



Petroleum Learning Centre  
центр профессиональной переподготовки  
специалистов нефтегазового дела

MSc Programs  
Магистерские программы



# Reservoir Simulation

## Gridding and Well Modelling

Sergey Kurelenkov  
2011  
(after Ken Sorbie)



# Outline

- Gridding in Reservoir Simulation
- Calculation of Block to Block Flows in Reservoir Simulators
- Wells in Reservoir Simulation



# Gridding in Reservoir Simulation

- **Reservoir Simulation Model**
  - **grid block** model of a petroleum reservoir...
- **Gridding**
  - process of dividing up a reservoir into grid blocks  
(**spatial discretization**)
- **Discretization**
  - process of dividing up a continuous quantity into discrete intervals

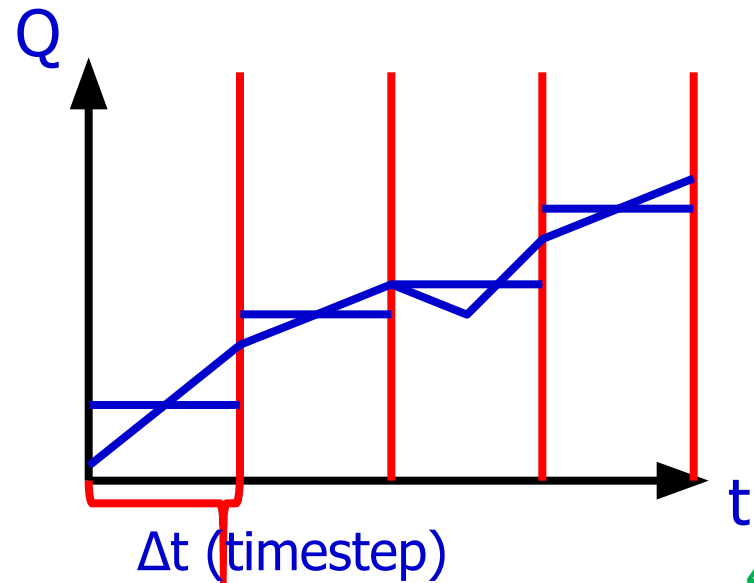
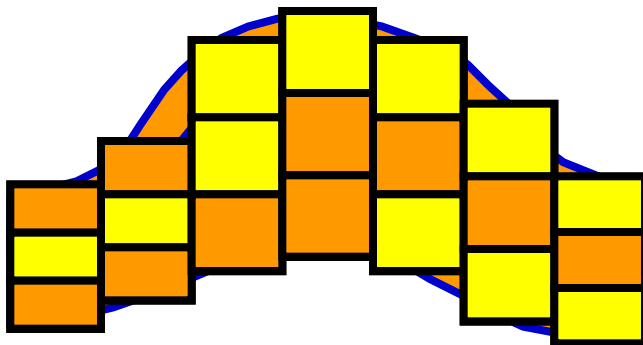


