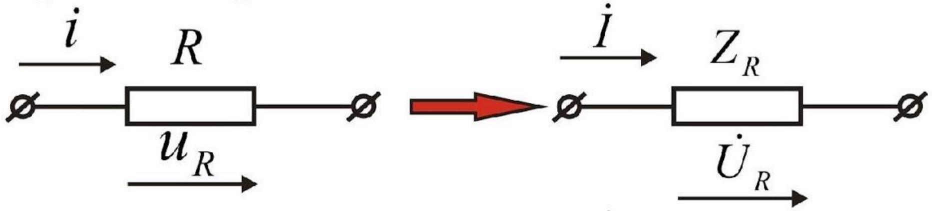
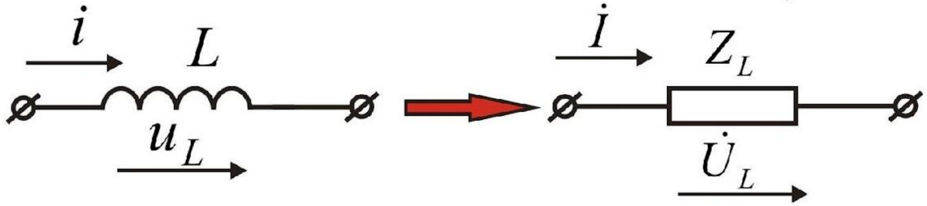
Расчет линейных цепей синусоидального тока комплексным методом

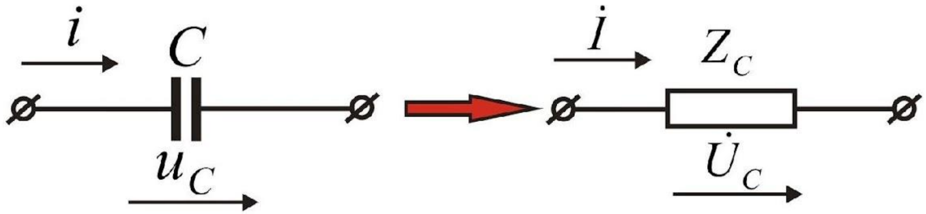
Правила перевода пассивных элементов и источников из временной области в комплексную



$$Z_R = R = R \cdot e^{j \cdot 0} \qquad j = \sqrt{-1} = e^{j \cdot 90}$$

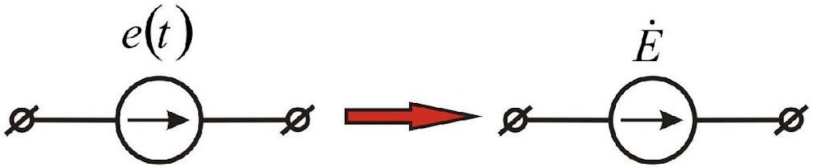


$$Z_L = j \cdot x_L = x_L \cdot e^{j \cdot 90^\circ};\ x_L = \omega L = |Z_L|$$



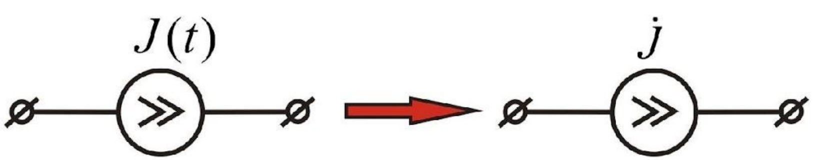
$$Z_C = -j \cdot x_C = x_C \cdot e^{-j \cdot 90^\circ};\ x_C = \frac{1}{\omega C} = |Z_C|$$

$$Z = r + j\left(\omega L - \frac{1}{\omega C}\right) = r + jx. \quad Z = ze^{j\varphi} = z\angle\varphi. \qquad z = |Z| \qquad z = \sqrt{r^2 + x^2};\ \varphi = arctg\,\frac{x}{r}.$$



$$e(t) = E_m \sin(\omega t + \varphi_e) = E\sqrt{2}\sin(\omega t + \varphi_e)$$

$$\dot{E} = E \cdot e^{j\varphi_e} = E \cdot \cos\varphi_e + jE \cdot \sin\varphi_e$$



$$J(t) = J_m \sin(\omega t + \varphi_J) = J\sqrt{2}\sin(\omega t + \varphi_J)$$

$$\dot{J} = J \cdot e^{j\varphi_J} = J \cdot \cos\varphi_J + jJ \cdot \sin\varphi_J$$

