Математика

Занятие 65. Простейшие задачи в координатах
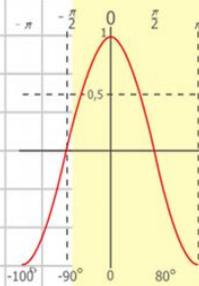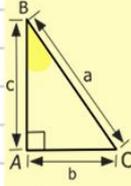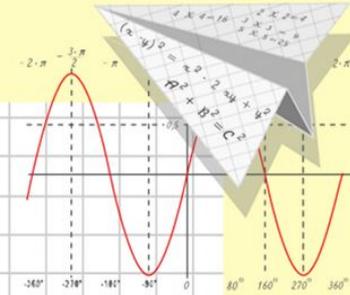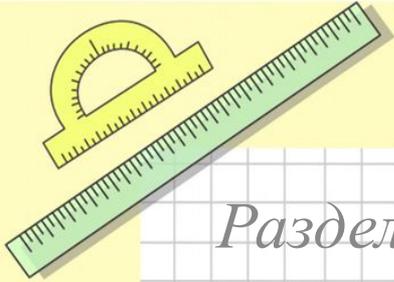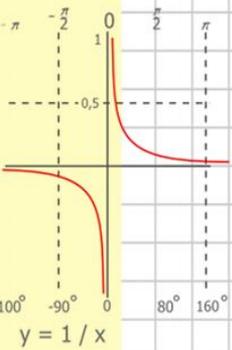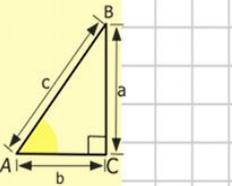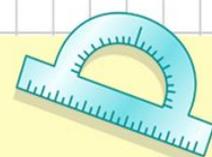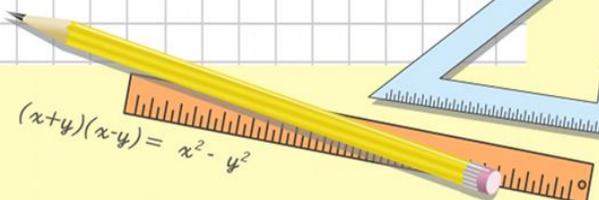
Простейшие задачи в координатах

Пусть даны $A = (x_1; y_1; z_1)$ $\qquad$ $B = (x_2; y_2; z_2)$

Тогда:

1. $\overrightarrow{AB} = (x_2 - x_1; y_2 - y_1; z_2 - z_1)$

2. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

3. *Если* $M = (x; y; z) - $ *середина* $AB,$ *то*

$$x = \frac{x_1 + x_2}{2}; \quad y = \frac{y_1 + y_2}{2}; \quad z = \frac{z_1 + z_2}{2}$$

4. *Если* $\vec{a} = (x; y; z),$ *то* $\quad \left|\vec{a}\right| = \sqrt{x^2 + y^2 + z^2}$

5. $\cos\varphi = \dfrac{\vec{a}\vec{b}}{\left|\vec{a}\right| \cdot \left|\vec{b}\right|},\quad$ *где* $\quad \varphi = \angle(\vec{a}; \vec{b})$
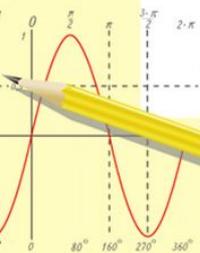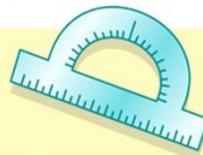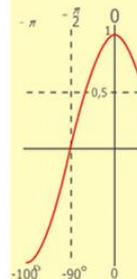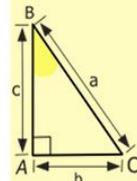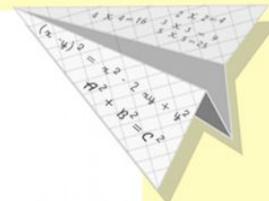
Простейшие задачи в координатах

Даны точки: $A = (4; 6; -1)$ $B = (0; -2; 5)$ $C = (3; 1; 1)$

1. Вычислить координаты векторов:

$$\overrightarrow{AB} = \begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 0-4 \\ -2-6 \\ 5-(-1) \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \\ 6 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \qquad \overrightarrow{AC} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 2 \end{pmatrix}$$

