Метод перемещений расчета на устойчивость

Пример расчета

Алгоритм метода перемещений расчета на устойчивость

1. Определение степени кинематической неопределимости

$$n = n_{\text{у}} + n_{\text{л}}$$

и выбор основной системы метода перемещений.

2. Формирование матрицы жесткости

$$\mathbf{R}(v) = \begin{bmatrix} r_{11}(v) & r_{12}(v) & \dots & r_{1n}(v) \\ r_{21}(v) & r_{22}(v) & \dots & r_{2n}(v) \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1}(v) & r_{n1}(v) & \dots & r_{nn}(v) \end{bmatrix}$$

2.1. Построение единичных эпюр моментов $\bar{M}_i \quad (i = 1, 2, \dots, n)$

2.2. Определение коэффициентов $r_{ij}(v) \quad (i, j = 1, 2, \dots, n)$.

3. Решение характеристического уравнения

$$D(v) = |\mathbf{R}(v)| = 0$$

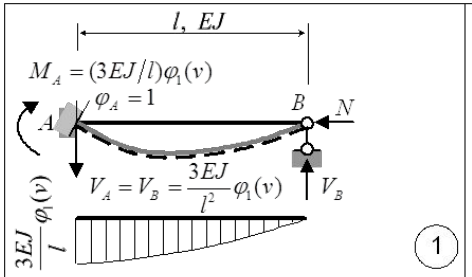
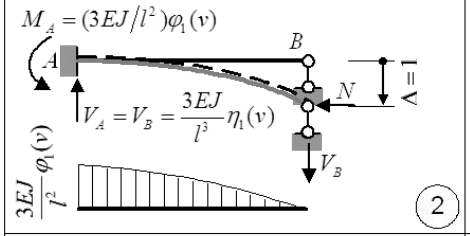
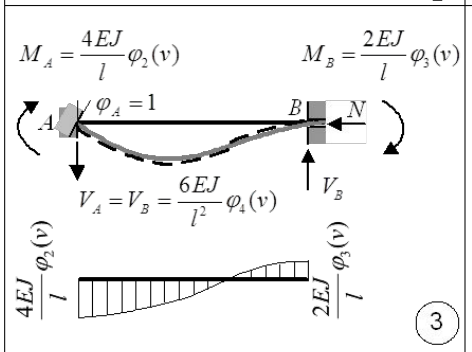
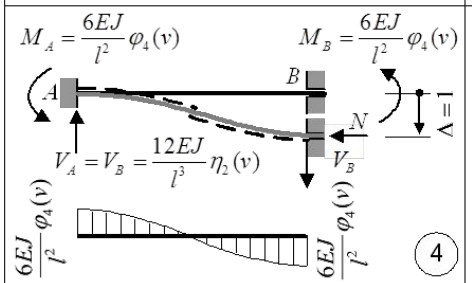
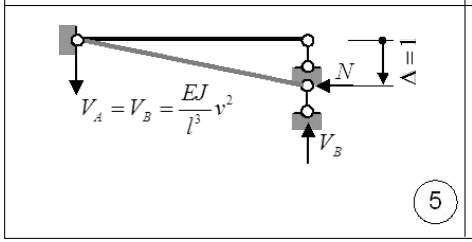
с целью определения минимального корня v_{cr}.

4. Определение критического параметра нагрузки F_{cr}.

5. Определение коэффициентов приведенных длин μ сжатых элементов.

6. Построение формы потери устойчивости

Таблица метода перемещений и специальные функции расчета на устойчивость

	$\varphi_1(v) = \frac{v^2}{3\left(1-\frac{v}{\operatorname{tg} v}\right)} = \frac{v^2}{3}\frac{\operatorname{tg} v}{\operatorname{tg} v - v}$
	$\eta_1(v) = \frac{v^2}{3\left(\frac{\operatorname{tg} v}{v}-1\right)} = \frac{v^3}{3(\operatorname{tg} v - v)}$
	$\varphi_2(v) = \frac{1-\frac{v}{\operatorname{tg} v}}{4\left(\frac{\operatorname{tg} v/2}{v/2}-1\right)} = \frac{v}{8\operatorname{tg} v}\frac{\operatorname{tg} v - v}{\operatorname{tg}\frac{v}{2}-\frac{v}{2}}$ $\varphi_3(v) = \frac{\frac{v}{\sin v}-1}{2\left(\frac{\operatorname{tg} v/2}{v/2}-1\right)} = \frac{v}{4\sin v}\frac{v-\sin v}{\operatorname{tg}\frac{v}{2}-\frac{v}{2}}$ $\varphi_4(v) = \varphi_1\left(\frac{v}{2}\right)$
	$\eta_2(v) = \eta_1\left(\frac{v}{2}\right)$
	$v = l\sqrt{\frac{N}{EJ}}$

$$\varphi 1(v) := \begin{cases} 1 & \text{if } v = 0 \\ \frac{v^2}{3}\cdot\frac{\tan(v)}{\tan(v)-v} & \text{otherwise} \end{cases}$$

$$\varphi 2(v) := \begin{cases} 1 & \text{if } v = 0 \\ \frac{v}{8\cdot\tan(v)}\cdot\frac{\tan(v)-v}{\tan\left(\frac{v}{2}\right)-\frac{v}{2}} & \text{otherwise} \end{cases}$$

$$\varphi 3(v) := \begin{cases} 1 & \text{if } v = 0 \\ \frac{v}{4\cdot\sin(v)}\cdot\frac{v-\sin(v)}{\tan\left(\frac{v}{2}\right)-\frac{v}{2}} & \text{otherwise} \end{cases}$$

