

$L(q(x), h, H)$  шекаралық  
есептің  
меншікті мәндері  
және  
 $\Delta(\lambda)$  характеристикалық  
функцияның нөлдері.

Соқмамен

$$\Delta(\lambda) = V(\psi) = -U(\psi) \quad (5)$$

$\Delta(\lambda) = \langle \psi, \psi \rangle$  - характеристикалык функция табалады

Т. 1.1. Характеристикалык  $\Delta(\lambda)$  функцияның нөз нөздері  $L$  операторының есентік  $\{\lambda_n\}$  нөздерімен беттеседі.  $\psi(x, \lambda_n)$ ,  $\psi(x, \lambda_n)$  - меншікті функциялар болып табалады және

$$\psi(x, \lambda_n) = \beta_n \varphi(x, \lambda_n), \quad \beta_n \neq 0, \quad \exists \{\beta_n\}_{n \geq 0} \quad (6)$$

$\square$  1)  $\lambda_0$ -сана  $\Delta(\lambda)$  функцисолгныг нөхцө, эгнэ  $\Delta(\lambda_0) = 0$  боловч. Сонго  $\langle \psi, \varphi \rangle$  Вронне кием  $x$ -кэ тэгүүлэгс болуогман

$$\Delta(\lambda_0) = 0 = \psi(x, \lambda_0) \cdot \varphi'(x, \lambda_0) - \psi'(x, \lambda_0) \varphi(x, \lambda_0) = 0 \Leftrightarrow$$

$$\psi(x, \lambda_0) = \frac{\psi'(x, \lambda_0)}{\varphi'(x, \lambda_0)} \varphi(x, \lambda_0), \text{ эгнэ}$$

$$B = \frac{\psi'(x, \lambda_0)}{\varphi'(x, \lambda_0)}, \text{ ага (3) тэгүүлэгс болуогман}$$

$$U(\varphi) := \varphi'(0, \lambda_0) - h \varphi(0, \lambda_0) = \begin{vmatrix} \varphi(0, \lambda_0) = 1 \\ \varphi'(0, \lambda_0) = h \end{vmatrix} = 0,$$

$$V(\psi) := \psi'(\bar{\pi}, \lambda_0) + H \psi(\bar{\pi}, \lambda_0) = \begin{vmatrix} \psi(\bar{\pi}, \lambda_0) = 1 \\ \psi'(\bar{\pi}, \lambda_0) = -H \end{vmatrix} = 0.$$

$\varphi(x, \lambda_0), \psi(x, \lambda_0)$  функцисолгоор (2) шөрөгтөл катартаа таныгсрагс  $\Rightarrow \lambda_0$ -мөншкүгс мөхө,  $\varphi(x, \lambda_0), \psi(x, \lambda_0)$ -мөншкүгс функцисолгоор.

2) Кери,  $\lambda_0$  - менейкти мөне (Шек. 1 есеи)

$\Delta y_0(x, \lambda_0)$  - сыйкес менейкти функция болсон. Сонда (2)  $\Rightarrow$

$$U(y_0) := y_0'(0) - h y_0(0) = 0, \quad y_0 = y_0(x, \lambda_0).$$

$$V(y_0) := y_0'(\bar{n}) + H y_0(\bar{n}) = 0,$$

Эрине,  $y_0(0) \neq 0$ , себеди, егер  $y_0(0) = 0$ , онда  $U(y_0) := y_0'(0) - h y_0(0) = 0 \Rightarrow y_0'(0) = 0$ , ан Коши есединин минимал теоремасын тугала теорема (!) бойынша  $y_0(x, \lambda_0) \equiv 0$

$$\left\{ \begin{array}{l} y_0'(0, \lambda_0) = 0 \\ y_0(0, \lambda_0) \neq 0 \end{array} \right. \Rightarrow y_0(x) = C; \quad y_0(\lambda_0) = 0.$$

Сондуктан  $y_0(0, \lambda_0) \neq 0$ . Исептешкүчтөр менен  $y_0(0, \lambda_0) = 1$  деп есептеуге болот.

