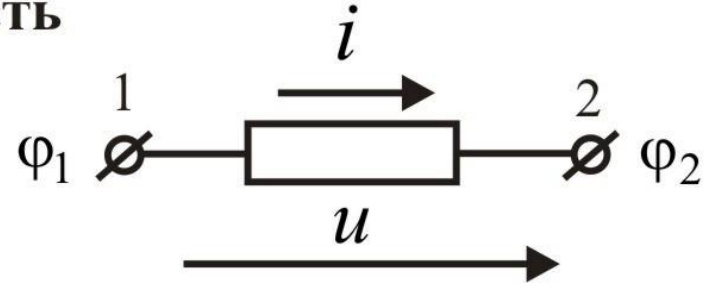


Методы расчета линейных электрических цепей постоянного тока

Теоретическая часть

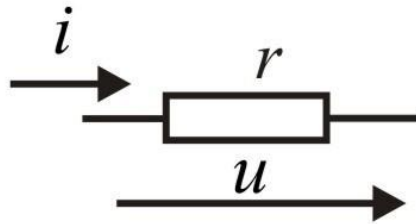
Ток [А] $i = \frac{dq}{dt}$, $q = q_+ + q_-$ $i > 0$ $i < 0$



Напряжение [В] $u_{12} = \varphi_1 - \varphi_2$ $u_{12} = -u_{21}$

Мощность [Вт] $dw = udq = uidt$. $p = \frac{dw}{dt} = ui$. $p > 0$ $p < 0$ $w = \int_{t_1}^{t_2} p dt$.

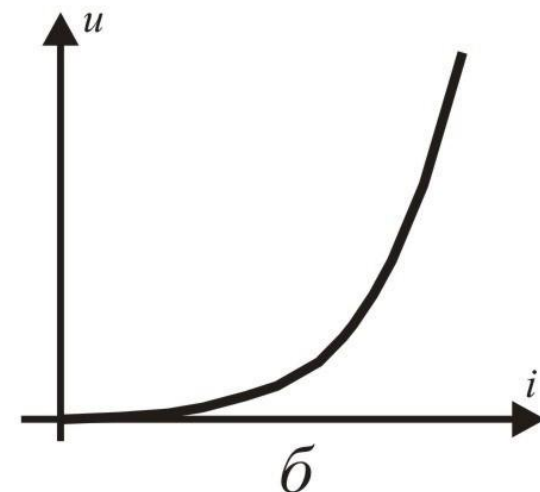
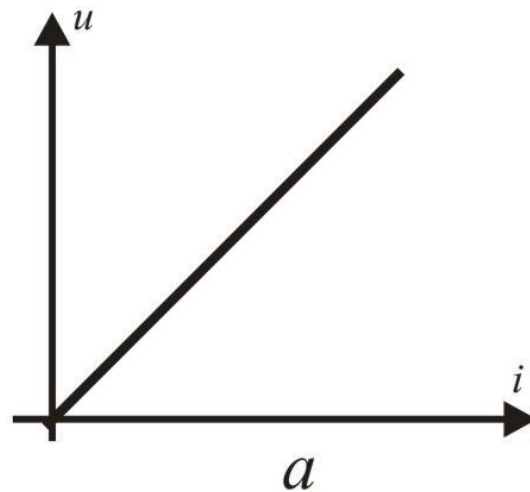
Сопротивление [Ом]



$$r = \frac{u}{i} > 0.$$

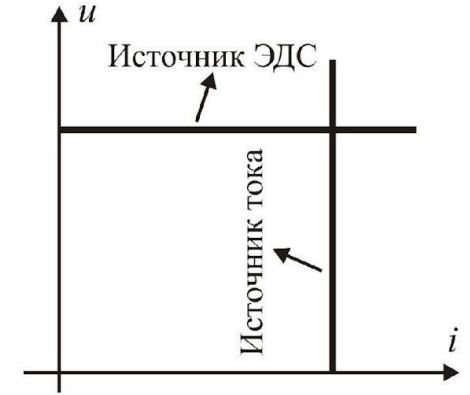
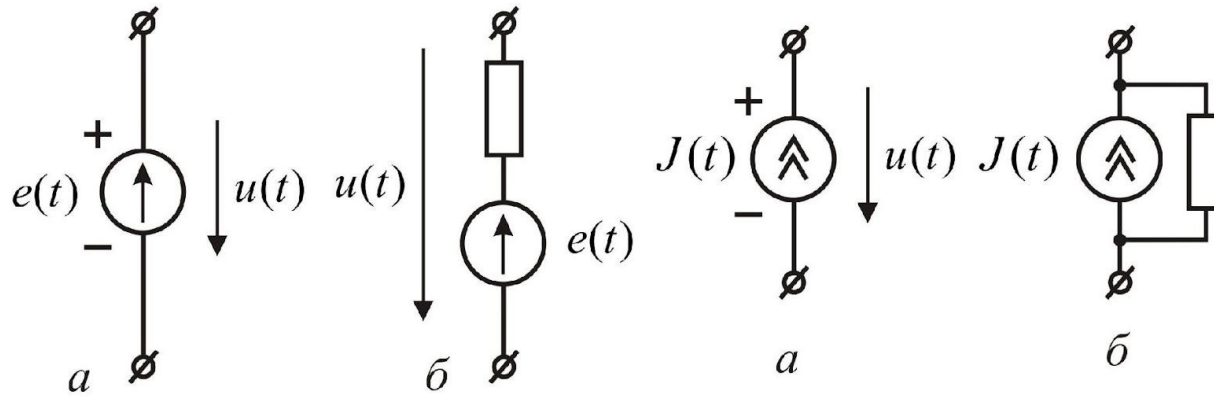
$$g = \frac{1}{r}$$