# Gridding in Reservoir Simulation

Discretization

Spatial

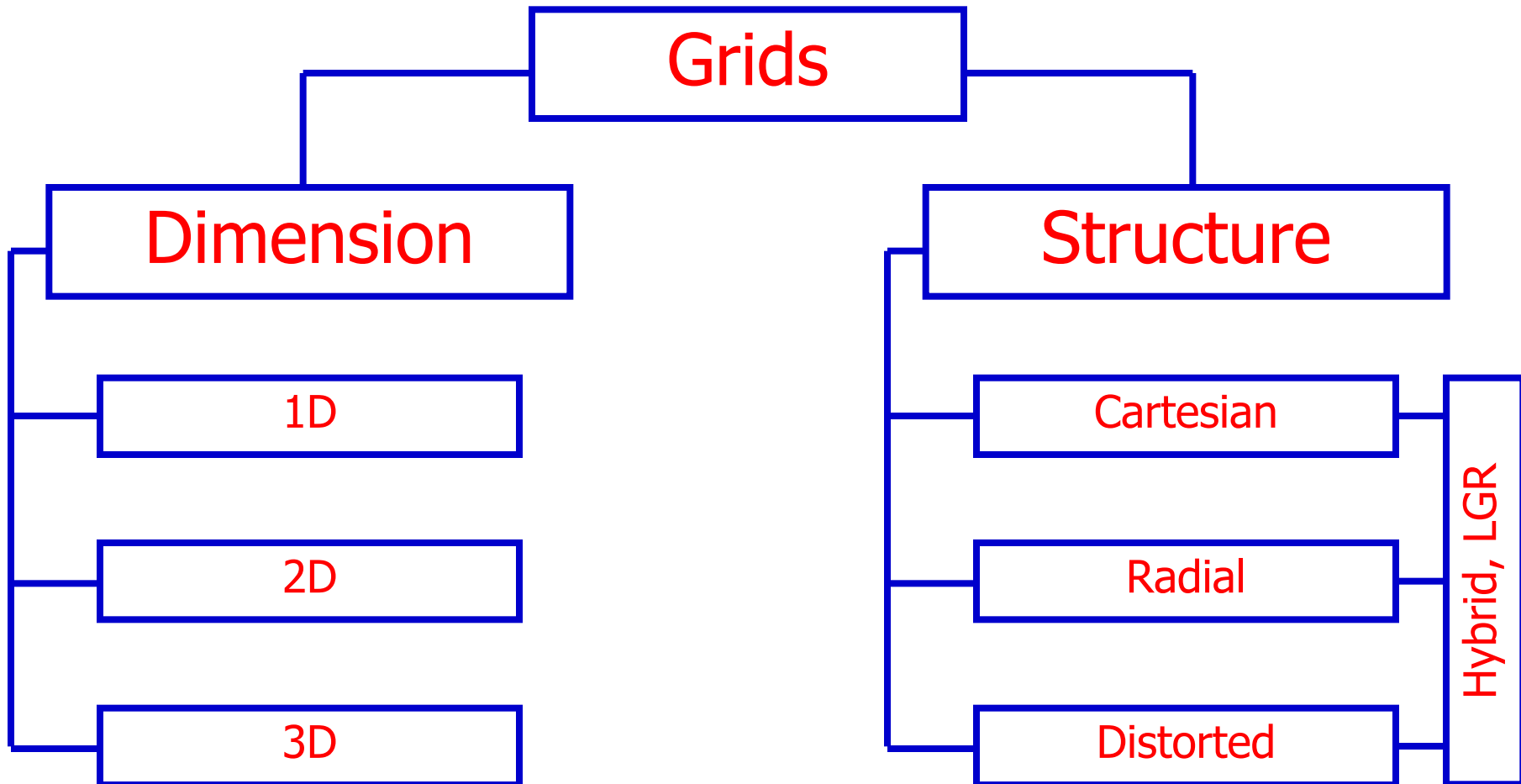
Temporal



$\Delta x, \Delta y, \Delta z$   
 $\phi, k, c, S, \rho$   
PVT

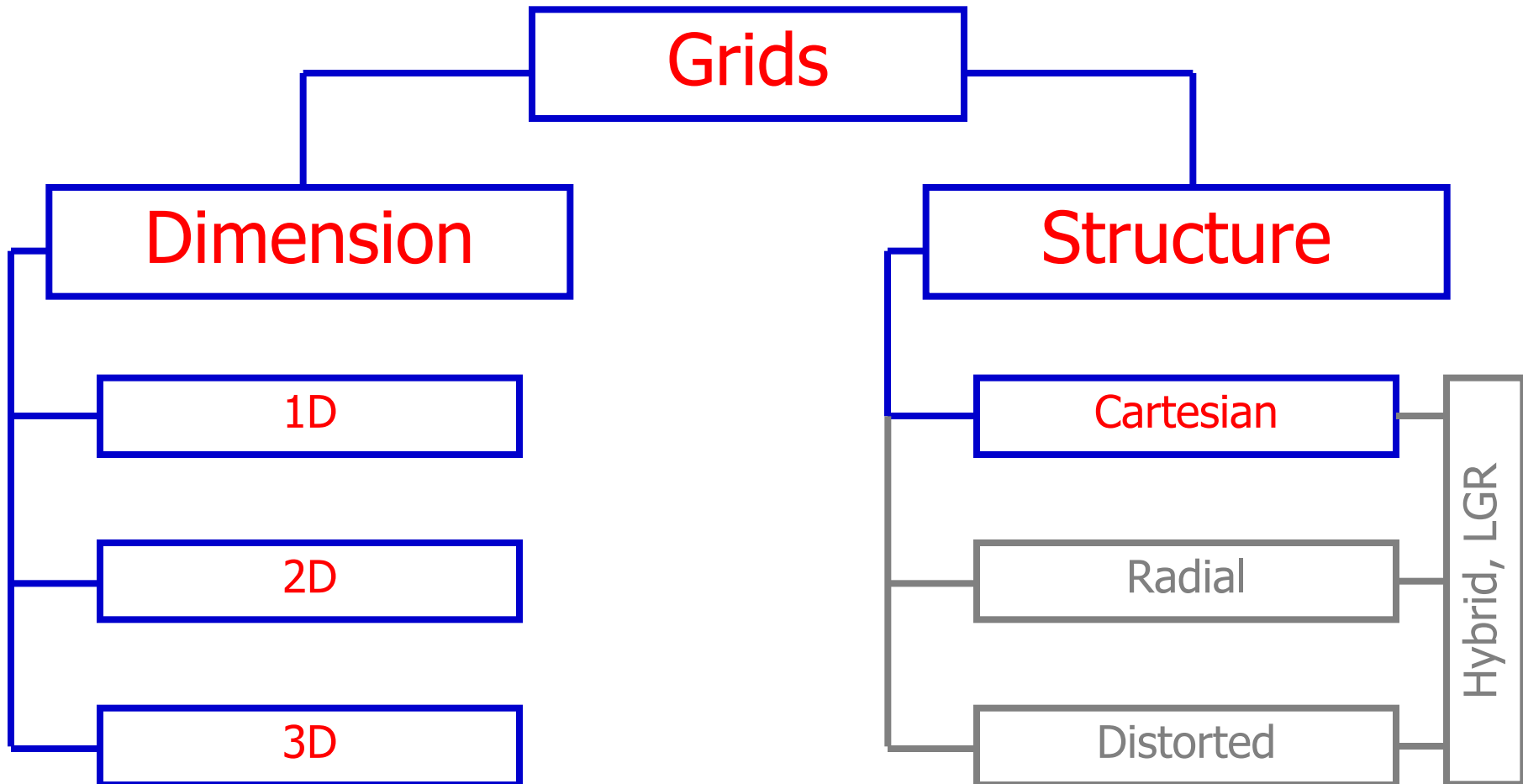


# Gridding in Reservoir Simulation





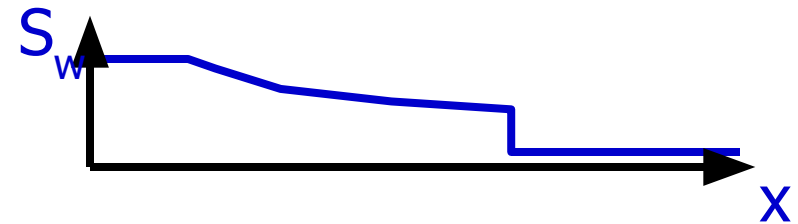
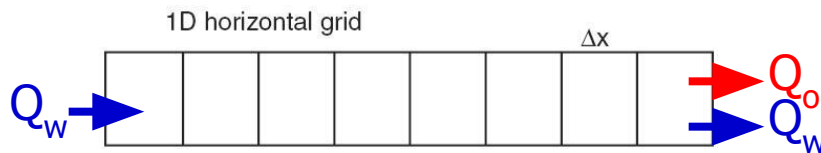
# Gridding in Reservoir Simulation



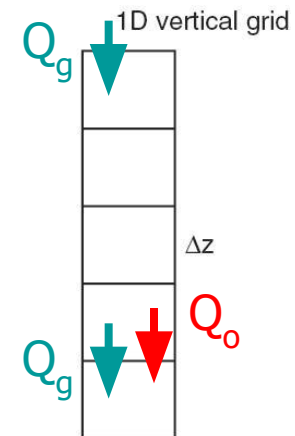


# Gridding in Reservoir Simulation

- 1D Cartesian (Linear) Grids
  - Horizontal
    - Buckley-Leverett type water displacement



- Vertical
  - gravity drainage
  - gravity stable gas displacement of oil



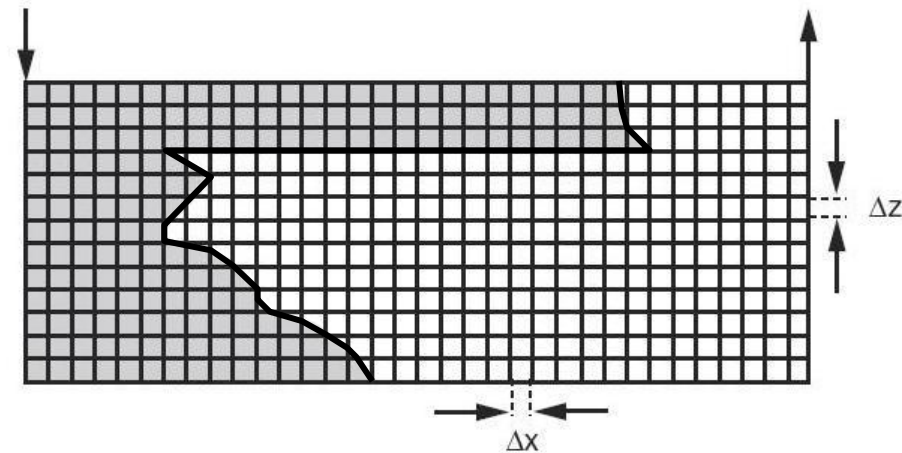


# Gridding in Reservoir Simulation

- **2D Cartesian Grids**
  - Cross-sectional (x/z)

- vertical sweep efficiency
- water/oil displacements in geostatistically generated
- generation of pseudo-relative permeabilities (2 phase upscaling)
- mechanism of gas displacement (importance of gravity)

2D (x/z) cross-sectional model showing a waterflood





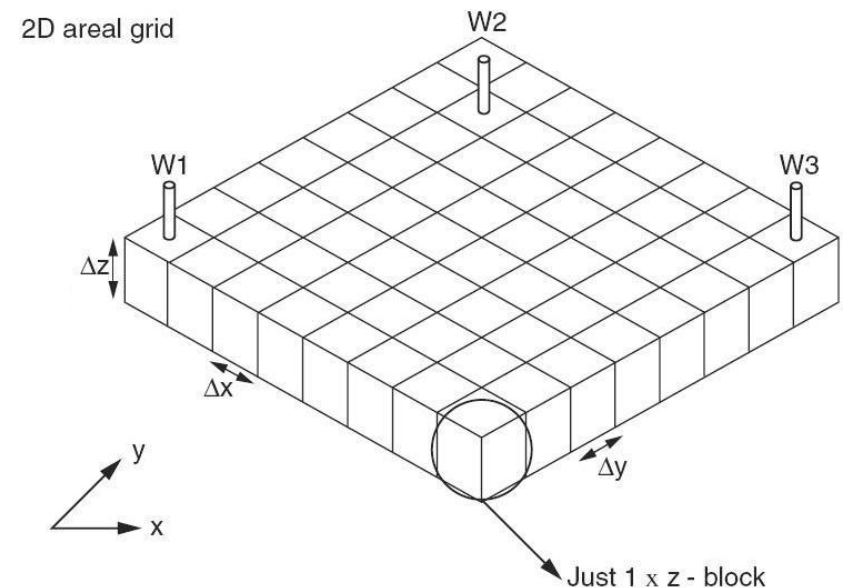


# Gridding in Reservoir Simulation

- 2D Cartesian Grids

  - Areal (x/y)

    - areal sweep efficiency (water/gas flooding)
    - benefits of infill drilling in an areal pattern flood
    - near-miscible gas injection in a homogeneous layer