$$\varphi 4(v) := \varphi 1\left(\frac{v}{2}\right)$$

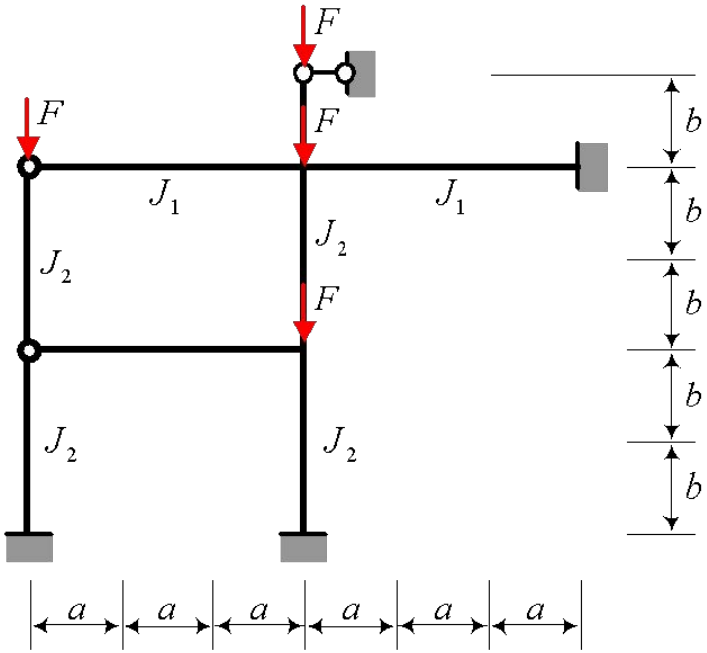
$$\eta 1(v) := \begin{cases} 1 & \text{if } v = 0 \\ \frac{v^3}{3\cdot(\tan(v)-v)} & \text{otherwise} \end{cases}$$

