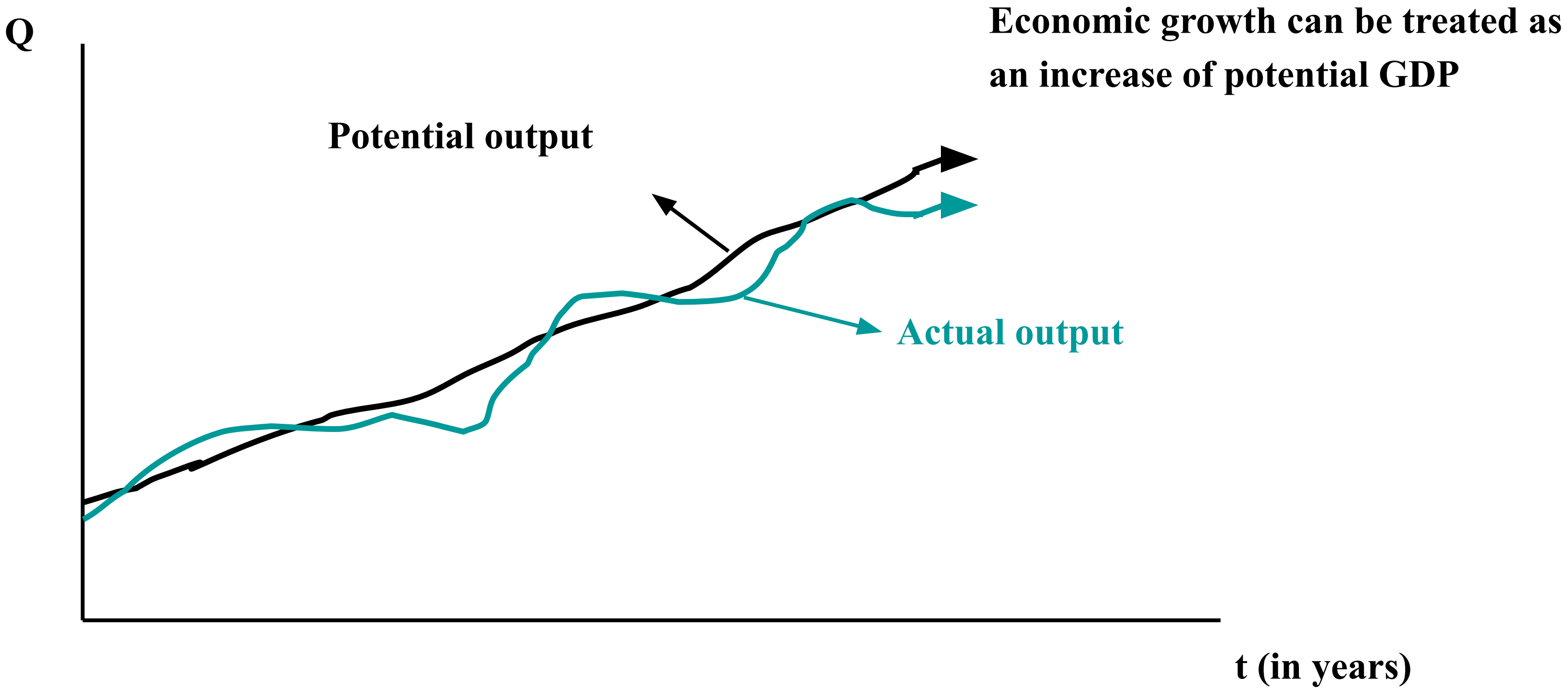
Macroeconomics

Lecture 7.

Long-run macroeconomic dynamics:

Solow model

Let remember one important picture…



Maddison growth data (view 1)

	1000	1500	1820	1870	1913	1950	1973	2001
Levels of per-capita GDP (1990 international dollars)								
Western Europe	400	771	1,204	1,960	3,458	4,579	11,416	19,256
Western offshoots	400	400	1,202	2,419	5,233	9,268	16,179	26,943
Japan	425	500	669	737	1,387	1,921	11,434	20,683
West	**405**	**702**	**1,109**	**1,882**	**3,672**	**5,649**	**13,082**	**22,509**
Asia (excluding Japan)	450	572	577	550	658	634	1,226	3,256
Latin America	400	416	692	681	1,481	2,506	4,504	5,811
E. Europe & f. USSR	400	498	686	941	1,558	2,602	5,731	5,038
Africa	425	414	420	500	637	894	1,410	1,489
Rest	**441**	**538**	**578**	**606**	**860**	**1,091**	**2,072**	**3,372**
World	**436**	**566**	**667**	**875**	**1,525**	**2,111**	**4,091**	**6,049**
Interregional spread	1.1:1	1.9:1	2.9:1	4.8:1	8.2:1	14.6:1	13.2:1	18.1:1
West/Rest spread	0.9:1	1.3:1	1.9:1	3.1:1	4.3:1	5.2:1	6.3:1	6.7:1

Perhaps, you heard in the introductory course about…

- ***hockey stick of economic progress***!

