Propositional Logic

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* I do not claim authorship for all texts and pictures in the presentation.

Summary of Previous Lecture

1. *What is Categorical Logic?*
2. *Elements of Categorical Logic*
3. *Quantity*
4. *Quality*
5. *Distribution*

- *Exercises*
- *Rephrase the following statements (When needed)into categorical proposition. Then individuate subject, predicate, copula, and quantifier. Say which type of statement and determine which term (if any) gets distributed*

1. All kittens are adorable

2. Some logic lessons are exciting

3. Some dogs are biters

4. All squirrels are nut-gatherers

5. Atlanta is in Georgia

6. Heights always make me nauseous

7. The mosquitos are everywhere

8. Newborn babies aren’t potty-trained

9. Birds are in the backyard

10. Several toys are still on the floor

TODAY’S PLAN

Propositional Logic

- Historical Antecedents
- Modern Development
- The Basic Unit of Propositional Logic
- Connectives (Logical Operators)
- How to Individuate the Top Level Operator
- Well formed propositions
- Translating propositions into propositional logic
- Exercises

Historical Antecedents

Stoic philosophy

The Stoics = influential and powerful school of thought

KEY concepts: **ataraxia** and **virtues**

KEY Proponents: Zeno, Crisippus, Epictetus, Cato, Seneca, Marcus Aurelio (yes, the Emperor!)

STOIC – someone who remain calm under difficult situations

Yet, the Stoic is not a passive individual –quite the opposite. Person set on a path of self improvement guided by 4 basic virtues:

- Wisdom
- Temperance
- Justice
- Courage

Stoicism inspired Mandela during his time in prison!

STOICS: Important Contributions to Logic

Crisippus / Aristotle rivalry

Difference: Stoic logic was based on the analysis of propositions rather than terms

The Fundamental Logical notion (atomic) for the Stoics was not that of term but rather that of **Assertible**

Assertible: 4 features

a. Sound uttered;
b. The actual things to which the sound referred;
c. Lekton, the sayable was conveyed by the language;
d. It possesses a truth value at any point in time

Atomic Units (Assertibles: ‘it is night’) can be enriched and expanded via connectives (as in propositional logic as we shall see)

Stoics: 3 connectives

Conditional (if)
Conjunctive (and)
Disjunctive (or)

Modern Propositional Logic we have 5

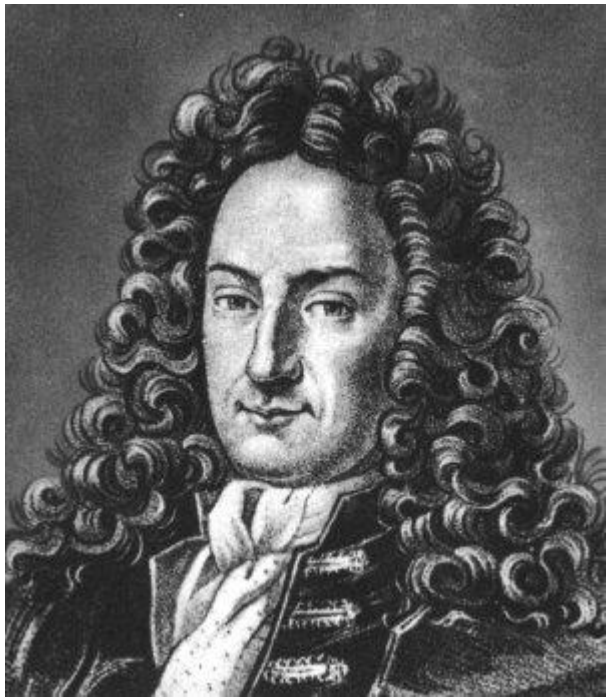
Stoic logic was largely forgotten (only rediscovered in modern times)

Modern Development : Leibniz

Gottfried Wilhelm Leibniz = philosopher and polymath

Contributions to physics, engineering, philosophy, mathematics, theology, history, law, politics, and philology..

He also discovered the binary number system and created the first calculator



Leibniz was born in Leipzig to prominent parents.

Age 12 = fluent in Latin and Greek
Age 13 = . he was deep into the works of Aristotle and scholastic philosophy,
Age 14 = entered the University of Leipzig,
Age 19 = Offered a professorship.