$$p_r = ui = ri^2 = gu^2.$$

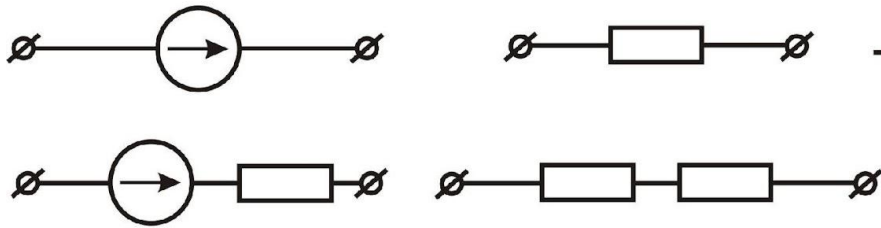


$$w_r = \int_0^t p_r dt = \int_0^t ri^2 dt = \int_0^t gu^2 dt.$$

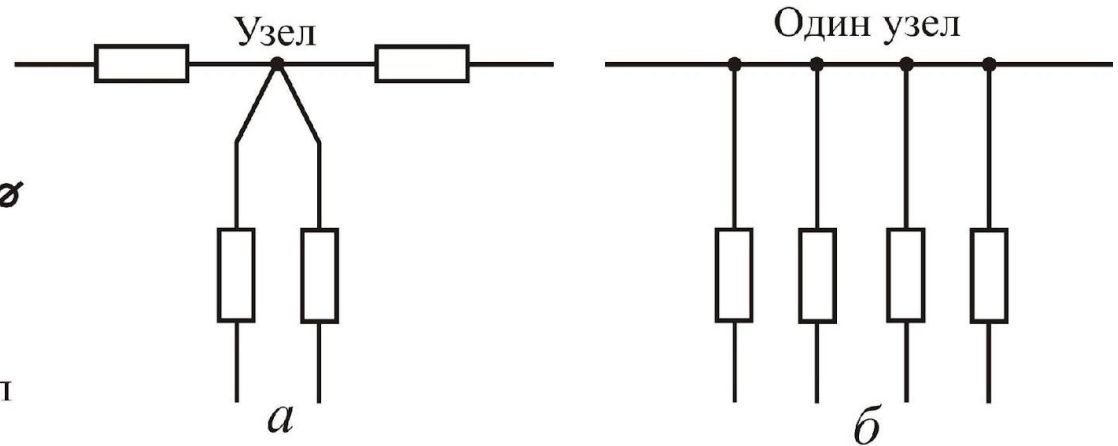
Источник ЭДС и источник тока



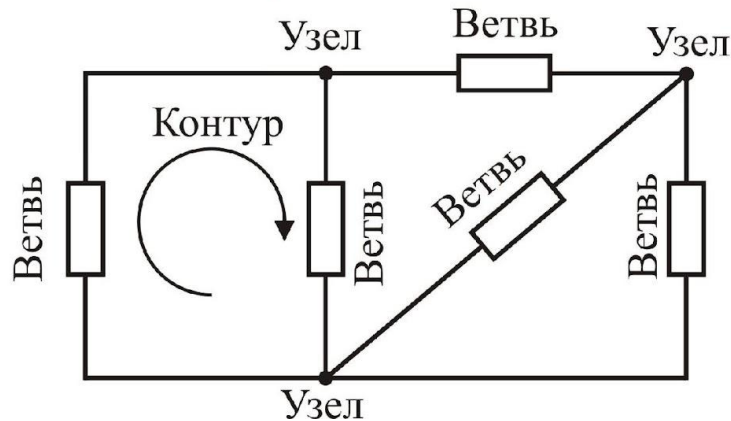
Изображения ветвей схемы



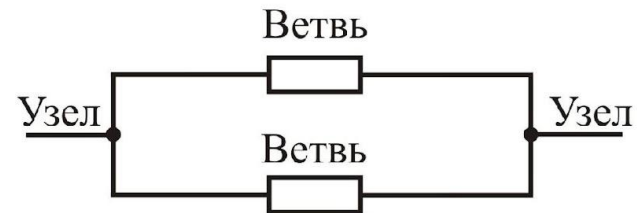
Изображение узла схемы



Электрическая схема

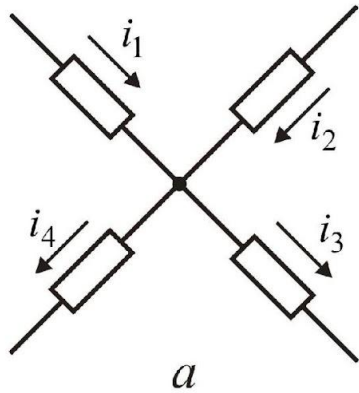


Параллельное соединение ветвей



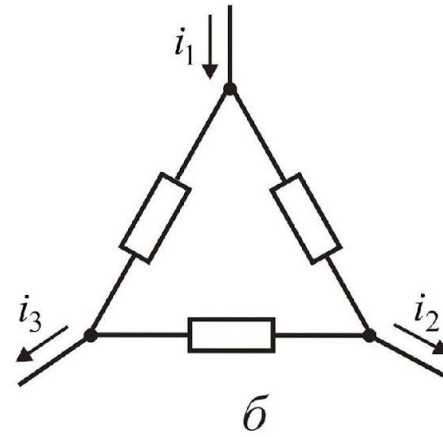
Законы Кирхгофа

Первый закон $\sum_{k=1}^n \pm i_k = 0.$