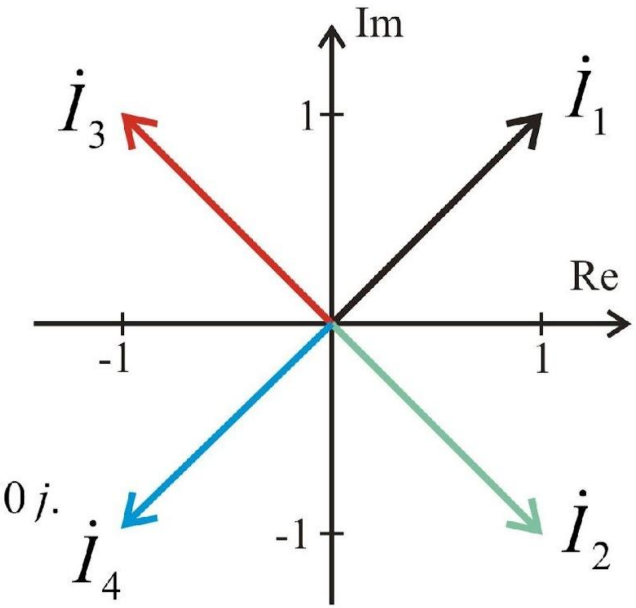
Перевести из временной области в комплексную

$$e(t)=100\sin\omega t \Rightarrow \dot{E}=\frac{100}{\sqrt{2}}\cdot e^{j0}=50\sqrt{2}(\cos 0+j\sin 0)=50\sqrt{2};$$

$$u(t)=100\sin(\omega t+\frac{\pi}{4})=\frac{100}{\sqrt{2}}\cdot e^{j\frac{\pi}{4}}=50\sqrt{2}\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)=50+50j;$$

$$i(t)=5\sqrt{2}\sin(\omega t-\frac{\pi}{2})=\frac{5\sqrt{2}}{\sqrt{2}}\cdot e^{-j\frac{\pi}{2}}=5(0-j\cdot 1)=-5j;$$

$$u(t)=100\sin(\omega t-135^\circ)=\frac{100}{\sqrt{2}}\cdot e^{-j135^\circ}=50\sqrt{2}\left(-\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}\right)=-50-50j.$$



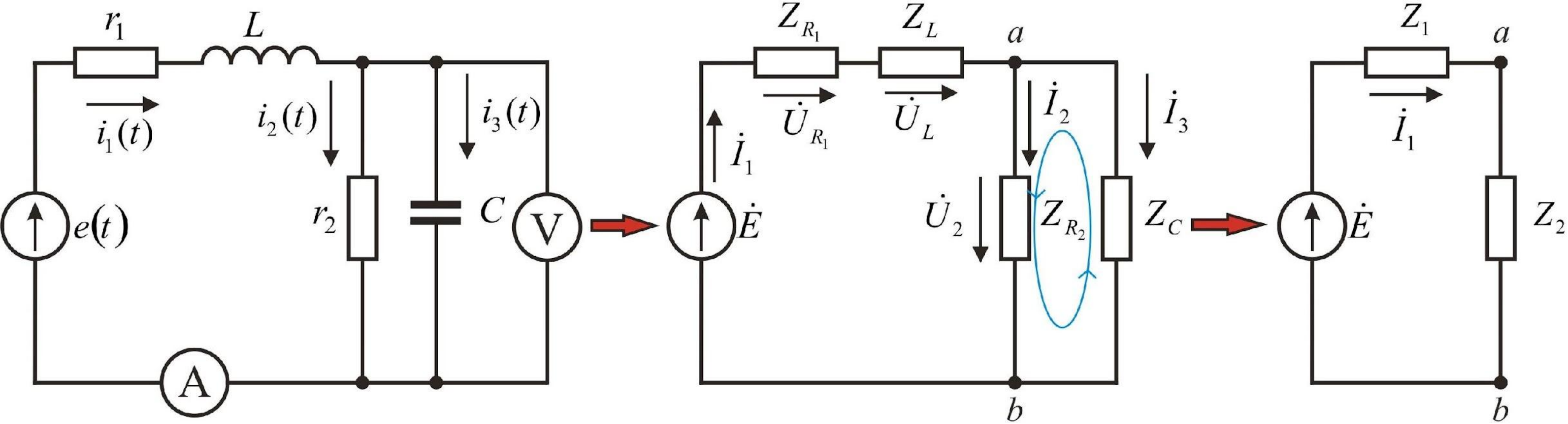
Перевести из комплексной области во временную