Τότε με βεβαιότητα  $y_0'(0, \lambda_0) - h y_0(0, \lambda_0) = 0$ ,

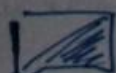
πολυπαι  $y_0'(0, \lambda_0) = h$ . Τακισι οδρροση

$$\begin{cases} y_0(0, \lambda_0) = 1, \\ y_0'(0, \lambda_0) = h, \end{cases} \text{ α πο γενοβιο} \quad \begin{cases} \varphi(0, \lambda) = 1, \\ \varphi'(0, \lambda_0) = h, \end{cases}$$

$\forall \lambda \in \mathbb{R}$  Οτιοδη  $y_0(x, \lambda_0) \equiv \varphi(x, \lambda_0)$ . Τότε

no (5):  $\Delta(\lambda) = V(\varphi(x, \lambda_0)) \Rightarrow$

$$\Delta(\lambda_0) = V(\varphi(x, \lambda_0)) = \bar{V}(y_0(x, \lambda_0)) = 0 \Rightarrow$$

$\lambda_0$  - τιμω χαριστουμετωρ εκκοφ φ γεκωμ  
 $\Delta(\lambda)$ . 

Εσκερσγ. Έστω  $y_0(0, \lambda_0) = c \neq 1$ , οπότε  
 $y_0'(0, \lambda_0) - h y_0(0, \lambda_0) = 0 \Rightarrow y_0'(0, \lambda_0) = c h.$

Ποσμε  $y_0(x, \lambda_0) = c \varphi(x, \lambda_0)$ , ετμε

$\forall \lambda \rightarrow !$  μεμικκι τυρεκεμυ  $\varphi(x, \lambda)$

τυρακτα καδευτικιμ ~~αφκ~~ εμεδριμεμ  
μεμταζ.

$$d_n := \int_0^{\bar{\pi}} \varphi^2(x, \lambda_n) dx. \quad (7).$$

def  $\{d_n\}_{n \geq 0}$  καταγορμν κατακεκτοκ  
καμερεμ, αε  $\{d_n, d_n\}_{n \geq 0}$  - μεμυτιαμν  
σπεκτρολεγι δεμεμεμ σ/αταλαγομ.

Лемма 1.1. Справедливо равенство

$$\beta_n \cdot \alpha_n = -\dot{\Delta}(\lambda_n), \quad (8)$$

где  $\beta_n = \frac{\psi(x, \lambda_n)}{\varphi(x, \lambda_n)}$ , а

$$\dot{\Delta}(\lambda_n) = \lim_{\lambda \rightarrow \lambda_n} \frac{\Delta(\lambda) - \Delta(\lambda_n)}{\lambda - \lambda_n}.$$



~~$$\frac{d}{dx} \langle \psi(x, \lambda_n), \psi(x, \lambda_n) \rangle = (\lambda - \lambda_n) \psi(x, \lambda) \psi(x, \lambda_n)$$~~

$$\frac{d}{dx} \langle \psi(x, \lambda), \psi(x, \lambda_n) \rangle = (\lambda - \lambda_n) \psi(x, \lambda) \cdot \psi(x, \lambda_n).$$

Докажем данное равенство.

$$\begin{aligned} \frac{d}{dx} \langle \psi(x, \lambda), \varphi(x, \lambda_n) \rangle &= (\psi(x, \lambda) \cdot \varphi'(x, \lambda_n) - \\ &- \psi'(x, \lambda) \varphi(x, \lambda_n))'_x = \underbrace{[\psi'(x, \lambda) \cdot \varphi'(x, \lambda_n) +} \\ &+ \psi(x, \lambda) \cdot \varphi''(x, \lambda_n)] - [\psi''(x, \lambda) \cdot \varphi(x, \lambda_n) +} \\ &+ \underbrace{\psi'(x, \lambda) \varphi'(x, \lambda_n)}] = \psi(x, \lambda) \varphi''(x, \lambda_n) - \\ &- \psi''(x, \lambda) \cdot \varphi(x, \lambda_n) = \left\{ \begin{array}{l} -\varphi''(x, \lambda_n) + q(x) \varphi(x, \lambda_n) = \lambda_n \varphi(x, \lambda_n) \\ -\psi''(x, \lambda) + q(x) \psi(x, \lambda) = \lambda \psi(x, \lambda) \end{array} \right\} \end{aligned}$$

