

Семинар 6

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Занятие 6

Прямая на плоскости

$$301] \ell \ni M_1(3; -7), \ell: \frac{x}{a} + \frac{y}{b} = 1$$

$$a = b$$

$$\text{Решение. } \begin{cases} \frac{3}{a} + \frac{-7}{b} = 1 \\ a = b \end{cases} \Rightarrow \frac{3}{b} - \frac{7}{b} = 1$$

$$\Rightarrow -\frac{4}{b} = 1 \Rightarrow b = a = -4$$

$$\Rightarrow \boxed{\ell: \frac{x}{-4} + \frac{y}{-4} = 1} \text{ или } \boxed{\ell: x + y + 4 = 0}$$

$$309] 1) \ell: \frac{3}{5}x - \frac{4}{5}y - 3 = 0$$

$$\cos \varphi = \frac{3}{5}, \sin \varphi = -\frac{4}{5} \Rightarrow \cos^2 \varphi + \sin^2 \varphi = \frac{9}{25} + \frac{16}{25} = 1$$

$p = 3$. Уравнение нормальное.

$$2) \ell: \frac{2}{5}x - \frac{3}{5}y - 1 = 0$$

$$\cos \varphi = \frac{2}{5}, \sin \varphi = -\frac{3}{5} \Rightarrow \cos^2 \varphi + \sin^2 \varphi = \frac{4}{25} + \frac{9}{25} \neq 1$$

Не является нормальным уравнением.

$$3) \ell: \frac{5}{13}x - \frac{12}{13}y + 2 = 0$$

$$\cos \varphi = \frac{5}{13}, \sin \varphi = -\frac{12}{13} \Rightarrow \cos^2 \varphi + \sin^2 \varphi = \frac{25}{169} + \frac{144}{169} = 1$$

но $p = -2$ - противоречие.

не является нормальным уравнением.

$$310 \quad 1) \ell: 4x - 3y - 10 = 0$$

$$\text{Решение: } \mu = \frac{1}{\sqrt{16+9}} = \frac{1}{5}$$

$$\Rightarrow \boxed{\ell: \frac{4}{5}x - \frac{3}{5}y - 2 = 0}$$

$$2) \ell: \frac{4}{5}x - \frac{3}{5}y + 10 = 0$$

$$\text{Решение: } \mu = \frac{-1}{\sqrt{\frac{16}{25} + \frac{9}{25}}} = \frac{-1}{1} = -1$$

$$\boxed{\ell: -\frac{4}{5}x + \frac{3}{5}y - 10 = 0}$$

$$312 \quad 1) A(2; -1) \quad \ell: 4x + 3y + 10 = 0$$

$$\delta(A, \ell) = ? \quad \rho(A, \ell) = ?$$

$$\text{Решение } \mu = \frac{-1}{\sqrt{16+9}} = -\frac{1}{5}$$

$$\ell: -\frac{4}{5}x - \frac{3}{5}y - 2 = 0$$

$$\Rightarrow \delta(A, \ell) = -\frac{4}{5} \cdot 2 - \frac{3}{5} \cdot (-1) - 2 = -1 - 2 = \boxed{-3}$$

$$\rho(A, \ell) = \boxed{3}$$

$$313 \quad M(1; -3), \quad O(0, 0)$$

$$1) \ell: 2x - y + 5 = 0$$

$$\text{Решение } \mu = \frac{-1}{\sqrt{4+1}} = -\frac{1}{\sqrt{5}}$$

$$\ell: \frac{-2x}{\sqrt{5}} + \frac{y}{\sqrt{5}} - \sqrt{5} = 0 \Rightarrow \delta(M, \ell) = -\frac{2}{\sqrt{5}} - \frac{3}{\sqrt{5}} - \sqrt{5} < 0$$

$\Rightarrow M$ и O по одну сторону от ℓ .

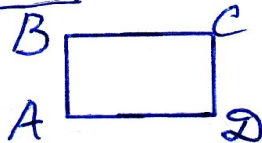
$$2) \ell_2: x - 3y - 5 = 0$$

Решение $\mu = \frac{1}{\sqrt{1+9}} = \frac{1}{\sqrt{10}}$

$$\Rightarrow \ell_2: \frac{x}{\sqrt{10}} - \frac{3y}{\sqrt{10}} - \frac{5}{\sqrt{10}} = 0 \Rightarrow \delta(M, \ell_2) = \frac{1}{\sqrt{10}}(1+9-5) > 0$$

$\Rightarrow M$ и O по разные стороны от ℓ_2 .

