THE FORMAL NORMAL FORM DEGENERATE SINGULAR POINTS IN THE CASE OF CASE OF FOCUS

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OBJECTIVE

- In the work we study the problem for the case $\lambda \notin \mathbb{R}, r = 2$. Have a normal form

$$(V_H + x^p \varphi(x) E_H) \cdot (1 + x^q \phi(x)),$$

where $E_H = (x, 2y), V_H = (y + (1+\lambda)x^2, -2\lambda x^2),$
Formal normal form takes the form

$$(y + (1+\lambda)x^2 + x^2\varphi(x), -2\lambda x^2 + 2xy\varphi(x)) \cdot (1 + \phi(x)),$$

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AIM OF WORK

- The direct study of a formal normal form with the feature type Bogdanov-Takens for a particular case, for the purpose of comparison with the results of Zolondek-Strozhinoi.

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PREFACE

- Takens in 1974 for the system of equations of the form

$$\dot{x} = y + \dots, \quad \dot{y} = \dots$$

got a fairly simple formal normal form

$$\dot{x} = y + a(x), \dot{y} = b(x)$$

but this formal normal form admits of further simplification.

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- General(for all cases) the formal normal form was obtained in 2015.
This Form looks very complicated and has the form:

$$\left(V_H + x^p \varphi(x) E_H\right) \cdot \left(1 + x^q \psi(x)\right)$$

THEOREM

- The field V (для случая $\lambda \notin \mathbb{Q}$) is formally equivalent to one of the following fields

  $N_{p;q}^{r;\lambda}: (V_H + x^p\varphi(x)E_H)\cdot(1 + x^q\psi(x))$, где
  $r-1 < p \le \infty, 0 < q \le \infty$, и
  $\varphi = \sum_{i\in\mathbb{Z}_+/l_{1+p}} a_i x^i, \psi = \sum_{i\in\mathbb{Z}_+/l_{1+q}} b_j x^j$,

  formal power series. Forms $N_{p;q}^{r;\lambda}$ are the only modulo substitutions

  $$(x, y) \longrightarrow (\alpha x, \alpha y), \alpha^{r-1} = 1..$$

DEFINITION

- Two analytical vector fields V, V_0 in $(\mathbb{C}^2; 0)$ are formally equivalent ,then and only then when there is a formal diffeomorphisms $H \in (\mathbb{C}^2; 0)$

$$H' \cdot V = V_0 \circ H$$

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THE FUNCTIONAL EQUATION OF EQUIVALENCE AND THE SCHEME OF REDUCTION TO SYSTEMS OF LINEAR EQUATIONS

$$H' \cdot V = V_0 \circ H$$

$$H(x,y)=(x+\sum c_{ij}x^i y^j, y + \sum d_{ij}x^i y^j)$$

$$V=(y+(1+\lambda)x^2 + \sum a_{ij}x^i y^j, -2\lambda x^2 + \sum b_{ij}x^i y^j)$$

$$V_0 = (y + (1+\lambda)x^2 + \sum \tilde{a}_{ij}x^i y^j, -2\lambda x^2 + \sum \widetilde{b_{ij}}x^i y^j)$$

Objective :to find a formal normal form as simple as possible species

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- We substitute the expansions H, V, V_0 in the main equation :

$$\begin{pmatrix} (1+2c_{20}x+c_{11}y++\ldots)(y+(1+\lambda)x^2+a_{11}xy+\ldots)+ \\ +(c_{11}x+2c_{02}y+c_{21}x^2++\ldots)((-2\lambda)x^2+b_{11}xy+\ldots) \\ (2d_{20}x+d_{11}x+3d_{30}x^2+\ldots)(y+(y+(1+\lambda)x^2++\ldots)+ \\ +(1+d_{11}x+2d_{02}y+d_{21}x^2+\ldots))((-2\lambda)x^2+b_{11}xy+\ldots) \end{pmatrix} =$$

$$\begin{pmatrix} (y+d_{20}x^2+\ldots)+(1+\lambda)(x^2+c_{20}^2x^4+\ldots) \\ -2\lambda(x^2+c_{20}^2x^4+\ldots) \end{pmatrix}$$

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	c_{20}	c_{11}	c_{02}	c_{30}	c_{21}	c_{12}	c_{03}	d_{20}	d_{11}	d_{02}	d_{30}	d_{21}	d_{12}	d_{03}
$1(2,0)$								-1						
$1(1,1)$	2								-1					
$1(0,2)$		1								-1				
$1(3,0)$		-2λ									-1			
$1(2,1)$	$2a_{11}$	$b_{20}--\mu$	-4λ	3								-1		
$1(1,2)$	$2a_{02}$	$a_{11}++b_{02}$	$2(b_{11}--\mu)$		2								-1	
$1(0,3)$		a_{02}	$2b_{02}$			1								-1
$2(2,0)$														
$2(1,1)$								2						
$2(0,2)$									1					
$2(3,0)$	4λ							2μ	-2λ					

tab1.

INFERENCE

- After we solve this system of equations the formal normal form takes the form

$$\begin{cases} \dot{x} = y + (1+\lambda)x^2 \\ \dot{y} = -2\lambda x^2 + \sum \alpha_k x^{3k} + \sum \beta_k y x^{3k} + \sum \gamma_k x^{3k+1} \quad k = 1, \ldots \end{cases}$$

We have 3 adjacent degree - 3 monoms from a formal normal form. Thus , our result is consistent with the result of Zolondek-Strozhinoi.

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REFERENCES

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Thank you for attention!

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