$$\begin{aligned} &= \underbrace{\psi(x, \lambda)} \left[ \underbrace{q(x) \varphi(x, \lambda_n)} - \lambda_n \varphi(x, \lambda_n) \right] - \\ &- \underbrace{\varphi(x, \lambda_n)} \cdot \left[ \underbrace{q(x) \cdot \psi(x, \lambda)} - \lambda \psi(x, \lambda) \right] = \lambda \psi(x, \lambda) \varphi(x, \lambda_n) - \\ &- \lambda_n \psi(x, \lambda) \cdot \varphi(x, \lambda_n) = (\lambda - \lambda_n) \psi(x, \lambda) \varphi(x, \lambda_n). \end{aligned}$$



В доказанном равенстве, рассмотрим обе части на  $[0, \pi]$ . Тогда

$$(\lambda - \lambda_n) \int_0^{\pi} \psi(x, \lambda) \varphi(x, \lambda_n) dx = \langle \psi(x, \lambda), \varphi(x, \lambda_n) \rangle \Big|_0^{\pi}.$$

$$\langle \psi(x, \lambda), \varphi(x, \lambda_n) \rangle \Big|_0^{\pi} = (\psi(\pi, \lambda) \varphi'(\pi, \lambda_n) - \psi'(\pi, \lambda) \varphi(\pi, \lambda_n)) -$$

$$- (\psi(0, \lambda) \varphi'(0, \lambda_n) - \psi'(0, \lambda) \varphi(0, \lambda_n)) =$$

$$= [\varphi'(\pi, \lambda_n) + H \varphi(\pi, \lambda_n)] - [\psi(0, \lambda) h - \psi'(0, \lambda)] =$$

$$= - \frac{h}{2} [\psi'(0, \lambda) - h \psi(0, \lambda)] + [\varphi'(\pi, \lambda_n) + H \varphi(\pi, \lambda_n)] =$$

$$= - (\Delta(\lambda) - \Delta(\lambda_n)).$$

$$(\lambda - \lambda_n) \int_0^{\pi} \psi(x, \lambda) \varphi(x, \lambda_n) dx = - (\Delta(\lambda) - \Delta(\lambda_n)) \Rightarrow$$

$$\Rightarrow \int_0^{\pi} \psi(x, \lambda) \varphi(x, \lambda_n) dx = - \frac{\Delta(\lambda) - \Delta(\lambda_n)}{\lambda - \lambda_n} \quad / \quad \lim_{\lambda \rightarrow \lambda_n} :$$

$$\lim_{\lambda \rightarrow \lambda_n} \int_0^{\pi} \psi(x, \lambda) \varphi(x, \lambda_n) dx = - \lim_{\lambda \rightarrow \lambda_n} \frac{\Delta(\lambda) - \Delta(\lambda_n)}{\lambda - \lambda_n} .$$

$$\int_0^{\pi} \psi(x, \lambda_n) \varphi(x, \lambda_n) dx = - \dot{\Delta}(\lambda_n) .$$

$$\text{T.K. } \psi(x, \lambda_n) = \beta_n \varphi(x, \lambda_n) \Rightarrow$$

$$\int_0^{\pi} \beta_n \cdot \varphi(x, \lambda_n) \cdot \varphi(x, \lambda_n) dx = -\dot{\Delta}(\lambda_n) \Rightarrow$$

$$\beta_n \int_0^{\pi} \varphi^2(x, \lambda_n) dx = -\dot{\Delta}(\lambda_n) \Rightarrow$$

$$\beta_n \cdot d_n = -\dot{\Delta}(\lambda_n)$$



Т. 1. 2.  $\{\lambda_n\}$  - меншикти ишдери  $\wedge$

$\varphi(x, \lambda_n), \psi(x, \lambda_n)$  - меншикти функциялар  
накди ишдери.