315 | $ABCD$ - прямоугольник, $A(-2; 1)$



$$\ell_1: 3x - 2y - 5 = 0$$

$$\ell_2: 2x + 3y + 7 = 0$$

Решение $A \notin \ell_1$ и $A \notin \ell_2$ и $\ell_1 \perp \ell_2$

Пусть $BC \in \ell_1$, $CD \in \ell_2$.

$$\mu_1 = \frac{1}{\sqrt{9+4}} = \frac{1}{\sqrt{13}} \Rightarrow \ell_1: \frac{1}{\sqrt{13}}(3x - 2y - 5) = 0$$

$$\Rightarrow \delta(A, \ell_1) = \frac{1}{\sqrt{13}}(3(-2) - 2(1) - 5) = -\frac{13}{\sqrt{13}} = -\sqrt{13}$$

$$\Rightarrow \rho(A, \ell_1) = \sqrt{13}$$

$$\rho(A, \ell_2) = \frac{|2 \cdot (-2) + 3 \cdot 1 + 7|}{\sqrt{4+9}} = \frac{6}{\sqrt{13}}$$

$$\Rightarrow S_{ABCD} = \rho(A, \ell_1) \cdot \rho(A, \ell_2) = \sqrt{13} \cdot \frac{6}{\sqrt{13}} = \boxed{6}$$

317 | $\ell: 2x - 3y + 6 = 0$ $M_1(-2; -3)$, $M_2(1; -2)$

$$\ell: \frac{1}{\sqrt{13}}(2x - 3y + 6) = 0 \quad \delta(M_1, \ell) = \frac{2 \cdot 2 - 3 \cdot 3 + 6}{\sqrt{13}} < 0$$

$\delta(M_2, \ell) = \frac{2 \cdot 1 + 3 \cdot (-2) - 6}{\sqrt{13}} < 0$ И так, M_1 и M_2 по одну сторону от ℓ . Значит пересечения нет.

$$\begin{aligned} 325 \quad & l_1: 10x + 15y - 3 = 0 \\ & l_2: 2x + 3y + 5 = 0 \\ & l_3: 2x + 3y - 9 = 0 \end{aligned} \quad l_1 \parallel l_2 \parallel l_3$$

Решение. Найдем точку $M_1 \in l_1$, положим

$$x=0 \Rightarrow y = \frac{1}{5} \Rightarrow M_1(0; \frac{1}{5}), M_2(0; -\frac{5}{2}), M_3(0, 3)$$

$$\mu_2 = \frac{-1}{\sqrt{4+9}} = \frac{-1}{\sqrt{13}} \Rightarrow l_2: \frac{1}{\sqrt{13}}(-2x - 3y - 5) = 0$$

$$\mu_3 = \frac{1}{\sqrt{4+9}} = \frac{1}{\sqrt{13}} \Rightarrow l_3: \frac{1}{\sqrt{13}}(2x + 3y - 9) = 0$$

$$\delta(M_1, l_2) = \frac{1}{\sqrt{13}}(-\frac{3}{5} - 5) = -\frac{28}{5\sqrt{13}} < 0 \Rightarrow \rho(M_1, l_2) = \frac{28}{5\sqrt{13}}$$

$$\delta(M_1, l_3) = \frac{1}{\sqrt{13}}(\frac{3}{5} - 9) = -\frac{42}{5\sqrt{13}} < 0 \Rightarrow \rho(M_1, l_3) = \frac{42}{5\sqrt{13}}$$