$$\eta 2(v) := \eta 1\left(\frac{v}{2}\right)$$

Пример

1. Выбор основной системы

Заданная рама

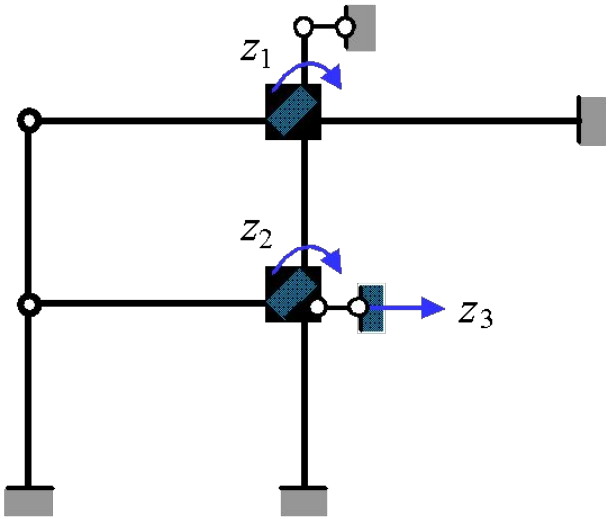


$b = 1.5\ м$

$a = 2\ м$

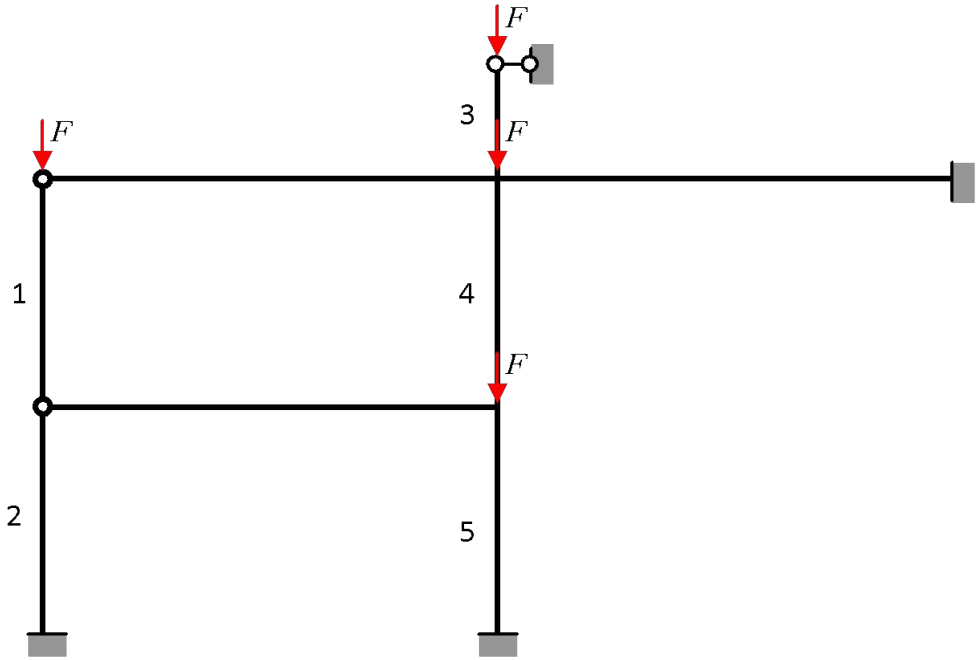
$J_1 / J_2 = k$

Основная система метода перемещений



2. Формирование матрицы жесткости

Определение параметра ν для каждого сжатого стержня



$$\nu_1 = 3\sqrt{\frac{\ }{\ }} = \frac{\xi\sqrt{3}}{3}$$

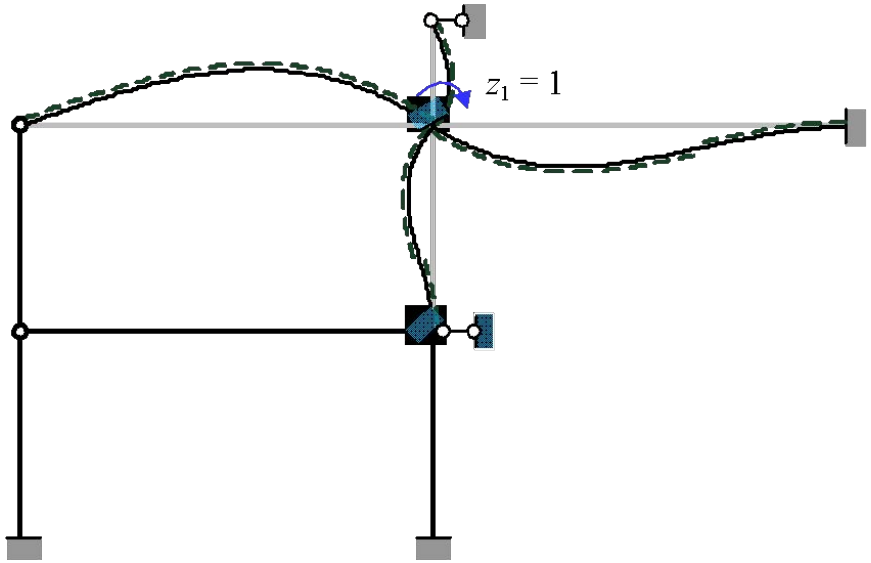
$$\nu_2 = 3\sqrt{\frac{\ }{\ }} = \frac{\xi\sqrt{3}}{3}$$

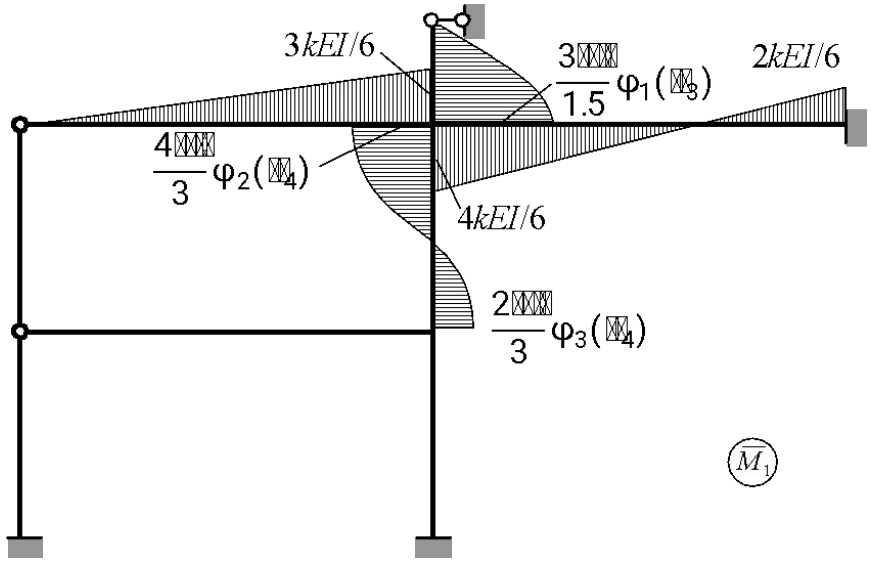
$$\nu_3 = 1.5\sqrt{\frac{\ }{\ }} = \frac{\xi\sqrt{3}}{6}$$

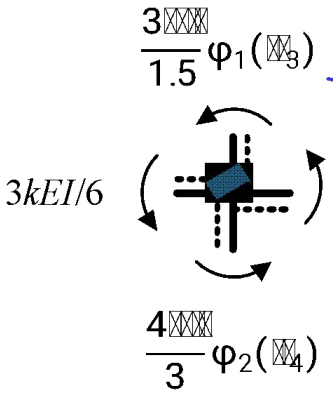
$$\nu_4 = 3\sqrt{\frac{2\ }{\ }} = \frac{\xi\sqrt{6}}{3}$$

$$\nu_5 = 3\sqrt{\frac{3\ }{\ }} =$$

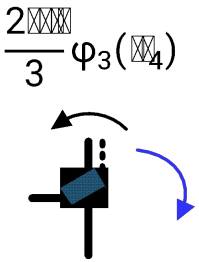
2.1 Построение единичных эпюр и
2.2. Определение коэффициентов матрицы жесткости



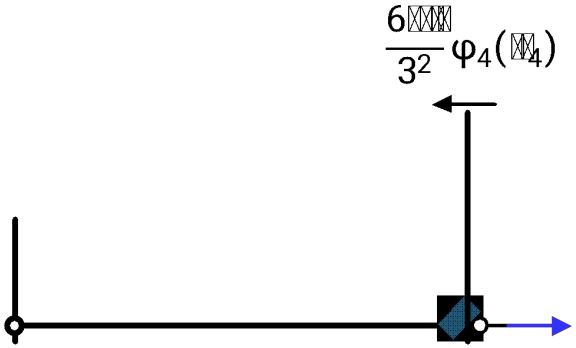




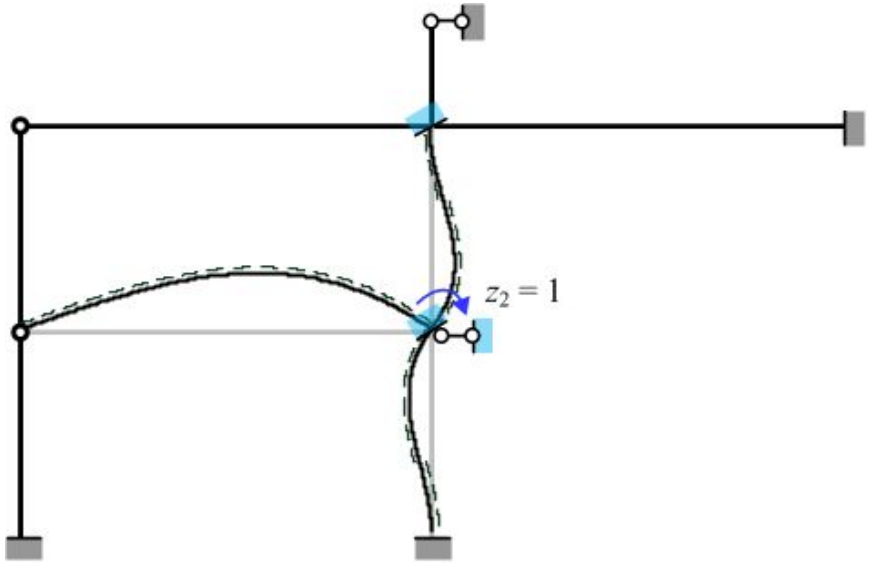
$$z_{11} = \frac{3}{1.5}\varphi_1(z_3) + \frac{4}{6} + \frac{4}{3}\varphi_2(z_4) + \frac{3}{6}$$