$\Delta(\lambda)$  характеристикалог функциялог  
барлог ишдери  $\{\lambda_n\}$  - жай ишдери,  
яни  $\Delta(\lambda_n) \neq 0$ .

Эртүрди меншикти ишдери  $\lambda_n \neq \lambda_k$   
сүйкөө  $\{\varphi(x, \lambda_n), \varphi(x, \lambda_k)\} \vee \psi$  меншикти  
функциялар өзара  $L^2(0; \pi)$  кеңис-  
тигиде ортогональ болот.

$\square \perp \lambda_n, \lambda_k$  ( $\lambda_n \neq \lambda_k$ ) - собственные значения и  
 $\lambda_n \leftrightarrow y_n(x), \lambda_k \leftrightarrow y_k(x)$ .

$$\int_0^{\pi} (Ly_n(x)) y_k(x) dx = \int_0^{\pi} y_n(x) (Ly_k(x)) dx.$$

$$Ly_n(x) := -y_n''(x) + q(x)y_n(x) \Rightarrow$$

$$\int_0^{\pi} [-y_n''(x) + q(x)y_n(x)] y_k(x) dx = - \int_0^{\pi} y_n''(x) y_k(x) dx +$$

$$+ \int_0^{\pi} q(x) y_n(x) y_k(x) dx = S_1 + S_2.$$

$$S_1 = \int_0^{\pi} y_n''(x) y_k(x) dx = \left| \begin{array}{l} u = y_k(x) \quad d\sigma = y_n''(x) \\ du = y_k'(x) dx \quad \sigma = y_n'(x) \end{array} \right|_0^{\pi} =$$

$$= y_k(x) \cdot y_n'(x) \Big|_0^{\pi} - \int_0^{\pi} y_k'(x) y_n'(x) dx = \left| \begin{array}{l} u = y_k'(x) \quad d\sigma = y_n'(x) \\ du = y_k''(x) dx \quad \sigma = y_n(x) \end{array} \right|_0^{\pi} =$$

$$= \left[ y_k(x) y_n'(x) \Big|_0^{\pi} - (y_k'(x) \cdot y_n(x)) \Big|_0^{\pi} \right] + \int_0^{\pi} y_k''(x) y_n(x) dx \Rightarrow$$

$$\begin{aligned}
 & \left[ y_k(x) y_n'(x) \right]_0^{\pi} - \left[ y_k'(x) y_n(x) \right]_0^{\pi} = \\
 & = \left( y_k(\pi) y_n'(\pi) - y_k'(0) y_n'(0) \right) - \left( y_k'(\pi) y_n(\pi) - \right. \\
 & \left. - y_k'(0) y_n(0) \right) = \left[ y_k(\pi) \underline{y_n'(\pi)} - \underline{y_k'(\pi)} y_n(\pi) \right] - \\
 & - \left[ y_k(0) \underline{y_n'(0)} - \underline{y_k'(0)} y_n(0) \right] = \begin{cases} \cancel{y_5 + H y} \\ y_5'(\pi) + H y_5(\pi) = 0 \\ y_5'(0) - h y_5(0) = 0 \end{cases} \Big|_x
 \end{aligned}$$

$$\begin{aligned}
 & = \left( y_k(\pi) \left[ -H y_k(\pi) \right] - \left[ -H y_k(\pi) \right] \cdot y_n(\pi) \right) - \\
 & - \left( y_k(0) \left[ h y_n(0) \right] - \left[ h y_k(0) \right] y_n(0) \right) = 0.
 \end{aligned}$$