$$\frac{\rho(M_1, l_3)}{\rho(M_1, l_2)} = \frac{42}{28} = \frac{3}{2}$$

$$331 \quad l: 3x - 4y - 10 = 0$$

$$l_1 - ?, \quad l_2 - ? : \rho(l, l_1) = 3, \quad \rho(l, l_2) = 3$$

Решение $\mu = \frac{1}{\sqrt{9+16}} = \frac{1}{5}$

$$l: \frac{3}{5}x - \frac{4}{5}y - 2 = 0$$

$$l_1: \frac{3}{5}x - \frac{4}{5}y - 2 = 3 \Rightarrow l_1: 3x - 4y - 25 = 0$$

$$l_2: \frac{3x - 4y - 10}{5} = -3 \Rightarrow l_2: 3x - 4y + 5 = 0$$

341 | $M(1; -2)$

1) $l_1: 2x - y - 5 = 0$

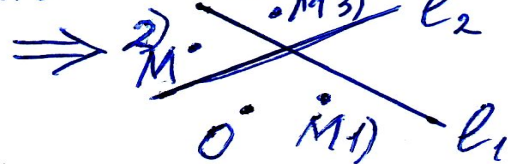
$l_2: 3x + y + 10 = 0$

$\delta(M, l_1) = \frac{1}{\sqrt{5}} (2 + 2 - 5) < 0$

$\delta(M, l_2) = \frac{1}{\sqrt{10}} (-3 + 2 - 10) < 0 \Rightarrow$

$l_1: \frac{1}{\sqrt{5}} (2x - y - 5) = 0$

$l_2: \frac{1}{\sqrt{10}} (3x + y + 10) = 0$



2) $l_1: 4x + 3y - 10 = 0$

$l_2: 12x - 5y - 5 = 0$

$l_1: \frac{1}{5} (4x + 3y - 10) = 0 \Rightarrow \delta(M, l_1) = \frac{1}{5} (4 - 6 - 10) < 0$

$l_2: \frac{1}{13} (12x - 5y - 5) = 0 \Rightarrow \delta(M, l_2) = \frac{1}{13} (12 + 10 - 5) > 0$

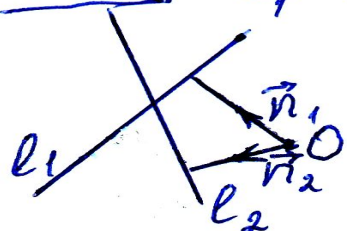
3) $l_1: x - 2y - 1 = 0 \Rightarrow l_1: \frac{1}{\sqrt{5}} (x - 2y - 1) = 0$

$l_2: 3x - y - 2 = 0 \Rightarrow l_2: \frac{1}{\sqrt{10}} (3x - y - 2) = 0$

$\Rightarrow \delta(M, l_1) = \frac{1}{\sqrt{5}} (1 + 4 - 1) > 0$

$\delta(M, l_2) = \frac{1}{\sqrt{10}} (3 + 2 - 2) > 0$

345 | $l_1: 3x - 2y + 5 = 0, l_2: 2x + y - 3 = 0$



какой угол тупой или острый
содержит $O(0,0)$

$l_1: -\frac{1}{\sqrt{13}} (3x - 2y + 5) = 0 \Rightarrow \vec{n}_1 = \left\{ \frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\}$

$l_2: \frac{1}{\sqrt{5}} (2x + y - 3) = 0 \Rightarrow \vec{n}_2 = \left\{ \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\}$

$\Rightarrow \vec{n}_1 \cdot \vec{n}_2 = \frac{-6}{\sqrt{65}} + \frac{2}{\sqrt{65}} = \frac{-4}{\sqrt{65}} < 0$ (\hat{n}_1, \hat{n}_2) - тупой

$\Rightarrow (l_1, l_2)$ - острый

$$349) \ell_1: x+2y-11=0, \ell_2: 3x-6y-5=0$$

$$M_0(1; -3) \ell_5 - ?$$

$$\text{Решение } \ell_1: \frac{1}{\sqrt{5}}(x+2y-11)=0$$

$$\ell_2: \frac{1}{3\sqrt{5}}(3x-6y-5)=0$$

$$\delta(M_0, \ell_1) = \frac{1}{\sqrt{5}}(1-6-11) < 0$$

$$\delta(M_0, \ell_2) = \frac{1}{3\sqrt{5}}(3+18-5) > 0 \Rightarrow M_0 \in \text{смежного угла}$$

1) \forall точки $M(x, y) \in \text{смежного угла} \Rightarrow$

$$\delta(M, \ell_1) > 0, \text{ а } \delta(M, \ell_2) < 0$$

или $\delta(M, \ell_1) < 0, \text{ а } \delta(M, \ell_2) > 0$, т.е. $\delta(M, \ell_1) \cdot \delta(M, \ell_2) < 0$