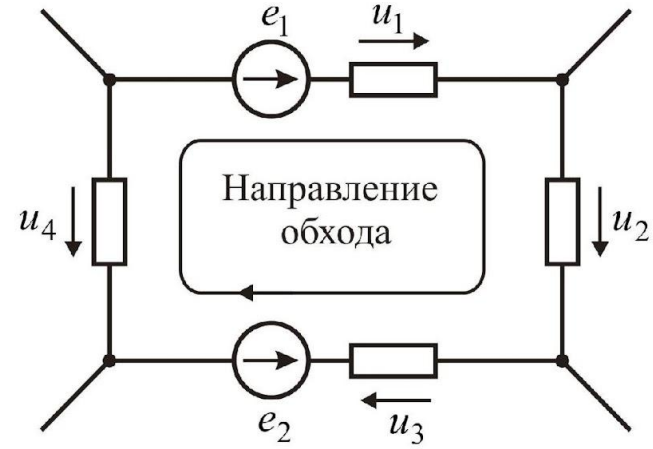
$$i_1 + i_2 - i_3 - i_4 = 0.$$

Второй закон



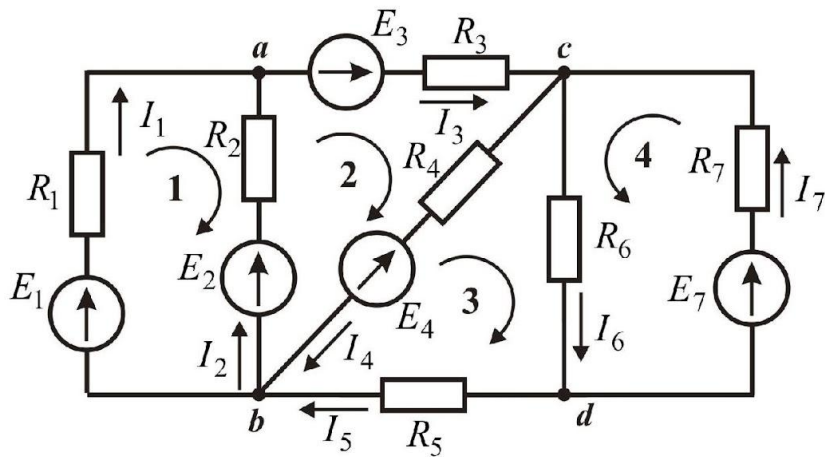
$$i_1 - i_2 - i_3 = 0.$$

$\sum_{k=1}^n \pm e_k = \sum_{k=1}^n \pm u_k$



$$e_1 - e_2 = u_1 + u_2 + u_3 - u_4.$$

Пример расчета по законам Кирхгофа



Первый закон

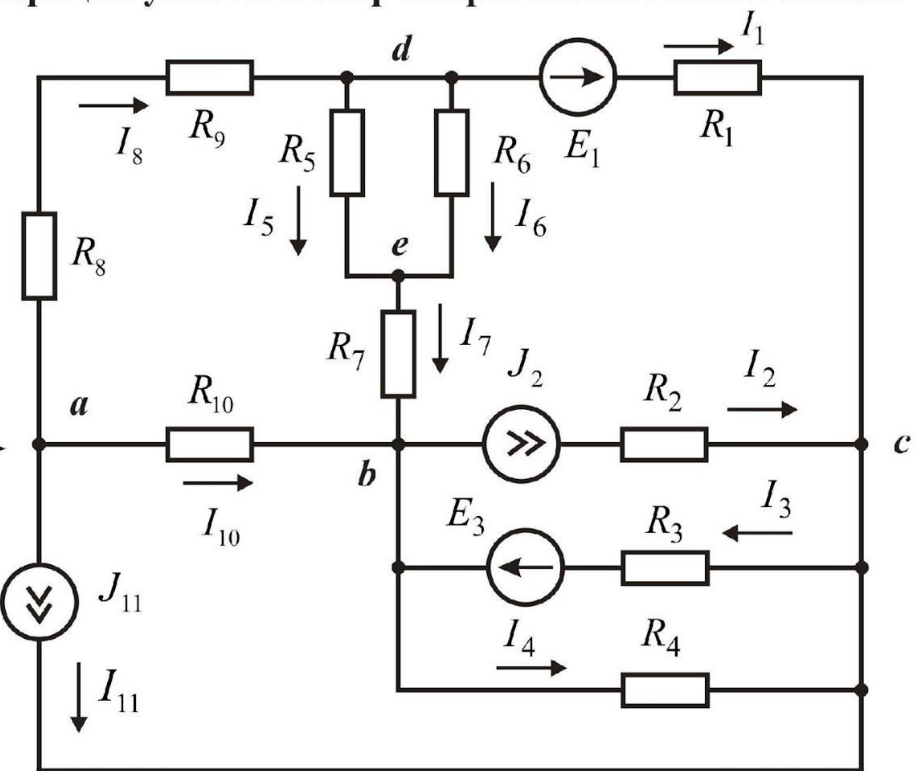
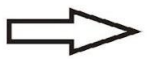
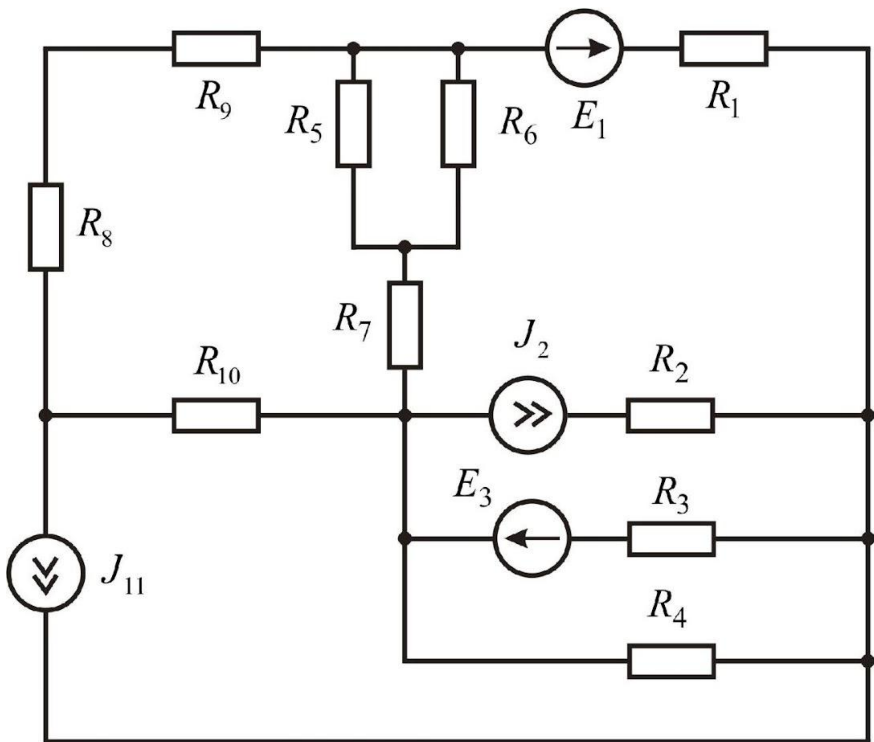
$a: I_1 + I_2 - I_3 = 0;$
 $b: -I_1 - I_2 + I_4 + I_5 = 0;$
 $c: I_3 - I_4 - I_6 + I_7 = 0;$
 $d: -I_5 + I_6 - I_7 = 0.$

Второй закон

Контур 1 $I_1 R_1 - I_2 R_2 = E_1 - E_2;$