Leibniz loved a luxurious life… ☺

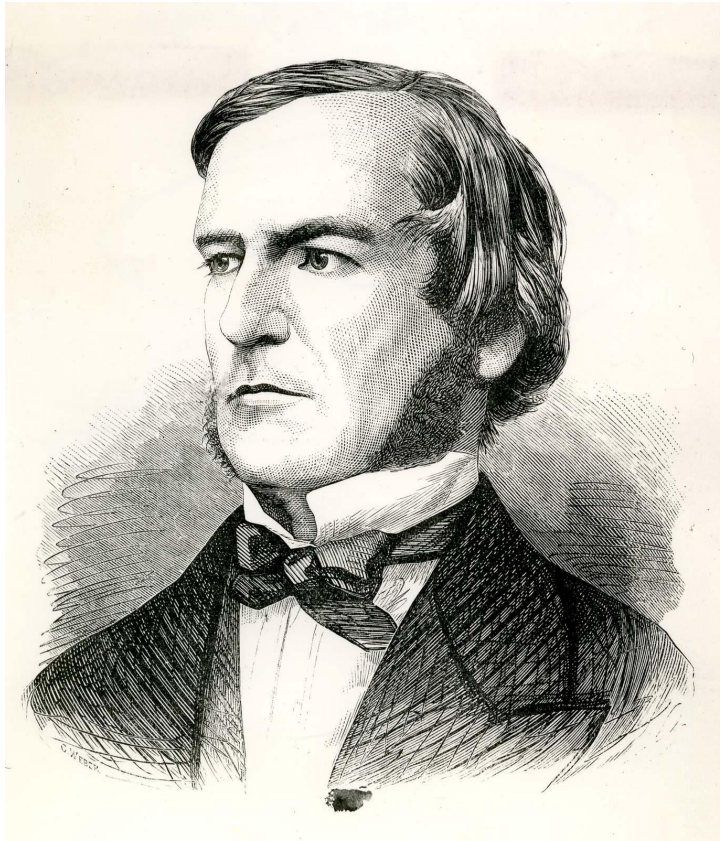
However, at age 18 he lost his huge inheritance , tricked by his uncle

To satisfy his expensive taste he decided to attach himself to rich people, who would pay for his lectures and intellectual company.

First he worked for the Duke Hannover, then worked for Peter the Great (he is the founder of the Russian Academy of Sciences) and even for Catherine the Great

Modern Development : Boole

English mathematician who **helped establish modern propositional logic and whose algebra of logic,** now called **Boolean algebra**, is basic to the design of digital computer circuits.



*Poor Family, unlike Leibnitz

Another genius …. who died because of pneumonia, which he caught because he delivered a lecture with wet clothes….

- The Basic Unit of Propositional Logic

Proposition!

A proposition refers to the meaning of a claim. So, an argument may have many propositions which are labelled by different letters or numbers (1,2, 3, etc)

Propositional Logic more natural tool for studying validity and truth of proposition than Categorical Logic

EXAMPLE: Prof Farina is a clown and Prof Farina likes little kids

How can symbolise this?

Prof Farina is a clown (proposition 1)

Prof Farina likes little kids (proposition 2)

And (connective, logical operator)

…

We need to introduce all possible connectives

CONNECTIVES

WHAT ARE THE DIFFERENT LOGICAL OPERATORS?*

Operator	Name	Logical function	Used to translate
~	tilde	negation	not, it is not the case that
•	dot	conjunction	and, also, moreover
∨	wedge	disjunction	or, unless
⊃	horseshoe	implication	if . . . then . . . , only if
≡	triple bar	equivalence	if and only if

DIFFERENCE BETWEEN SIMPLE STATEMENTS AND COMPOUND STATEMENT?

So, negation can exist and be found in simple statements (statements that have 1 proposition)

Whereas the other 4 logical operators can only exist in compound sentences (that is statements that consist of two or more propositions)

The great thing about propositional logic is precisely that allows us to study the level of truth and validity of complex statements

Translation Examples

It is not the case that *A*.
D and *C*.
Either *P* or *E*.
If *N* then *F*.
B if and only if *R*.

IMPORTANT: Basic way to learn how to use logical operators and build complex statements

Rolex does not make computers.	$\sim R$
It is not the case that Rolex makes computers.	$\sim R$
It is false that Rolex makes computers.	$\sim R$

WHAT HAPPENS WHEN THERE ARE TWO CONNECTIVES IN THE SAME STATEMENTS?

