Detection of streaks of faint space objects

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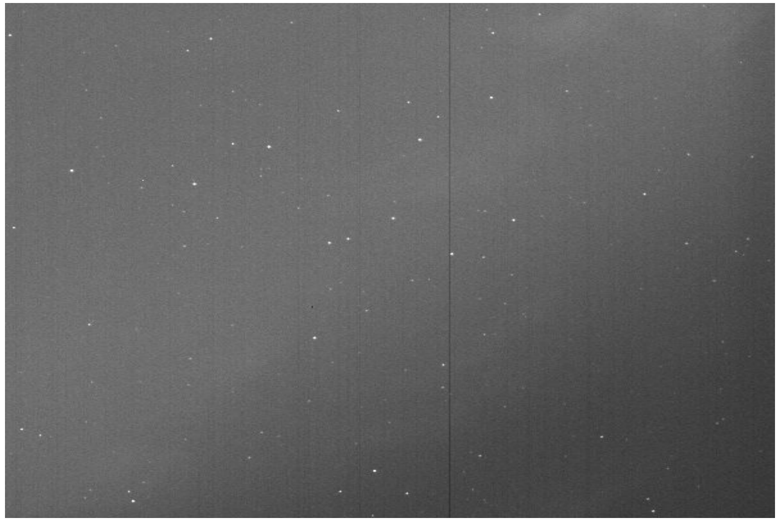
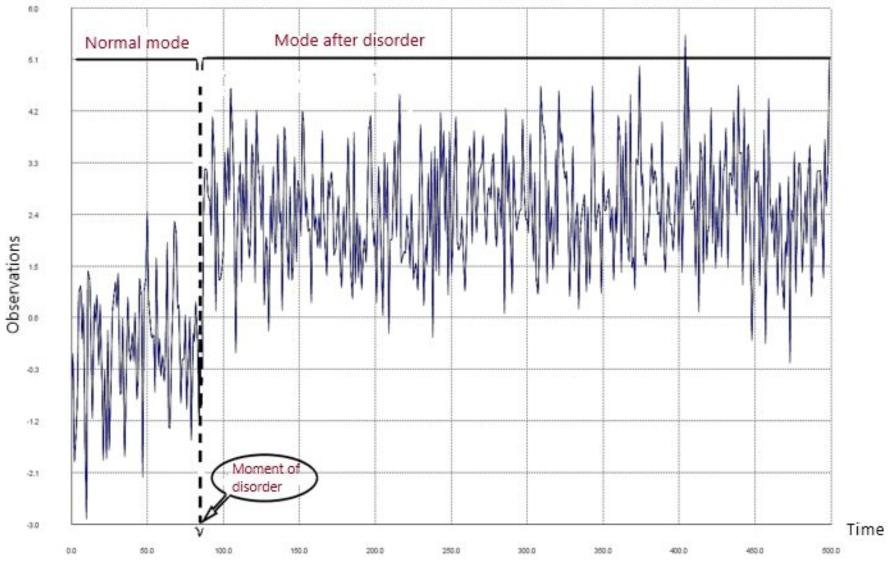
Introduction

- A problem of joint detection and estimation of parameters of faint space object streaks in digital images (frames) is considered.
- Approaches implementing signal thresholding and further grouping of detections in “hot” pixels gives unsatisfactory results due to a considerable increase in the number of false detections.
- Generalized Likelihood Ratio Hypothesis Test (GLRT) requires testing of a huge number of hypotheses associated with an unknown number of objects and parameters of their streaks, which is problematic for frames of large sizes even with a supercomputer.
- An effective two-stage algorithm for detecting faint streaks is proposed.
  - **First stage:** a sequential change detection method is used to detect abrupt changes and localize the object position.
  - **Second stage:** maximal likelihood test is used to estimate more precisely the position of the streak in the selected direction.





Models of signals

Background clutter will be suppressed after preprocessing. Discuss it later.

