Macroeconomics

Lecture 8.

Long-run macroeconomic dynamics:

Selected Post-Solow models

Solow model vs. Endogenous growth theory

- Solow model:
  - sustained growth in living standards is due to tech progress
  - the rate of technological progress is exogenous
- Endogenous growth theory:
  - a set of models in which the growth rate of productivity and living standards is endogenous

Externalities concerned with accumulation of capital

- In the Solow model, firms are able to capture all of the returns to investment.
- However, ***it seems reasonable that there might be externalities in capital formation so that the social return might be higher than the private rate of return.***
- These externalities could arise because workers move between firms taking their knowledge of the production process with them (***learning by doing***).
- In an extreme case this might lead to there being constant returns to capital.

AK model as the simplest endogenous growth model (Part 1)

- Production function: $Y = AK$
  where A is the amount of output for each unit of capital (A is exogenous & constant)
- Key difference between this model & Solow:
  MPK is constant here, diminishes in Solow
- Investment: sY
- Depreciation: δK
- Equation of motion for total capital:

$$\Delta K = sY - \delta K$$

AK model as the simplest endogenous growth model (Part 2)

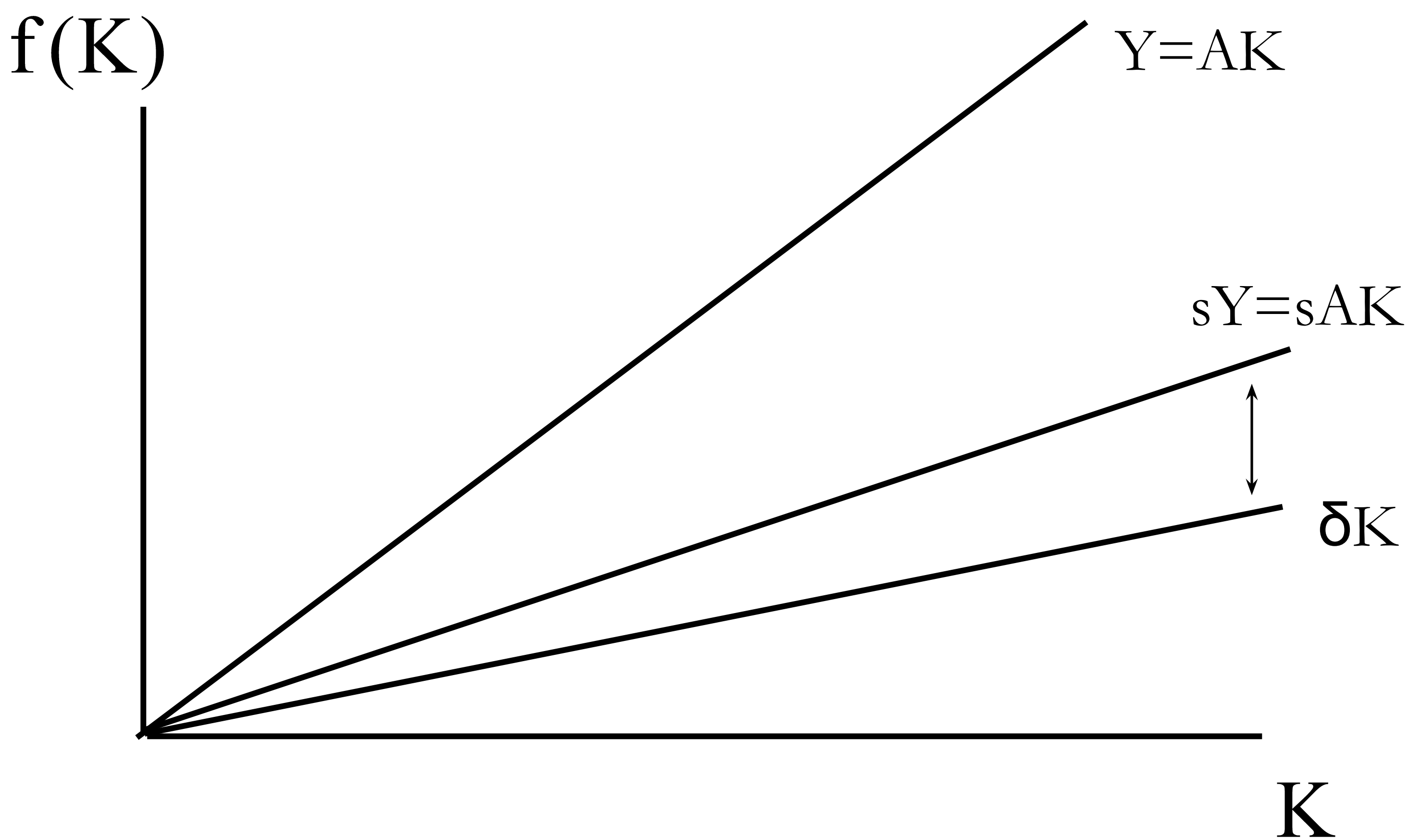
$$\Delta K = sY - \delta K$$

- Divide through by K and use $Y = AK$, get:

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = sA - \delta$$

- If $sA > \delta$, then income will grow forever, and investment is the **"engine of growth."**
- Here, the permanent growth rate depends on s. In Solow model, it does not.

AK model graphically



AK model implications

- The growth rate of an AK economy is an increasing function of the saving rate, so a government policy to raise the saving rate will raise the growth rate.
- ***The growth rate of an AK economy does not depend upon its initial capital stock, so there is no convergence between economies with different initial capital stocks even if they have the same saving rates, levels of technology and depreciation rates***.
- Technical progress and population growth are not necessary to generate per capita growth.

Differences between predictions of Solow model and AK model

- The Solow model has two main predictions:
  - For countries with the same steady-state, poor countries should grow faster than rich ones.
  - An increase in investment raises the growth rate temporarily as the economy moves to a new steady-state. But once the new higher steady-state level of income is reached, the growth rate returns to its previous level.
- However, the AK model yields the opposite predictions – there is no convergence, and policy changes can have permanent effects.

R&D model as another version of endogenous growth theory (Part 1)

- Two sectors:
  - manufacturing firms produce goods
  - research universities produce knowledge that increases labor efficiency in manufacturing
- u = fraction of labor in research (u is exogenous)
- Mfg prod func: $Y = F[K, (1-u)EL]$
- Research prod func: $\Delta E = g(u)E$
- Capital accumulation: $\Delta K = sY - \delta K$

R&D model as another version of endogenous growth theory (Part 2)

- In the steady state, mfg output per worker and the standard of living grow at rate $\Delta E/E = g(u)$.
- Key variables:

  s: affects the level of income, but not its growth rate (same as in Solow model)

  u: affects level and growth rate of income

- Question:
  Would an increase in ***u*** be unambiguously good for the economy?

R&D model as another version of endogenous growth theory (Part 3)

1. Much research is done by firms seeking profits.
2. Firms profit from research because
   - new inventions can be patented, creating a stream of monopoly profits until the patent expires
   - there is an advantage to being the first firm on the market with a new product
3. Innovation produces externalities that reduce the cost of subsequent innovation.

Much of the new endogenous growth theory attempts to incorporate these facts into models to better understand technological progress.

R&D sector and externalities

- The existence of positive externalities in the creation of knowledge suggests that the private sector is not doing enough R&D.
- But, there is much duplication of R&D effort among competing firms.
- Estimates: The social return to R&D is at least 40% per year.

Thus, many believe government should encourage R&D.

Basic ideas of Lucas (1988) model of growth and human capital accumulation

Human capital – the Lucas model

- Lucas defines human capital as the skill embodied in workers
- Constant number of workers in economy is N
- Each one has a human capital level of h
- Human capital can be used either to produce output (proportion *u*)
- Or to accumulate new human capital (proportion *1-u*)
- Human capital grows at a constant rate

$$dh/dt = h(1-u)$$

The basic relationships of Lucas (1988) model

- The production depends on the human capital stock and the share of working time spent on production
- The human capital stock depends on the share of working time spent on the higher education.

Galor – Zeira model as a growth model including both human capital accumulation and role of income distribution

Consider a small open economy in a one-good world. The good can be used for either consumption or investment. The good can be produced by two technologies, one which uses skilled labour and capital and the other using unskilled labour only. Production in the skilled labour sector is described by:

$$Y_t^s = F(K_t, L_t^s), \tag{1}$$

where Y_t^s is output in this sector at time t, K_t is the amount of capital and L_t^s is labour input. F is a concave production function with constant returns to scale. It is assumed that investment in human capital and in physical capital is made one period in advance. For the sake of simplicity it is assumed that there are no adjustment costs to investment and no depreciation of capital. Production in the unskilled labour sector is described by:

$$Y_t^n = w_n \cdot L_t^n, \tag{2}$$

Utility function

$$u = \alpha \log c + (1 - \alpha) \log b, \quad (3)$$

where c is consumption in second period, b is bequest, and $0 < \alpha < 1$. Notice that all individuals are born with the same potential abilities and with the same preferences. They differ only in the amounts they inherit from their parents.