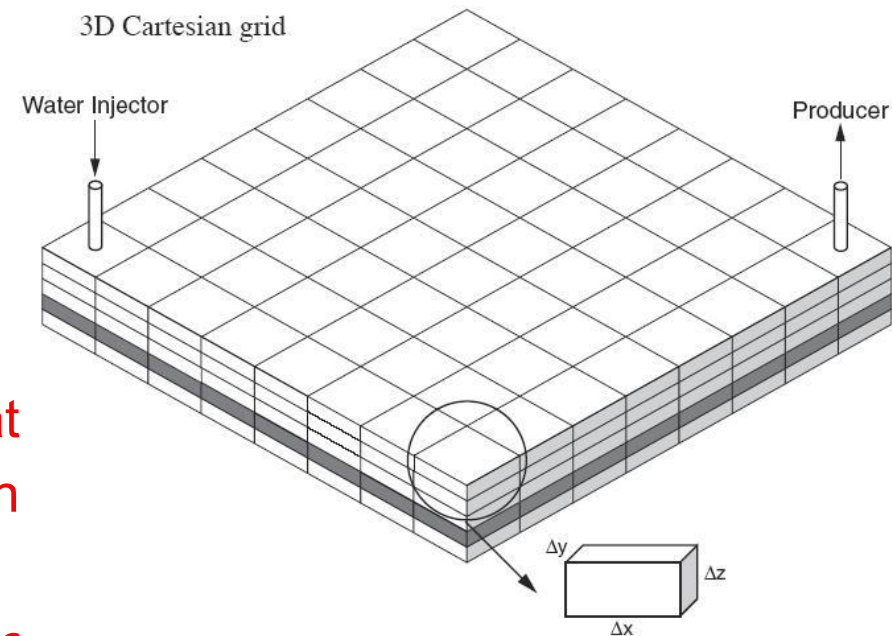


# Gridding in Reservoir Simulation

- **3D Cartesian Grids**

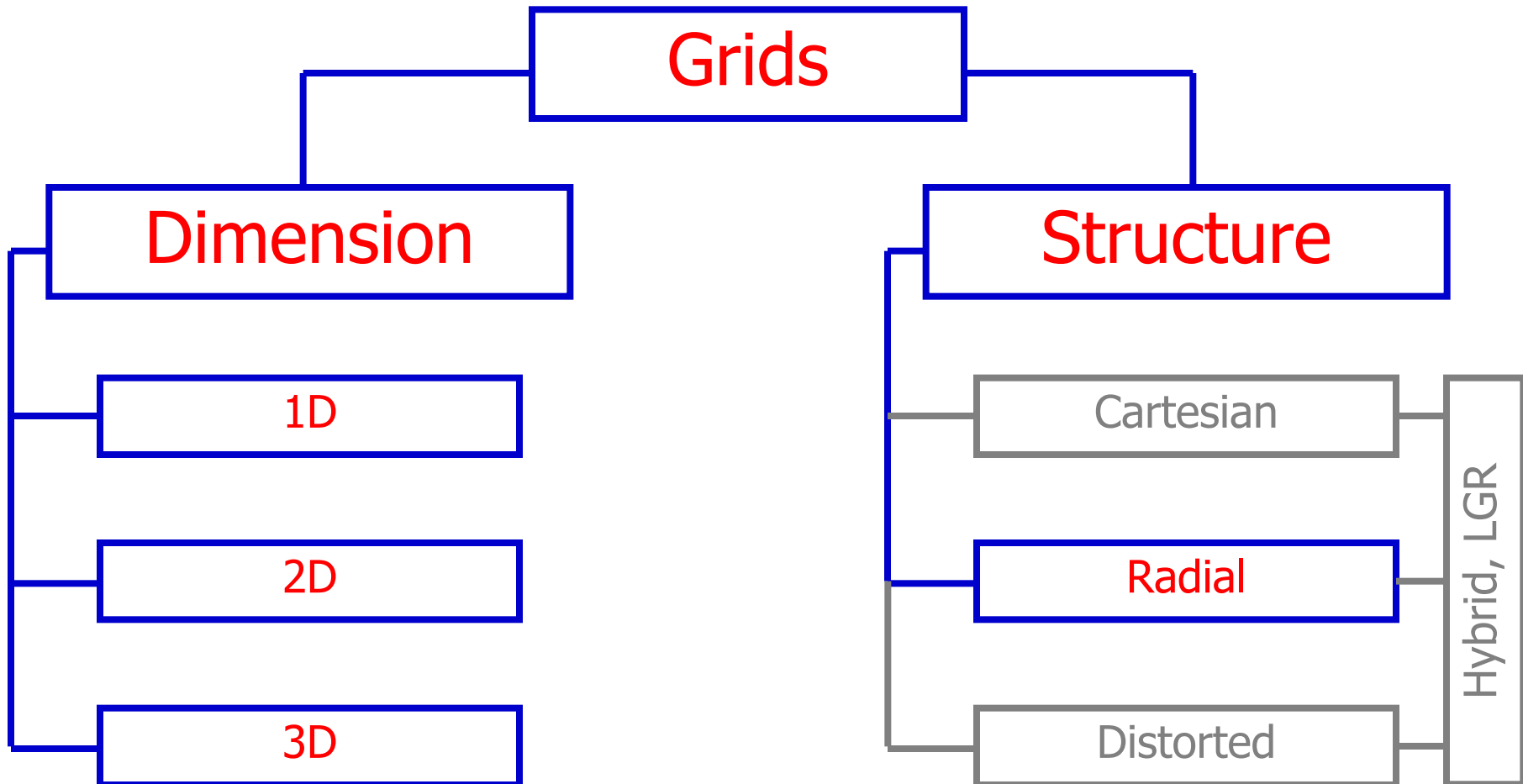
- **Default grid type**

- **easy to set up**
- **less time consuming simulation**  
(equations are derived for the grid)
- **wide range of field wide reservoir  
production processes (full field simulation of  
water/gas flooding and etc.)**





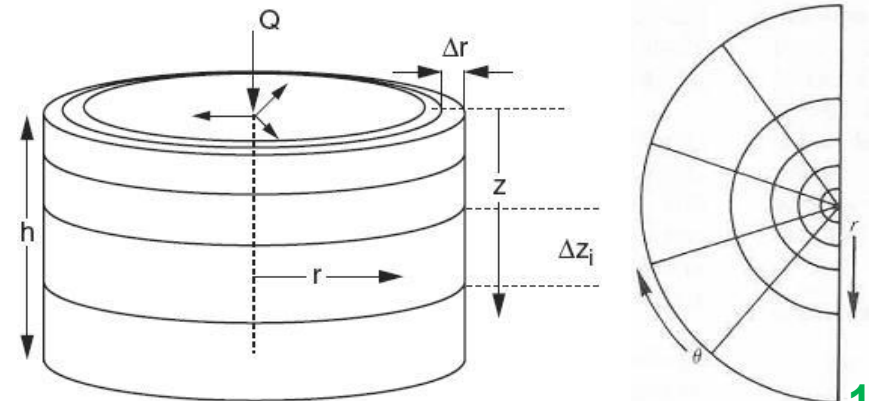
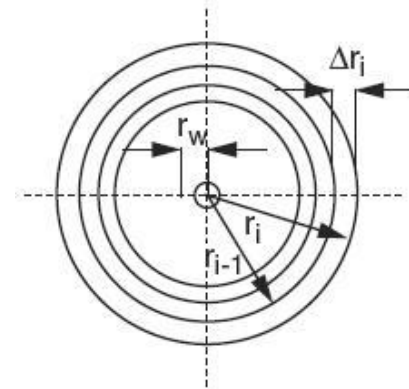
# Gridding in Reservoir Simulation





# Gridding in Reservoir Simulation

- Radial Grids
  - 1D
    - pressure front propagation
  - 2D (r/z), 3D
    - near wellbore processes (water/gas coning)





