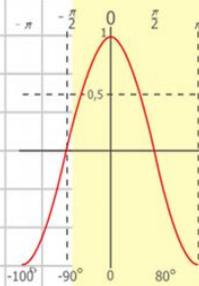
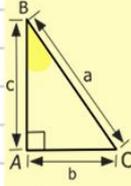
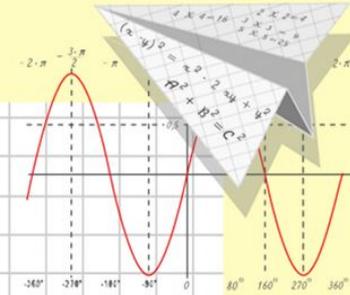
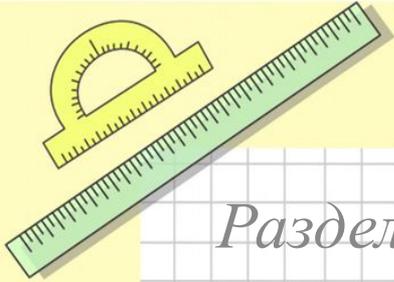


# Математик

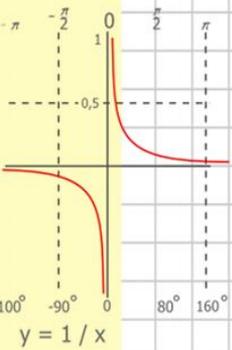
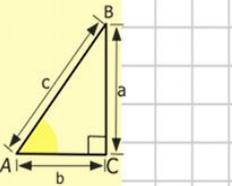
Раздел 6. Метод <sup>а</sup> координат в пространстве

## Занятие 65.

# Простейшие задачи в координатах



- $y = \cos x$
- $2 \times 2 = 4$
  - $3 \times 3 = 9$
  - $4 \times 4 = 16$
  - $5 \times 5 = 25$
  - $6 \times 6 = 36$
  - $7 \times 7 = 49$
  - $8 \times 8 = 64$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$

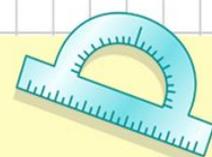


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

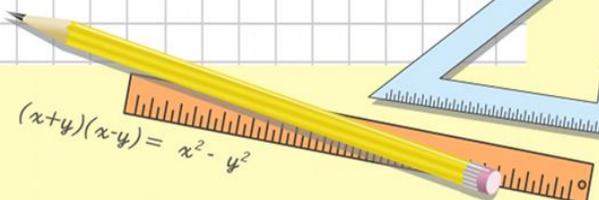
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$



$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \\ y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$



$$(x+y)(x-y) = x^2 - y^2$$

# Простейшие задачи в

## координатах

Пусть даны

$$A = (x_1; y_1; z_1) \quad B = (x_2; y_2; z_2)$$

Тогда:

$$1. \quad \vec{AB} = (x_2 - x_1; y_2 - y_1; z_2 - z_1)$$

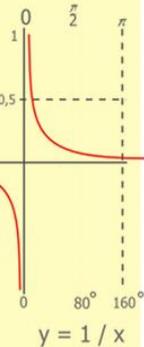
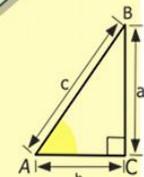
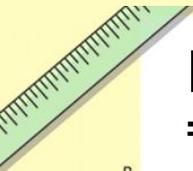
$$2. \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3. Если  $M = (x; y; z)$  – середина  $AB$ , то

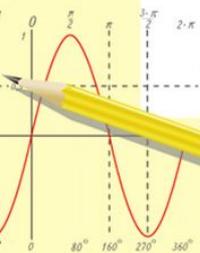
$$x = \frac{x_1 + x_2}{2}; \quad y = \frac{y_1 + y_2}{2}; \quad z = \frac{z_1 + z_2}{2}$$

$$4. \quad \text{Если } \vec{a} = (x; y; z), \text{ то } |\vec{a}| = \sqrt{x^2 + y^2 + z^2}$$

$$5. \quad \cos \varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}, \quad \text{где } \varphi = \angle(\vec{a}; \vec{b})$$