Marginal productivity of skilled labor

$$F_K(K_t, L_t^s) = r. \quad (4)$$

Hence, there is a constant capital-labour ratio in this sector, which determines the wage of skilled labour w_s, which is constant as well. This wage w_s depends on r and on technology only.

We further assume that both labour markets and the good market are perfectly competitive and expectations are fully rational.

Asymmetric information on credit market

Let us first examine the capital market equilibrium for individual borrowers. It is clear that lenders to individuals must have positive costs of keeping track of each borrower, since otherwise everyone defaults. Hence, the individual must borrow at a rate higher than r, to cover these tracking costs. An individual who borrows an amount d pays an interest rate i_d which covers lenders' interest rate and lenders' costs z, as competitive financial intermediation operates on zero profits:

$$d \cdot i_d = d \cdot r + z. \tag{5}$$

Lenders choose z to be high enough to make evasion disadvantageous:

$$d(1 + i_d) = \beta z. \tag{6}$$

This is an incentive compatibility constraint. Equations (5) and (6) determine i_d:

$$i_d = i = \frac{1 + \beta r}{\beta - 1} > r. \tag{7}$$

The borrowing interest rate i is, therefore, independent of the amount borrowed d, as tracking costs rise with the amount borrowed d. This result is quite intuitive: as the amount borrowed increases, the incentive to default rises and hence tracking costs rise.

Decisions to invest or not to invest in human capital and consequences

We now turn to describe individual optimal decisions. Consider an individual who inherits an amount x in first period of life. If this individual decides to work as unskilled and not invest in human capital, his (her) lifetime utility is:

$$U_n(x) = \log\left[(x + w_n)(1 + r) + w_n\right] + \varepsilon, \tag{8}$$

where:

$$\varepsilon = \alpha \log \alpha + (1 - \alpha) \log (1 - \alpha).$$

This unskilled worker is a lender who leaves a bequest of size:

$$b_n(x) = (1 - \alpha)\left[(1 + r)(x + w_n) + w_n\right]. \tag{9}$$

An individual with inheritance $x \geq h$, who invests in human capital, is a lender with utility:

$$U_s(x) = \log\left[w_s + (x - h)(1 + r)\right] + \varepsilon, \tag{10}$$

and a bequest of:

$$b_s(x) = (1 - \alpha)\left[w_s + (x - h)(1 + r)\right]. \tag{11}$$

An individual who invests in human capital but has inheritance x smaller than h is a borrower, with lifetime utility:

$$U_s(x) = \log\left[w_s + (x - h)(1 + i)\right] + \varepsilon, \tag{12}$$

and a bequest of:

$$b_s(x) = (1 - \alpha)\left[w_s + (x - h)(1 + i)\right] \tag{13}$$

The condition for a refusal to invest in human capital

It is clear that if $w_s - h(1+r) < w_n(2+r)$ all individuals prefer to work as unskilled.

$$w_s - h(1+r) \geq w_n(2+r). \qquad (14)$$

The condition for investing in human capital

Hence, as investment in human capital pays back more than unskilled labour, lenders prefer to invest in human capital, as is seen from equations (8) and (10). Borrowers invest in human capital as long as $U_s(x) \geqq U_n(x)$, that is as long as:

$$x \geqq f = \frac{1}{i-r}[w_n(2+r) + h(1+i) - w_s]. \qquad (15)$$

Individuals who inherit an amount smaller than f would prefer not to invest in human capital but work as unskilled. Education is, therefore, limited to individuals with high enough initial wealth, due to a higher interest rate for borrowers.

Distribution of inheritances and its role

The amount an individual inherits in first period of life, therefore, fully determines his (her) decisions whether to invest in human capital or work as unskilled, and how much to consume and bequeath. Let D_t be the distribution of inheritances by individuals born in period t. This distribution satisfies:

$$\int_0^\infty dD_t(x_t) = L. \tag{16}$$

The distribution D_t, therefore, fully determines economic performance in period t. It determines the amount of skilled labour:

$$L_t^s = \int_f^\infty dD_t(x_t) \tag{17}$$

and unskilled labour:

$$L_t^n = \int_0^f dD_t(x_t). \tag{18}$$

The conditions for accumulation of human capital

The distribution of wealth not only determines equilibrium in period t, but also determines next period distribution of inheritances D_{t+1}:

$$x_{t+1} = \begin{cases} b_n(x_t) = (1-\alpha)[(x_t + w_n)(1+r) + w_n], & \text{if } x_t < f \\ b_s(x_t) = (1-\alpha)[w_s + (x_t - h)(1+i)], & \text{if } f \leq x_t < h \\ b_s(x_t) = (1-\alpha)[w_s + (x_t - h)(1+r)], & \text{if } h \leq x_t. \end{cases} \qquad (19)$$

In order to illustrate the dynamic evolution of wealth distribution through time we present in Figure 1 the curves b_n and b_s which describe the dynamic relationships between inheritance and bequest for unskilled and skilled workers, respectively. Notice that f is determined by the intersection of b_n and b_s.

