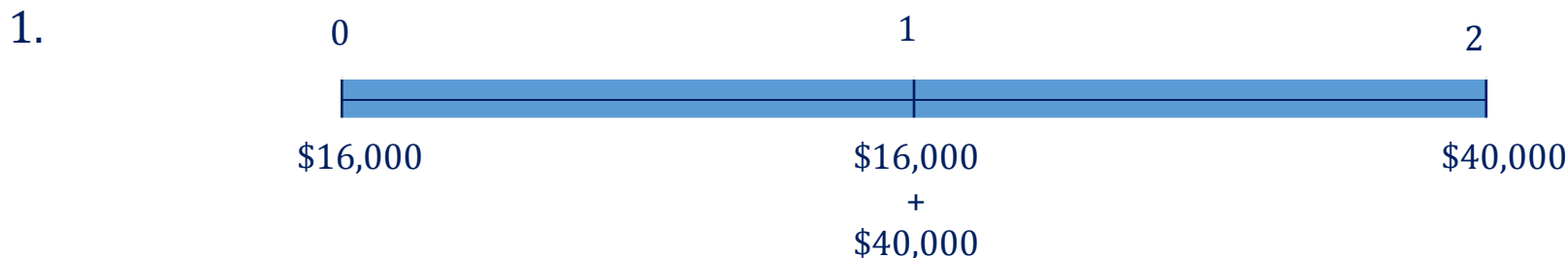
What is the present value of the cost of MBA?

2. *Assume that you will work 30 years after graduation, and that the salary differential (\$14000 = \$54000-\$40000) will continue through this period.*

What is the present value of the Benefits Before Taxes of entering the program?

What is the present value of getting an MBA (benefits – costs)?

## Solution for Exercise 4: How much is an MBA worth?



The MBA program is over 2 years starting from now. Today, I pay \$16,000 and I pay the same amount at the beginning of the next year. If I accept the MBA, I will not work, which means that I will not obtain my salary, which means a cost of \$40,000 per year (See timeline above).

$$\square \text{ PV Of Cost Of MBA} = \$16,000 + (16,000 + 40,000)/1.08 + 40,000/1.08^2 = \$102,145$$

Other solution: I can consider an annuity of \$40,000 per year over two years:

$$\square \text{ PV of cost of MBA} = \$16,000 + \frac{\$16,000}{1.08} + \$40,000 \left[ \frac{1 - \frac{1}{1.08^2}}{0.08} \right] = \$102,145$$

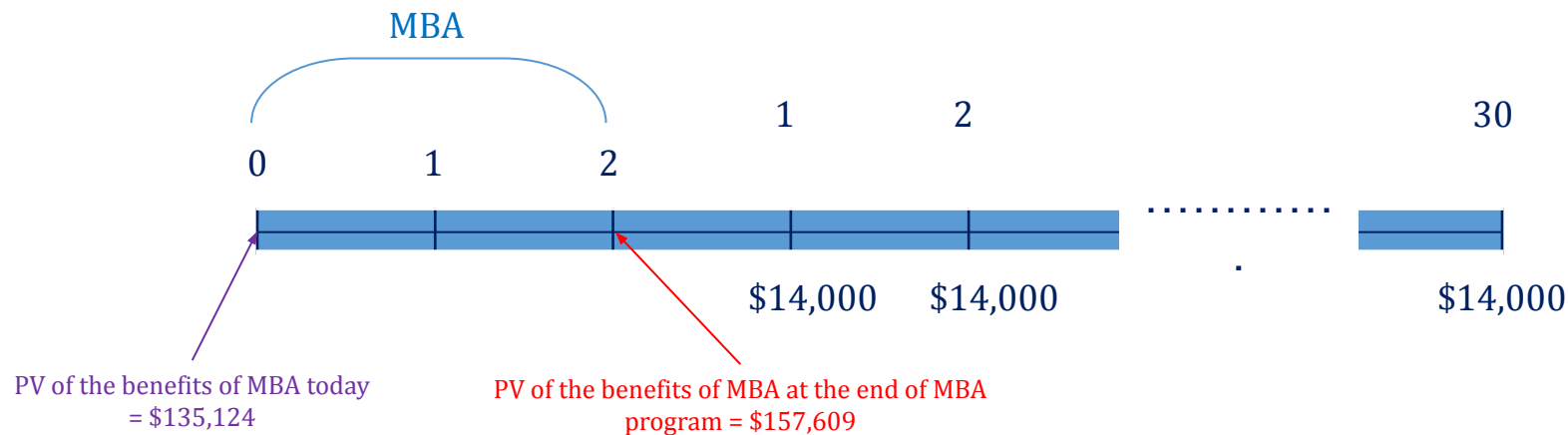
## Solution for Exercise 4: How much is an MBA worth?