Maddison growth data (view 2)

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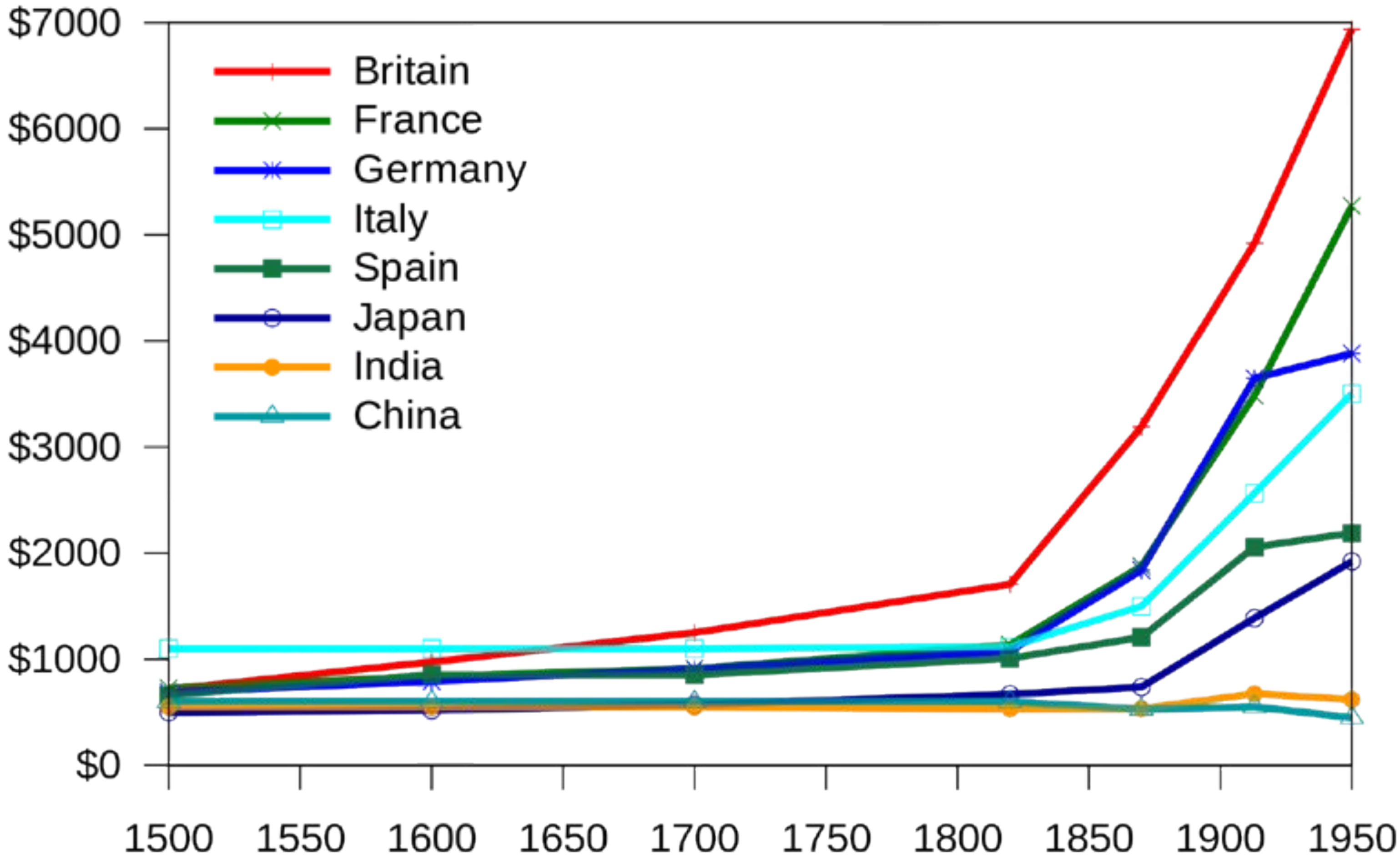


Economic Theory of the XIX Century: An example of "Outdated" View on Growth – part 1

- Ricardo (1817) and Mill (1848) believed that rate of economic growth decreases in the course of time.
- Ricardo and Mill developed theory based on "the Law of Diminishing Fertility of the Soil" or growth in the conditions of decreasing returns to scale.
- As a result, an economy is characterized by a tendency to a stagnation.

Economic Theory of the XIX Century: An example of "Outdated" View on Growth – part 2

- Marx (1867, 1885, 1894) believed also that rate of economic growth decreases in the course of time.
- Marx developed theory that predicts enrichment of the large/successful capitalists and immiserization of all others. In particular, workers are displaced from the production process by "labor-augmented" technical progress.
- As a result, the economy is characterized a tendency to the social revolution…

Towards the contemporary theory of growth – part 1

- Harrod (1939) and Domar (1946) laid the foundation of the contemporary theory of growth:
- They assumed away the Law of Diminishing Fertility of the Soil" and believed that population growth does not depend on the difference between the actual wage rate and the minimal wage rate.