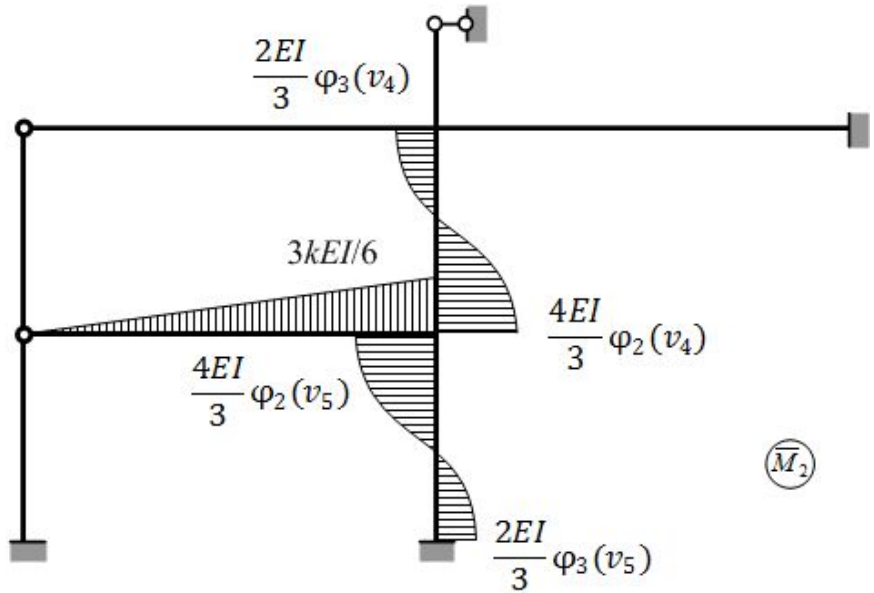


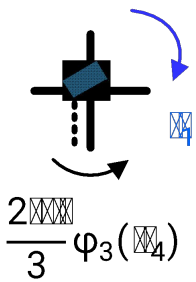
$$z_{21} = \frac{2}{3}\varphi_3(z_4)$$



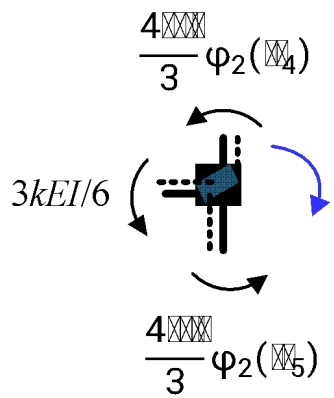
$$z_{31} = \frac{6}{3^2}\varphi_4(z_4)$$



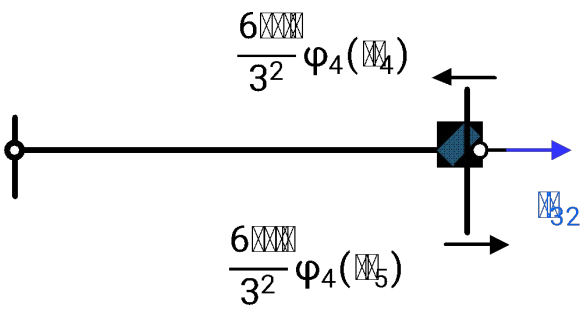




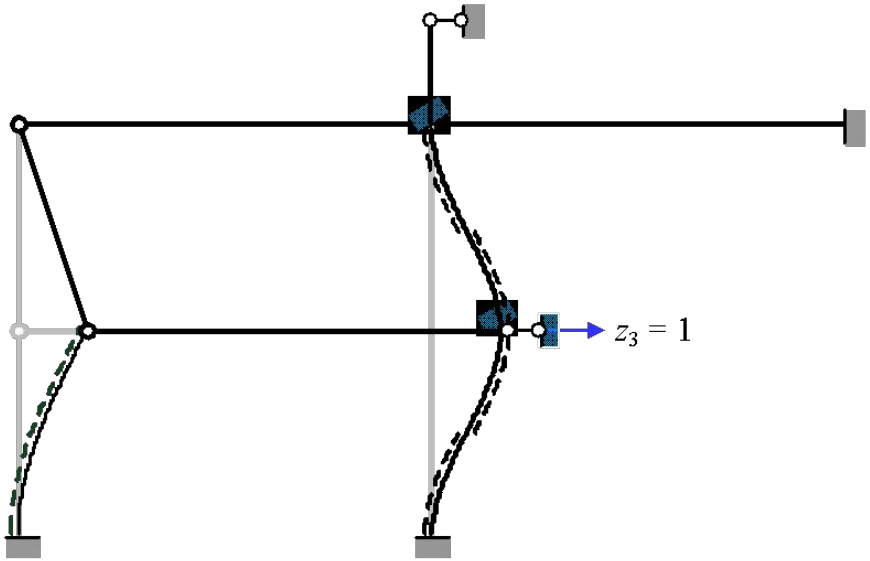
$$r_{12} = \frac{2EI}{3}\varphi_3(v_4)$$