$$\dot{I}=10+20j \Rightarrow I=\sqrt{10^2+20^2}=22{,}36 \Rightarrow I_m=I\sqrt{2}=31{,}5;\varphi=\mathrm{arctg}\frac{20}{10}\Rightarrow i(t)=31{,}5\sin(\omega t+\mathrm{arctg}2);$$

$$\dot{U}=-10j \Rightarrow U=\sqrt{0^2+(-10)^2}=10 \Rightarrow U_m=10\sqrt{2};\varphi=\mathrm{arctg}\frac{-10}{0}=-\frac{\pi}{2}\Rightarrow u(t)=10\sqrt{2}\sin(\omega t-\frac{\pi}{2});$$

$$\dot{I}_1=1+j;\dot{I}_2=1-j;\dot{I}_3=-1+j;\dot{I}_4=-1-j \Rightarrow I_1=I_2=I_3=I_4=\sqrt{2};I_{1m}=I_{2m}=I_{3m}=I_{4m}=2;$$

$$i_1(t)=2\sin(\omega t+45);i_2(t)=2\sin(\omega t-45);i_3(t)=2\sin(\omega t+135);i_4(t)=2\sin(\omega t-135).$$



$$e(t)=446\sin(1000t+18{,}5^\circ);\qquad \dot{E}=\frac{446}{\sqrt{2}}\cdot e^{j18,5}=316{,}2(\cos 18{,}5+j\sin 18{,}5)=300+100j$$

$$r_1=10;\ r_2=40;\ L=0{,}01;\ C=25\cdot 10^{-6}\qquad Z_{R_1}=10;\ Z_{R_2}=40;\ x_L=\omega L=10\Rightarrow Z_L=10j;\ x_C=\frac{1}{\omega C}=40\Rightarrow Z_C=-40j.$$

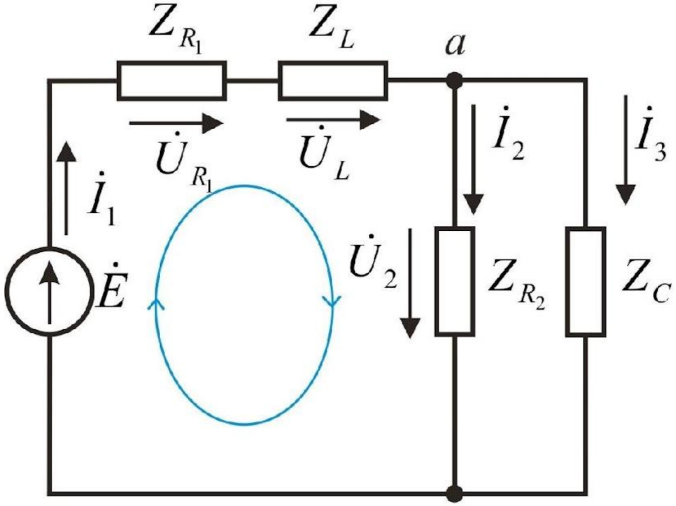
$$Z_1=Z_{R_1}+Z_L=10+10j\qquad Z_2=\frac{Z_{R_2}\cdot Z_C}{Z_{R_2}+Z_C}=\frac{40\cdot(-40j)}{40-40j}=\frac{(-40j)}{1-j}=\frac{(-40j)(1+j)}{(1-j)(1+j)}=\frac{40-40j}{2}=20-20j$$

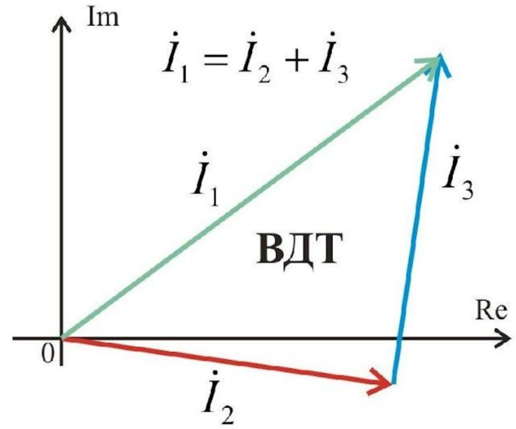
$$\sum_k \pm\dot{U}_k=\sum_k \pm\dot{E}_k\Rightarrow \dot{I}_1\cdot Z_1+\dot{I}_1\cdot Z_2=\dot{E}\Rightarrow \dot{I}_1=\frac{\dot{E}}{Z_1+Z_2}=\frac{300+100j}{30-10j}=\frac{(30+10j)(3+j)}{(3-j)(3+j)}=8+6j$$

$$\dot{I}_1=\dot{I}_2+\dot{I}_3;\ \dot{I}_2\cdot Z_{R_2}-\dot{I}_3\cdot Z_C=0\Rightarrow \dot{I}_2=\dot{I}_1\frac{Z_C}{Z_{R_2}+Z_C}=(8+6j)\frac{(-40j)}{40-40j}=7-j;\ \dot{I}_3=\dot{I}_1-\dot{I}_2=1+7j.$$