2.

$$\text{PV Of benefits before taxes} = \$14,000 \left[ \frac{1 - \frac{1}{1.08^{30}}}{0.08} \right] = \$157,609$$

This has to be discounted back two years:  $\$157,609 / 1.08^2 = \$135,124$

□ The present value of getting an MBA =  $\$135,124 - \$102,145 = \$32,979$



## Exercise 5: Valuing a Straight Bond

1. You are trying to value a straight bond with a fifteen year maturity, a 10.75% coupon rate and a face value of \$ 1,000. The current interest rate on bonds of this risk level is 8.5%.

PV of cash flows on bond =

2. If interest rates rise to 10%,

PV of cash flows on bond =

Percentage change in price =

3. If interest rates fall to 7%,

PV of cash flows on bond =

Percentage change in price = +

## Solution for Exercise 5: Valuing a Straight Bond

1. You are trying to value a straight bond with a fifteen year maturity, a 10.75% coupon rate and a face value of \$ 1,000. The current interest rate on bonds of this risk level is 8.5%.

$$\text{PV of cash flows on bond} = 107.50 * \text{PV}(\text{Annuity}) + 1000 / 1.085^{15} = \$ 1186.85$$

2. If interest rates rise to 10%,

PV of cash flows on bond =

Percentage change in price =

3. If interest rates fall to 7%,

PV of cash flows on bond =

Percentage change in price =



## Solution for Exercise 5: Valuing a Straight Bond

1. You are trying to value a straight bond with a fifteen year maturity, a 10.75% coupon rate and a face value of \$ 1,000. The current interest rate on bonds of this risk level is 8.5%.

$$\text{PV of cash flows on bond} = 107.50 * \text{PV}(\text{Annuity}) + 1000 / 1.085^{15} = \$ 1186.85$$

2. If interest rates rise to 10%,

$$\text{PV of cash flows on bond} = 107.50 * \text{PV}(\text{Annuity}) + 1000 / 1.10^{15} = \$1,057.05$$

$$\text{Percentage change in price} = -10.94\%$$

3. If interest rates fall to 7%,

$$\text{PV of cash flows on bond} = 107.50 * \text{PV}(\text{Annuity}) + 1000 / 1.07^{15} = \$1,281.26$$

$$\text{Percentage change in price} = 18.92\%$$

## Solution for Exercise 5: Valuing a Straight Bond

1. You are trying to value a straight bond with a fifteen year maturity, a 10.75% coupon rate and a face value of \$ 1,000. The current interest rate on bonds of this risk level is 8.5%.

$$\text{PV of cash flows on bond} = 107.50 * \text{PV}(\text{Annuity}) + 1000 / 1.085^{15} = \$ 1186.85$$

2. If interest rates rise to 10%,

$$\text{PV of cash flows on bond} = 107.50 * \text{PV}(\text{Annuity}) + 1000 / 1.10^{15} = \$1,057.05$$

$$\text{Percentage change in price} = -10.94\%$$

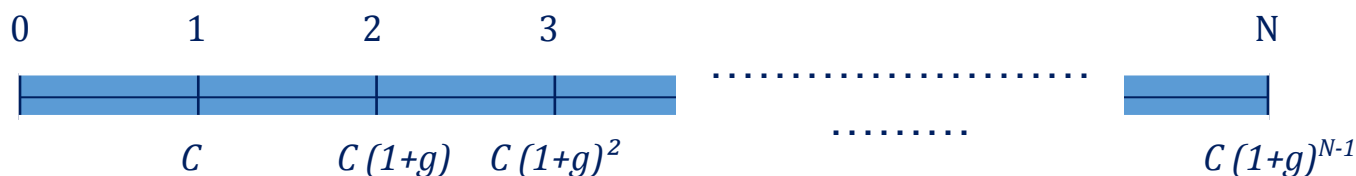
3. If interest rates fall to 7%,

$$\text{PV of cash flows on bond} = 107.50 * \text{PV}(\text{Annuity}) + 1000 / 1.07^{15} = \$1,341.55$$

$$\text{Percentage change in price} = +13.03\%$$

## 1.5. Growing Annuity

1. A growing annuity is a cash flow growing at a constant rate for a specified period of time.
2. If  $C$  is the cash flow at the end of period 1, and  $g$  is the expected growth rate, the time line for a growing annuity looks as follows:



## 1.6. Present Value of a Growing Annuity

The present value of a growing annuity can be estimated in all cases, but one - where the growth rate is equal to the discount rate, using the following model:

$$PV = C \times \frac{1}{r - g} \left[ 1 - \left( \frac{1 + g}{1 + r} \right)^N \right] \quad (5)$$

## Exercise 6: The Value of a Gold Mine

Consider the example of a gold mine, where you have the rights to the mine for the next 20 years, over which period you plan to extract 5,000 ounces of gold every year.