Streak motion model

$$x_\tau = x_0 + v_x\tau,\ y_\tau = y_0 + v_y\tau$$

$$v_x = (x_T - x_0)/T,\ v_y = (y_T - y_0)/T$$

Streak of the space object is determined by the vector

$$X = (x_0, y_0, x_T, y_T)$$

x, y - position in the frame (for a unit of length for each coordinate, the pixel size in the corresponding direction is taken)

$\varepsilon_{i,j}$ - Gaussian noise with zero mean and known (estimated empirically) local variance σ^2 (after preprocessing)

$$Y_{i,j} = AS_{i,j}(X) + B_{i,j} + \sigma\varepsilon_{i,j}$$

$$S_{i,j} = S_{i,j}(x_0, y_0, v_x, v_y) =$$

$$\int_0^T \left\{ \int_i^{i+1}\int_j^{j+1} F(x - x_0 - v_x\tau, y - y_0 - v_y\tau)\,dxdy \right\} d\tau$$

$Y_{i,j}$ - signal from one object in pixel (i,j) of the frame

T - exposure time

A - signal “amplitude”

$F(u,v)$ - Point Spread Function (PSF)

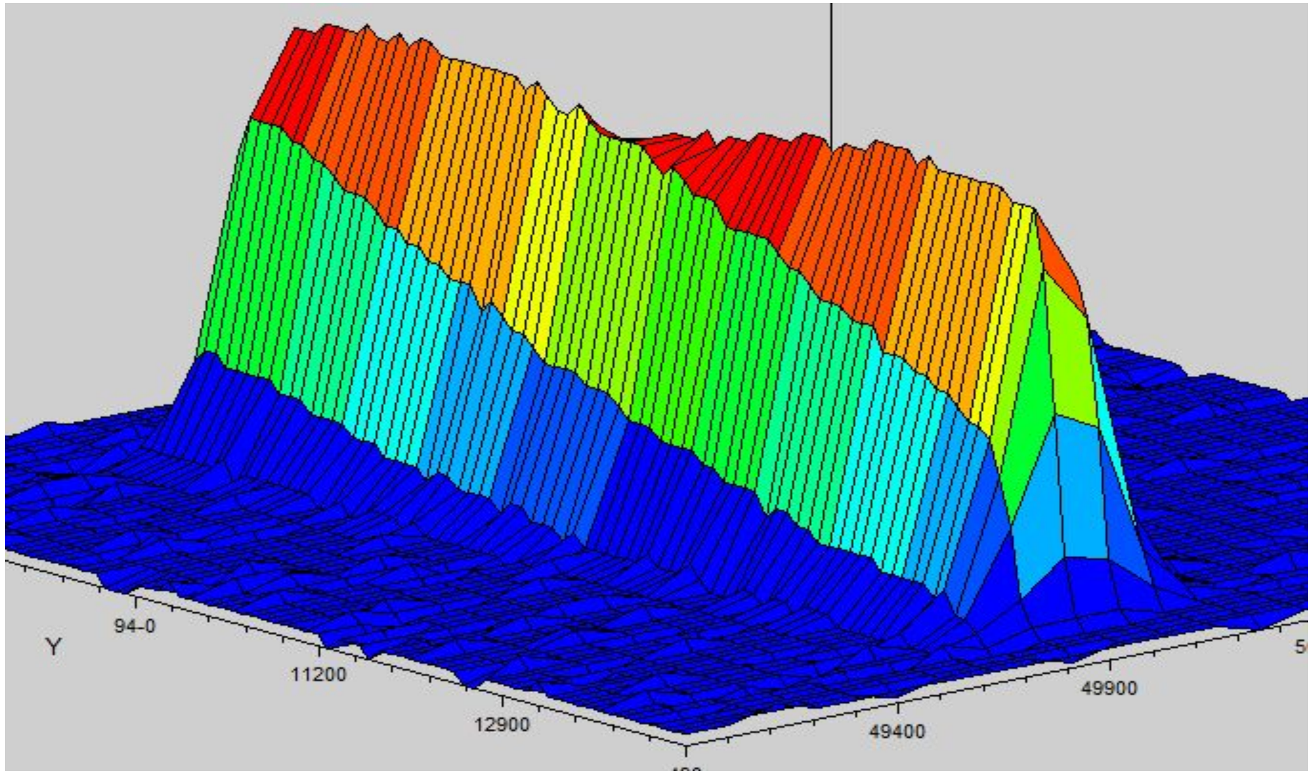
The problem (joint detection & estimation)

Hypotheses

H_0 - no streaks in the search area

H_1 - the streak exists in the search area with certain position X and amplitude A

The problem is to make a decision which of two hypotheses is valid and estimate unknown position X