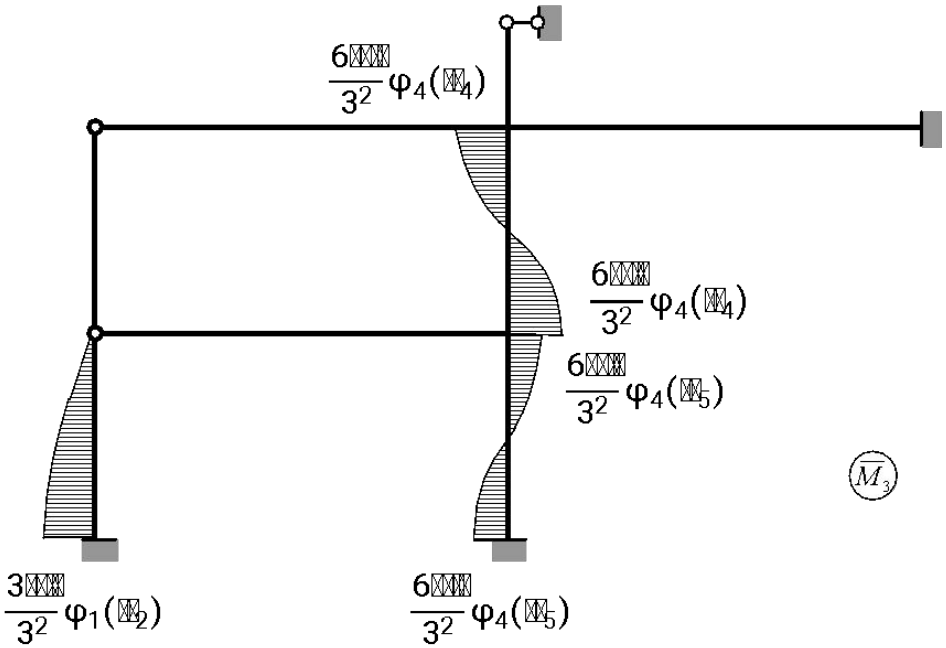


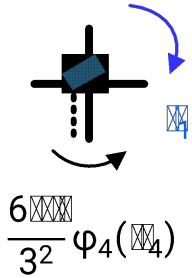
$$r_{22} = \frac{4EI}{3}\varphi_2(v_4) + \frac{3kEI}{6} + \frac{4EI}{3}\varphi_2(v_5)$$

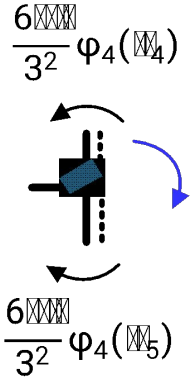


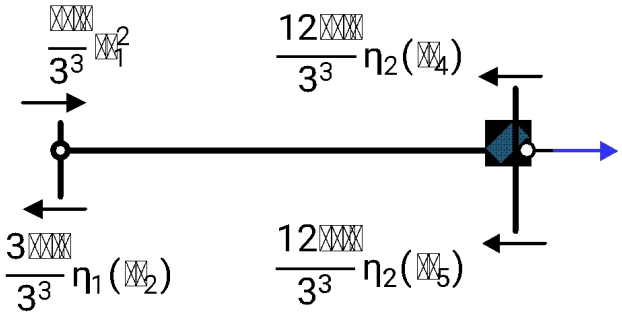
$$r_{32} = \frac{6EI}{3^2}\varphi_4(v_4) - \frac{6EI}{3^2}\varphi_4(v_5)$$







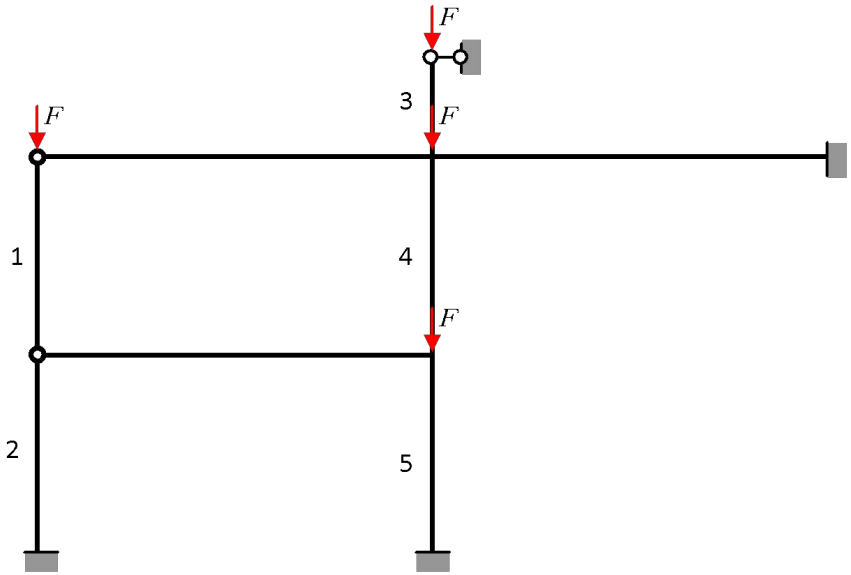




$$\eta_{33}EI = \frac{12EI}{3^3}\eta_2(z_4) + \frac{12EI}{3^3}\eta_2(z_5) + \frac{3EI}{3^3}\eta_1(z_2) - \frac{EI}{3^3}z_1^2$$

Реализация в среде MathCad

(MathCad-текст оконтурен)