 Контур 2 $I_2 R_2 + I_3 R_3 + I_4 R_4 = E_2 + E_3 - E_4;$
 Контур 3 $-I_4 R_4 + I_6 R_6 + I_5 R_5 = E_4;$
 Контур 4 $I_7 R_7 + I_6 R_6 = E_7.$

Пример расчета

Нумерация узлов и выбор направлений токов в ветвях



$J_{11} = 1; J_2 = 2; E_1 = 25; E_3 = 40; R_1 = 14; R_2 = 18;$
 $R_3 = R_4 = R_8 = R_9 = 10; R_5 = 20; R_6 = 180; R_7 = 12; R_{10} = 50.$

Выполняем эквивалентные преобразования

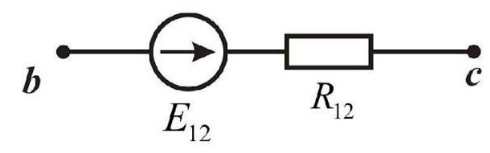
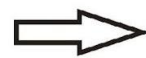
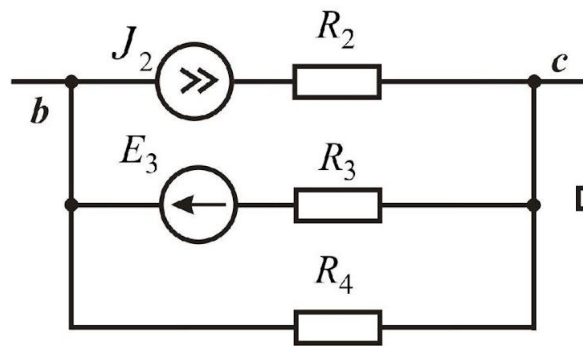
$$R_{89} = R_8 + R_9 = 20$$