$$S_1 = - \int_0^{\bar{1}} y_F''(x) y_n(x) dx \Rightarrow$$

$$\int_0^{\bar{1}} \ell y_n(x) y_F(x) dx = \int_0^{\bar{1}} -y_F''(x) y_n(x) dx + \int_0^{\bar{1}} q(x) y_F(x) y_n(x) dx =$$

$$= \int_0^{\bar{1}} y_n(x) [-y_F''(x) + q(x) y_F(x)] dx =$$

$$= \int_0^{\bar{1}} y_n(x) \ell y_F(x) dx.$$

$$\int_0^{\bar{1}} \ell y_n(x) y_F(x) dx = \int_0^{\bar{1}} y_n(x) \ell y_F(x) dx$$

пов-во доказано

$$L y_n(x) := -y_n''(x) + q(x)y_n(x) \Rightarrow (1) \Rightarrow$$

$$L y_n(x) = \lambda_n y_n(x), \quad \wedge \quad L y_k(x) = \lambda_k y_k(x) \Rightarrow$$

$$\lambda_n \int_0^{\pi} y_n(x) y_k(x) dx = \lambda_k \int_0^{\pi} y_n(x) y_k(x) dx \Leftrightarrow$$

$$(\lambda_n - \lambda_k) \int_0^{\pi} y_n(x) y_k(x) dx = 0$$

$$\lambda_n \neq \lambda_k \Leftrightarrow \lambda_n - \lambda_k \neq 0 \Leftrightarrow$$

$$\int_0^{\pi} y_n(x) y_k(x) dx = 0.$$



def.  $y_n(x) \perp y_k(x)$  в  $L^2(0; \pi) \Leftrightarrow$

$$(y_n(x), y_k(x))_{L^2(0; \pi)} = 0,$$

$$(y_n(x), y_k(x))_{L^2(0; \pi)} \stackrel{\text{def}}{=} \int_0^{\pi} y_n(x) y_k(x) dx \Leftrightarrow$$

$y_n(x) \perp y_k(x)$  в  $L^2(0; \pi)$ . 3) - го к-но.

Пусть  $\lambda^0 = u + i v, v \neq 0; \lambda^0 \in \mathbb{C}$

$$\lambda^0 \leftrightarrow y^0(x) \neq 0.$$

$$g(x), h, H \in \mathbb{R} \Rightarrow \bar{\lambda}^0 = u - i v, \bar{\lambda}^0 - \text{c.3.}$$

$$\bar{\lambda}^0 \leftrightarrow \bar{y}^0(x). \quad \lambda^0 \neq \bar{\lambda}^0 \Leftrightarrow$$

$$\|y^0\|_{L^2(0; \pi)} \stackrel{\text{def}}{=} \int_0^{\pi} y^0(x) \bar{y}^0(x) dx = 0 \Leftrightarrow$$

$$y^0(x) \equiv 0. \quad \Leftrightarrow \lambda^0 = \bar{\lambda}^0 \Leftrightarrow v = 0!$$

Отсюда также следует, что собственные функции  $\varphi(x, \lambda_n)$ ,  $\psi(x, \lambda_n)$  — вещественны.

Пусть  $\varphi(x, \lambda_n) = u(x, \lambda_n) + i v(x, \lambda_n)$  комплекснозначная функция. Тогда

$$\varphi''(x, \lambda_n) = u''(x, \lambda_n) + i v''(x, \lambda_n) \text{ и}$$

$$-\varphi''(x, \lambda_n) + q(x) \varphi(x, \lambda_n) = \lambda_n \varphi(x, \lambda_n), \text{ следовательно}$$

$\bar{\varphi}(x, \lambda_n) = u(x, \lambda_n) - i v(x, \lambda_n)$  также собственная функция. Докажем.

$$-\varphi''(x, \lambda_n) + q(x) \varphi(x, \lambda_n) = \lambda_n \varphi(x, \lambda_n) \Rightarrow$$

$$\begin{cases} -u''(x, \lambda_n) + q(x) u(x, \lambda_n) = \lambda_n u(x, \lambda_n) \\ -v''(x, \lambda_n) + q(x) v(x, \lambda_n) = \lambda_n v(x, \lambda_n) \end{cases}$$

