

§ 6. Важнейшие уравнения математической физики.

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \quad (6.1)$$

$$\frac{\partial^2 u}{\partial t^2} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (6.2)$$

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \quad (6.3)$$

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (6.4)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (6.5)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad (6.6)$$

§ 7. Уравнение теплопроводности (уравнение Фурье)

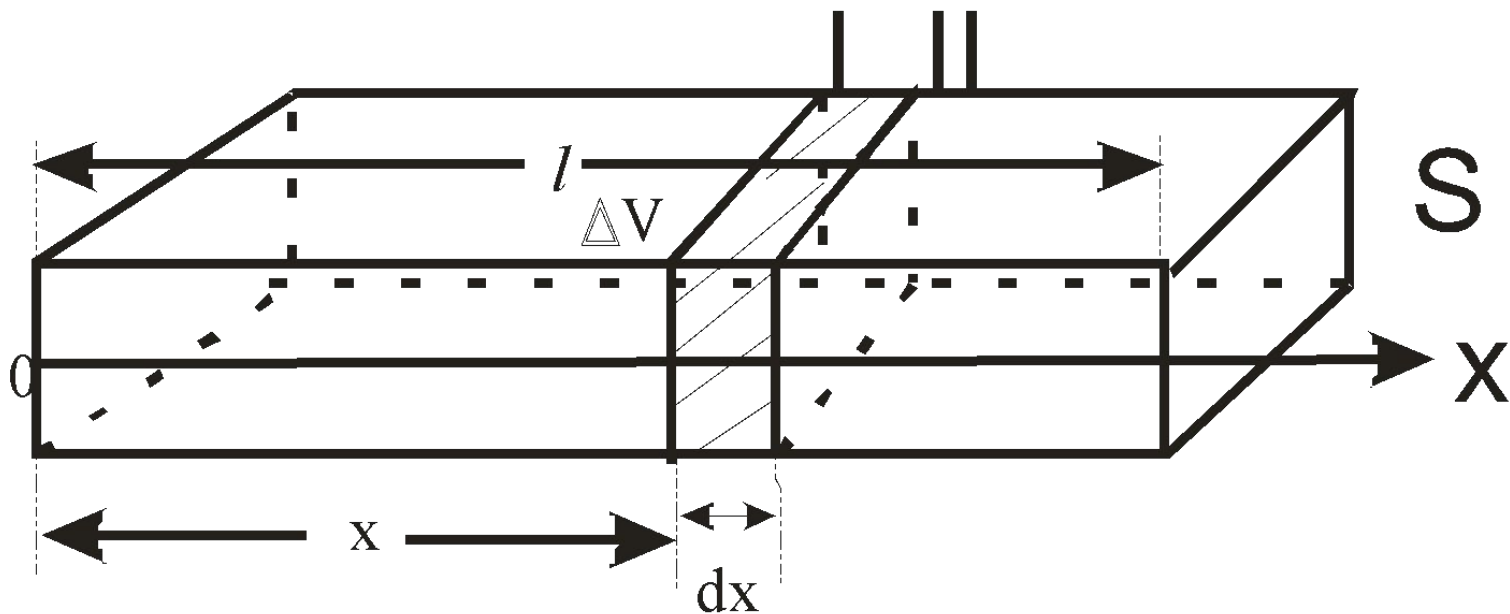


Рис. 1

$$u = u(x, t) \quad (0 \leq x \leq l, 0 \leq t < +\infty)$$

$$\rho = \text{const}$$

$$c = \text{const}$$

$$k = \text{const}$$

$$\Phi(x, t)$$

$$\text{Дж}/(\text{кг} \cdot \text{K}^0)$$

$$1^0\text{K} = 1^0\text{C}$$

$$dQ_x = -k \frac{\partial u}{\partial x} S dt, \quad (7.1) \quad \frac{\partial u}{\partial x} > 0$$

$$\Delta V$$

$$I \text{ и } II$$

$$x \text{ и } x + dx$$

$$dQ$$

$$dt$$

$$\rho S \Phi(x, t) dx dt \quad (7.2.)$$

$$dQ_x = -k \frac{\partial u}{\partial x} S dt, \quad (7.3)$$

$$dQ = -k \frac{\partial u}{\partial x} \Big|_x S dt + k \frac{\partial u}{\partial x} \Big|_{x+\Delta x} S dt + \rho S \Phi(x, t) dx dt . \quad (7.4)$$

$$f(x + \Delta x) \approx f(x) + f'(x)dx \quad (7.5)$$

$$\frac{\partial u}{\partial x} \Big|_{x+\Delta x} = \frac{\partial u}{\partial x} \Big|_x + \frac{\partial^2 u}{\partial x^2} \Big|_x dx. \quad (7.6)$$

$$dQ = -k \frac{\partial u}{\partial x} \Big|_x S dt + k \frac{\partial u}{\partial x} \Big|_x S dt +$$

$$+ k \frac{\partial^2 u}{\partial x^2} \Big|_x S dx dt + \rho S \Phi(x, t) dx dt \quad (7.7)$$

$$dQ = k \frac{\partial^2 u}{\partial x^2} S dx dt + \rho S \Phi(x, t) dx dt. \quad (7.8)$$

$$dQ = Sdxdt(k \frac{\partial^2 u}{\partial x^2} + \rho\Phi(x,t)) \quad (7.9)$$

$$\Delta u = \frac{\partial u}{\partial t} dt \quad \Delta V \text{ равна } \rho Sdx$$

$$dQ = c\rho Sdx \frac{\partial u}{\partial t} dt. \quad (7.10)$$

$$k \frac{\partial^2 u}{\partial x^2} Sdxdt + \rho S\Phi(x,t)dxdt = c\rho Sdx \frac{\partial u}{\partial t} dt. \quad (7.11)$$

$$Sdxdt(k \frac{\partial^2 u}{\partial x^2} + \rho\Phi(x,t)) = c\rho Sdx \frac{\partial u}{\partial t} dt. \quad (7.12)$$

$$k \frac{\partial^2 u}{\partial x^2} + \rho\Phi(x,t) = c\rho \frac{\partial u}{\partial t}. \quad (7.13)$$

$$c\rho \quad \frac{k}{c\rho} = a^2, \quad (7.14)$$

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + \frac{1}{c} \Phi(x, t) \quad (7.15)$$

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}. \quad (7.16)$$

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (7.17)$$

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (7.18)$$

$$\frac{\partial u}{\partial t} = a^2 \Delta u, \quad (7.19)$$

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \quad (7.20)$$

§ 8. Решение задачи о распределении температуры

в ограниченном стержне

$$u = u(x, t) \quad \frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \quad (0 < x < l). \quad (8.1)$$

$$u(x, 0) = f(x) \quad x = 0 \text{ и } x = l$$

$$u(0, t) = 0, \quad u(l, t) = 0 \text{ при любом } t \geq 0$$