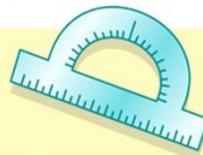
$$\begin{array}{r} 1 \\ \times 2500 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

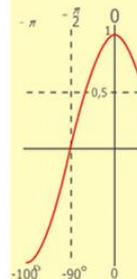
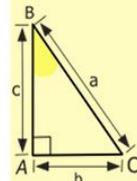
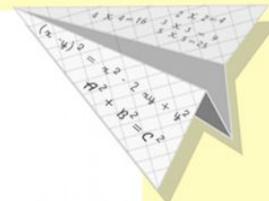


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

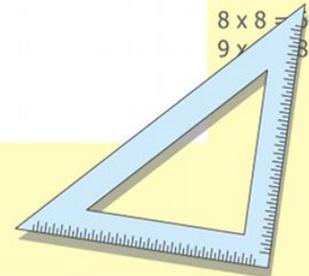
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



# Простейшие задачи в

## координатах

Даны  $A = (4; 6; -1)$   $B = (0; -2; 5)$   $C = (3; 1; 1)$

Точки:  
1. Вычислить координаты

векторов:

$$\vec{AB} = \begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 0-4 \\ -2-6 \\ 5-(-1) \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \\ 6 \end{pmatrix}$$

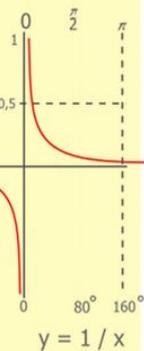
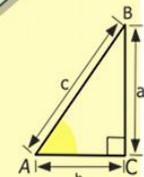
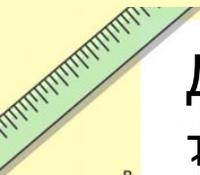
$$\vec{BC} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 2 \end{pmatrix}$$

2. Вычислить длины

векторов:

$$|\vec{AB}| = \sqrt{(-4)^2 + (-8)^2 + 6^2} = \sqrt{16 + 64 + 36} = \sqrt{116} = 2\sqrt{29}$$

$$|\vec{AC}| = \sqrt{(-1)^2 + (-5)^2 + 2^2} = \sqrt{1 + 25 + 4} = \sqrt{30}$$



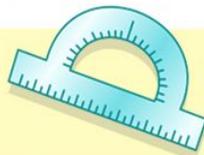
$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ 10500 \\ \hline \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

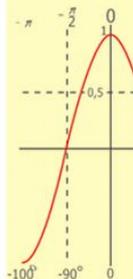
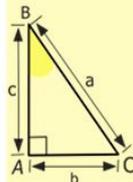
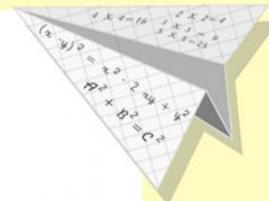


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

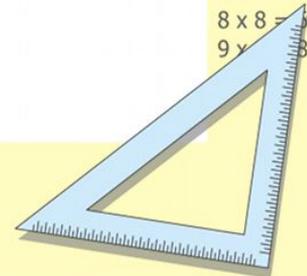
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$



$$y = \cos$$

$$\begin{array}{l} 2 \times 2 = 4 \\ 3 \times 3 = 9 \\ 4 \times 4 = 16 \\ 5 \times 5 = 25 \\ 6 \times 6 = 36 \\ 7 \times 7 = 49 \\ 8 \times 8 = 64 \\ 9 \times 9 = 81 \end{array}$$



# Простейшие задачи в координатах

Даны  $A = (4; 6; -1)$   $B = (0; -2; 5)$   $C = (3; 1; 1)$

точки:

3. Вычислить длину медианы  $CM$  из

$\triangle ABC$

Если  $CM$  медиана, то  $M = (x; y; z)$  середина

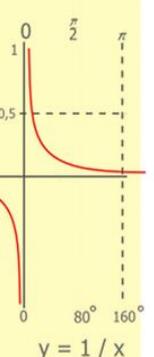
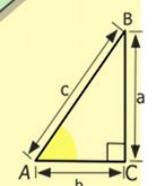
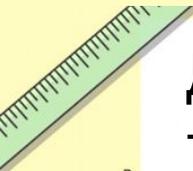
$AB$

$$x = \frac{4+0}{2} = 2; \quad y = \frac{6+(-2)}{2} = 2; \quad z = \frac{-1+5}{2} = 2$$

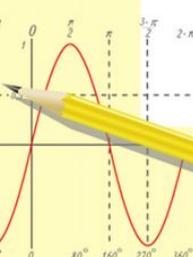
т.е.  $M = (2; 2; 2)$

2)

$$CM = \sqrt{(2-3)^2 + (2-1)^2 + (2-1)^2} = \sqrt{1+1+1} = \sqrt{3}$$



$$\begin{array}{r} 2500 \\ \times 42 \\ \hline 2100 \\ + 8400 \\ \hline 105000 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

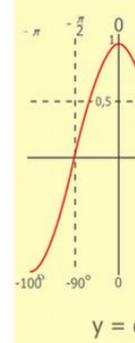
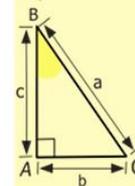
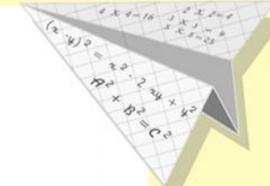
$$\sin 90^\circ = 1$$



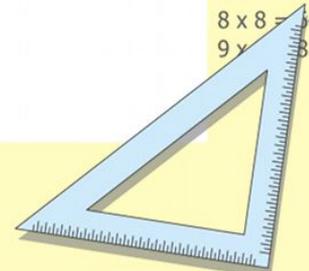
$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$



|            |
|------------|
| 2 x 2 = 4  |
| 3 x 3 = 9  |
| 4 x 4 = 16 |
| 5 x 5 = 25 |
| 6 x 6 = 36 |
| 7 x 7 = 49 |
| 8 x 8 = 64 |
| 9 x 9 = 81 |



# Простейшие задачи в

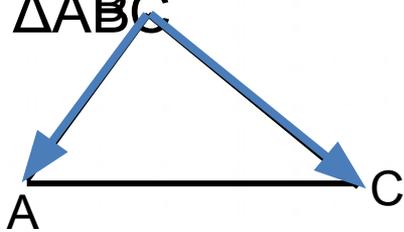
## координатах

Даны  $A = (2; 1; -1)$   $B = (4; 0; -2)$   $C = (-1; 3; 2)$

Точки:

4. Вычислить  $\angle B$  в

$\triangle ABC$



$$\cos B = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}| \cdot |\overrightarrow{BC}|}$$

$$\overrightarrow{BA} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \quad \overrightarrow{BC} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix}$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix} = 10 + 3 + 4 = 17$$

$$|\overrightarrow{BA}| = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$$

$$|\overrightarrow{BC}| = \sqrt{(-5)^2 + 3^2 + 4^2} = \sqrt{25 + 9 + 16} = \sqrt{50} = 5\sqrt{2}$$

$$\cos B = \frac{17}{\sqrt{6} \cdot 5\sqrt{2}} = \frac{17}{5\sqrt{12}} = \frac{17}{10\sqrt{3}}$$

$$\angle B = \arccos \frac{17}{10\sqrt{3}}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