George is a clown and doesn’t like to wear hats

It is not the case the George is a clown and he likes hats

What is the top logic operator?

$$[(R \supset T) \vee (S \supset U)] \cdot [(W \equiv X) \vee (Y \equiv Z)]$$

$\sim C \vee \sim D$

$(F \cdot H) \vee (\sim K \cdot \sim L)$

$\sim [S \cdot (T \supset U)] \vee \sim [X \cdot (Y \equiv Z)]$

$H \supset \sim J$

$(A \vee C) \supset \sim (D \cdot E)$

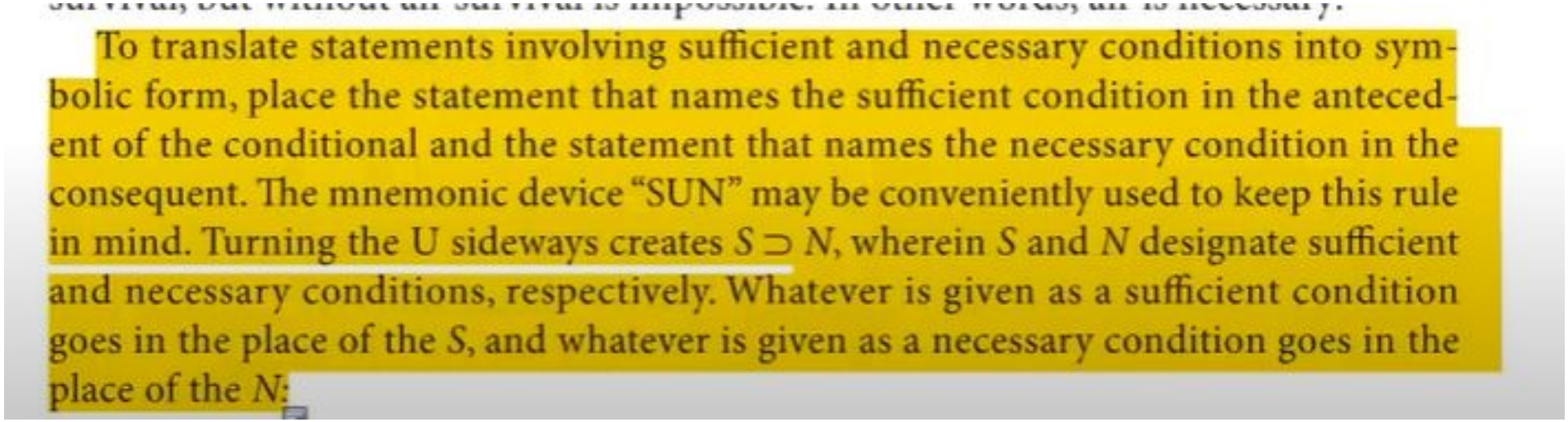
$[K \vee (S \cdot \sim T)] \supset [\sim F \vee (M \cdot O)]$

A condition *A* is said to be *necessary* for a condition *B*, if (and only if) the falsity (/nonexistence /non-occurrence) [as the case may be] of *A* guarantees (or brings about) the falsity (/nonexistence /non-occurrence) of *B*

A condition *A* is said to be *sufficient* for a condition *B*, if (and only if) the truth (/existence /occurrence) [as the case may be] of *A* guarantees (or brings about) the truth (/existence /occurrence) of *B*.

[Air is necessary for human life] –> necessary but not sufficient
[John is a king" implies that John is male. So knowing that John is a king is sufficient to knowing that he is a male.]

See:
https://www.sfu.ca/~swartz/conditions1.htm#:~:text=Examples%208.2%20%2D%20The%20first%20is,%2C%20for%20being%20a%20male.%22&text=%22Winning%20a%20lottery%20is%20a,%2C%20for%20having%20a%20ticket.%22



