

# Решение тригонометрических уравнений

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**К простейшим тригонометрическим уравнениям относятся уравнения:**

$$\sin x = a;$$

$$\cos x = a;$$

$$\operatorname{tg} x = a;$$

$$\operatorname{ctg} x = a;$$

$\sin x = a$ , при  $|a| \leq 1$ :

$$x = (-1)^n \arcsin a + \pi n;$$

$$x = \arcsin a + 2\pi k;$$

$$x = \pi - \arcsin a + 2\pi k;$$

$$\sin x = a, \quad \text{при } |a| \leq 1:$$

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$\sin x = a$ , при  $|a| \leq 1$ :

$\sin x = a;$   
 $\cos x = a;$  } **не имеют решений;**

$\operatorname{tg} x = a$  имеет решение

$$x = \operatorname{arctg} a + \pi n;$$

$\operatorname{ctg} x = a$  имеет решение

$$x = \operatorname{arcctg} a + \pi n;$$

## Частные случаи:

	$a = 0$	$a = -1$	$a = 1$
$\sin x = a$	$x = \pi n$	$x = -\frac{\pi}{2} + 2\pi n$	$x = \frac{\pi}{2} + 2\pi n$
$\cos x = a$	$x = \frac{\pi}{2} + \pi n$	$x = \pi + 2\pi n$	$x = 2\pi n$



$\sin x = a, \quad \text{при } |a| \leq 1:$

**Решение.**

$\sin x = a, \quad \text{при } |a| \leq 1:$

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$\sin x = a, \quad \text{при } |a| \leq 1: \implies \sin x = a, \quad \text{при } |a| \leq 1:$

$x = (-1)^n \arcsin a + \pi n;$

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$$x = (-1)^n \frac{\pi}{16} + \frac{\pi n}{4}.$$

**Ответ:**  $x = (-1)^n \frac{\pi}{16} + \frac{\pi n}{4}. \quad \blacktriangleleft$

**Замечание.**

$\sin x = a, \quad \text{при } |a| \leq 1: \implies \sin x = a, \quad \text{при } |a| \leq 1:$

$$\sin x = a, \quad \text{при } |a| \leq 1:$$

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Решение.

$$\sin x = a, \quad \text{при } |a| \leq 1: \quad x = (-1)^n \frac{\pi}{16} + \frac{\pi n}{4};$$

$$\sin x = a, \quad \text{при } |a| \leq 1: \quad x = (-1)^0 \frac{\pi}{16} + 0 = \frac{\pi}{16}; \quad \sin x = a, \quad \text{при } |a| \leq 1:$$

$$\sin x = a, \quad \text{при } |a| \leq 1: \quad x = (-1)^1 \frac{\pi}{16} + \frac{\pi \cdot 1}{4} = -\frac{\pi}{16} + \frac{\pi}{4} = \frac{3\pi}{16}; \quad \sin x = a, \quad \text{при } |a| \leq 1:$$

$$\sin x = a, \quad \text{при } |a| \leq 1: \quad x = (-1)^2 \frac{\pi}{16} + \frac{\pi}{2} = \frac{\pi}{16} + \frac{\pi}{2} = \frac{9\pi}{16}; \quad \sin x = a, \quad \text{при } |a| \leq 1:$$

$$\sin x = a, \quad \text{при } |a| \leq 1: \quad x = (-1)^{-1} \frac{\pi}{16} - \frac{\pi}{4} = -\frac{\pi}{16} - \frac{\pi}{4} = -\frac{5\pi}{16}; \quad \sin x = a, \quad \text{при } |a| \leq 1:$$

$$\sin x = a, \quad \text{при } |a| \leq 1: \quad \blacksquare$$

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Решение.

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$$y^2 - 3y + 2 = 0;$$

$$y = 1; y = 2;$$

$$\sin x = a, \quad \text{при } |a| \leq 1; \quad \sin x = a, \quad \text{при } |a| \leq 1:$$

$$\operatorname{tg} t \cdot \operatorname{ctg} t = 1$$

$\operatorname{tg} x = a$  имеет решение

$$x = \operatorname{arctg} a + \pi n;$$

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$$\operatorname{tg} t \cdot \operatorname{ctg} t = 1$$

$$\sin x = a, \quad \text{при } |a| \leq 1:$$

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Решение.

$$\sin 7x (2 \cos x - 1) = 0;$$

$$\sin 7x = 0; \quad 2 \cos x - 1 = 0;$$

$$2 \cos x = 1;$$

$$\sin x = a, \quad \text{при } |a| \leq 1:$$

	$a = 0$	$a = -1$	$a = 1$
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t	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$
cost	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

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Замечание.

$$f_1(x) \cdot f_2(x) = 0;$$

$$\sin x = a, \quad \text{при } |a| \leq 1:$$

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Решение.

$$\operatorname{ctg} x = 0; \quad \cos x - 1 = 0;$$

$$\cos x = 1;$$

t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
ctg t	-	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

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$x = 2\pi n$  – это **посторонний корень**;

$$\sin x = a, \quad \text{при } |a| \leq 1: \quad \blacktriangleleft$$

Решить уравнения самостоятельно в тетради

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$$\sin x = a, \quad \text{при } |a| \leq 1:$$



# Ответы

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$$\sin x = a, \quad \text{при } |a| \leq 1:$$