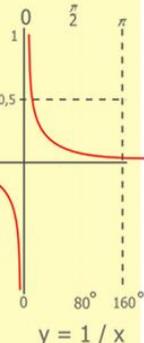
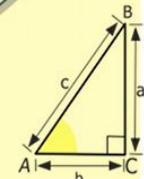
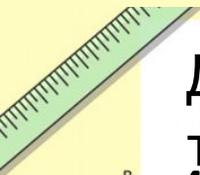
$$\sin 90^\circ = 1$$

$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

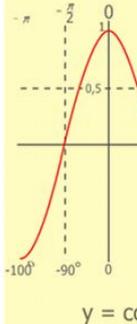
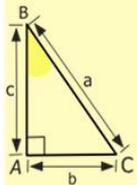
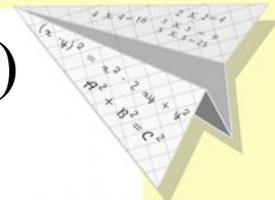
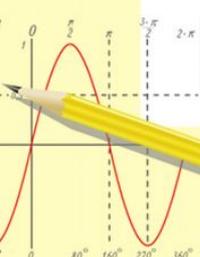
$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

$$x = 70$$

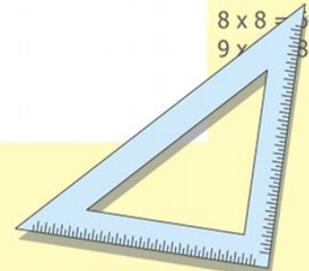
$$(x+y)(x-y) = x^2 - y^2$$



$$\begin{array}{r} 1 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$



- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
- 6 x 6 = 36
- 7 x 7 = 49
- 8 x 8 = 64
- 9 x 9 = 81



# Простейшие задачи в

## координатах

5.  $AB_1C_1D_1$  – прямоугольный параллелепипед,  $AB=1$ ,  $BC=2$ ,  $AA_1=3$ .  
Найти угол между прямыми  $AC$  и  $B_1D$

Решение  $A = (1; 0; 0)$       $C = (0; 2; 0)$

:  $D = (1; 2; 0)$       $B_1 = (0; 0; 3)$

$$\cos \varphi = \frac{\overrightarrow{AC} \cdot \overrightarrow{B_1D}}{|\overrightarrow{AC}| \cdot |\overrightarrow{B_1D}|}$$

$$\overrightarrow{AC} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \quad \overrightarrow{B_1D} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

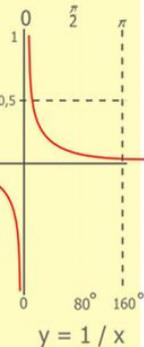
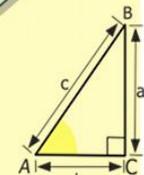
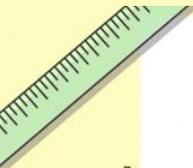
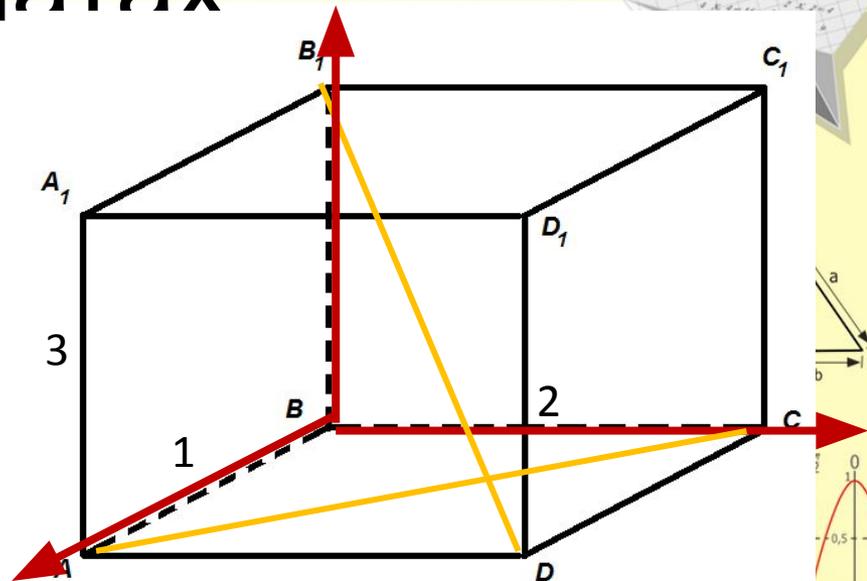
$$\overrightarrow{AC} \cdot \overrightarrow{B_1D} = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = -1 + 4 + 0 = 3$$

$$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 2^2 + 0^2} = \sqrt{1 + 4 + 0} = \sqrt{5}$$

