# Integration as a Process of Summation 

(using integration to calculate the area under a curve between two $x$ values)

## Consider the area under a curve

- The area under a curve can be thought of as a series of strips with width $\delta x$ and height $y$ (=f(x))

- So the area of one strip is: $\delta A \cong y \delta x$


## Consider the area under a curve

- The total area between the boundary values of $x=a$ and $x=b$ is:

$$
A \cong \sum_{a}^{b} y \delta x
$$

- Since the strip is an approximation of the area under the curve, the area of the strip approaches the area under the curve as $\delta x \rightarrow 0$
- So,

$$
A={ }_{\delta x} \underline{\lim }_{0} \sum_{a}^{b} y \delta x
$$

## Consider the area under a curve

- Now consider an alternative expression for A:
- Considering the same starting point of: $\delta A \cong y \delta x$
- Then: $\delta A$

$$
\frac{\delta x}{\delta x} \cong y
$$

- Again this approximation becomes more accurate as: $\delta x \rightarrow 0$
- So,

$$
{ }_{\delta x} \lim _{0} \frac{\delta A}{\delta x}=y
$$

## Consider the area under a curve

- So, $\delta_{\delta x} \lim _{0} \frac{\delta A}{\delta x}=y$
- But, ${ }_{\delta x} \xrightarrow{\lim _{0}} \frac{\delta A}{\delta x}=\frac{d A}{d x}$

$$
\text { So, } \quad \frac{d A}{d x}=y
$$

$$
\text { Therefore, } \quad A=\int y d x
$$

- So between $x=a$ and $x=b$ :

$$
A=\int_{a}^{b} y d x
$$

## Consider the area under a curve

- Putting the two alternative equations together:

$$
A={ }_{\delta x} \lim _{0} \sum_{a}^{b} y \delta x=\int_{a}^{b} y d x
$$



- And therefore integration is a process of summation.