Algorithm: Stage 1 – Localization

$$T_a^M = \inf\{n \geq M: \sum_{t=n-M+1}^{n} \log[\frac{p_{Y|H_1,X,A}(y)}{p_{Y|H_0}(y)}] \geq a\}$$

$$p_{Y|H_0}(y) = c\exp\left\{-\frac{1}{2\sigma^2}\sum_{(i,j)\in M} y_{i,j}^2\right\}$$

$$p_{Y|H_1,X,A}(y) = c\exp\left\{-\frac{1}{2\sigma^2}\sum_{(i,j)\in M}\left[y_{i,j} - AS_{i,j}(x)\right]^2\right\}$$

Sliding window rule (Moving average test)

$$R_{M_{N,K}}(y) = \sum_{(i,j)\in M_{N,K}} y_{i,j}S_{i,j}(x) \geq h_{\alpha,\sigma,M}$$

N,K – length and width (depends on σ_{PSF}),

$S_{i,j}(x)$ – model profile values (calculated in advance)

The following maximin problem statement is the most suitable for streak detection:

$$\inf_{\nu\geq 0} P_\nu(T_{opt} \leq \nu + M | T_{opt} > \nu) = \sup_{T\in C(m,\alpha)} \inf_{\nu\geq 0} P_\nu(\nu < T \leq \nu + M | T > \nu)$$

in class

$$C(m,\alpha) = \{T: \sup_{l\geq 0} P_\infty(T \leq l+m | T > l) \leq \alpha\}$$

The solution of this optimality problem is open.

CONJECTURE: The proposed test is asymptotically nearly optimal for α goes to 0.

Algorithm: Stage 2 – Position estimation

Maximal likelihood method

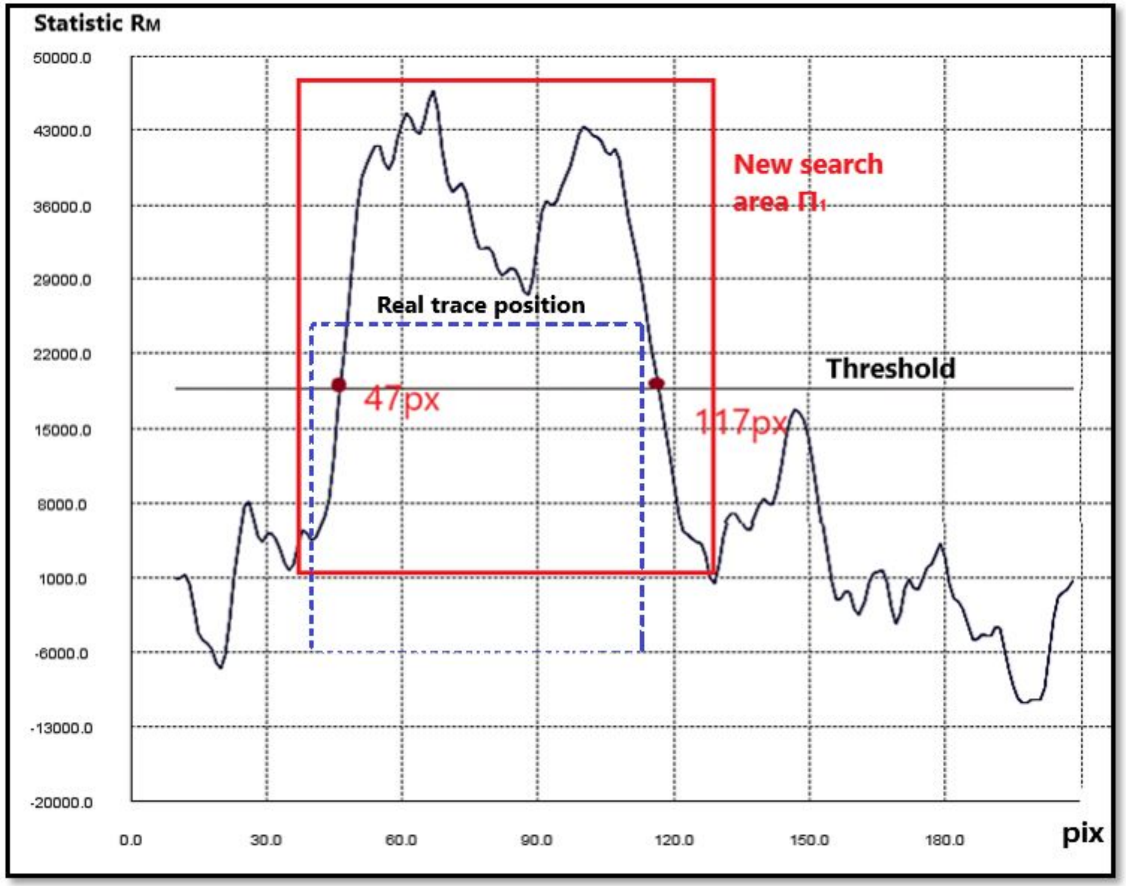
$$\max_{x,a} \exp\left\{\frac{1}{2\sigma^2}\left[\sum_{(i,j)\in\Pi_1} 2y_{i,j}AS_{i,j}(x) - A^2S^2{}_{i,j}(x)\right]\right\}$$