# Gridding in Reservoir Simulation

## • Radial Grids

- pressure change near well is steep

$$\Delta P(r) \sim \ln\left(\frac{r}{r_w}\right)$$

- logarithmically-spaced grid

- grid is divided such that

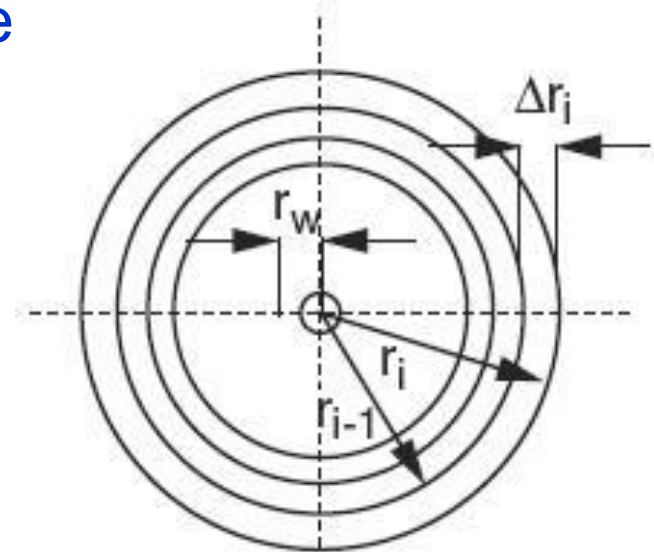
$$\Delta r_i = r_i - r_{i-1}$$

$$P(r_i) - P(r_{i-1}) = \text{const}$$

$$P(r_i) - P(r_{i-1}) \sim \ln\left(\frac{r_i}{r_{i-1}}\right)$$

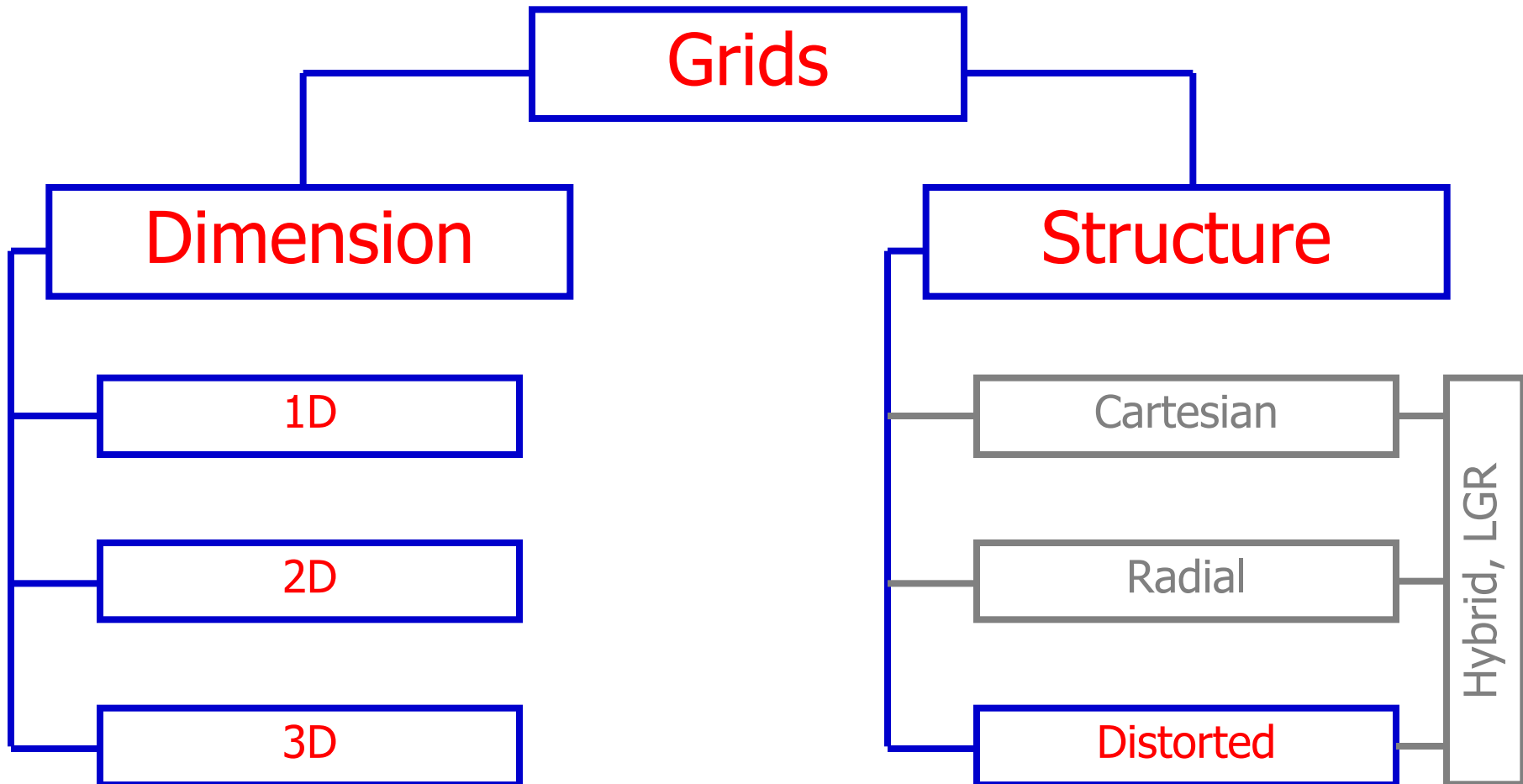


$$\frac{r_i}{r_{i-1}} = \text{const}$$





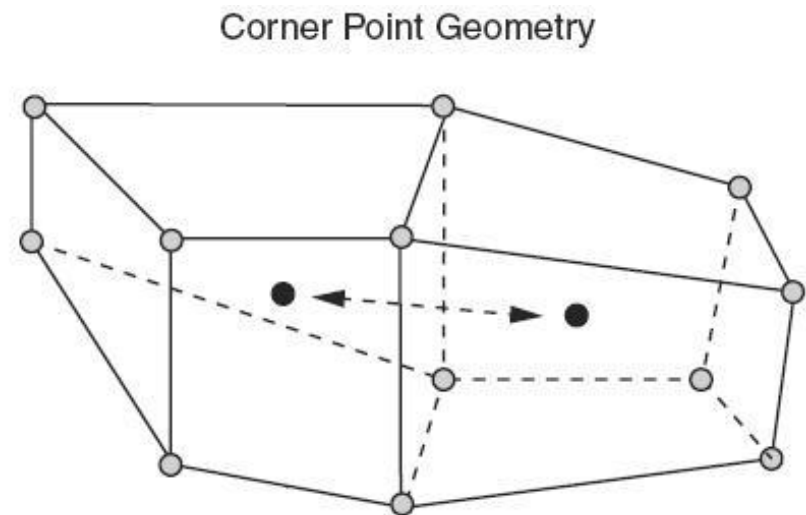
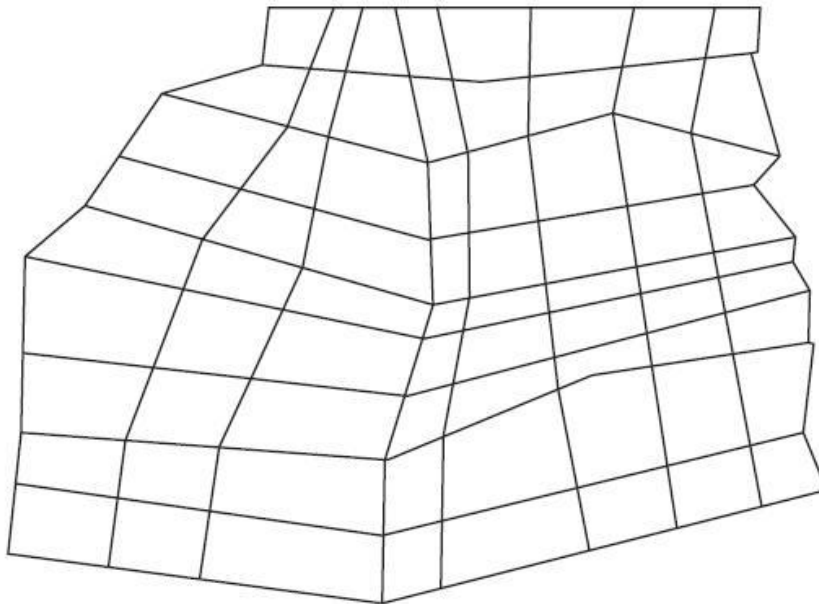
# Gridding in Reservoir Simulation





# Gridding in Reservoir Simulation

- **Distorted grids**
  - **Corner Point**
    - **account structure features (faults)**