The price per ounce is \$300 currently, but it is expected to increase 3% a year. The appropriate discount rate is 10%.

## Solution for Exercise 6: The Value of a Gold Mine

The present value of the gold that will be extracted from this mine can be estimated as follows (using formula IV.5):

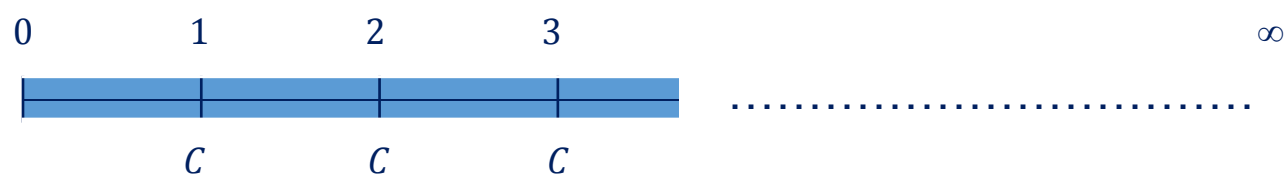
$$PV = \underbrace{\$1,500,000}_{\substack{\text{5000 ounces per year, the price per} \\ \text{ounce is \$300} \square \$1,500,000}} \times \frac{1}{0.10 - 0.03} \left[ 1 - \left( \frac{1.03}{1.10} \right)^{20} \right] = \$15,675,708.72$$

# 2. Perpetuities

□ A perpetuity is a constant cash flow at regular intervals forever.

## 2.1. Present Value of a Perpetuity

$$\text{PV of Perpetuity} = \frac{C}{r} \qquad (6)$$



## Exercise 7: Valuing a Console Bond

1. A console bond is a bond that has no maturity and pays a fixed coupon.
2. Assume that you have a 6% coupon console bond (face value = \$ 1,000).  
Compute the value of this bond if the interest rate is 9%,

## Solution for Exercise 7: Valuing a Console Bond

The value of this bond, if the interest rate is 9%, is as follows:

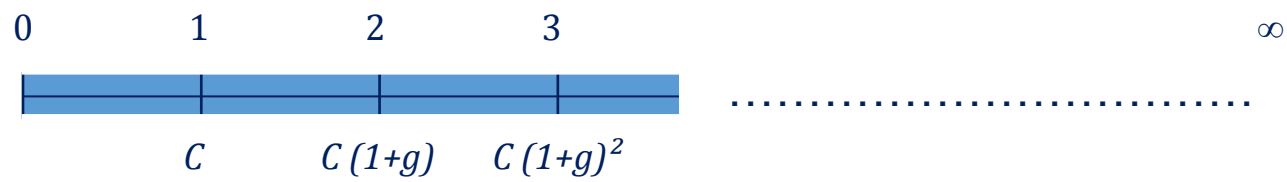
$$\text{Value of Console Bond} = \$60 / 0.09 = \$667$$

## 2.2. Growing Perpetuities

1. A growing perpetuity is a cash flow that is expected to grow at a constant rate forever.

2. The present value of a growing perpetuity is:

$$PV = \frac{C}{(r - g)} \quad (7)$$





## Exercise 8: Valuing a Stock with Growing Dividends

1. Southwestern Bell paid dividends per share of \$2.73 in 2016.
2. Its earnings and dividends have grown at 6% a year between 2012 and 2016, and are expected to grow at the same rate in the long term.
3. The rate of return required by investors on stocks of equivalent risk is 12.23%.

## Solution for Exercise 8: Valuing a Stock with Growing Dividends

The rate of return required by investors on stocks of equivalent risk is 12.23%.

Current Dividends per share = \$2.73

Expected Growth Rate in Earnings and Dividends = 6%

Discount Rate = 12.23%

Value of Stock =  $\$2.73 \times 1.06 / (.1223 - .06) = \$46.45$