$x \in \Pi_1$ (New search area after stage 1)

$$J(a,x) = \sum_{(i,j)\in\Pi}\left[y_{i,j} - AS_{i,j}(x)\right]^2$$

$$J(\hat{a},\hat{x}) = \min_x \min_a J(A,x)$$

$$J(x) = \sum_{(i,j)\in\Pi_1}\left[y_{i,j} - \frac{\sum_{(i,j)\in\Pi_1} y_{i,j}S_{i,j}(x)}{\sum_{(i,j)\in\Pi_1} S^2_{i,j}(x)} S_{i,j}(x)\right]^2$$



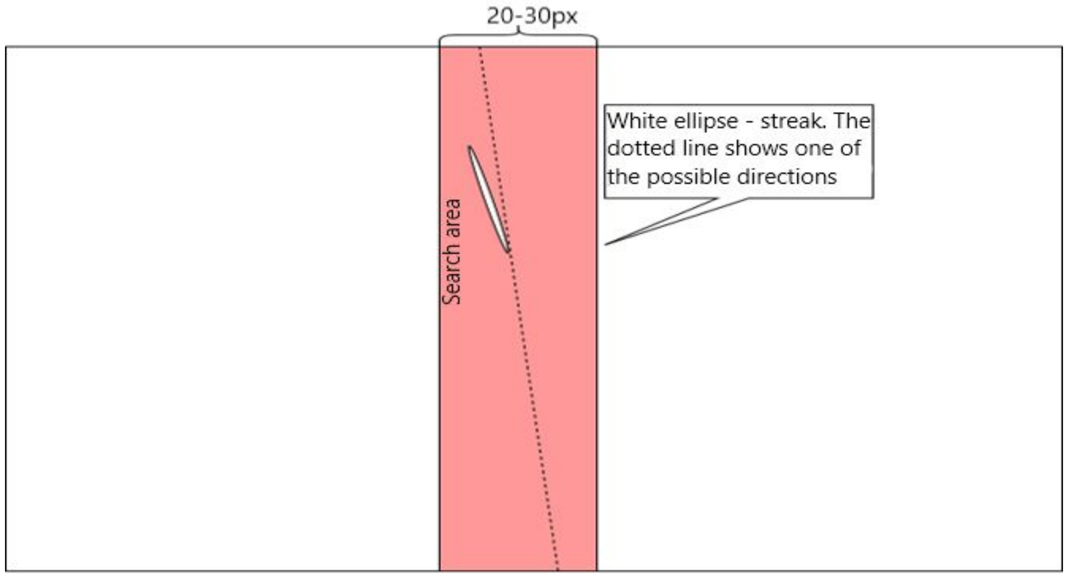
Find: $X = (x_0, y_0, x_1, y_1)$, $x_0, y_0, x_1, y_1 \in \Pi_1$