στοιχα, γενικαμεν βτοχαε ροβεκετοε  
με (-i), i.e.

$$\begin{cases} -u''(x, \lambda_n) + q(x) u(x, \lambda_n) = \lambda_n u(x, \lambda_n), \\ i v''(x, \lambda_n) - i q(x) v(x, \lambda_n) = -i \lambda_n v(x, \lambda_n), \end{cases}$$

zeteu αλοκωε u x, uo μyzeu.

$$\begin{aligned} & -u''(x, \lambda_n) + i v''(x, \lambda_n) + q(x) (u(x, \lambda_n) - i v(x, \lambda_n)) = \\ & = \lambda_n (u(x, \lambda_n) - i v(x, \lambda_n)). \end{aligned}$$

$$\rightarrow [u''(x, \lambda_n) - i v''(x, \lambda_n)] + q(x) \bar{\varphi}(x, \lambda_n) = \lambda_n \bar{\varphi}(x, \lambda_n)$$

$$- \bar{\varphi}''(x, \lambda_n) + q(x) \bar{\varphi}(x, \lambda_n) = \lambda_n \bar{\varphi}(x, \lambda_n).$$

$\bar{\varphi}(x, \lambda_n) \leftrightarrow \lambda_n^*$  - собственное значение.

По доказанному равенству.

$$(\lambda_n - \lambda_n^*) \int_0^{\pi} \varphi(x, \lambda_n) \bar{\varphi}(x, \lambda_n) dx = 0, \text{ а}$$

$$\int_0^{\pi} \varphi(x, \lambda_n) \bar{\varphi}(x, \lambda_n) dx = \|\varphi(x, \lambda_n)\|_{L^2(0; \pi)}^2 = 0 \Rightarrow$$

$\varphi(x, \lambda_n) \equiv 0$ , что невозможно.

Тогда  $\lambda_n - \lambda_n^* = 0 \Leftrightarrow \lambda_n = \lambda_n^*$  так как

устойчивое, с тем, что  $\Delta(\lambda)$  имеет

простые нули, т.е.  $\lambda_n = \lambda_n^*$  не м/д.

$$\text{Так как } \alpha_n = \int_0^{\bar{x}} \varphi_1^2(x, \lambda_n) dx \neq 0 \text{ и}$$

$$\beta_n \neq 0, \text{ то}$$

$$\dot{\Delta}(\lambda_n) \neq 0.$$

Пример 1.1.  $\downarrow q(x) \equiv 0, h=H=0,$

$\lambda = p^2$ . Тогда

$$C(x, \lambda) = \varphi(x, \lambda) = \cos px,$$

$$S(x, \lambda) = \frac{\sin px}{p}$$

$$\Psi(x, \lambda) = \cos p(\bar{x} - x)$$

$$\Delta(\lambda) = -p \sin p\bar{x};$$

$$\lambda_n = n^2 \quad (n \geq 0).$$

$$\varphi(x, \lambda_n) = \cos nx;$$

$$\beta_n = (-1)^n.$$

$$\alpha_n = ?$$

Решение.

$$-y''(x) + p(x)y(x) = \lambda y(x), \quad p(x) = 0 \Leftrightarrow$$

$$-y''(x) = p^2 y(x) \Leftrightarrow$$

$-y''(x) - p^2 y(x) = 0$ . Характеристическое уравнение  $\text{генер. ур.}$

$$-(y''(x) + p^2 y(x)) = 0 \Leftrightarrow k^2 + p^2 = 0 \Leftrightarrow$$

$$k^2 - (ip)^2 = 0 \Leftrightarrow k = \pm ip \Leftrightarrow |k = \pm ip \Leftrightarrow$$

$$y(x) = e^{ix} (C_1 \cos px + C_2 \sin px), \quad \text{а т.к. } i^2 = -1$$

$$y(x) = C_1 \cos px + C_2 \sin px.$$

Т.к.  $h = H = 0 \Leftrightarrow U(y) = y'(0) = 0, \bar{V}(y) = y'(\bar{\pi}) = 0.$

$y'(x) = -\rho C_1 \sin \rho x + \rho C_2 \cos \rho x$ , а т.к.