Towards the contemporary theory of growth – part 2

- So, Harrod and Domar created the models in which the main aggregate macroeconomic variables – output (GDP), capital, labor, consumption – grow with constant rate.
- But they assumed an absence of substitutability between capital and labor.
- So, in these models the equilibrium growth is unstable.

Kaldor’s (1961) stylised facts

- ***Per capita output grows over time and its growth rate does not tend to diminish;***
- Physical capital per worker grows over time;
- The rate of return to capital is nearly constant;
- The ratio of physical capital to output is nearly constant;
- The shares of labour and physical capital in national income are nearly constant;
- ***The growth rate of output per worker differs substantially across countries.***

Solow (1956) growth model: the general description

- ***Solow model is the starting point of contemporary economic analysis of growth***
- Assumption and conclusions:

- Constant returns to scale
- Presence of factor substitutability (due to both technical aspects and incentives)
- ***The economy generates constant rate of growth of output and some other important variables***
- ***This equilibrium growth is stable one.***

A general production function in the Solow growth model

- Consider a general production function

$$Y = F(L, K)$$

- This is a "neoclassical" production function if there are positive and diminishing returns to K and L; if there are constant returns to scale (CRS); and if it obeys the Inada conditions:

$$f(0) = 0;\ f'(0) = \infty;\ \lim_{k \to \infty} f'(k) = 0$$

- with CRS, we have output per worker of

$$Y / L = F(1, K / L)$$

If we write K/L as k and Y/L as y, then in *intensive form*:

$$y = f(k)$$

The Cobb-Douglas production function

- One simple production function that provides – as many economists believe – a reasonable description of actual economies is the ***Cobb-Douglas***:

$$Y = AK^{\alpha}L^{1-\alpha}$$

where A>0 is the level of technology and α is a constant with 0<α<1.
The CD production function can be written in *intensive form* as

$$y = Ak^{\alpha}$$

The marginal product can be found from the derivative:

$$\mathrm{MPK} = \frac{\partial Y}{\partial K} = \alpha AK^{\alpha-1}L^{1-\alpha} = \alpha\frac{AK^{\alpha}L^{1-\alpha}}{K} = \alpha\frac{Y}{K} = \alpha \mathrm{APK}$$

Results for distribution of income

- If firms pay workers a wage of w, and pay r to rent a unit of capital for one period, profit-maximizing firms should maximise:

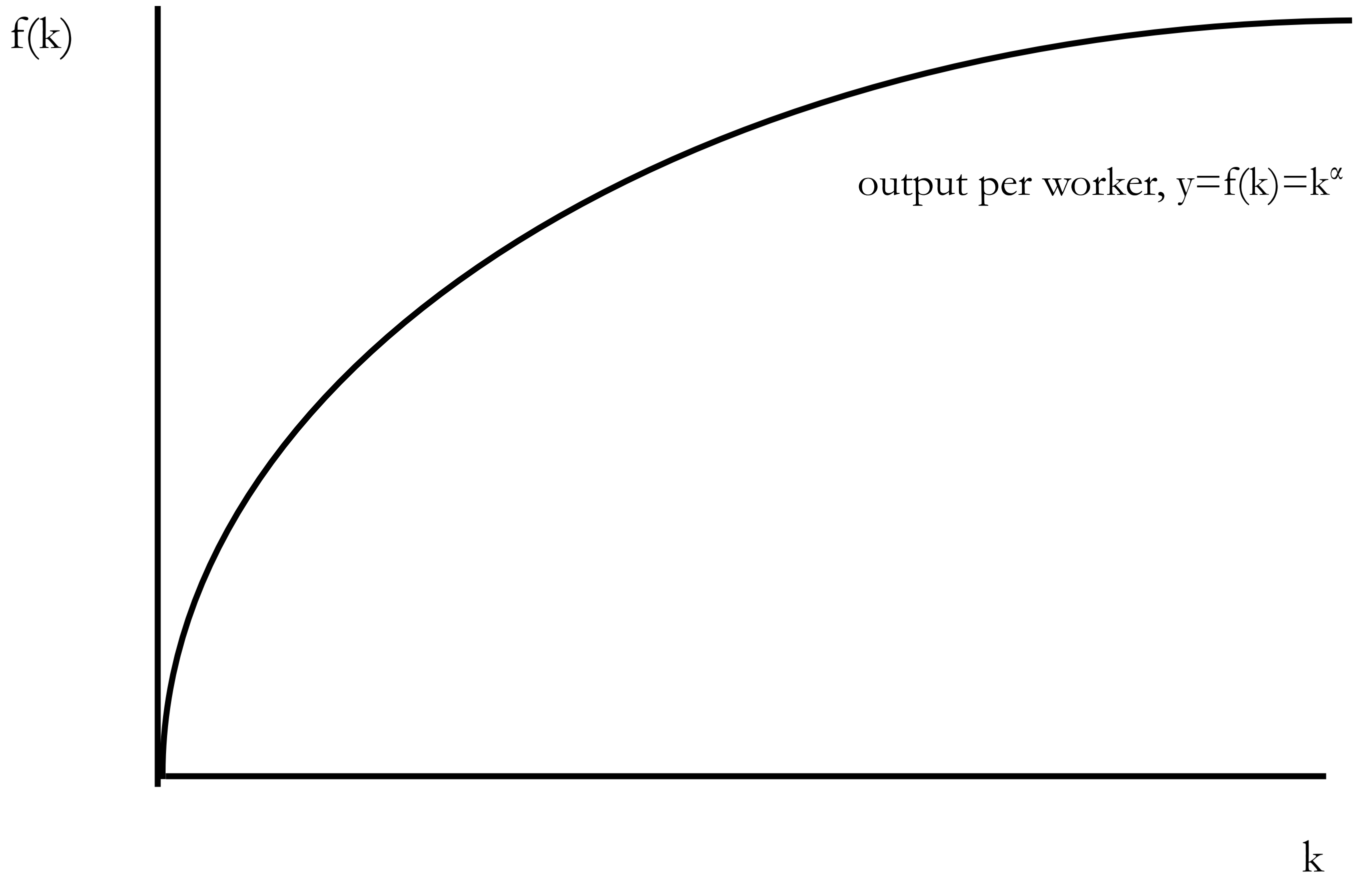
$$\max_{K,L} F(K,L) - rK - wL$$

- Under perfect competition firms are price-takers so they employ workers and rent capital until w and r are equal to the marginal products of labour and capital

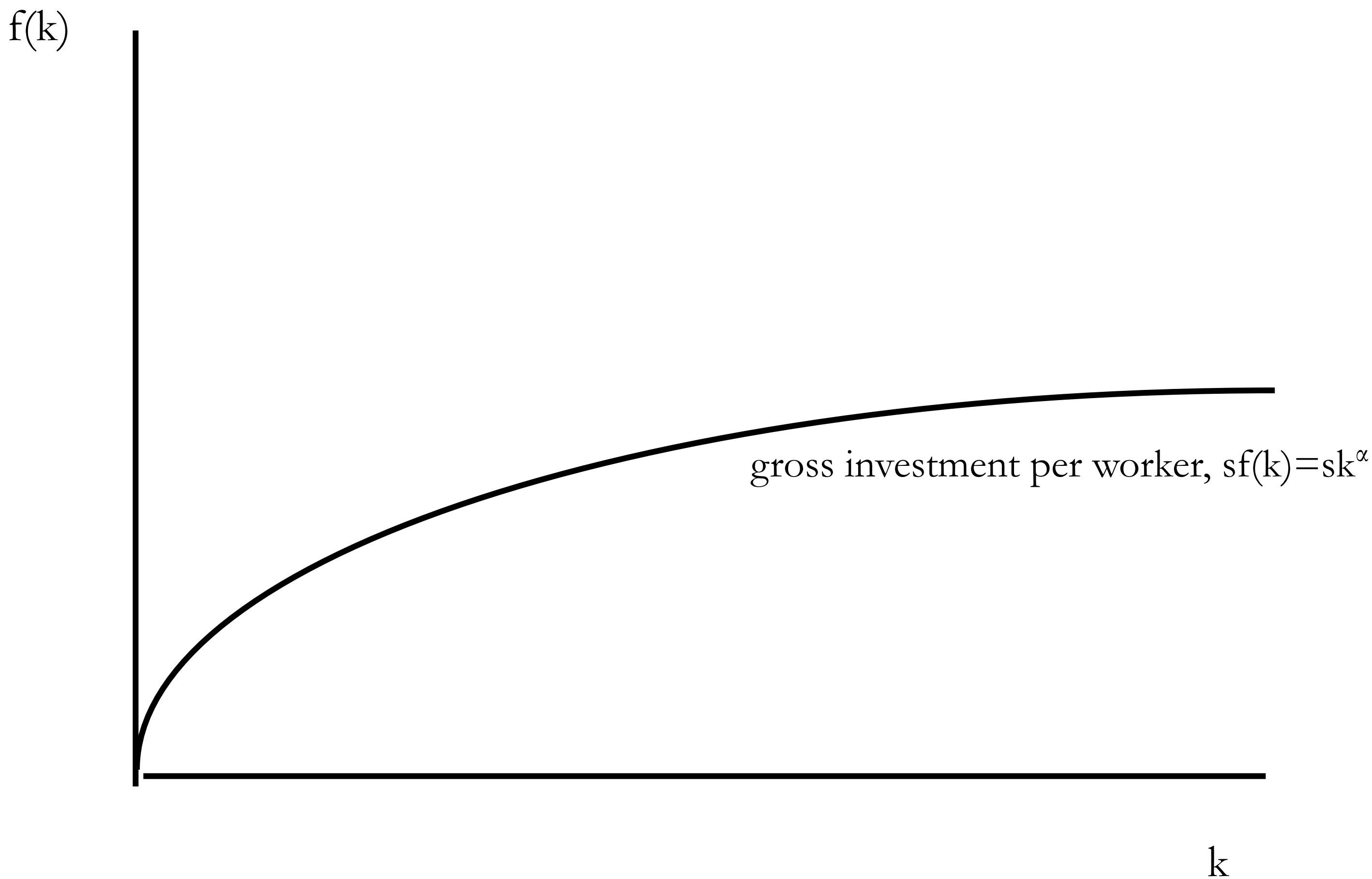
$$w = \frac{\partial F}{\partial L} = (1-\alpha)\frac{Y}{L};\ r = \frac{\partial F}{\partial K} = \alpha\frac{Y}{K}$$