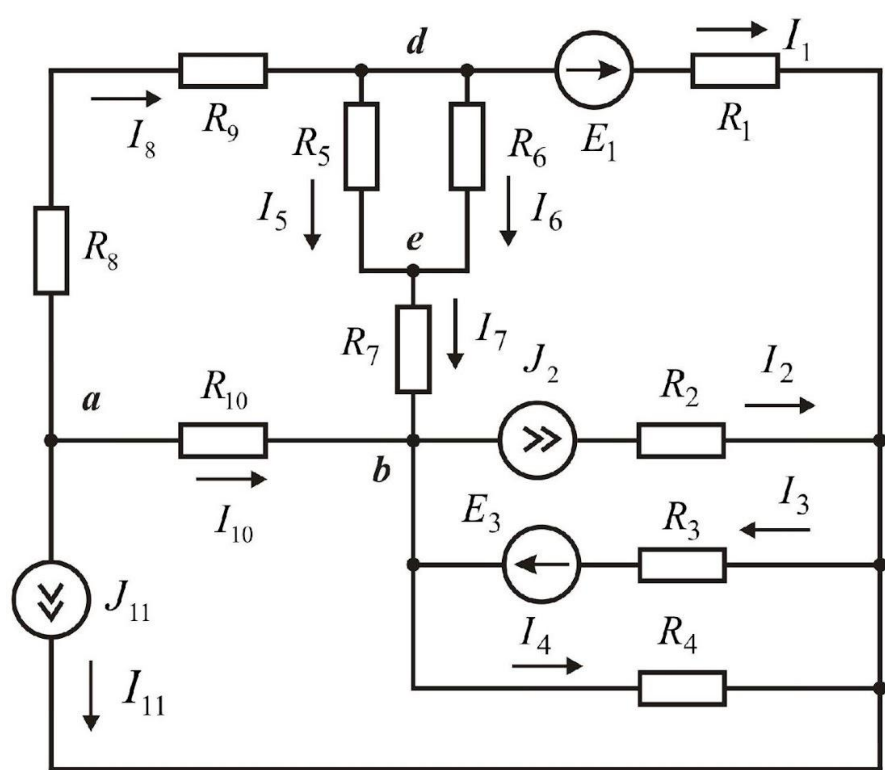
$$\frac{1}{R_{56}} = \frac{1}{R_5} + \frac{1}{R_6} \Rightarrow R_{56} = \frac{R_5 \cdot R_6}{R_5 + R_6} = 18$$

$$R_{567} = R_{56} + R_7 = 30$$

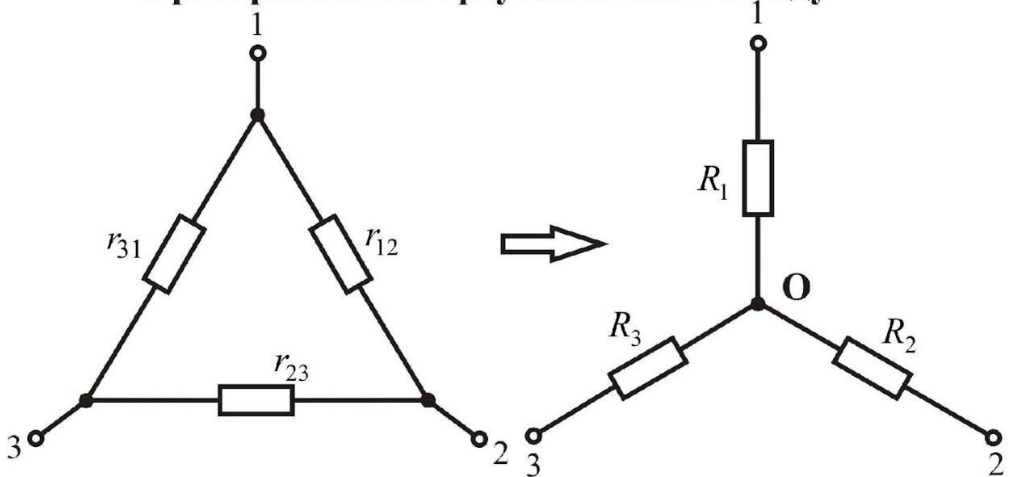
$$\frac{1}{R_{12}} = \frac{1}{R_2 + \infty} + \frac{1}{R_3 + 0} + \frac{1}{R_4} \Rightarrow R_{12} = 5$$

$$E_{12} = \frac{J_2 - \frac{E_3}{R_3}}{\frac{1}{R_{12}}} \Rightarrow E_{12} = -15$$

