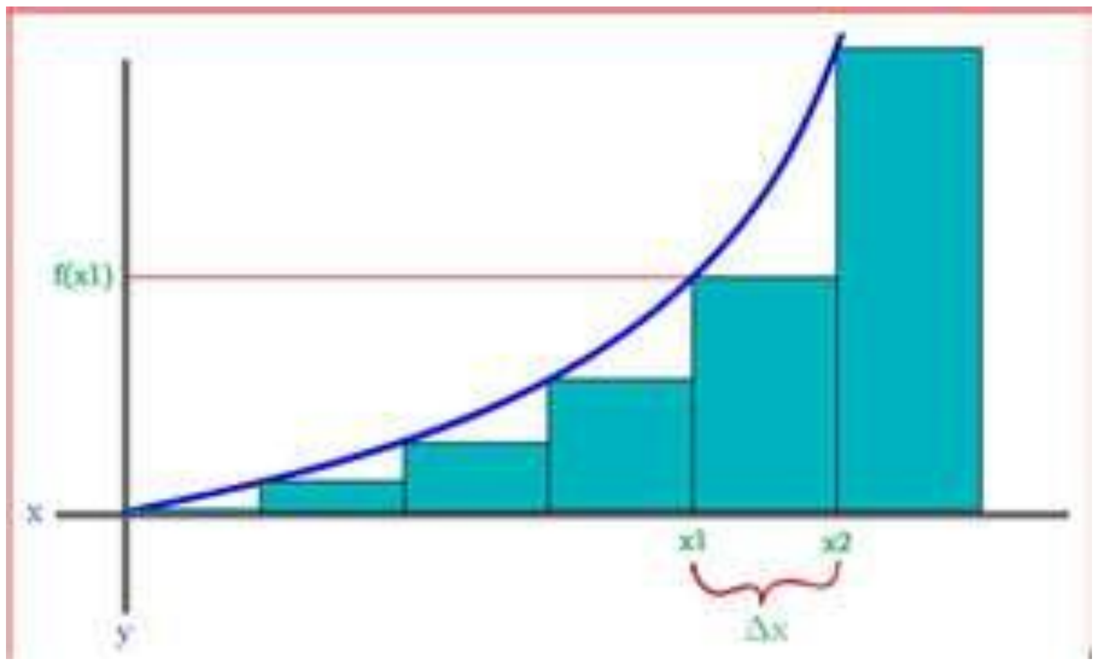
Integration as a Process of Summation

(using integration to calculate the area under a curve between two x values)

Consider the area under a curve

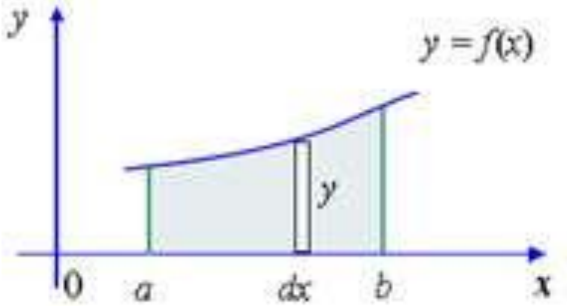
- The area under a curve can be thought of as a series of strips with width δx and height y (=f(x))



- So the area of one strip is: $\delta A \cong y\delta x$

Consider the area under a curve

- Now consider the graph:



- The total area between the boundary values of x=a and x=b is:

$$A \cong \sum_{a}^{b} y\delta x$$

- Since the strip is an approximation of the area under the curve, the area of the strip approaches the area under the curve as $\delta x \to 0$
- So,

$$A = \lim_{\delta x \to 0} \sum_{a}^{b} y\delta x$$

Consider the area under a curve

- Now consider an alternative expression for A:
- Considering the same starting point of: $\delta A \cong y\delta x$
- Then: $\dfrac{\delta A}{\delta x} \cong y$
- Again this approximation becomes more accurate as: $\delta x \to 0$
- So,

$$\lim_{\delta x \to 0} \frac{\delta A}{\delta x} = y$$

Consider the area under a curve

- So, $$\lim_{\delta x \to 0} \frac{\delta A}{\delta x} = y$$
- But, $$\lim_{\delta x \to 0} \frac{\delta A}{\delta x} = \frac{dA}{dx}$$

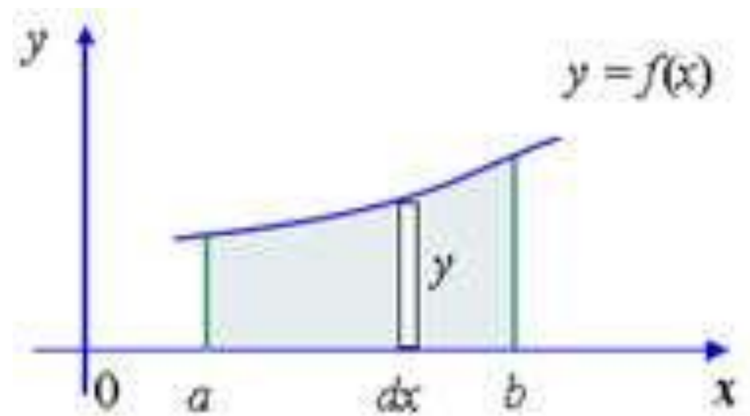
  $$So, \quad \frac{dA}{dx} = y$$

  $$Therefore, \quad A = \int y\,dx$$
- So between x=a and x=b: $$A = \int_a^b y\,dx$$

Consider the area under a curve

- Putting the two alternative equations together:

$$A = \lim_{\delta x \to 0} \sum_{a}^{b} y\delta x = \int_{a}^{b} y\,dx$$



- And therefore <u>integration is a process of summation</u>.