- Notice that wL+rK=Y, that is, payments to inputs completely exhaust output so economic profits are zero.

Diminishing returns to capital



The economy is saving and investing a constant fraction of income…



What is “labor-augmenting technical progress”?

- This is technical progress that increases contribution of labor into output!

If we take into account “labor-augmenting technical progress” that

- We now write the production function as:

$$Y = F(K, L \times E)$$

- where $L \times E$ = the number of effective workers.
  - Hence, increases in labor efficiency have the same effect on output as increases in the labor force.

Production function with technical progress in the intensive form

- Notation:

  $y = Y/LE$ = output per effective worker

  $k = K/LE$ = capital per effective worker

- Production function per effective worker:

  $y = f(k)$

- Saving and investment per effective worker:

  $s\,y = s\,f(k)$

What is break-even investment?

$(\delta + n + g)k$ = break-even investment: the amount of investment necessary to keep k constant.

Consists of:

- δk to replace depreciating capital
- $n k$ to provide capital for new workers
- $g k$ to provide capital for the new "effective" workers created by technological progress

Derivation of equilibrium capital per effective worker

$$\because \Delta K \text{ (net investment)} = I \text{ (gross investment)} - \delta \cdot K \text{ (depreciation}$$

$$\Rightarrow \frac{\Delta K}{L \cdot E} = \frac{I}{L \cdot E} - \frac{\delta K}{L \cdot E} \equiv i - \delta \cdot k = s \cdot f(k) - \delta \cdot k$$

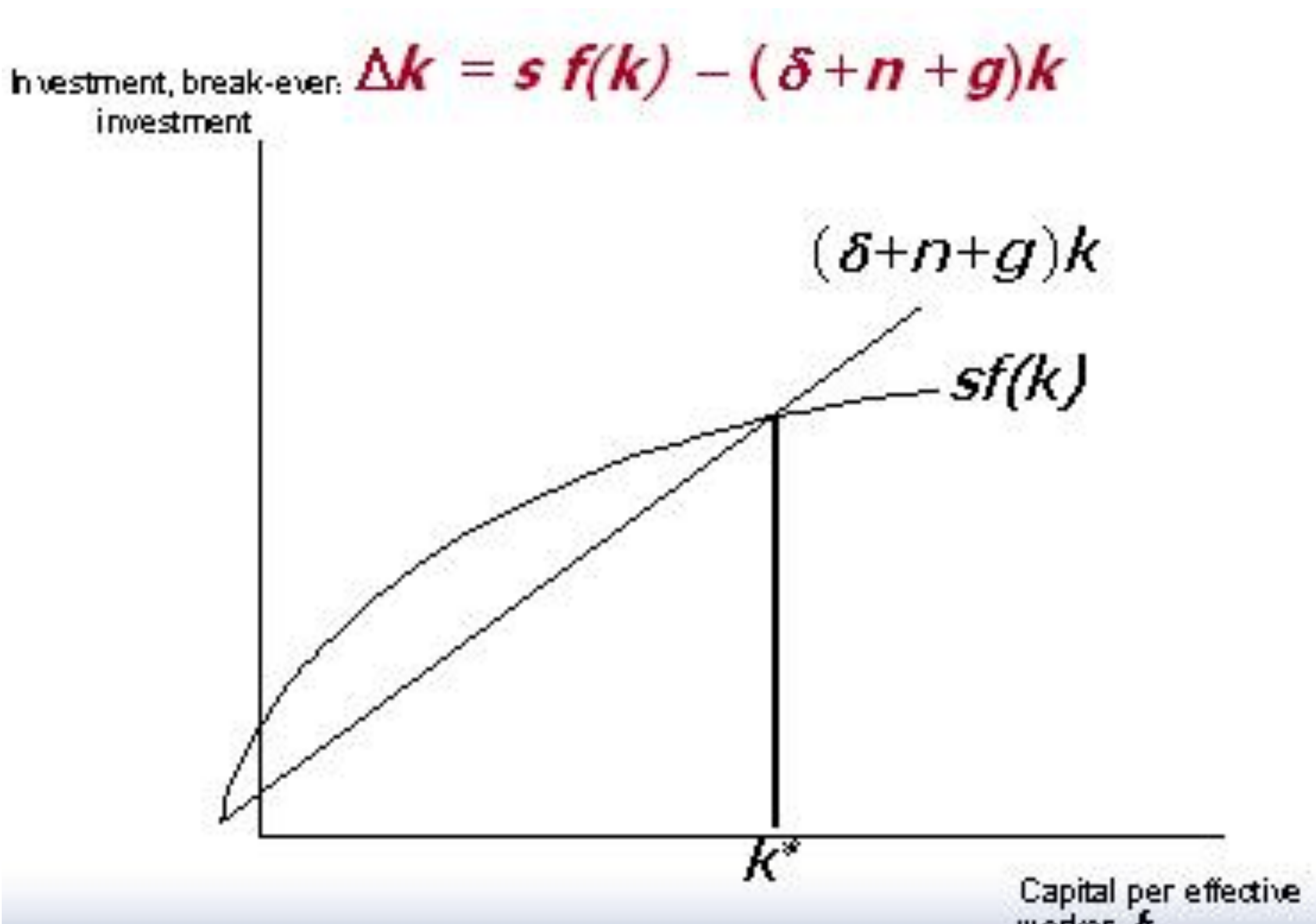
$$\text{Since } \Delta k = \Delta\left(\frac{K}{L \cdot E}\right) = \frac{(L \cdot E)\Delta K - K(L \cdot \Delta E + E \cdot \Delta L)}{(L \cdot E)^2}$$