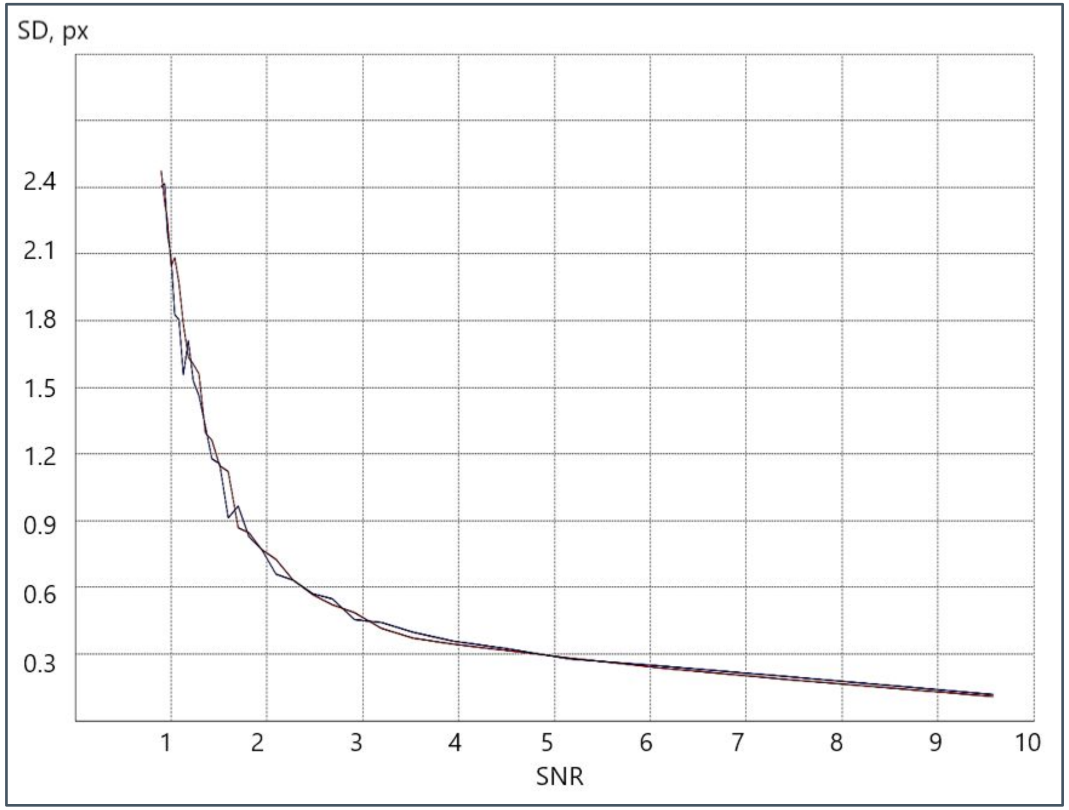
for $J(x)$ minimization

After the stage 2 position of streak will be estimated up to 1-2 pixels

Simulation

- Algorithm was tested in solving the real problem of detecting streaks in digital frames taken with a telescope, situated at the equator.
- The search area is bounded by a rectangle with a width of several tens of pixels and located in the middle of the frame.
- **The first stage** of the algorithm solves the problem of finding the most suitable directions and the approximate location of the streak on each of them inside the search area.
- **The second stage** of the algorithm makes a decision about the right direction and estimation of the streak position.
- Using simulated frames **1000x500** in size with white Gaussian noise and streaks (length = 50px), the dependence of the standard deviation on SNR was obtained.
- Probability of detection (PD) = 0.9 – 0.95 when FA = 0.001 (down to SNR = 1)





Simulation: real frames

- When operating with real frames it is crucial to get rid of strong discrete clutter generated by stars and background.

- The simplest method of clutter suppression is subtraction of two sequential frames, which however leads to an increase in noise variance.

Spatiotemporal regression is proposed as a proper method for image whitening:

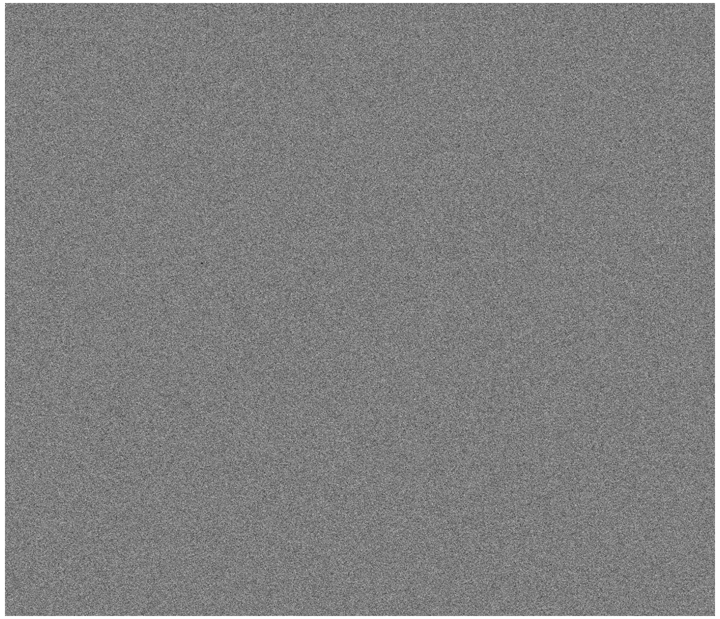
$Y_t^{i,j} = S_t^{i,j} + B_t^{i,j} + \xi_t^{i,j}$ – input image model