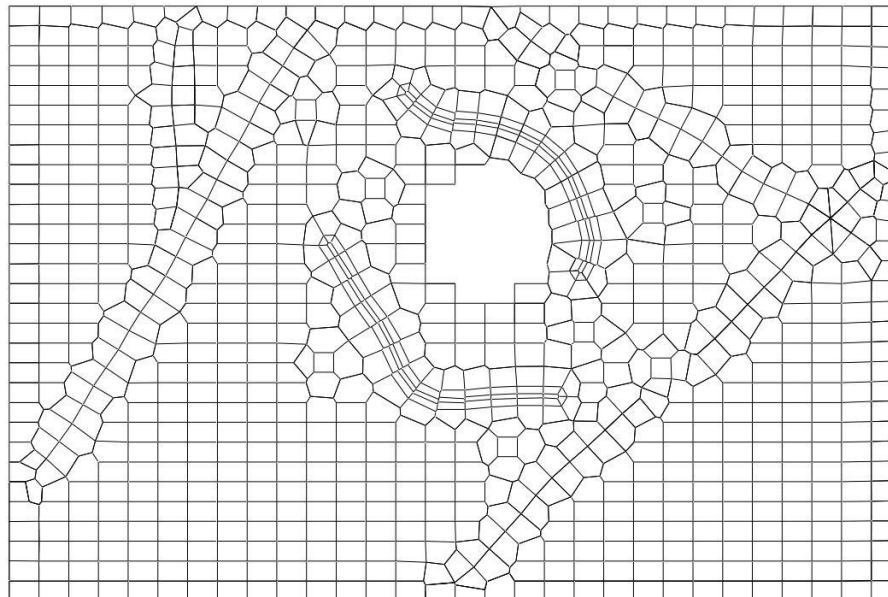


8 corners



# Gridding in Reservoir Simulation

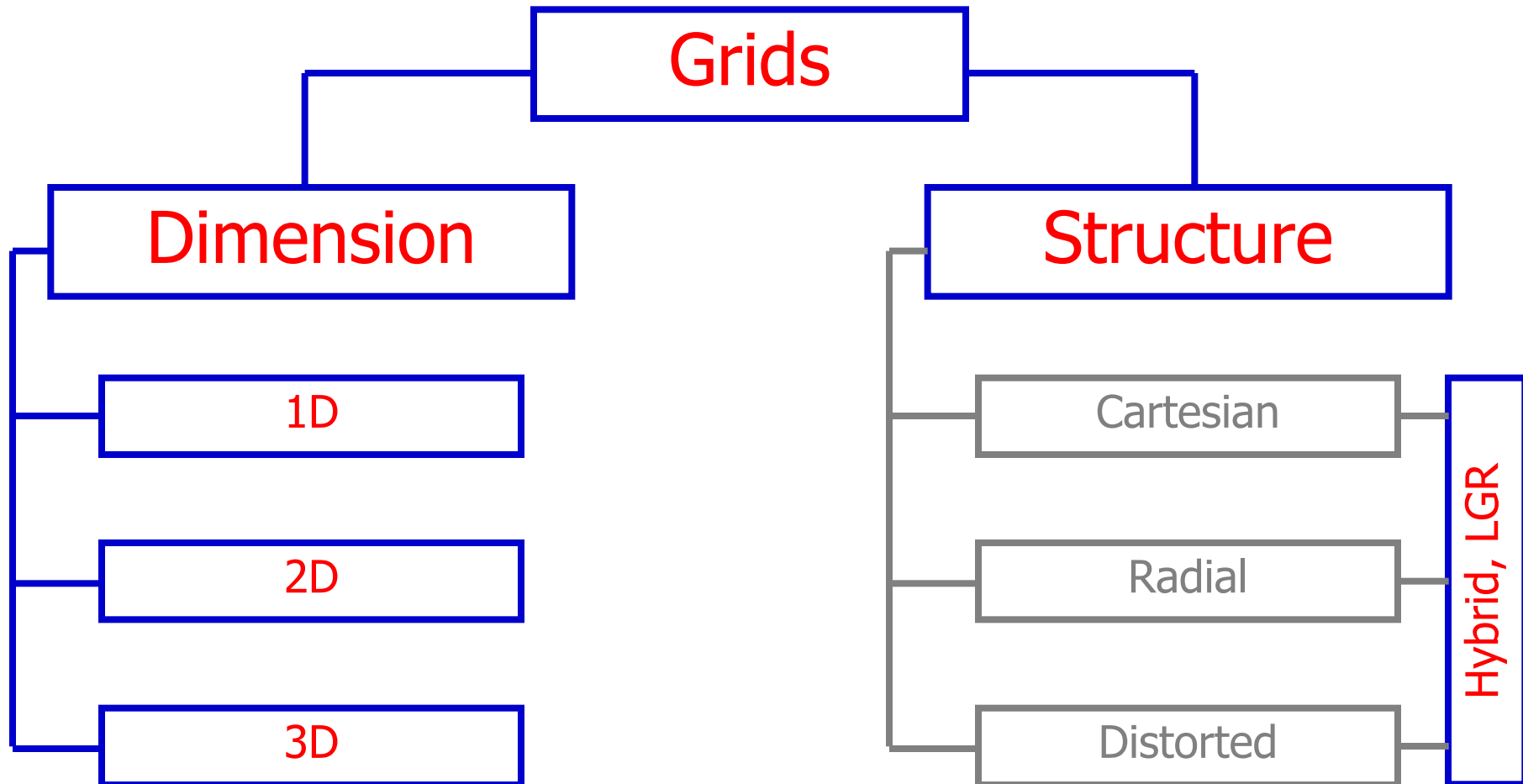
- **Distorted grids**
  - Perpendicular Bisector (PEBI), Voronoi
    - account structure features (faults)
    - horizontal well modelling







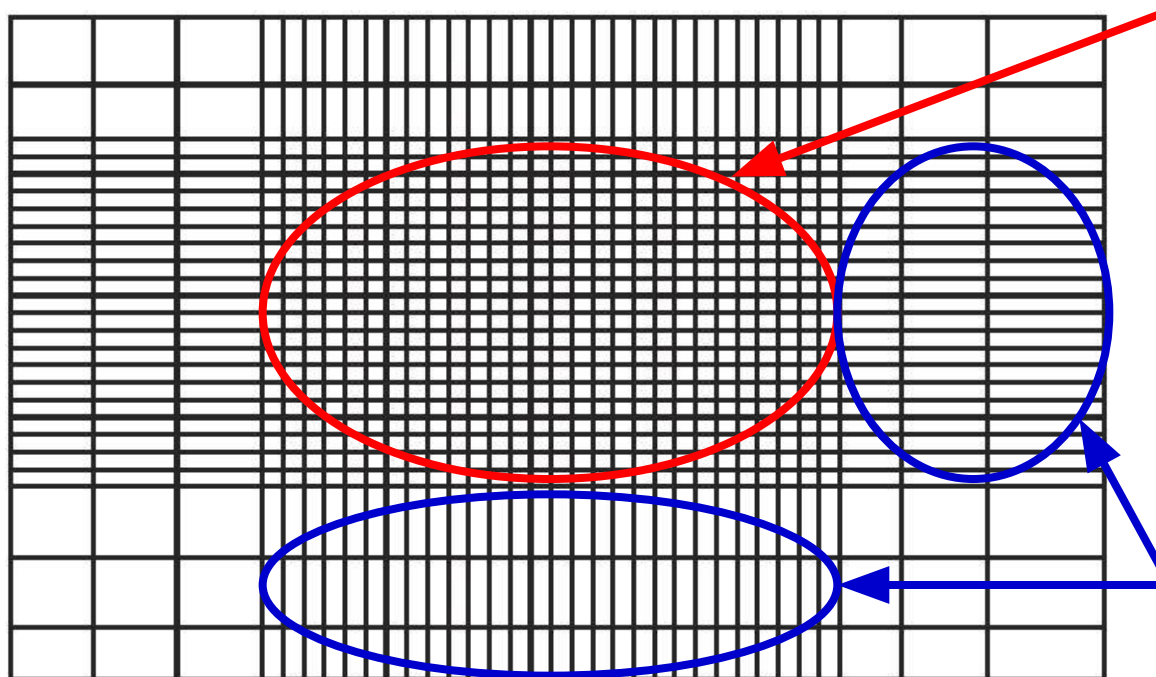
# Gridding in Reservoir Simulation





# Gridding in Reservoir Simulation

- Local Grid Refinement (LGR)
  - Simple grid refinement



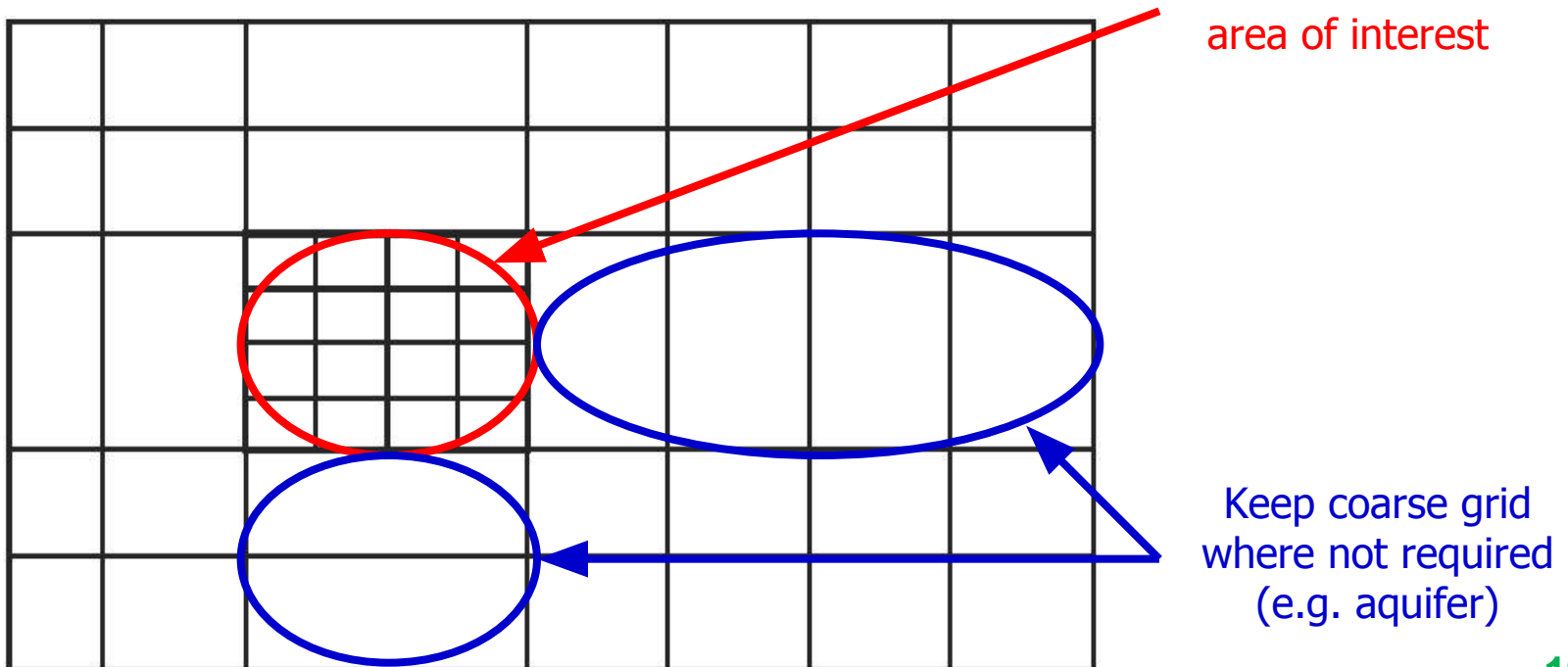
Refinement in  
area of interest

Refinement where not  
required (e.g. aquifer)