$$= \frac{\Delta K}{L \cdot E} - \frac{K}{L \cdot E}\left(\frac{\Delta L}{L} + \frac{\Delta E}{E}\right)$$

$$= s \cdot f(k) - \delta \cdot k - k(n + g), \text{ where } \frac{\Delta L}{L} \equiv n, \text{ and } \frac{\Delta E}{E} \equiv g$$

$$\Rightarrow \Delta k = s \cdot f(k) - (\delta + n + g)k$$

Equilibrium as a situation of steady-state growth



Dynamics of parameters on the steady-state

variable	symbol	Steady-state growth rate
Capital per effective worker	$k = K/(L \times E)$	0
Output per effective worker	$y = Y/(L \times E)$	0
Capital per worker	$(K/L) = k \times E$	g
Output per worker	$(Y/L) = y \times E$	g
Total output	$Y = y \times E \times L$	$n + g$

Balanced growth

Solow model's steady state exhibits **balanced growth** - many variables grow at the same rate.

- Solow model predicts ***Y/L*** and ***K/L*** grow at same rate (***g***), so that ***K/Y*** should be constant.

  This is true in the real world.

- Solow model predicts real wage grows at same rate as ***Y/L***, while real rental price is constant. Also true in the real world.

Growth in steady state and outside steady state

- ***In the steady state –*** when actual investment per "effective worker" = break-even investment ***- the rate of economic growth will be equal to the sum of rate of population growth and rate of technical progress = n+g.***
- If "initial" capital stock is less than steady state capital stock, then the rate of economic growth will be more than *n+g*.

Unconditional convergence

- Solow model predicts that, other things equal, "poor" countries (with lower Y/L and K/L) should grow faster than "rich" ones.
- If true, then the income gap between rich & poor countries would shrink over time, and living standards "converge."
- In real world, many poor countries do NOT grow faster than rich ones. Does this mean the Solow model fails?

Conditional convergence

- No, because "other things" aren't equal.
  - In samples of countries with similar savings & pop. growth rates, income gaps shrink about 2%/year.
  - In larger samples, if one controls for differences in saving, population growth, and human capital, incomes converge by about 2%/year.
- What the Solow model *really* predicts is **conditional convergence** - countries converge to their own steady states, which are determined by saving, population growth, and education. And this prediction comes true in the real world.

The concept of the Golden Rule

To find the Golden Rule capital stock, express c^* in terms of k^*:

$$\begin{aligned} c^* &= y^* - i^* \\ &= f(k^*) - (\delta + n + g)k^* \end{aligned}$$

c^* is maximized when

$$MPK = \delta + n + g$$

or equivalently,

$$\boxed{MPK - \delta = n + g}$$

In the Golden Rule Steady State, the marginal product of capital net of depreciation equals the pop. growth rate plus the rate of tech progress.

The Golden Rule – for what?