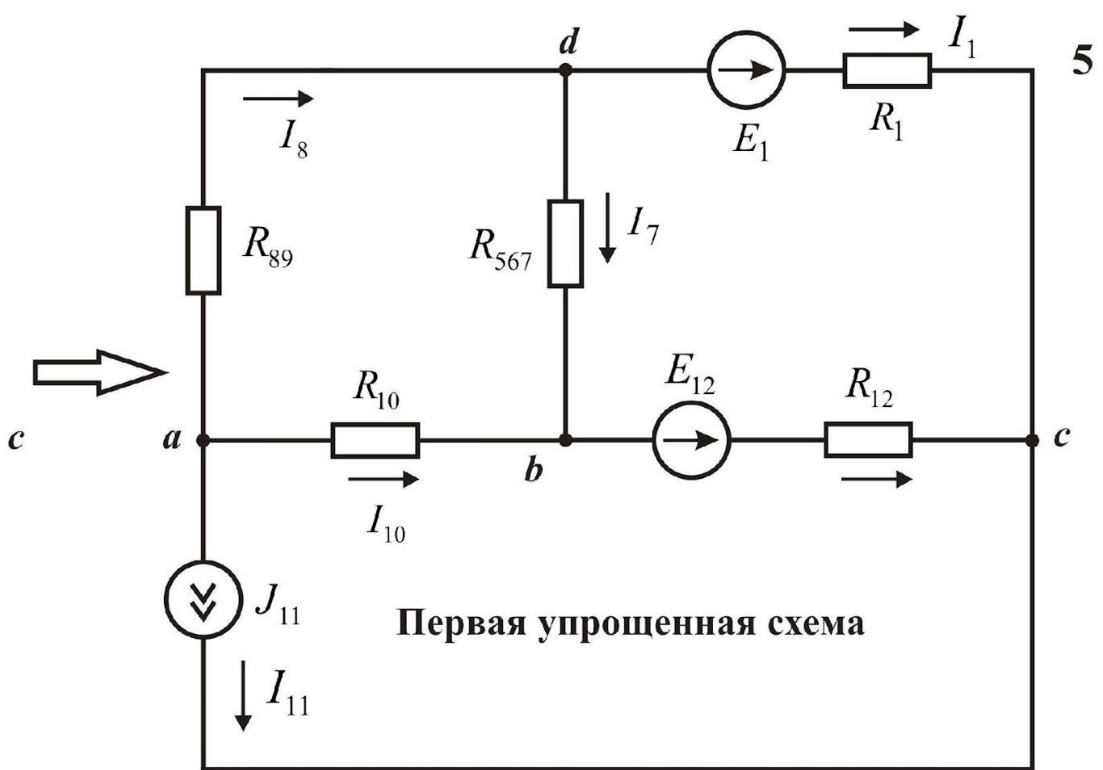




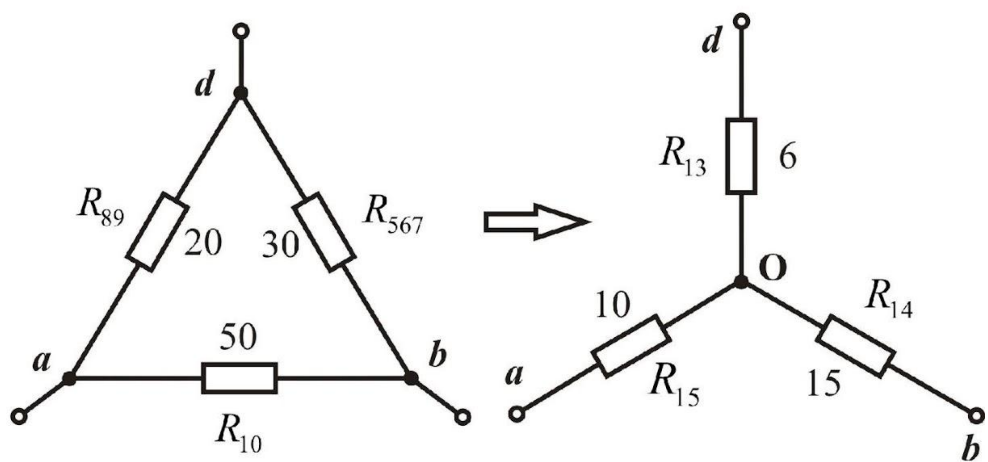
Преобразование треугольника в звезду

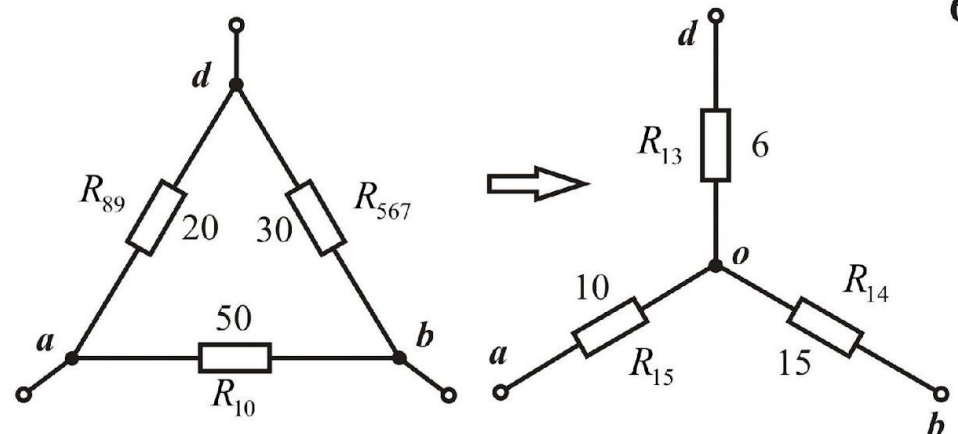
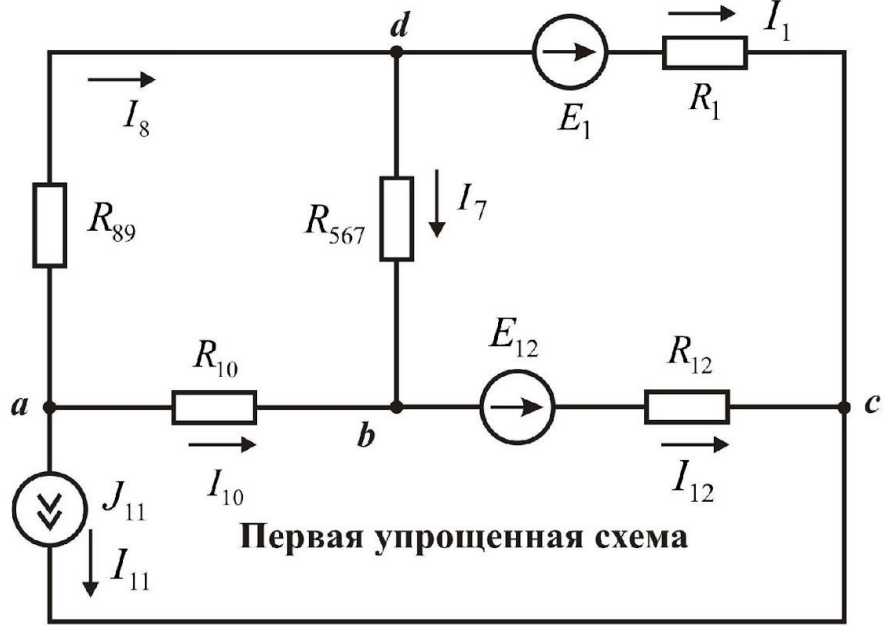


$$R_1 = \frac{r_{12}r_{31}}{r_{12} + r_{23} + r_{31}}; \quad R_2 = \frac{r_{12}r_{23}}{r_{12} + r_{23} + r_{31}}; \quad R_3 = \frac{r_{23}r_{31}}{r_{12} + r_{23} + r_{31}}.$$