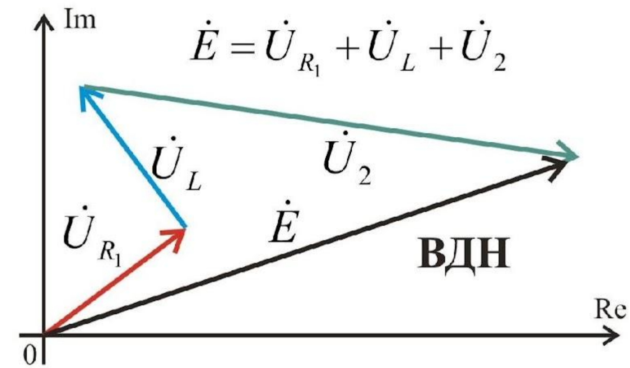
$$\dot{U}_{R_1}=\dot{I}_1\cdot Z_{R_1}=80+60j;\ \dot{U}_L=\dot{I}_1\cdot Z_L=-60+80j;\ \dot{U}_2=\dot{I}_2\cdot Z_{R_2}=\dot{I}_3\cdot Z_C=280-40j;$$

$$A=I_1=\left|\dot{I}_1\right|=\sqrt{8^2+6^2}=10;\ V=U_2=\left|\dot{U}_2\right|=\sqrt{280^2+40^2}=282{,}8$$





$\dot{I}_1 = 8 + 6j; \dot{I}_2 = 7 - j; \dot{I}_3 = 1 + 7j.$



$\dot{E} = 300 + 100j; \dot{U}_{R_1} = 80 + 60j;$

$\dot{U}_L = -60 + 80j; \dot{U}_2 = 280 - 40j.$

Баланс мощности в комплексной форме

$$\sum \tilde{S}_{\text{ист}} = \sum \tilde{S}_{\text{потр}} \Rightarrow \sum P_{\text{ист}} = \sum P_{\text{потр}};\ \sum Q_{\text{ист}} = \sum Q_{\text{потр}}.$$

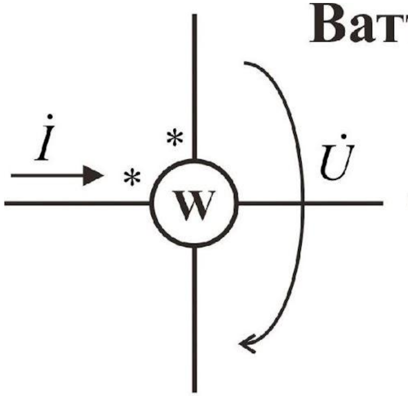
$$\left\{ \tilde{S}_{\text{ист}} = \sum_{k=1}^{n} \underset{\longrightarrow}{\dot{E}_k} \cdot \underset{\longrightarrow}{\overset{*}{I}_k} + \sum_{k=1}^{n} \underset{\longleftarrow}{\dot{U}_k} \cdot \underset{\longrightarrow}{\overset{*}{I}_k} \right\} = \left\{ \tilde{S}_{\text{потр}} = \sum_{k=1}^{n} \underset{\longleftarrow}{\dot{E}_k} \cdot \underset{\longrightarrow}{\overset{*}{I}_k} + \sum_{k=1}^{n} \underset{\longrightarrow}{\dot{U}_k} \cdot \underset{\longrightarrow}{\overset{*}{I}_k} + \sum_{k=1}^{n} I_k^2 \cdot Z_k \right\}. \qquad I_k^2 = \dot{I}_k \cdot \overset{*}{I}_k.$$