# Gridding in Reservoir Simulation

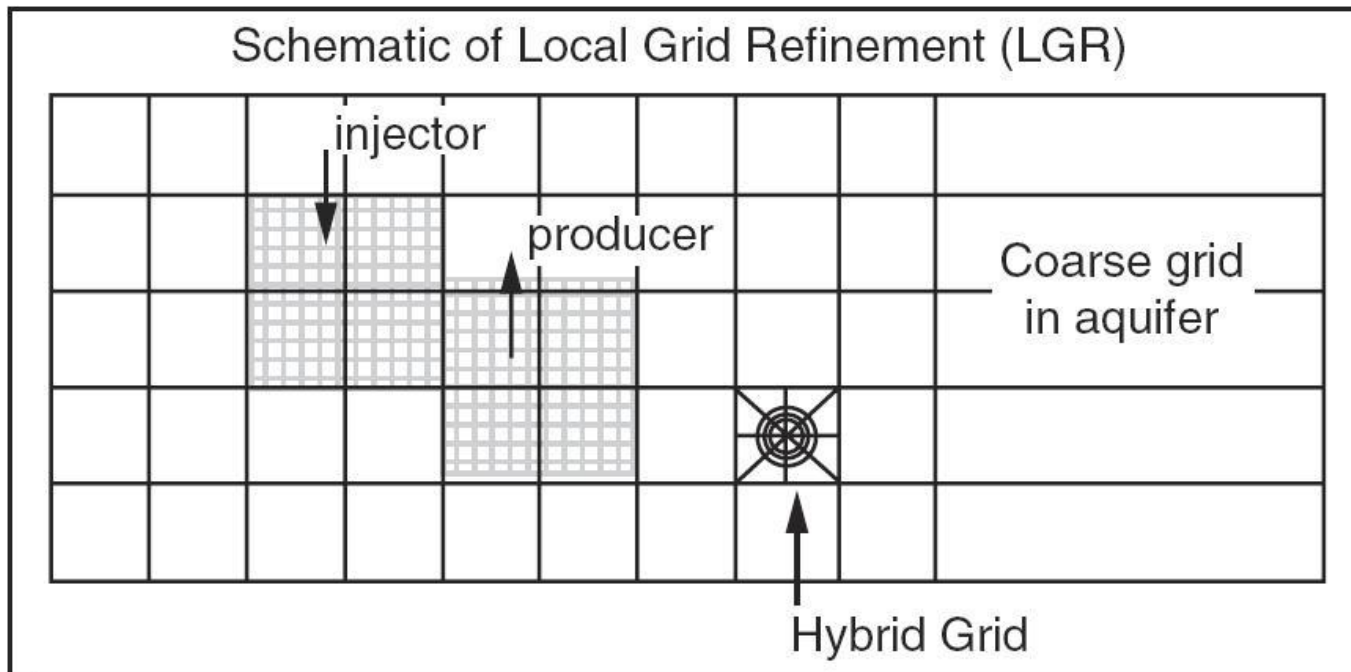
- Local Grid Refinement (LGR)
  - Local Grid Refinement





# Gridding in Reservoir Simulation

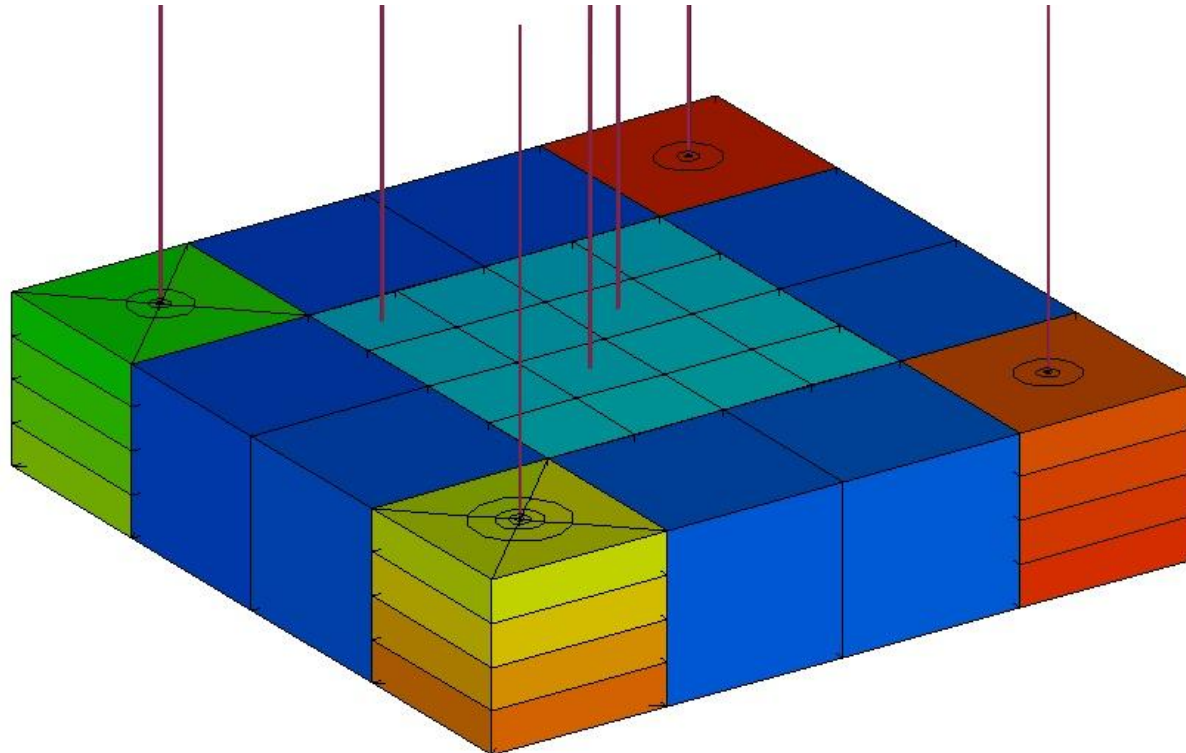
- **Local Grid Refinement (LGR)**
  - Hybrid Grid Local Grid Refinement





# Gridding in Reservoir Simulation

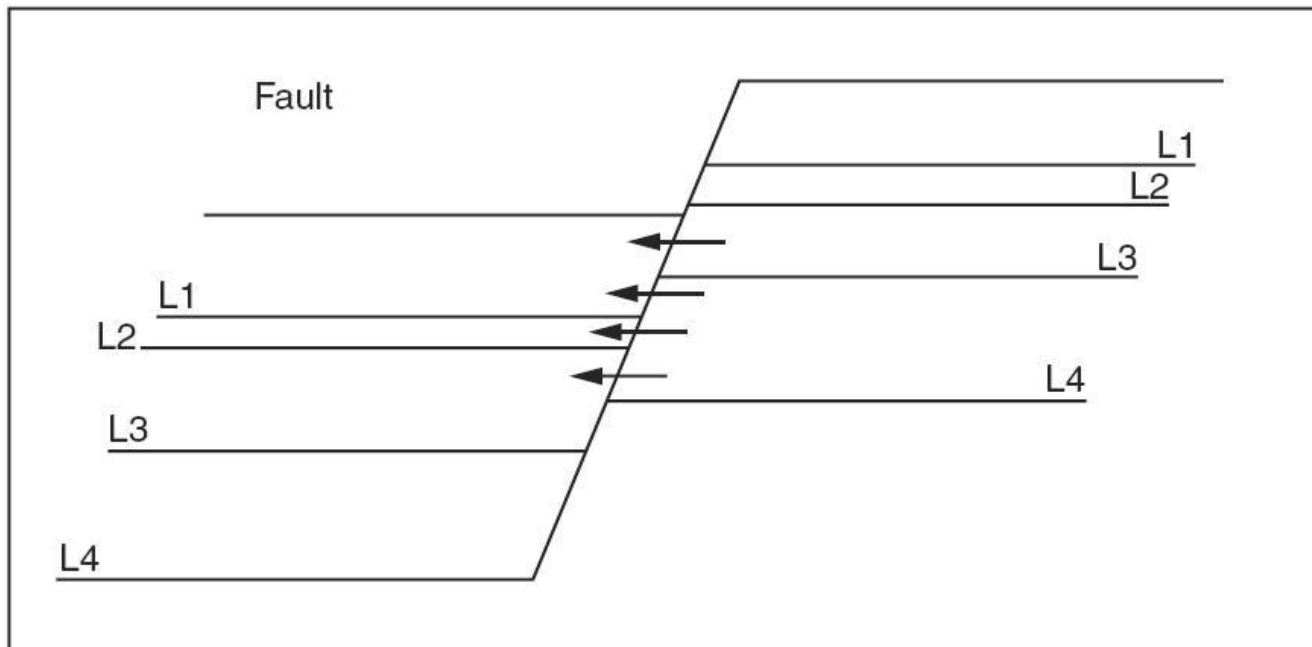
- Local Grid Refinement (LGR)
  - Hybrid Grid Local Grid Refinement