- Use the Golden Rule to determine whether our saving rate and capital stock are too high, too low, or about right.
- To do this, we need to compare $(\text{MPK} - \boldsymbol{\delta})$ to $(\boldsymbol{n} + \boldsymbol{g})$.
- If $(\text{MPK} - \boldsymbol{\delta}) > (\boldsymbol{n} + \boldsymbol{g})$, then we are below the Golden Rule steady state and should increase $\boldsymbol{s}$.
- If $(\text{MPK} - \boldsymbol{\delta}) < (\boldsymbol{n} + \boldsymbol{g})$, then we are above the Golden Rule steady state and should reduce $\boldsymbol{s}$.

The U.S. Golden Rule – Estimation (Part 1)

To estimate $(\text{MPK} - \delta)$, we use three facts about the U.S. economy:

1. $\boldsymbol{k} = 2.5\,\boldsymbol{y}$
   The capital stock is about 2.5 times one year's GDP.
2. $\delta \boldsymbol{k} = 0.1\,\boldsymbol{y}$
   About 10% of GDP is used to replace depreciating capital.
3. $\text{MPK} \times \boldsymbol{k} = 0.3\,\boldsymbol{y}$
   Capital income is about 30% of GDP

The U.S. Golden Rule – Estimation (Part 2)

1. $k = 2.5\ y$
2. $\delta k = 0.1\ y$
3. $\text{MPK} \times k = 0.3\ y$

To determine δ, divided 2 by 1:

$$\frac{\delta k}{k} = \frac{0.1\, y}{2.5\, y} \quad \Rightarrow \quad \delta = \frac{0.1}{2.5} = 0.04$$

The U.S. Golden Rule – Estimation (Part 3)

1. $\boldsymbol{k} = 2.5\ \boldsymbol{y}$
2. $\delta \boldsymbol{k} = 0.1\ \boldsymbol{y}$
3. $\text{MPK} \times \boldsymbol{k} = 0.3\ \boldsymbol{y}$

To determine MPK, divided 3 by 1:

$$\frac{\text{MPK} \times \boldsymbol{k}}{\boldsymbol{k}} = \frac{0.3\,\boldsymbol{y}}{2.5\,\boldsymbol{y}} \quad \Rightarrow \quad \text{MPK} = \frac{0.3}{2.5} = 0.12$$

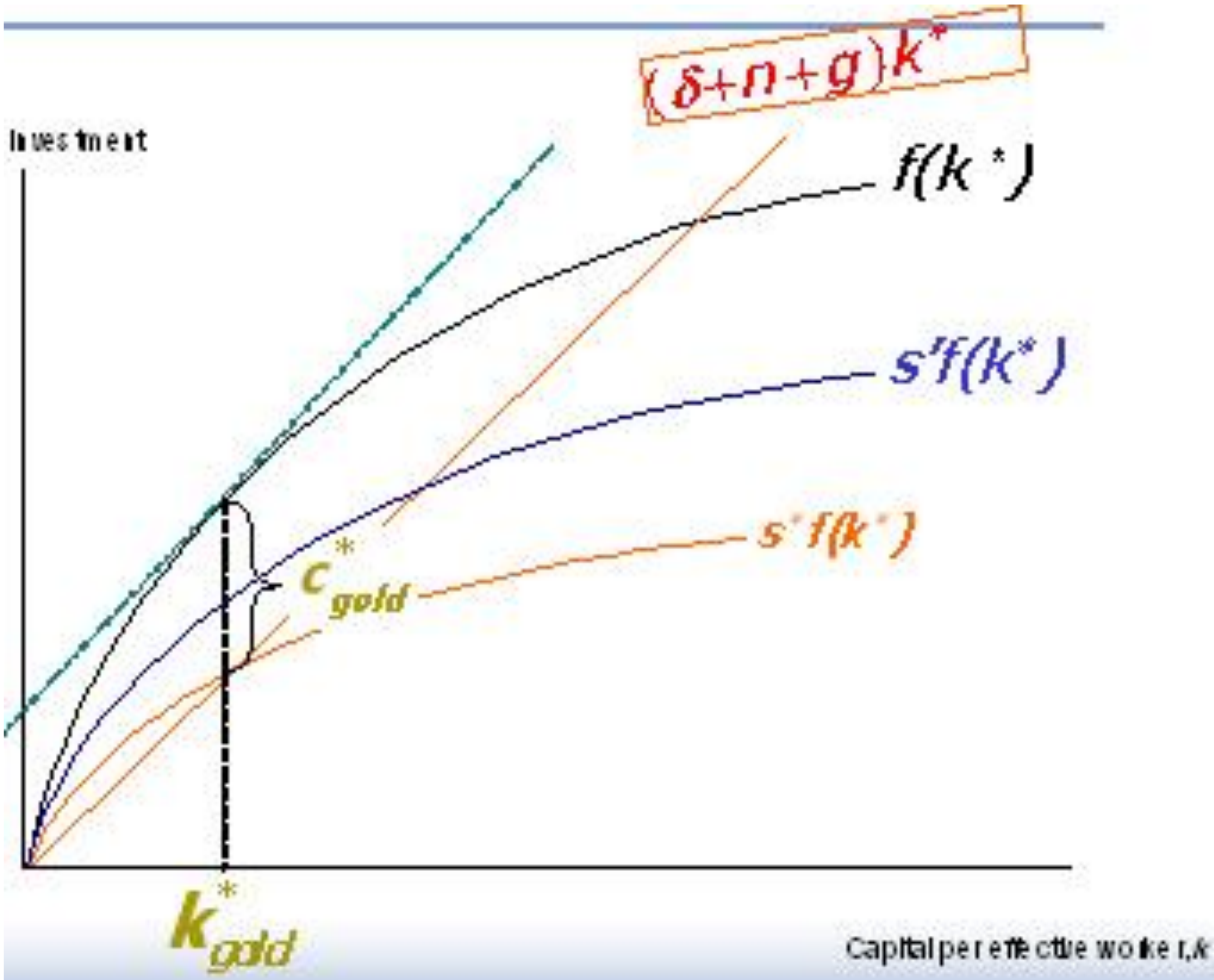
Hence, $\text{MPK} - \delta = 0.12 - 0.04 = 0.08$