ГОСТ-26020-83

$I1 := 10400 \cdot 10^{-8}$ 30Ш1 - сечение ригеля

$I2 := 3820 \cdot 10^{-8}$ 20К1 - сечение стоек

$a := 2$ $b := 1.5$

$L := 3 \cdot a = 6$ $h := 2 \cdot b = 3$

$k := \frac{I1}{I2} = 2.723$

$EI := 1$

$v1 := 3 \cdot \sqrt{\frac{F}{EI}}$ $v2 := 3 \cdot \sqrt{\frac{F}{EI}}$ $v3 := \frac{3}{2} \cdot \sqrt{\frac{F}{EI}}$ $v4 := 3 \cdot \sqrt{\frac{2 \cdot F}{EI}}$ $v5 := 3 \cdot \sqrt{\frac{3 \cdot F}{EI}}$

v5 = v

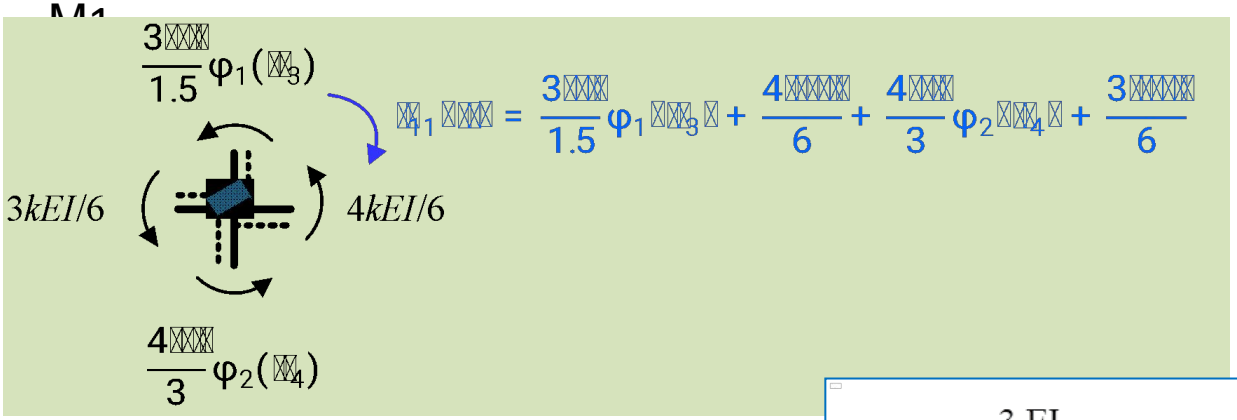
$\frac{v1}{v5} \rightarrow \frac{\sqrt{3}}{3}$ $v1(v) := \frac{\sqrt{3}}{3} \cdot v$ $\frac{v2}{v5} \rightarrow \frac{\sqrt{3}}{3}$ $v2(v) := \frac{\sqrt{3}}{3} \cdot v$

$\frac{v3}{v5} \rightarrow \frac{\sqrt{3}}{6}$ $v3(v) := \frac{\sqrt{3}}{6} \cdot v$ $\frac{v4}{v5} \rightarrow \frac{\sqrt{2} \cdot \sqrt{3}}{3}$ $v4(v) := \frac{\sqrt{6}}{3} \cdot v$ $v5(v) := v$

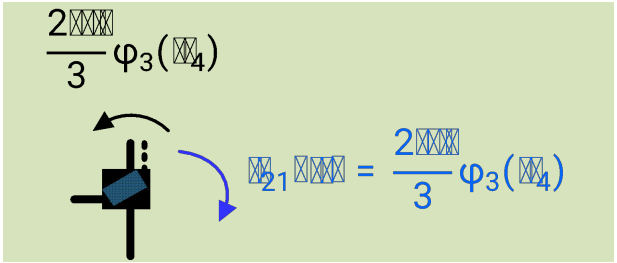
$\boxed{} = \boxed{} \sqrt{\frac{\boxed{}}{\boxed{}}}$

2. Формирование матрицы жесткости (МЖ)

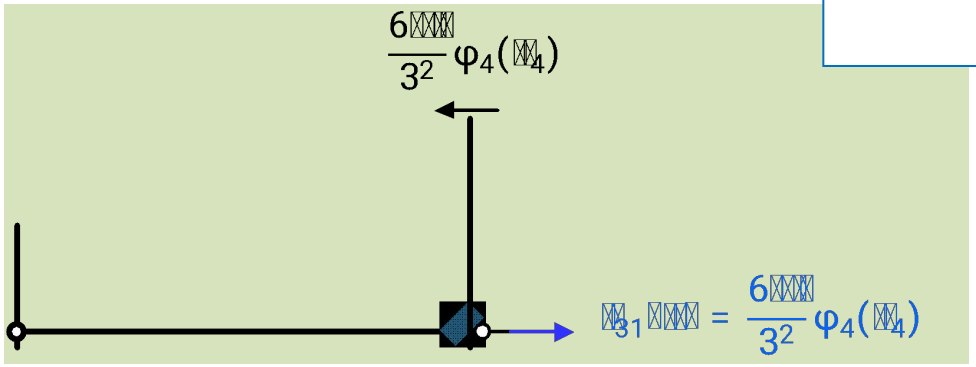
Определение коэффициентов первого столбца МЖ по эп. М1



$$r_{11}(v) = \frac{3EI}{1.5}\varphi_1(v_3) + \frac{4kEI}{6} + \frac{4EI}{3}\varphi_2(v_4) + \frac{3kEI}{6}$$



$$r_{21}(v) = \frac{2EI}{3}\varphi_3(v_4)$$



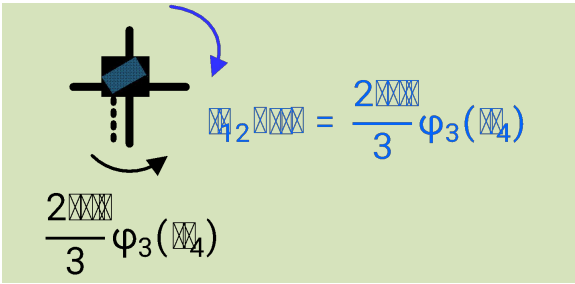
$$r_{31}(v) = \frac{6EI}{3^2}\varphi_4(v_4)$$

$$r11(v) := \frac{3\cdot EI}{b}\cdot\varphi1(v3(v)) + \frac{4\cdot k\cdot EI}{L} + \frac{4\cdot EI}{h}\cdot\varphi2(v4(v)) + \frac{3\cdot k\cdot EI}{L}$$