$\hat{b}_t^{i,j} = \sum_{p=-L}^{L} \sum_{q=-L}^{L} \sum_{\tau=1}^{T} \beta_\tau^{p,q} y_{t-\tau}^{i+p,j+q}$ – background estimation as a linear combination

L – space memory
T – number of frames

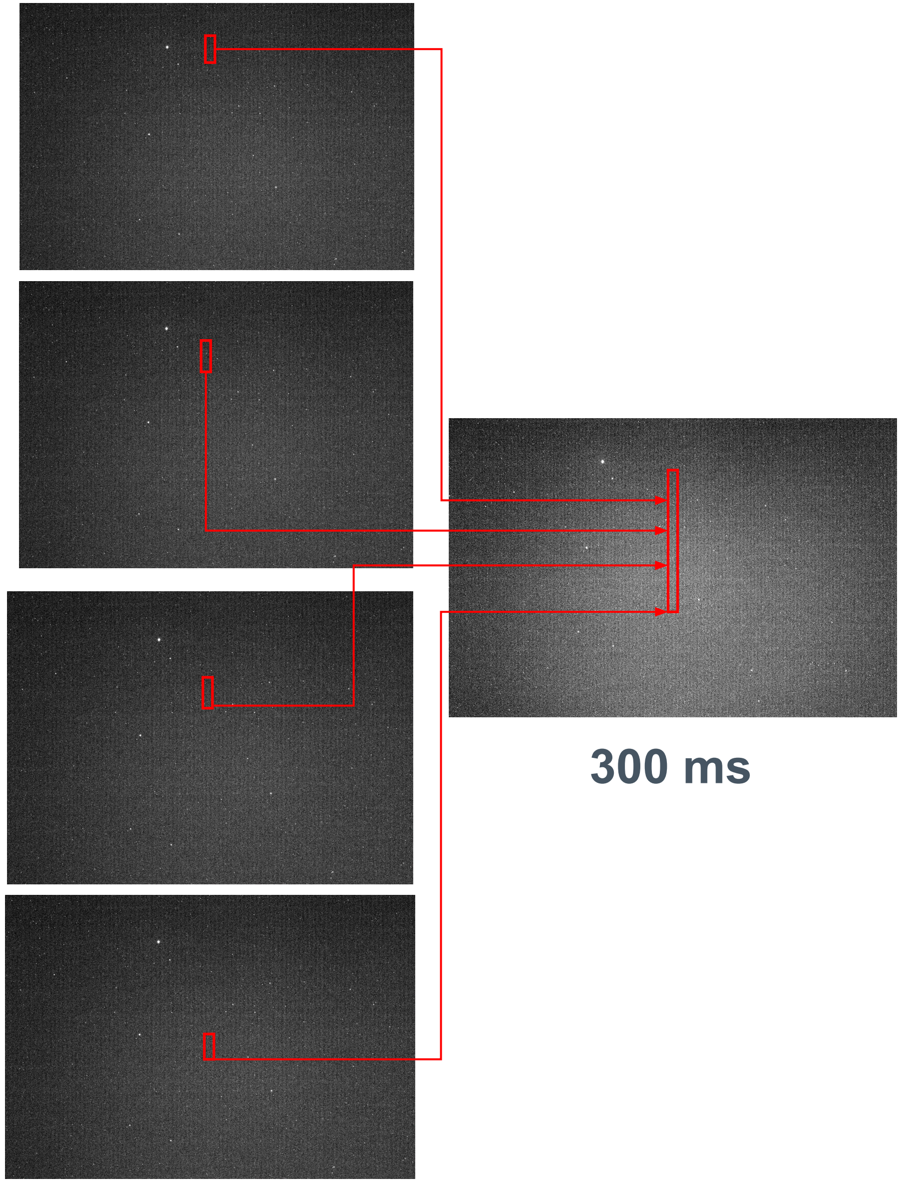
$$\Delta^2 = \sum_i \sum_j \sum_\tau \left(y_{t-\tau}^{i,j} - \sum_{p=-L}^{L} \sum_{q=-L}^{L} \sum_{\tau=1}^{T} \beta_\tau^{p,q} y_{t-\tau}^{i+p,j+q} \right)^2 \to \min_{\beta_\tau^{p,q}} \Delta^2$$

