Volumes of Revolution 2

Rotating about a line parallel to the x or y axis;

Rotating between two curves about the x or y axis;

Rotating between two curves about a line parallel to the x or y axis;

Rotating areas about lines above or lines to the right of the curve(s)

Rotating about a line parallel to the x or y axis

- The area to be rotated can be translated so that the axis of rotation lies upon the x or y axis.
- If the area is translated then the equation of the curve must be translated too.

Example 1

- Consider the region bound by:
  - $y = 4x - x^2$
  - $y = 3$
- Find the volume of the solid formed by revolving this region 360° about the line $y = 3$.

Rotating between two curves about the x or y axis

- We use the same theory we used to find the area between two curves.
- The solid formed will have a hole in the middle.
- The volume of the “hole” is subtracted from the total volume of the solid.

Example 2

- Consider the region bound by:
  - $y = 4x - x^2$
  - $y = 3$
- Find the volume of the solid formed by revolving this region 360° about the x axis.

Rotating between two curves about a line parallel to the x or y axis

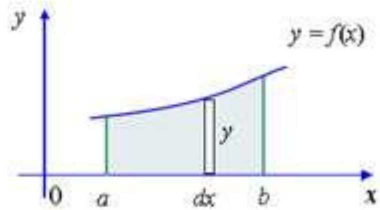
- We now combine the above two ideas.
- First we must translate the region to be rotated so that the axis of rotation lies on the x or y axis.
- Second the solid formed will have a hole in the middle so the volume of the “hole” must be subtracted from the total volume of the solid.

Example 3

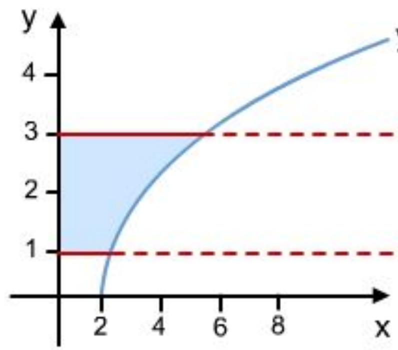
- Consider the region in the first quadrant bound by:
  - $y = x^2 + 1$
  - $y = 5$
  - $x = 0$
- Find the volume of the solid formed by revolving this region 360° about the line:
  - $x = -2$

Rotating areas about lines above or lines to the right of the curve(s)

- So far the process of integration has found areas below the curve or areas to the left of the curve;
- i.e. areas bound with the x axis that is below the curve:

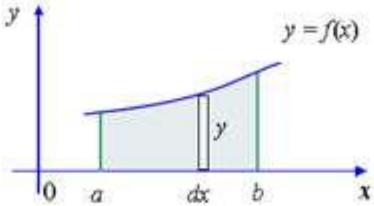


- or areas bound with the y axis that is to the left of the curve:

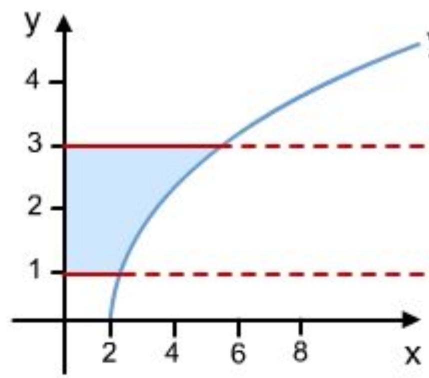


Rotating areas about lines above or lines to the right of the curve(s)

- Following on to volumes of revolution, this means that the axis of rotation has either been below the region being revolved:



- Or the axis of rotation has been to the left of the region being revolved:



Rotating areas about lines above or lines to the right of the curve(s)

- Now consider when the axis of rotation is above or to the right of the region being revolved.
- We must first reflect the region in the axis of rotation so that the curve is on the "correct" side of the axis of rotation.
- Therefore we must also reflect the equation of the curve.

Example 4

- Consider the region in the first quadrant bound by:
  - $y = x^2 + 1$
  - $y = 5$
  - $x = 0$
- Find the volume of the solid formed by revolving this region 360° about the line:
  - $x = 3$