$x=0 \Rightarrow y'(0) = -\rho C_1 \sin 0 + \rho C_2 \cos 0 = \rho C_2 = 0. C_2 = 0.$

\* Тогда  $y(x) = C_1 \cos \rho x$ , без ограничения общности  $y(x) = \cos \rho x$ , а т.к.

$y(0) = \cos 0 = 1$ , а  $y(x) = C(x, \lambda)$ , т.е.

$y'(0) = -\rho \sin \rho \cdot 0 = 0$ , то

$C(x, \lambda) = \cos \rho x.$

Связи с условием

$\varphi(x, \lambda) = C(x, \lambda) = \cos \rho x.$

$\varphi(x, \lambda) :$

$\begin{cases} \varphi(0, \lambda) = 1 \\ \varphi'(0, \lambda) = h = 0. \end{cases}$  то

$$y(x) = C_1 \cos px + C_2 \sin px.$$

$$S(x, \lambda) : \begin{cases} S(0, \lambda) = 0; \\ S'(0, \lambda) = 1. \end{cases}$$

$$y(0) = C_1 \cos 0 + C_2 \sin 0 = 0 \Rightarrow C_1 = 0.$$

$$y'(x) = -C_1 p \sin px + p C_2 \cos px \Rightarrow$$

$$y'(0) = -C_1 p \sin 0 + p C_2 \cos 0 \stackrel{!}{=} 1 \Rightarrow$$

$$\Rightarrow p C_2 = 1 \Rightarrow C_2 = \frac{1}{p}.$$

$$\text{T.O. } C_1 = 0; C_2 = \frac{1}{p} \Rightarrow$$

$$y(x) = \frac{1}{p} \sin px = S(x, \lambda).$$



$$y(x) = C_1 \cos px + C_2 \sin px.$$

$$\Psi(x, \lambda): \begin{cases} \Psi(\pi, \lambda) = 1; \\ \Psi'(\pi, \lambda) = -H = 0. \end{cases}$$

$$y(\pi) = C_1 \cos p\pi + C_2 \sin p\pi = 1.$$

$$y'(x) = -p C_1 \sin px + p C_2 \cos px \Rightarrow$$

$$y'(\pi) = -p C_1 \sin p\pi + p C_2 \cos p\pi = 0.$$

$$\begin{cases} C_1 \cos p\pi + C_2 \sin p\pi = 1. \\ -C_1 \sin p\pi + C_2 \cos p\pi = 0. \end{cases}$$

$$C_2 = \frac{C_1 \sin p\bar{n}}{\cos p\bar{n}} \Rightarrow$$

$$C_1 \cos p\bar{n} + \frac{C_1 \sin^2 p\bar{n}}{\cos p\bar{n}} = 1 \Rightarrow$$

$$\frac{C_1 \cos^2 p\bar{n} + C_1 \sin^2 p\bar{n}}{\cos p\bar{n}} = 1 \Leftrightarrow C_1 = \cos p\bar{n} \Rightarrow$$

$$C_2 = \sin p\bar{n} \Rightarrow$$

$$\begin{aligned} \psi(x, \lambda) &= \cos p\bar{n} \cdot \cos px + \sin p\bar{n} \sin px = \\ &= \cos p(\bar{n} - x). \end{aligned}$$