# Gridding in Reservoir Simulation

- Distorted grids
  - Faults modelling
    - Non-neighbor connections (NNC)

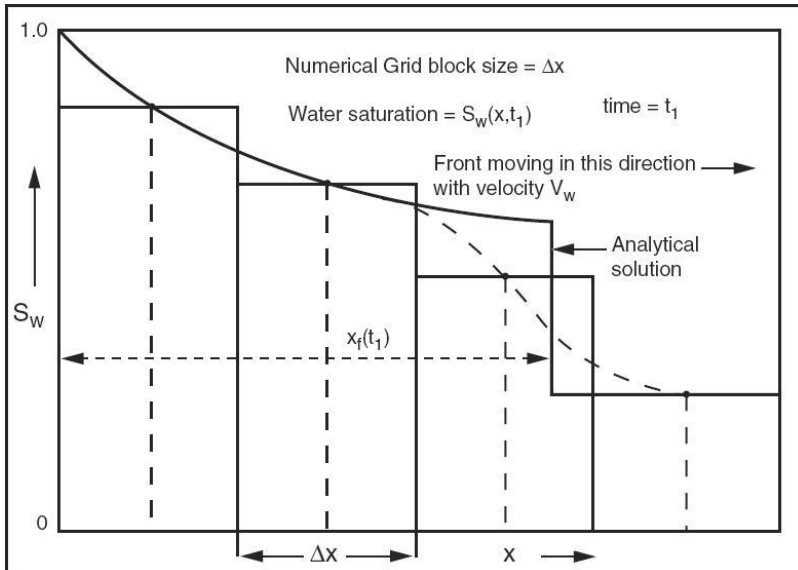




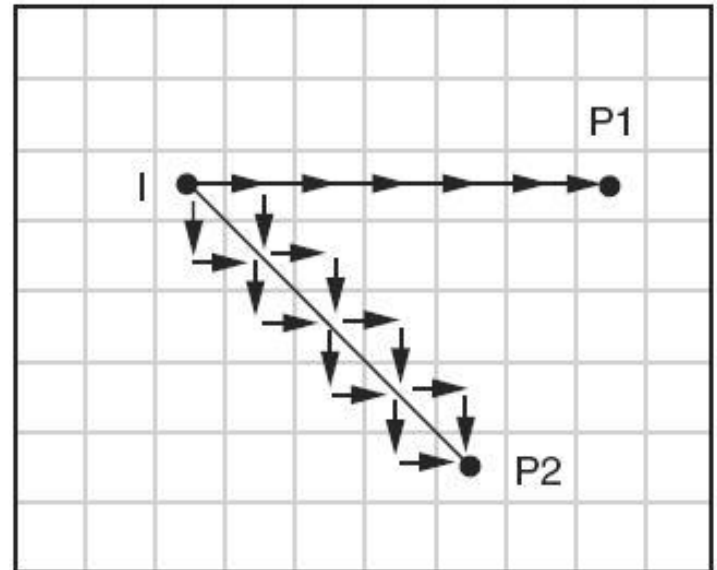
# Gridding in Reservoir Simulation

## Numerical problems

### Numerical dispersion



### Grid orientation

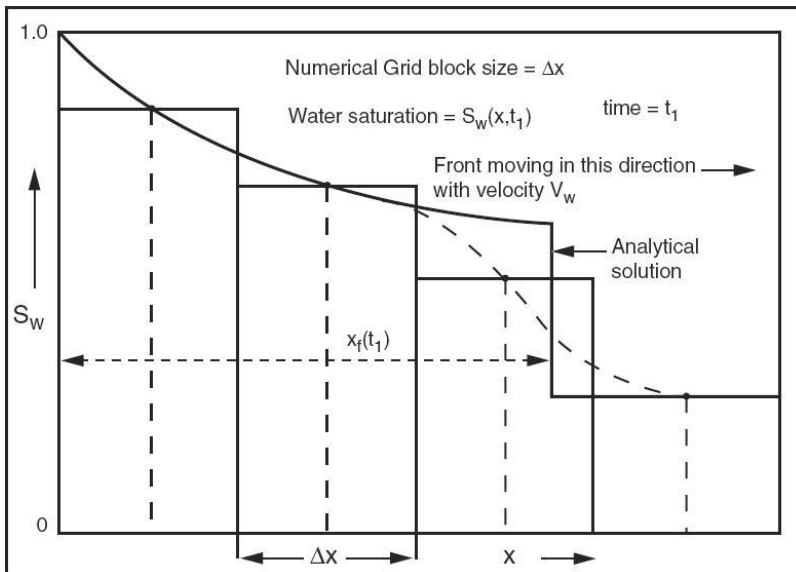




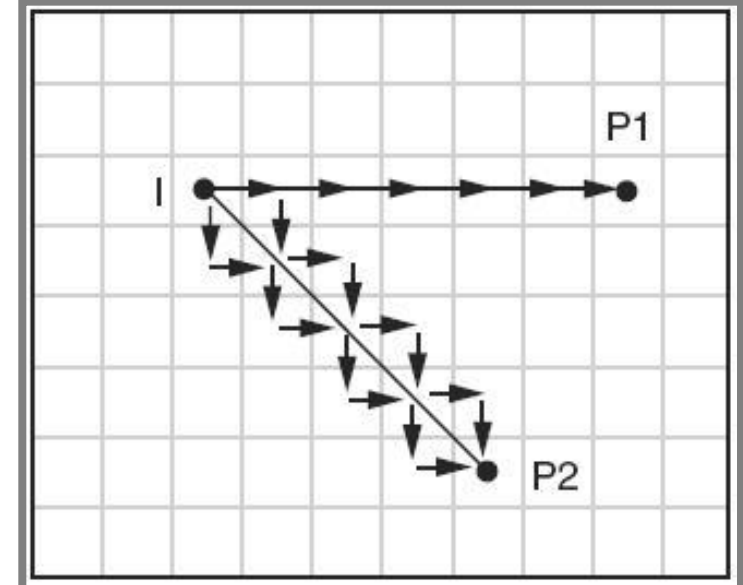
# Gridding in Reservoir Simulation

## Numerical problems

### Numerical dispersion



### Grid orientation



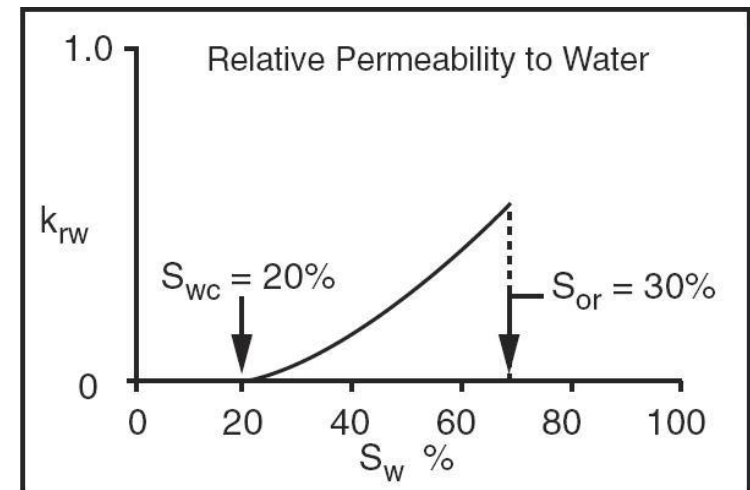
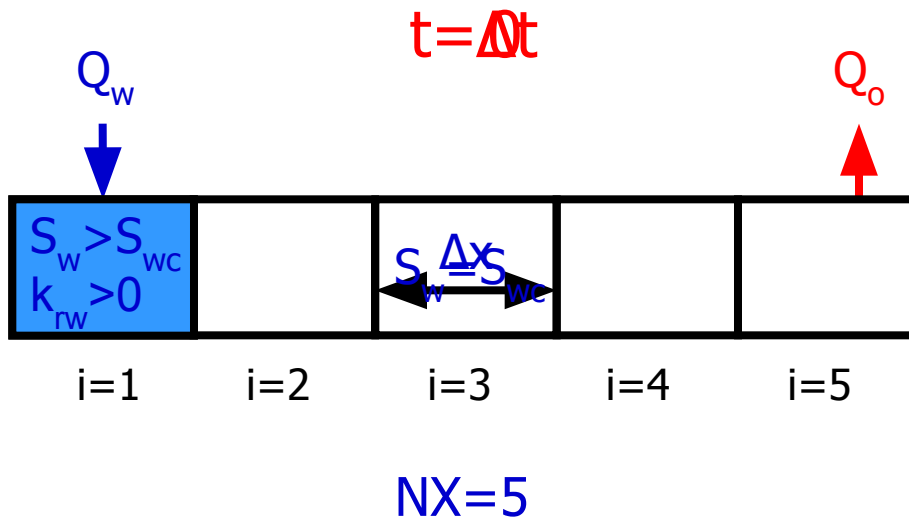