2. Вычислить длины векторов:

$$\left| \overrightarrow{AB} \right| = \sqrt{(-4)^2 + (-8)^2 + 6^2} = \sqrt{16 + 64 + 36} = \sqrt{116} = 2\sqrt{29}$$

$$\left| \overrightarrow{AC} \right| = \sqrt{(-1)^2 + (-5)^2 + 2^2} = \sqrt{1 + 25 + 4} = \sqrt{30}$$
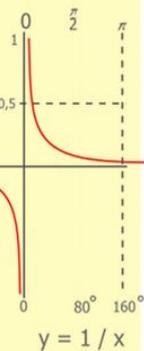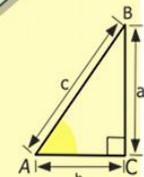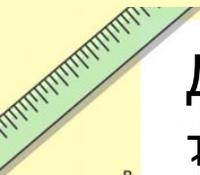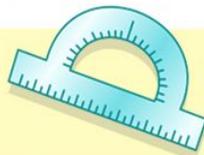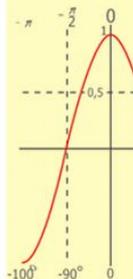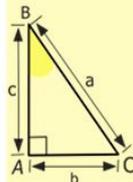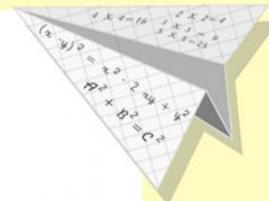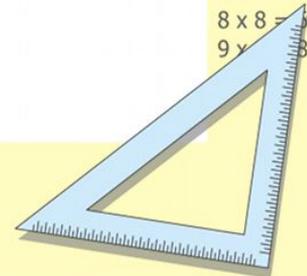
Простейшие задачи в координатах

Даны точки: $A = (4; 6; -1)$ $B = (0; -2; 5)$ $C = (3; 1; 1)$

3. Вычислить длину медианы *CM* из △*ABC*

Если CM медиана, то M = *(x; y; z)* середина AB

$$x = \frac{4+0}{2} = 2; \quad y = \frac{6+(-2)}{2} = 2; \quad z = \frac{-1+5}{2} = 2$$

т.е. M = *(2; 2; 2)*

$$CM = \sqrt{(2-3)^2 + (2-1)^2 + (2-1)^2} = \sqrt{1+1+1} = \sqrt{3}$$
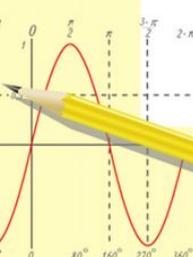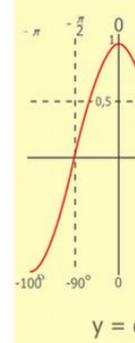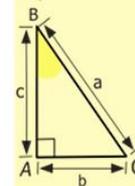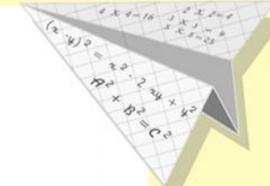
Простейшие задачи в координатах

Даны точки: $A = (2; 1; -1)$ $B = (4; 0; -2)$ $C = (-1; 3; 2)$

4. Вычислить $\angle B$ в $\triangle ABC$

$$\cos B = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\left|\overrightarrow{BA}\right| \cdot \left|\overrightarrow{BC}\right|}$$

$$\overrightarrow{BA} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \overrightarrow{BC} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix}$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix} = 10 + 3 + 4 = 17$$

$$\cos B = \frac{17}{\sqrt{6} \cdot 5\sqrt{2}} = \frac{17}{5\sqrt{12}} = \frac{17}{10\sqrt{3}}$$

$$\left|\overrightarrow{BA}\right| = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$$

$$\angle B = \arccos \frac{17}{10\sqrt{3}}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{(-5)^2 + 3^2 + 4^2} = \sqrt{25 + 9 + 16} = \sqrt{50} = 5\sqrt{2}$$
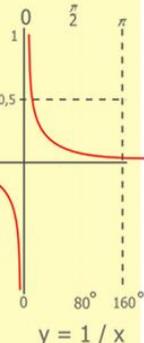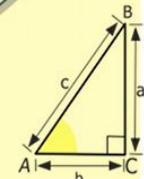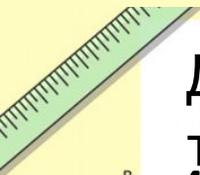
Простейшие задачи в координатах

