Integration as a Process of Summation

(using integration to calculate the area under a curve between two x values)

• The area under a curve can be thought of as a series of strips with width $\delta \! x$ and height y



• So the area of one strip is: $\delta A \cong y \delta x$

(=f(x))

• Now consider the graph:



- The total area between the boundary values of x=a and x=b is: $A \cong \sum_{i=1}^{b} y \delta x$
- Since the strip is an approximation of the area under the curve, the area of the strip approaches the area under the curve as $\delta x \rightarrow 0$

• So,
$$A =_{\delta x} \underline{\lim}_{0} \sum_{a}^{b} y \delta x$$

- Now consider an alternative expression for A:
- Considering the same starting point of: $\delta A \cong y \delta x$
- Then: $\frac{\delta A}{\delta x} \cong y$
- Again this approximation becomes more accurate as: $\delta x \rightarrow 0$
- So,

$$\int_{\delta x} \underline{\lim}_{0} \frac{\delta A}{\delta x} = y$$

• So,
$$_{\delta x} \underline{\lim}_{0} \frac{\delta A}{\delta x} = y$$

• But, $_{\delta x} \underline{\lim}_{0} \frac{\delta A}{\delta x} = \frac{dA}{dx}$
 $So, \quad \frac{dA}{dx} = y$
Therefore, $A = \int y dx$
• So between x=a and x=b: $A = \int_{0}^{b} y dx$

• Putting the two alternative equations together:



 And therefore <u>integration is a process of</u> <u>summation</u>.