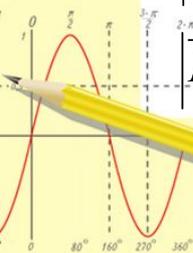
$$|\overrightarrow{B_1D}| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

$$\cos B = \frac{3}{\sqrt{5} \cdot \sqrt{14}} = \frac{3}{\sqrt{70}}$$

$$\angle B = \arccos \frac{3}{\sqrt{70}}$$



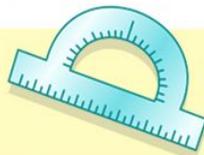
$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 10500 \end{array}$$



$$\frac{a}{A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$

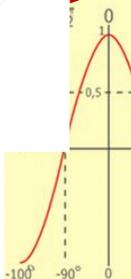
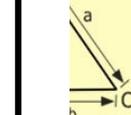


$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \end{cases}$$

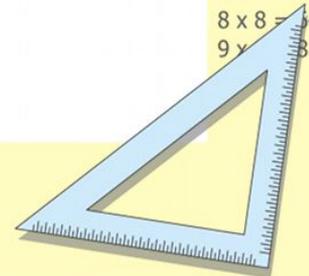
$$x = 70$$

$$(x+y)(x-y) = x^2 - y^2$$

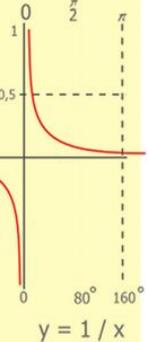
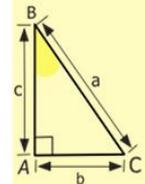
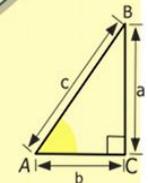
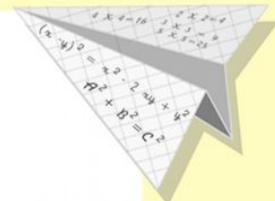
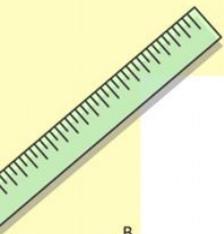


$$y = \cos$$

- 2 x 2 = 4
- 3 x 3 = 9
- 4 x 4 = 16
- 5 x 5 = 25
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- 8 x 8 = 64
- 9 x 9 = 81

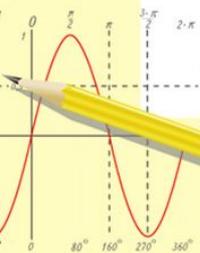


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Юрьевич  
преподаватель математики  
ГБПОУ ЗКНО  
Москва, 2021г.



$$\begin{array}{r} 1 \\ 2500 \\ \times 42 \\ \hline 210 \\ + 84 \\ \hline 105000 \end{array}$$

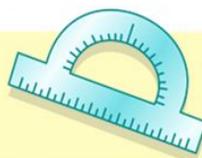
- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$
- $6 \times 6 = 36$
- $7 \times 7 = 49$
- $8 \times 8 = 64$
- $9 \times 9 = 81$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\sin 90^\circ = 1$$



$$\begin{cases} y = \sin 90 \\ x = 25y + 45 \end{cases}$$

$$\begin{cases} y = 1 \\ x = 25 + 45 \\ \hline x = 70 \end{cases}$$

$$(x+y)(x-y) = x^2 - y^2$$