5. $ABCDA_1B_1C_1D_1$ – прямоугольный параллелепипед, $AB=1$, $BC=2$, $AA_1=3$. Найти угол между прямыми AC и B_1D

Решение:

$$A = (1; 0; 0) \qquad C = (0; 2; 0)$$

$$D = (1; 2; 0) \qquad B_1 = (0; 0; 3)$$

$$\cos\varphi = \frac{\overrightarrow{AC} \cdot \overrightarrow{B_1D}}{\left|\overrightarrow{AC}\right| \cdot \left|\overrightarrow{B_1D}\right|}$$

$$\overrightarrow{AC} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \qquad \overrightarrow{B_1D} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

$$\overrightarrow{AC} \cdot \overrightarrow{B_1D} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = -1 + 4 + 0 = 3$$

$$\left|\overrightarrow{AC}\right| = \sqrt{(-1)^2 + 2^2 + 0^2} = \sqrt{1+4+0} = \sqrt{5}$$

$$\left|\overrightarrow{B_1D}\right| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$\cos B = \frac{3}{\sqrt{5} \cdot \sqrt{14}} = \frac{3}{\sqrt{70}}$$

$$\angle B = \arccos \frac{3}{\sqrt{70}}$$
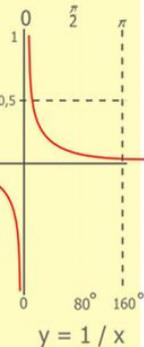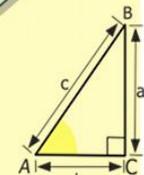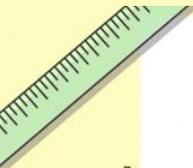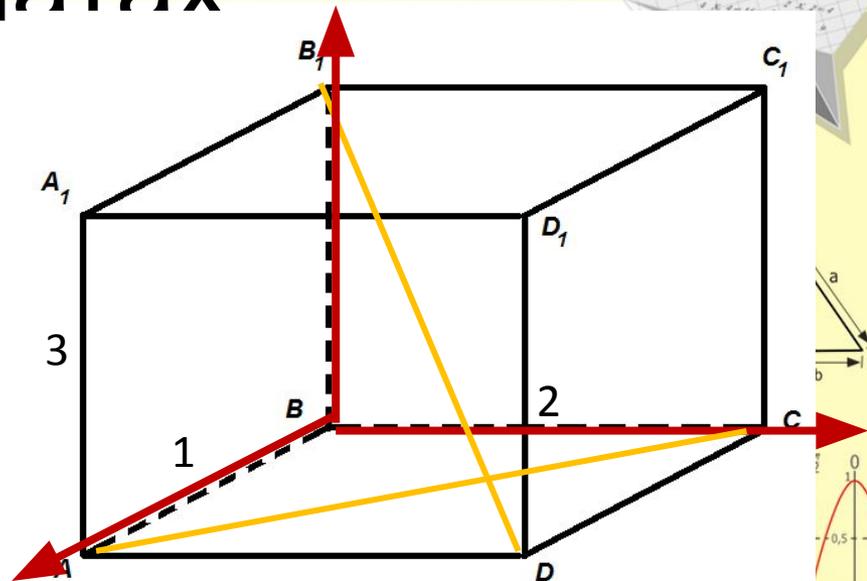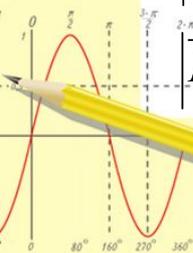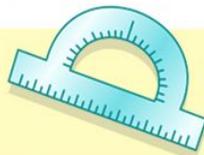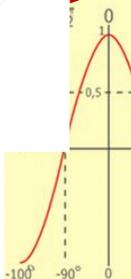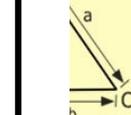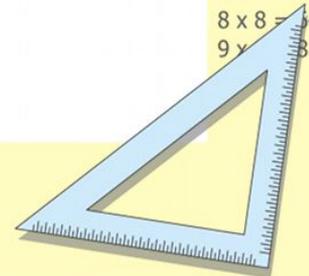
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   преподаватель математики
   ГБПОУ ЗКНО
   Москва, 2021г.