Input frame

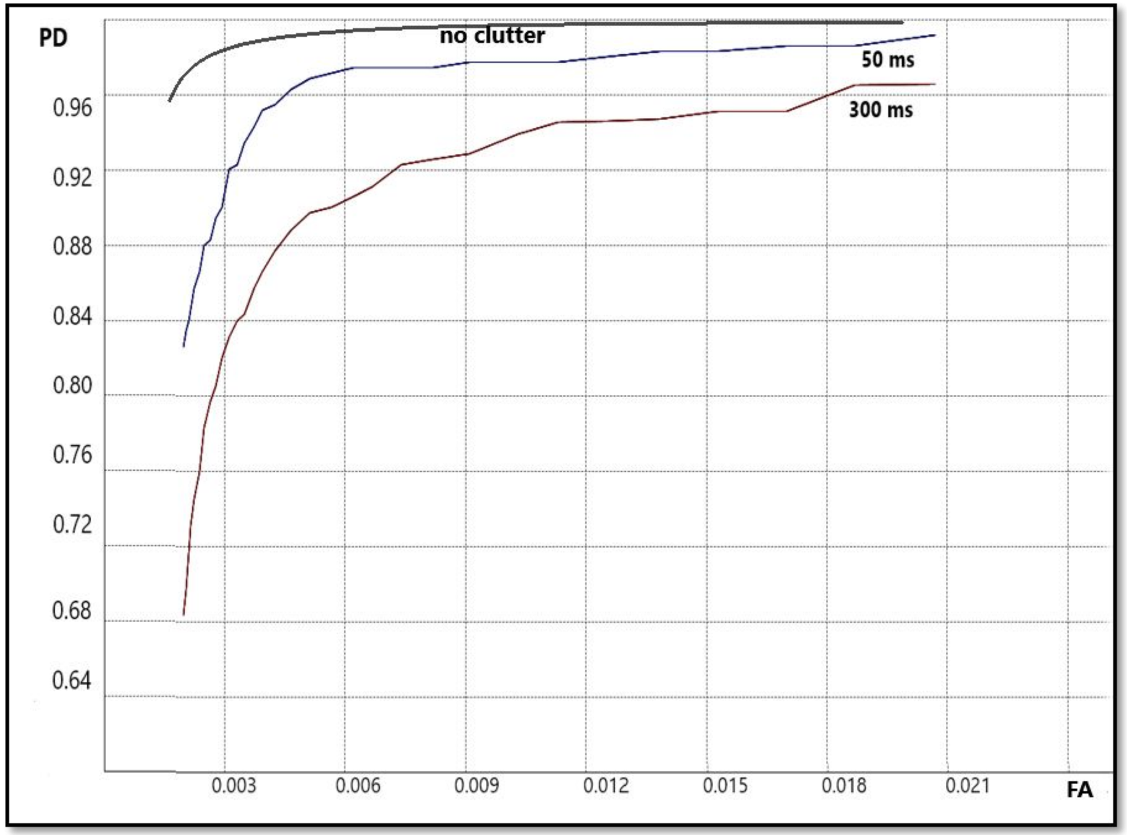


Frame after regression

Simulation: spatiotemporal regression:


300 ms

50 ms



Detection probability (PD) as function of probability of false alarm (PFA), SNR = 0.5.

When FA = 0.005, SD = 7-10 px (length = 60px)

Conclusion and future work

- ✔ We proposed an effective two-stage algorithm which significantly reduces the number of hypotheses that have to be tested and the time of processing compared to the popular GLRT.
- ✔ Testing showed that the algorithm is capable of detecting streaks of space objects and accurately estimating their parameters with a signal-to-noise ratio near 1 both on simulated frames and on real data.
- ✔ The algorithm also showed good results in detection of faint streaks (down to SNR = 0.5) on real frames after background clutter suppression using spatiotemporal regression approach.
- ✔ In the future, it is planned to test the algorithm using other clutter filtering methods, as well as compare our trace detection approach with, for instance, Radon transform.

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Thank you!
Questions?