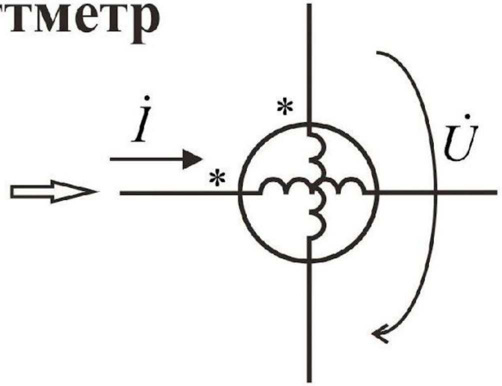
$$\tilde{S}_{\text{ист}} = \dot{E} \cdot \overset{*}{I}_1 = (300 + 100j) \cdot (8 - 6j) = 3000 - 1000j$$

$$I_1^2 = (8 + 6j)(8 - 6j) = 100;\ I_2^2 = (7 - j)(7 + j) = 50;\ I_3^2 = (1 + 7j)(1 - 7j) = 50$$

$$\tilde{S}_{\text{потр}} = I_1^2 \cdot Z_{R_1} + I_1^2 \cdot Z_L + I_2^2 \cdot Z_{R_2} + I_3^2 \cdot Z_C = 100 \cdot 10 + 100 \cdot 10j + 50 \cdot 40 + 50 \cdot (-40j) = 3000 - 1000j$$

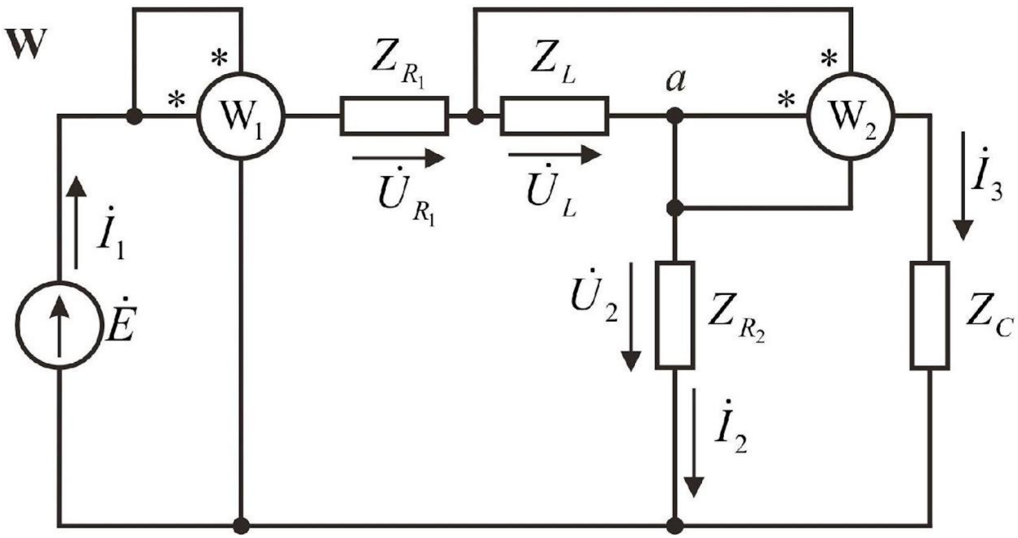
Ваттметр





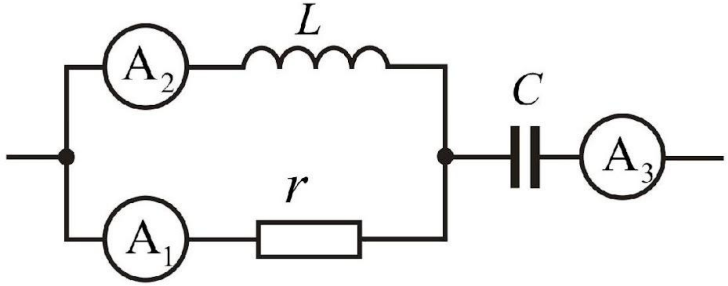
$$\tilde{S} = \dot{U} \cdot \overset{*}{I} = P + jQ; P = U \cdot I \cdot \cos\varphi;$$