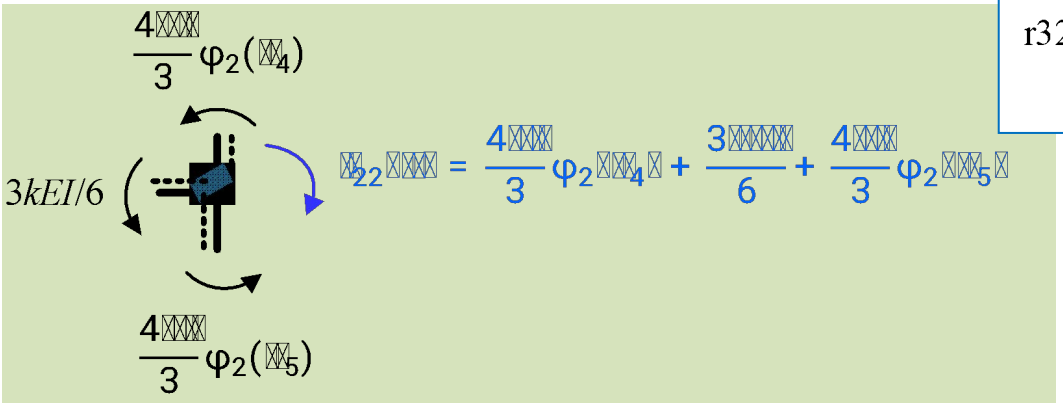
$$r21(v) := \frac{2\cdot EI}{h}\cdot\varphi3(v4(v))$$

$$r31(v) := \frac{6\cdot EI}{h^2}\cdot\varphi4(v4(v))$$

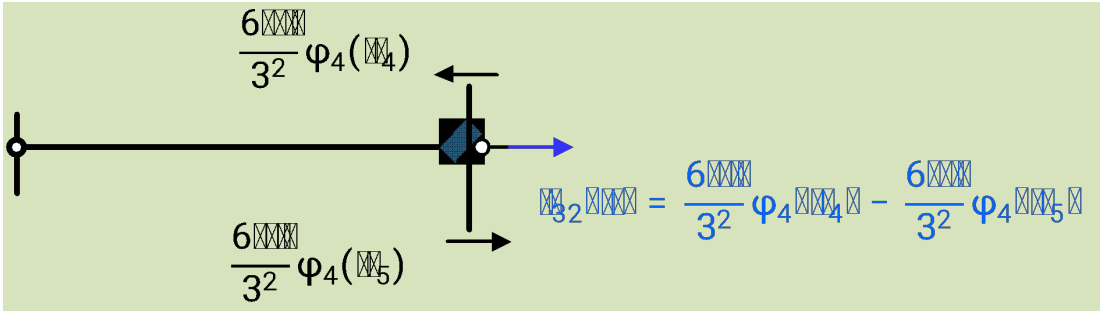
Определение коэффициентов второго столбца МЖ по эп. М2



$$r_{12}(v_4) = \frac{2EI}{3}\varphi_3(v_4)$$



$$r_{22}(v_4) = \frac{4EI}{3}\varphi_2(v_4) + \frac{3kEI}{6} + \frac{4EI}{3}\varphi_2(v_5)$$



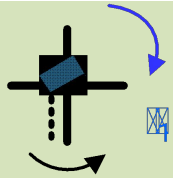
$$r_{32}(v_4) = \frac{6EI}{3^2}\varphi_4(v_4) - \frac{6EI}{3^2}\varphi_4(v_5)$$

$$r12(v) := \frac{2\cdot EI}{h}\cdot\varphi3(v4(v))$$

$$r22(v) := \frac{4\cdot EI}{h}\cdot\varphi2(v4(v)) + \frac{3\cdot k\cdot EI}{L} + \frac{4\cdot EI}{h}\cdot\varphi2(v5(v))$$

$$r32(v) := \frac{6\cdot EI}{h^2}\cdot\varphi4(v4(v)) - \frac{6\cdot EI}{h^2}\cdot\varphi4(v5(v))$$

Определение коэффициентов третьего столбца МЖ по эп.

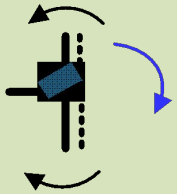


$$r_{13}(v) = \frac{6EI}{h^2}\varphi_4(v_4)$$

$$\frac{6EI}{h^2}\varphi_4(v_4)$$

$$r_{23}(v) = \frac{6EI}{h^2}\varphi_4(v_4) - \frac{6EI}{h^2}\varphi_4(v_5)$$

$$\frac{6EI}{h^2}\varphi_4(v_4)$$



$$\frac{6EI}{h^2}\varphi_4(v_5)$$

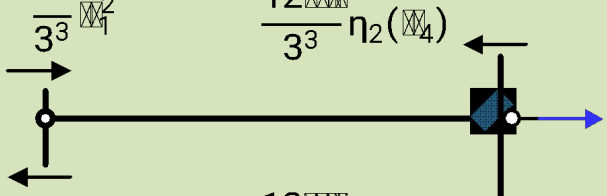
$$r_{13}(v) := \frac{6\cdot EI}{h^2}\cdot\varphi4(v4(v))$$

$$r23(v) := \frac{6\cdot EI}{h^2}\cdot\varphi4(v4(v)) - \frac{6\cdot EI}{h^2}\cdot\varphi4(v5(v))$$

$$r33(v) := \frac{12\cdot EI}{h^3}\cdot\eta2(v4(v)) + \frac{12\cdot EI}{h^3}\cdot\eta2(v5(v)) + \frac{3\cdot EI}{h^3}\cdot\eta1(v2(v)) - \frac{EI}{h^3}\cdot v1(v)^2$$