The essential diagram

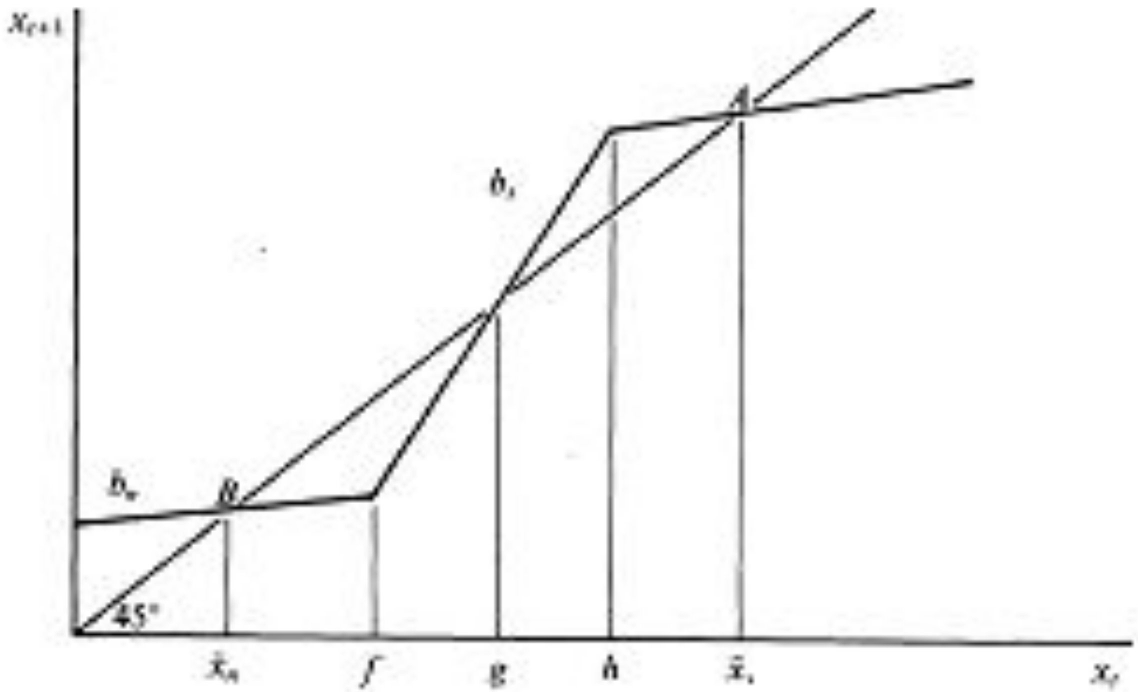


FIGURE 1

The description of choices

Individuals who inherit less than f work as unskilled and so are their descendants in all future generations. Their inheritances converge to a long-run level $\bar{x}_n$:

$$\bar{x}_n = \frac{1-\alpha}{1-(1-\alpha)(1+r)} w_n(2+r). \tag{20}$$

Individuals who inherit more than f invest in human capital but not all their descendants will remain in the skilled labour sector in future generations. The critical point is g in Figure 1:

$$g = \frac{(1-\alpha)[h(1+i)-w_s]}{(1+i)(1-\alpha)-1}. \tag{21}$$

Individuals who inherit less than g in period t may invest in human capital, but after some generations their descendants become unskilled workers and their inheritances converge to $\bar{x}_n$. Individuals who inherit more than g invest in human capital and so do their descendants, generation after generation. Their bequests converge to $\bar{x}_s$:

$$\bar{x}_s = \frac{1-\alpha}{1-(1-\alpha)(1+r)}[w_s - h(1+r)]. \tag{22}$$

Thus, dynasties in this economy are concentrated in the long run in two groups: rich dynasties, where generation after generation invests in human capital, and poor ones, where generation after generation are unskilled workers.

Some conclusions regarding Galor – Zeira model

The long-run equilibrium in this model, therefore, depends on the initial distribution of wealth and is as a result historically dependent. There are multiple long-run equilibria and the specific one the economy converges to depends on the initial distribution of wealth.