$$Q = U \cdot I \cdot \sin\varphi; \mathrm{W} = P = \mathrm{Re}(\tilde{S})$$

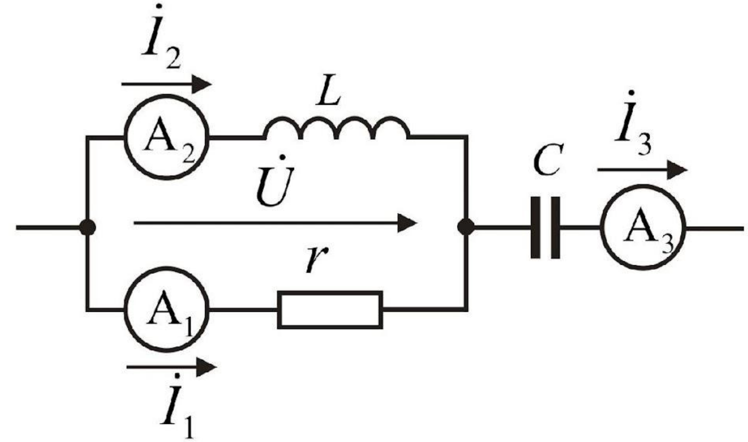


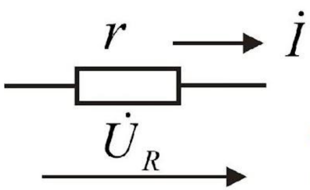
$$\mathrm{W}_1 = \mathrm{Re}(\tilde{S}_1) = \mathrm{Re}(\dot{E} \cdot \overset{*}{I}_1)$$

$$\mathrm{W}_2 = \mathrm{Re}(\tilde{S}_2) = \mathrm{Re}(\dot{U}_L \cdot \overset{*}{I}_3)$$

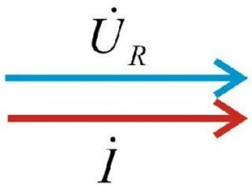


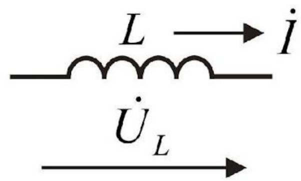
$A_1 = 3;\ A_3 = 5;\ A_2 = ?$



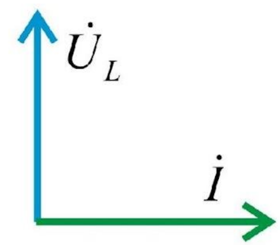


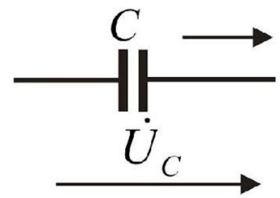
$\dot{U}_R = \dot{I} \cdot r$



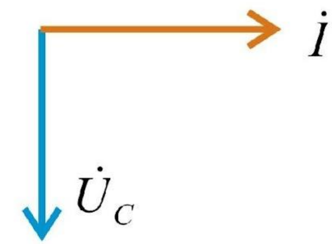


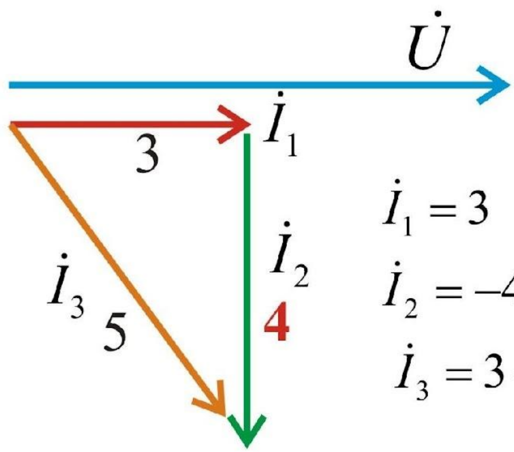
$\dot{U}_L = \dot{I} \cdot jx_L$





$\dot{U}_C = \dot{I} \cdot (-jx_C)$

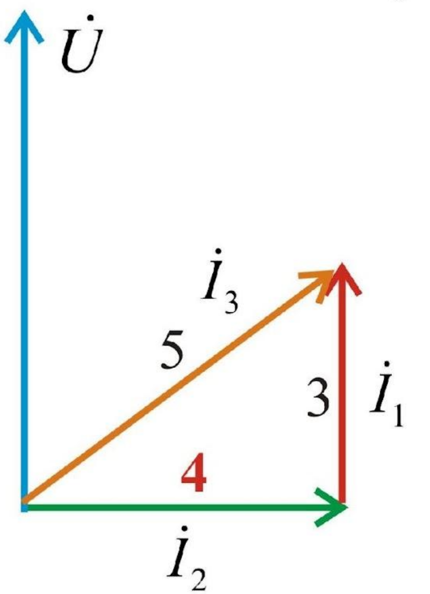




$\dot{I}_1 = 3$

$\dot{I}_2 = -4j$

$\dot{I}_3 = 3 - 4j$



$\dot{I}_1 = 3j$

$\dot{I}_2 = 4$

$\dot{I}_3 = 4 + 3j$