$$\frac{EI}{h^3}v_1^2$$

$$\frac{12EI}{h^3}\eta_2(v_4)$$



$$\frac{3EI}{h^3}\eta_1(v_2)$$

$$\frac{12EI}{h^3}\eta_2(v_5)$$

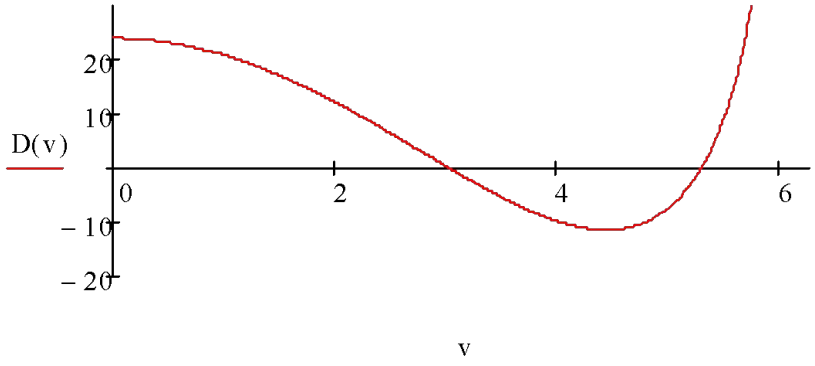
$$r_{33}(v) = \frac{12EI}{h^3}\eta_2(v_4) + \frac{12EI}{h^3}\eta_2(v_5) + \frac{3EI}{h^3}\eta_1(v_2) - \frac{EI}{h^3}v_1^2$$

Матрица жесткости

$$R(v) := \begin{pmatrix} r11(v) & r12(v) & r13(v) \\ r21(v) & r22(v) & r23(v) \\ r31(v) & r32(v) & r33(v) \end{pmatrix}$$

3. Решение характеристического уравнения

$D(v) := |R(v)|$



$vcr := root(D(v), v, 2, 3.5)$ $vcr = 3.034$

Интерфейс с функцией MathCad-а root()

root(f(var), var, [a, b])

Возвращает значение переменной var, для которого функция f принимает нулевое значение. Если указаны параметры a и b, функция root находит значение var на этом интервале. В противном случае var необходимо определить с помощью начального приближения до вызова функции root.

4. Определение критического параметра нагрузки F_{cr}

$$Fcr := \frac{vcr^2 \cdot EI}{3^2 \cdot 3} = 0.341$$

поскольку $v = v5 = 3 \cdot \sqrt{\frac{3 \cdot F}{EI}}$

$$EI := 2 \cdot 10^{11} \cdot I2 = 7.64 \times 10^6$$

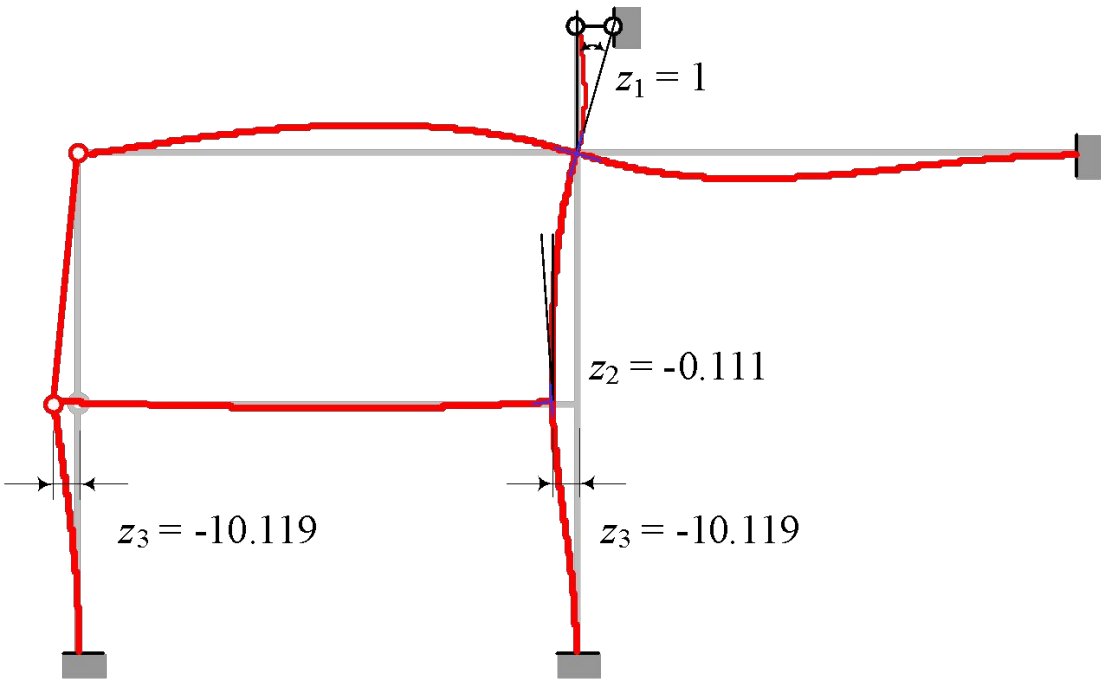
$$Fcr := \frac{vcr^2 \cdot EI}{h^2 \cdot 3} = 2.605 \times 10^6$$

5. Определение коэффициентов приведенных длин μ сжатых элементов

$$\mu 1 := \frac{\pi}{v1(vcr)} = 1.793 \qquad \mu 2 := \frac{\pi}{v2(vcr)} = 1.793 \qquad \mu 3 := \frac{\pi}{v3(vcr)} = 3.586$$

$$\mu 4 := \frac{\pi}{v4(vcr)} = 1.268 \qquad \mu 5 := \frac{\pi}{v5(vcr)} = 1.035$$

6. Построение формы потери устойчивости



$z1 := 1 \qquad z2 := 0 \qquad z3 := 0$

Given

$$r12(vcr)\cdot z2 + r13(vcr)\cdot z3 = -(r11(vcr)\cdot z1)$$

$$r22(vcr)\cdot z2 + r23(vcr)\cdot z3 = -(r21(vcr)\cdot z1)$$

$$\begin{pmatrix} z2 \\ z3 \end{pmatrix} := \mathrm{Find}(z2, z3) = \begin{pmatrix} -0.111 \\ -10.119 \end{pmatrix}$$