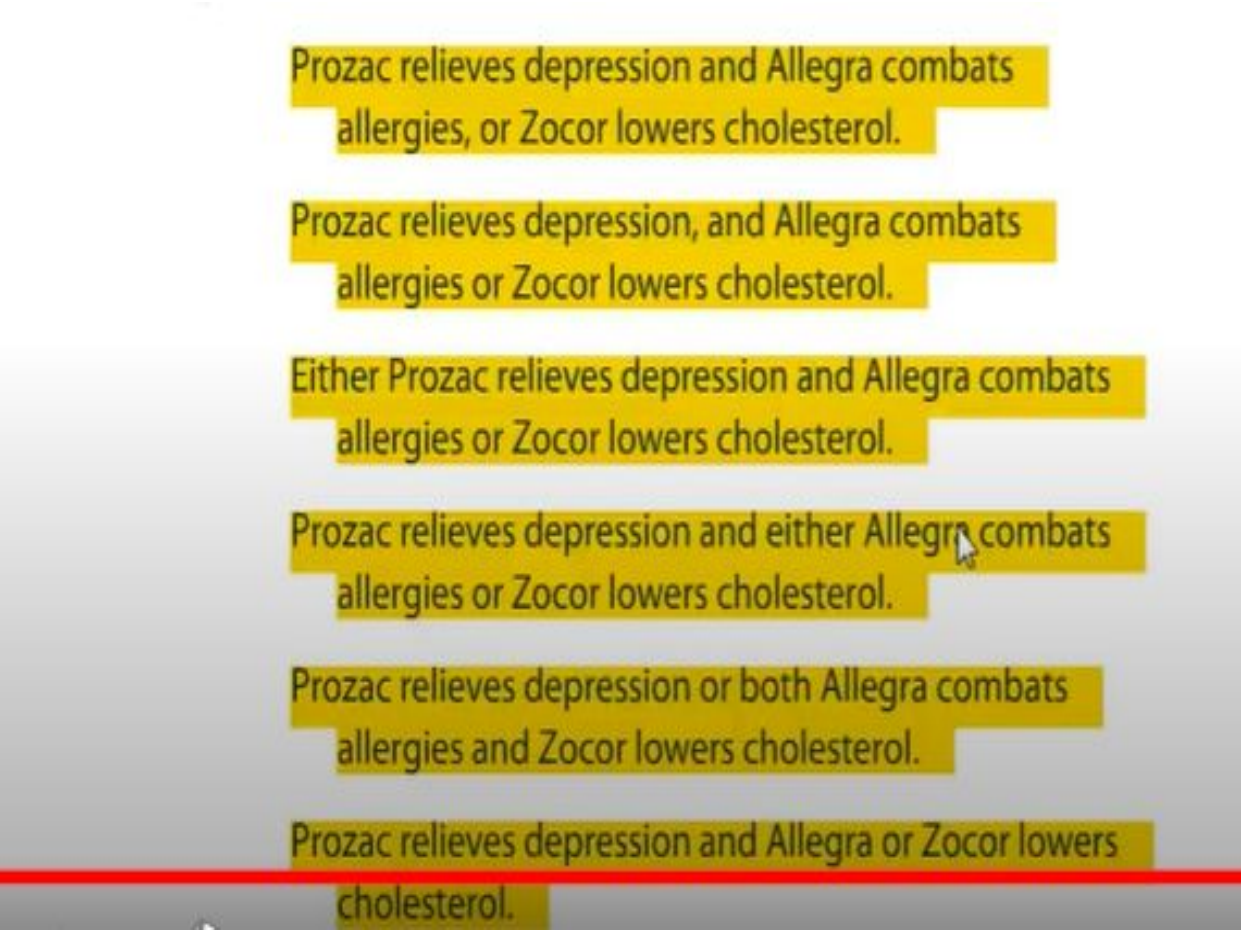
To translate statements involving sufficient and necessary conditions into symbolic form, place the statement that names the sufficient condition in the antecedent of the conditional and the statement that names the necessary condition in the consequent. The mnemonic device "SUN" may be conveniently used to keep this rule in mind. Turning the U sideways creates $S \supset N$, wherein S and N designate sufficient and necessary conditions, respectively. Whatever is given as a sufficient condition goes in the place of the S, and whatever is given as a necessary condition goes in the place of the N:

EXAMPLE:

Hilton's opening a new hotel is a sufficient condition for Marriott's doing so

Hilton's opening a new hotel is a necessary condition for Marriott's doing so

Like in Algebra we need to be careful where we put our parenthesis and brackets



EXERCISES

If MIG changing its logo implies that Sukhoi increases sales, then Antonov will reorganise

If Gazprom and Lukoil lower prices or Rosneft downsizes, then Novatek will expand production

I will not pass this class unless I go to class every day and do all of the homework exercises.

I lock the doors and close the windows whenever I leave to go to work.

Getting up on time and getting ready quickly is sufficient for arriving at work on time.

Practicing an hour a day and getting private lessons twice a week is necessary for playing in the wind ensemble.

WHAT YOU LEARNED TODAY?

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