2) \forall точки $M(x, y) \in \text{биссектрисе} \Rightarrow$

$$|\delta(M, \ell_1)| = |\delta(M, \ell_2)|$$

$$\text{Из 1) и 2) } \Rightarrow \delta(M, \ell_1) = -\delta(M, \ell_2)$$

$$\Rightarrow \frac{1}{\sqrt{5}}(x+2y-11) = -\frac{1}{3\sqrt{5}}(3x-6y-5)$$

$$\Rightarrow (3+3)x + (6-6)y + (-33-5) = 0$$

$$\boxed{\ell_5: 3x - 19 = 0}$$

351) $l_1: 3x + 4y - 5 = 0, l_2: 5x - 12y + 3 = 0$

l_5 - ? острого угла

Решение $l_1: \frac{1}{5}(3x + 4y - 5) = 0, l_2: \frac{1}{13}(5x - 12y + 3) = 0$

$\Rightarrow \vec{n}_1 = \left\{ \frac{3}{5}; \frac{4}{5} \right\}, \vec{n}_2 = \left\{ -\frac{5}{13}; \frac{12}{13} \right\}$

$\Rightarrow \vec{n}_1 \cdot \vec{n}_2 = -\frac{15}{65} + \frac{48}{65} = \frac{33}{65} > 0$ ($\vec{n}_1 \cdot \vec{n}_2$) - острый

$\Rightarrow O \in$ тупому углу

$\Rightarrow l_5 \in$ смежному углу

В силу N 349 $\Rightarrow \forall$ точки $M(x, y) \in l_5$

$\Rightarrow \delta(M, l_1) = -\delta(M, l_2)$

$\Rightarrow \frac{1}{5}(3x + 4y - 5) = +\frac{1}{13}(5x - 12y + 3)$

$\Rightarrow 39x + 52y - 65 = 25x - 60y + 15$

$\Rightarrow l_5: 14x + 112y - 80 = 0$

$\Rightarrow \boxed{l_5: 7x + 56y - 40 = 0}$

359)  $AB: x + 2y - 1 = 0$

$BC: 5x + 4y - 17 = 0$

$AC: x - 4y + 11 = 0$

AM - ?, BN - ?, CH - ? - высоты

1) $AM: \alpha(x + 2y - 1) + \beta(x - 4y + 11) = 0 \quad AM \perp BC$

$AM: (\alpha + \beta)x + (2\alpha - 4\beta)y + (11\beta - \alpha) = 0$

$\vec{n}_{AM} \perp \vec{n}_{BC} \Rightarrow 5(\alpha + \beta) + 4(2\alpha - 4\beta) = 0 \Rightarrow 13\alpha = 11\beta$

Положим $\beta = 13 \Rightarrow \alpha = 11$

$\Rightarrow AM: 24x - 30y + 132 = 0$

$\boxed{AM: 4x - 5y + 22 = 0}$

$$2) BN: \alpha(x+2y-1) + \beta(5x+4y-17) = 0$$

$$BN \perp AC$$

$$BN: (\alpha+5\beta)x + (2\alpha+4\beta)y + (-\alpha-17\beta) = 0$$

$$\vec{n}_{AC} \perp \vec{n}_{BN} \Rightarrow (\alpha+5\beta) - 4(2\alpha+4\beta) = 0$$

$$\Rightarrow -7\alpha - 11\beta = 0 \quad 7\alpha = -11\beta$$

$$\beta := 7 \Rightarrow \alpha = -11$$

$$BN: 24x + 6y - 108 = 0$$

$$\Rightarrow \boxed{BN: 4x + y - 18 = 0}$$

$$3) CH: \alpha(5x+4y-17) + \beta(x-4y+11) = 0$$

$$AB \perp CH \quad CH: (5\alpha+\beta)x + (4\alpha-4\beta)y + (11\beta-17\alpha) = 0$$

$$(5\alpha+\beta) + 2(4\alpha-4\beta) = 0$$

$$13\alpha - 7\beta = 0 \Rightarrow 13\alpha = 7\beta$$

$$\beta := 13 \Rightarrow \alpha = 7$$

$$CH: 48x - 24y + 24 = 0$$

$$\boxed{CH: 2x - y + 1 = 0}$$

Дома к. 305, 309(4,5,6), 310(3,4,5), 312(3),
313(3,5), 318, 323, 335, 342, 346, 350,
352, 361.