The U.S. Golden Rule – Estimation (Part 4)

- From the last slide: $\text{MPK} - \boldsymbol{\delta} = 0.08$
- U.S. real GDP grows an average of 3%/year, so $\boldsymbol{n} + \boldsymbol{g} = 0.03$
- Thus, in the U.S.,

$$\text{MPK} - \boldsymbol{\delta} = 0.08 > 0.03 = \boldsymbol{n} + \boldsymbol{g}$$

When saving rate is too much high



Accounting of growth in Solow model (Part 1)

- Production Function (CRTS)

$$Y = F(K, L)$$

- Increases in Capital and Labor

$$\Delta Y = (MPK \times \Delta K) + (MPL \times L)$$

$$\Rightarrow \quad \frac{\Delta Y}{Y} = \alpha \frac{\Delta K}{K} + (1 - \alpha) \frac{\Delta L}{L}$$

where α is capital's share and $(1 - \alpha)$ is labor's share.

Accounting of growth in Solow model (Part 2)

- Production Function with Technology

$$Y = AF(K, L)$$

- Technological Progress

$$\frac{\Delta Y}{Y} = \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} + \frac{\Delta A}{A}$$

Growth in output = Contribution of capital + Contribution of labor + Growth in Total Factor Productivity

The above is the growth-accounting equation.

Accounting of growth in Solow model (Part 3)

- **Solow Residual**

$$\frac{\Delta A}{A} = \frac{\Delta Y}{Y} - \alpha \frac{\Delta K}{K} - (1-\alpha)\frac{\Delta L}{L}$$

- $\Delta A/A$ is the change in output that cannot be explained by changes in inputs. Thus, the growth in total factor productivity is computed as a residual – that is, as the amount of output growth that remains after we have accounted for the determinants of growth that we can measure. Indeed, **$\Delta A/A$ is sometimes called the *Solow residual***, after Robert Solow, who first showed how to compute it.

Accounting of growth in the U.S. economy In the end of the XX century

(average percentage, %, increase per year) Years	$\begin{pmatrix} \text{Output Growth} \\ \Delta Y / Y \end{pmatrix}$ =	SOURCE OF GROWTH $\begin{pmatrix} \text{Capital} \\ \alpha \Delta K / K \end{pmatrix}$ +	$\begin{pmatrix} \text{Labor} \\ (1-\alpha) \Delta L / L \end{pmatrix}$ +	$\begin{pmatrix} \text{Total Factor Productivity} \\ \Delta A / A \end{pmatrix}$
1950-1960	3.5	1.1	0.8	1.6
1960-1970	4.1	1.2	1.3	1.7
1970-1980	3.1	0.9	1.6	0.5
1980-1990	2.9	0.8	1.3	0.8
1990-1996	2.2	0.6	0.8	0.8
1950-1996	3.2	0.9	1.2	1.1

Accounting of growth among "Asian Tigers" In the end of the XX century

%	Hong Kong (1966-1991)	Singapore (1966-1990)	South Korea (1966-1990)	Taiwan (1966-1990)
GDP per capita growth	5.7	6.8	6.8	6.7
TFP* growth	2.3	0.2	1.7	2.6
Δ% labor force participation	38→49	27→51	27→36	28→37
Δ% secondary education or higher	27.2→71.4	15.8→66.3	26.5→75.0	25.8→67.6

*TFP: total factor productivity

Source: Alwyn Young, "The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience," *Quarterly Journal of Economics*, August 1995.

Solow model vs. Endogenous growth theory

- Solow model:
  - sustained growth in living standards is due to tech progress
  - the rate of technological progress is exogenous
- Endogenous growth theory:
  - a set of models in which the growth rate of productivity and living standards is endogenous