Первая упрощенная схема

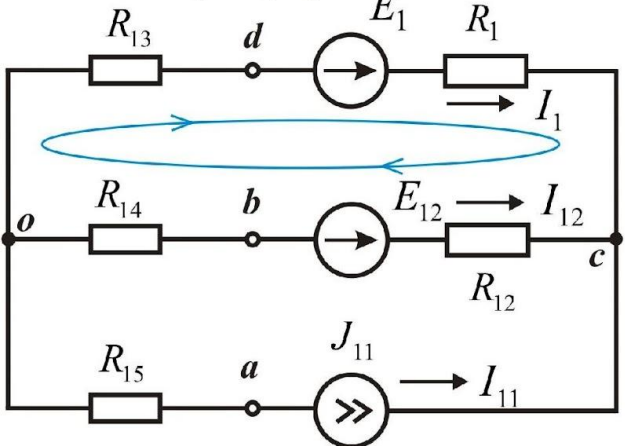




$$I_{10} \cdot R_{10} - U_{ab} = 0 \Rightarrow I_{10} = \frac{U_{ab}}{R_{10}}$$

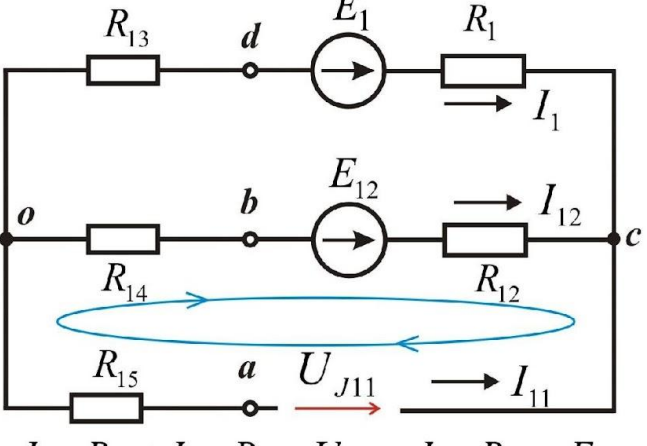
$$I_7 = \frac{U_{db}}{R_{567}}$$

Вторая упрощенная схема



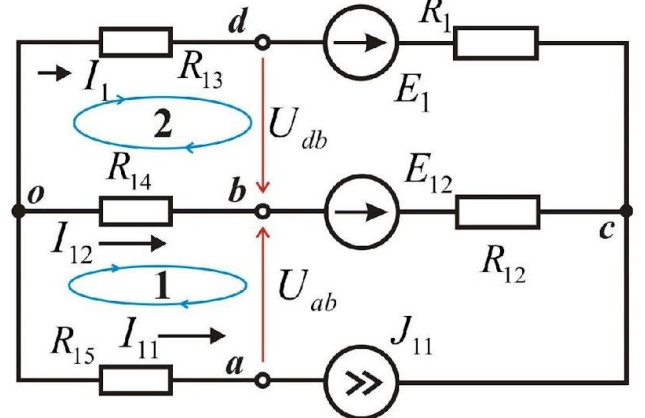
$$I_{11} = J_{11} = 2 \quad I_1 + I_{12} + I_{11} = 0$$

$$I_1 \cdot R_{13} + I_1 \cdot R_{11} - I_{12} \cdot R_{12} - I_{12} \cdot R_{14} = E_1 - E_{12} \Rightarrow I_1 = 0; I_{12} = -2; I_{11} = 2$$



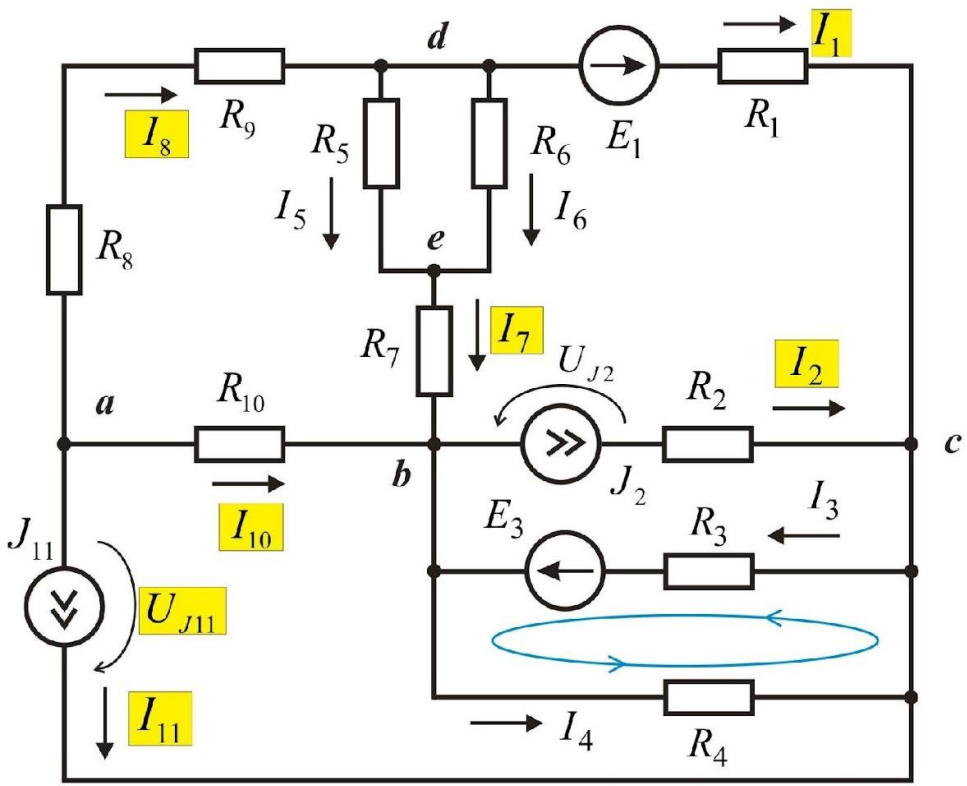
$$I_{12} \cdot R_{14} + I_{12} \cdot R_{12} - U_{J11} - I_{11} \cdot R_{15} = E_{12}$$

