§ 6. Важнейшие уравнения математической физики.

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (6.1)$$

$$\frac{\partial^2 u}{\partial t^2} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (6.2)$$

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (6.3)$$

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (6.4)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (6.5)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \qquad (6.6)$$

§ 7. Уравнение теплопроводности (уравнение Фурье)

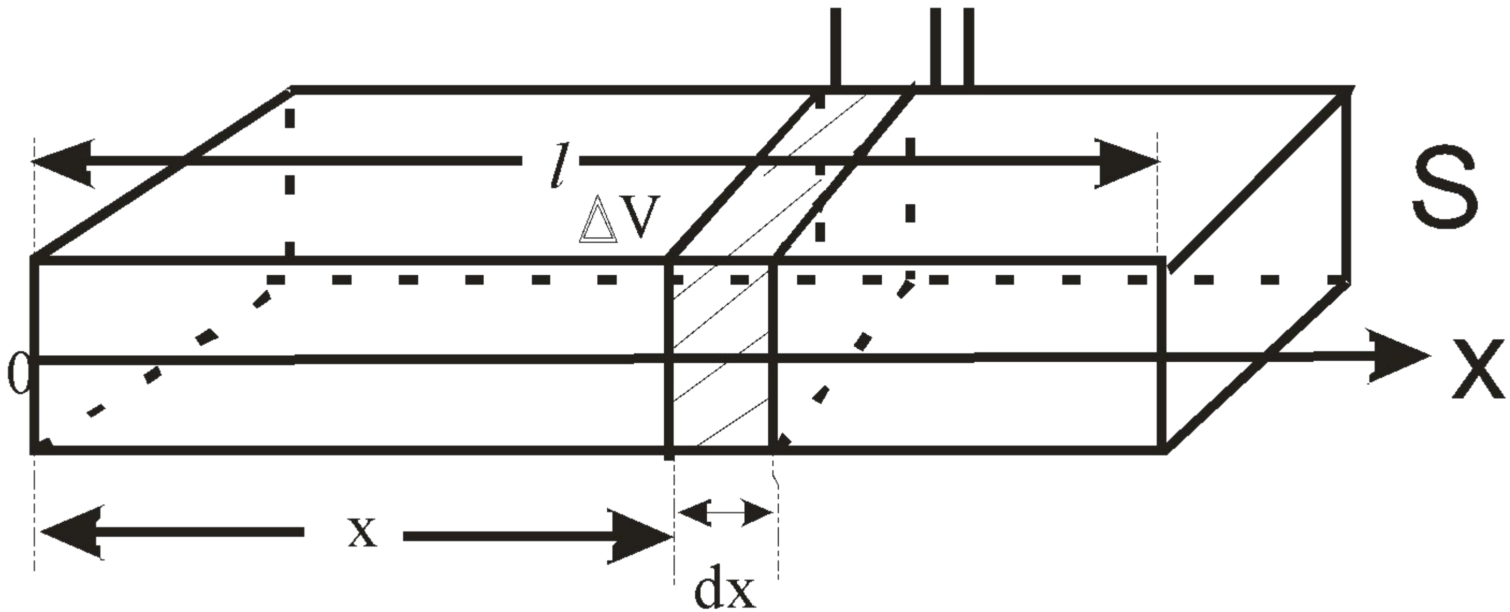

Рис. 1

$u = u(x, t)$ $(0 \le x \le l,\ 0 \le t < +\infty)$

$\rho = const$ $c = const$ $k = const$

$\Phi(x, t)$ *Дж/(кг·К⁰)* $1^0K = 1^0C$

$$dQ_x = -k\frac{\partial u}{\partial x}Sdt, \quad (7.1)$$ $$\frac{\partial u}{\partial x} > 0$$

ΔV I и II x и $x + dx$

dQ dt

$$\rho S\Phi(x, t)dxdt \quad (7.2.)$$

$$dQ_x = -k\frac{\partial u}{\partial x}Sdt, \quad (7.3)$$

$$dQ = -k\frac{\partial u}{\partial x}\Big|_x Sdt + k\frac{\partial u}{\partial x}\Big|_{x+\Delta x} Sdt + \rho S\Phi(x, t)\, dxdt\,. \quad (7.4)$$

$$f(x + \Delta x) \approx f(x) + f'(x)dx \quad (7.5)$$

$$\frac{\partial u}{\partial x}\Big|_{x+\Delta x} = \frac{\partial u}{\partial x}\Big|_x + \frac{\partial^2 u}{\partial x^2}\Big|_x dx. \quad (7.6)$$

$$dQ = -k\frac{\partial u}{\partial x}\Big|_x Sdt + k\frac{\partial u}{\partial x}\Big|_x Sdt +$$

$$+ k\frac{\partial^2 u}{\partial x^2}\Big|_x S\, dx\, dt + \rho S\Phi(x,t)\, dxdt \quad (7.7)$$

$$dQ = k\frac{\partial^2 u}{\partial x^2} Sdxdt + \rho S\Phi(x,t)dxdt. \quad (7.8)$$

$$dQ = Sdxdt\left(k\frac{\partial^2 u}{\partial x^2} + \rho\Phi(x,t)\right) \quad (7.9)$$

$$\Delta u = \frac{\partial u}{\partial t}dt$$

ΔV равна ρSdx

$$dQ = c\rho Sdx\frac{\partial u}{\partial t}dt. \quad (7.10)$$

$$k\frac{\partial^2 u}{\partial x^2}Sdxdt + \rho S\Phi(x, t)dxdt = c\rho Sdx\frac{\partial u}{\partial t}dt. \quad (7.11)$$

$$Sdxdt\left(k\frac{\partial^2 u}{\partial x^2} + \rho\Phi(x, t)\right) = c\rho Sdx\frac{\partial u}{\partial t}dt. \quad (7.12)$$

$$k\frac{\partial^2 u}{\partial x^2} + \rho\Phi(x, t) = c\rho\frac{\partial u}{\partial t}. \quad (7.13)$$

cρ

$$\frac{k}{c\rho} = \mathrm{a}^2, \quad (7.14)$$

$$\frac{\partial u}{\partial t} = \mathrm{a}^2 \frac{\partial^2 u}{\partial x^2} + \frac{1}{c}\Phi(\mathrm{x}, \mathrm{t}) \quad (7.15)$$

$$\frac{\partial u}{\partial t} = \mathrm{a}^2 \frac{\partial^2 u}{\partial x^2}. \quad (7.16)$$

$$\frac{\partial u}{\partial t} = \mathrm{a}^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (7.17)$$

$$\frac{\partial u}{\partial t} = \mathrm{a}^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad (7.18)$$

$$\frac{\partial u}{\partial t} = a^2 \Delta u, \qquad (7.19)$$

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \qquad (7.20)$$

§ 8. Решение задачи о распределении температуры в ограниченном стержне

$u = u(x,t)$

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \quad (0 < x < l). \qquad (8.1)$$

$u(x, 0) = f(x)$

$x = 0$ и $x = l$

$u(0, t) = 0$, $u(l, t) = 0$ при любом $t \geq 0$