$$\begin{aligned}
\Delta(\lambda) &= \langle \Psi, \Phi \rangle = \Psi(x, \lambda) \Phi'(x, \lambda) - \Psi'(x, \lambda) \cdot \Phi(x, \lambda) = \\
&= \left| \begin{array}{l} \Psi(x, \lambda) = \cos p(\pi - x) \Rightarrow \Psi'(x, \lambda) = p \sin p(\pi - x) \\ \Phi(x, \lambda) = \cos px \Rightarrow \Phi'(x, \lambda) = -p \sin px \end{array} \right| = \\
&= \cos p(\pi - x) \cdot (-p \sin px) \stackrel{+}{=} p \sin p(\pi - x) \cdot \cos px = \\
&= -p \left[ (\cos p\pi \cdot \cos px + \sin p\pi \sin px) \sin px + \right. \\
&\quad \left. + (\sin p\pi \cos px - \cos p\pi \cdot \sin px) \cos px \right] = -p \cdot \\
&\cdot \left( \underline{\cos p\pi \cos px \sin px} + \sin p\pi \sin^2 px + \sin p\pi \cos^2 px - \right. \\
&\quad \left. - \underline{\cos p\pi \cdot \sin px \cdot \cos px} \right) = -p \cdot (\sin p\pi \sin^2 px + \sin p\pi \cos^2 px) \\
&= -p \sin p\pi.
\end{aligned}$$

$$\beta_n = \frac{\psi(x, \lambda_n)}{\varphi(x, \lambda_n)} = \left| \lambda_n = n^2 \right| =$$

$$= \frac{\cos n(\pi - x)}{\cos nx} = \frac{\cos n\pi \cdot \cos nx + \overset{=0}{\sin n\pi} \cdot \sin nx}{\cos nx} =$$

$$= \left| \forall n \in \mathbb{Z} \Rightarrow \sin n\pi = 0 \right| = \frac{\cos n\pi \cos nx}{\cos nx} =$$

$$= \cos n\pi = (-1)^n.$$

$$\beta_n = (-1)^n.$$

$$\alpha_n = \int_0^{\pi} \varphi^2(x, \lambda_n) dx = \int_0^{\pi} \cos^2 nx dx =$$

$$= \int_0^{\pi} \frac{1}{2} [1 + \cos 2nx] dx = \frac{1}{2} \int_0^{\pi} dx +$$

$$+ \frac{1}{2} \int_0^{\pi} \cos 2nx dx = \frac{1}{2} x \Big|_0^{\pi} +$$

$$+ \frac{1}{2} \cdot \frac{1}{2n} \cdot \sin 2nx \Big|_0^{\pi} = \frac{1}{2} \pi + \frac{1}{4n} (\sin 2n\pi - 0) =$$

$$= \frac{1}{2} \pi.$$

$$\lambda = p^2$$

$$\begin{aligned} C(x, \lambda) = \varphi(x, \lambda) &= \cos px; \\ S(x, \lambda) &= \frac{1}{p} \sin px; \\ \Psi(x, \lambda) &= \cos p(\bar{\pi} - x) \end{aligned} \quad \left| \quad \Delta(\lambda) = -p \sin p\bar{\pi}$$

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$$\lambda_n = n^2 \quad (n \geq 0)$$

$$C(x, \lambda_n) = \cos nx; \quad S(x, \lambda_n) = \frac{1}{n} \sin nx;$$

$$\Psi(x, \lambda) = \cos n(\bar{\pi} - x) = (-1)^n \cdot \cos nx;$$

$$\Delta(\lambda_n) = -n \cdot \sin n\bar{\pi} = 0. \quad \lambda_n - \text{узлы.}$$

$$\beta_n = (-1)^n; \quad \alpha_n = \frac{1}{2} \bar{\pi} = \text{const.}$$

## Задача 2.

Определить Вронскиан  
для решений  
диф. ур.  
 $y_1(x), y_2(x)$

$$1) \quad y'' + (3x - 1)y' + 2y = 0$$

$$2) \quad y'' + (5x + 1)y' - 2y = 0$$

$$3) y'' + (x^2 + 2)y' + y = 0$$

$$4) y'' - (2x + 5)y' - y = 0.$$

применить формулу

Остроградского - Лиувилля