$$U_{J11} = -45$$



$$I_{12} \cdot R_{14} - U_{ab} - I_{11} \cdot R_{15} = 0 \Rightarrow U_{ab} = -50 \Rightarrow I_{10} = -1$$

$$I_1 \cdot R_{13} + U_{db} - I_{12} \cdot R_{14} = 0 \Rightarrow U_{db} = -30 \Rightarrow I_7 = -1 \quad I_8 = -1$$



$$I_5 + I_6 = I_7 = -1; I_5 \cdot R_5 - I_6 \cdot R_6 = 0 \Rightarrow I_5 = -0,9; I_6 = -0,1$$

$$I_1 + I_2 + I_{11} + I_4 - I_3 = 0 \quad I_3 \cdot R_3 + I_4 \cdot R_4 = E_3 \quad I_4 = 0,5; I_3 = 3,5$$

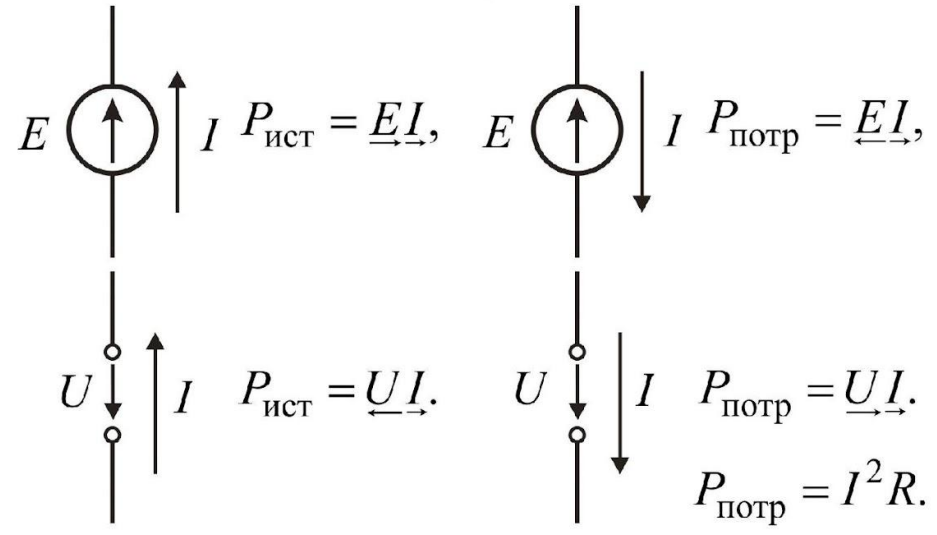
$$U_{J2} + I_4 \cdot R_4 - I_2 \cdot R_2 = 0 \Rightarrow U_{J2} = 13$$

$$P_{ист} = E_1 \cdot I_1 + E_3 \cdot I_3 + U_{J2} \cdot I_2 = 25 \cdot 0 + 40 \cdot 3,5 + 13 \cdot 1 = 153$$

$$P_{потр} = U_{J11} \cdot I_{11} + I_1^2 \cdot R_1 + I_2^2 \cdot R_2 + I_3^2 \cdot R_3 + I_4^2 \cdot R_4 + I_5^2 \cdot R_5 + I_6^2 \cdot R_6 + I_7^2 \cdot R_7 + I_8^2 \cdot (R_8 + R_9) + I_{10}^2 \cdot R_{10} = -45 \cdot 2 + 0 + 1 \cdot 18 + 12,25 \cdot 10 + 0,25 \cdot 10 + 0,81 \cdot 20 + 0,01 \cdot 180 + 1 \cdot 12 + 1 \cdot 20 + 1 \cdot 50 = 153$$

Баланс мощности
Источники

$$\sum P_{ист} = \sum P_{потр}$$



$$\left\{ P_{ист} = \sum_{k=1}^n E_k \cdot \underline{I}_k + \sum_{k=1}^n U_k \cdot \underline{I}_k \right\} =$$

$$\left\{ P_{потр} = \sum_{k=1}^n E_k \cdot \underline{I}_k + \sum_{k=1}^n U_k \cdot \underline{I}_k + \sum_{k=1}^n I_k^2 \cdot R_k \right\}$$