# Gridding in Reservoir Simulation

- Numerical Dispersion**

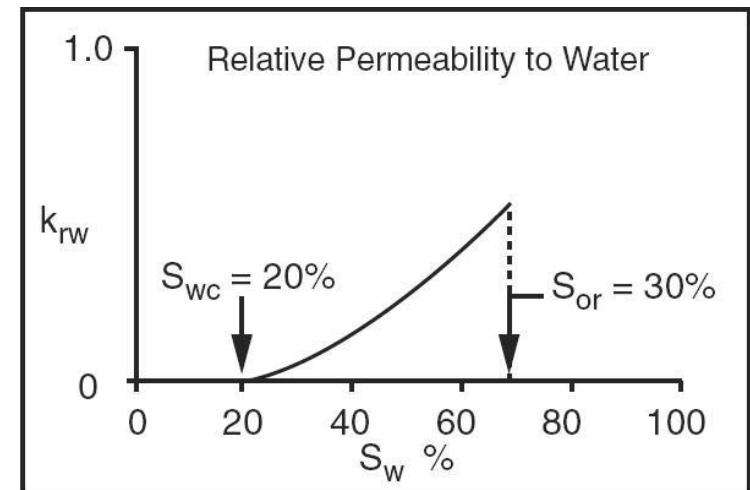
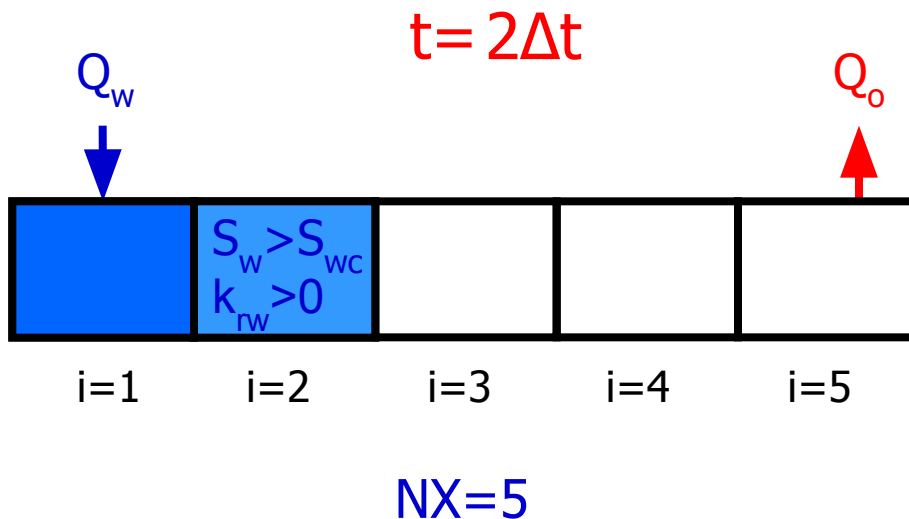
- numerical error due to grid block approximation for solving the flow equations





# Gridding in Reservoir Simulation

- Numerical Dispersion
  - numerical error due to grid block approximation for solving the flow equations

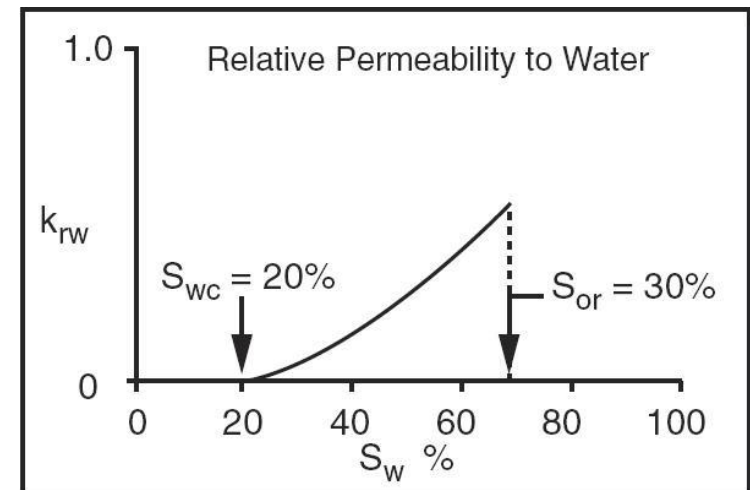
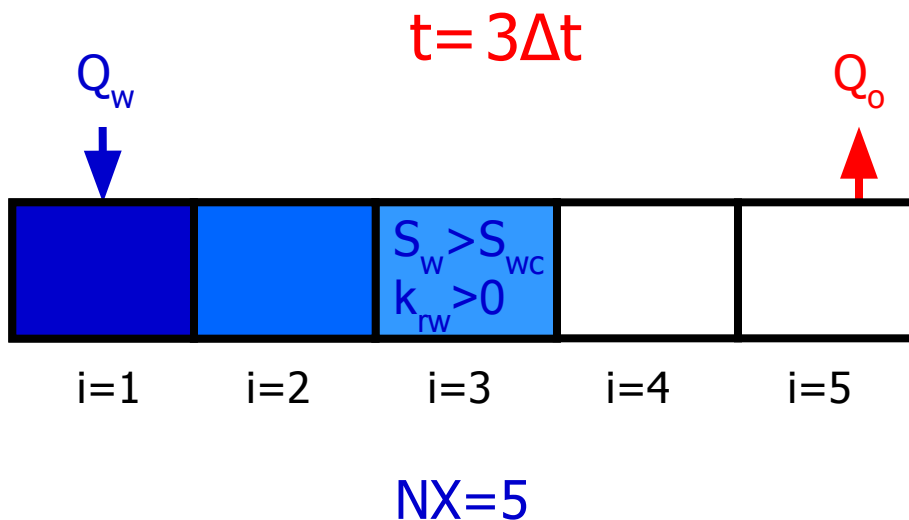




# Gridding in Reservoir Simulation

- Numerical Dispersion

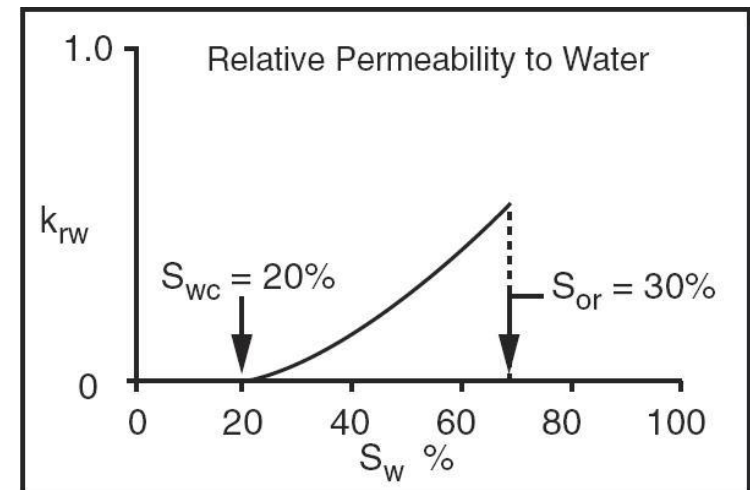
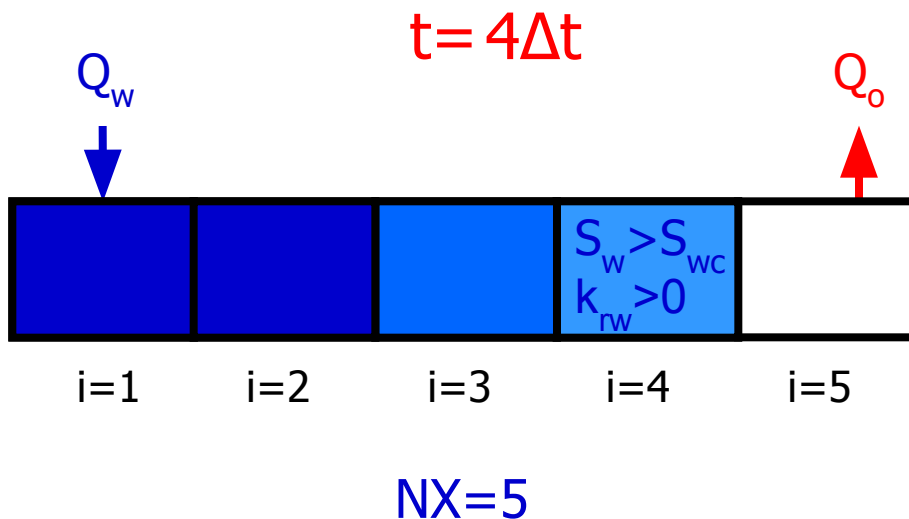
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# Gridding in Reservoir Simulation

- Numerical Dispersion
  - numerical error due to grid block approximation for solving the flow equations

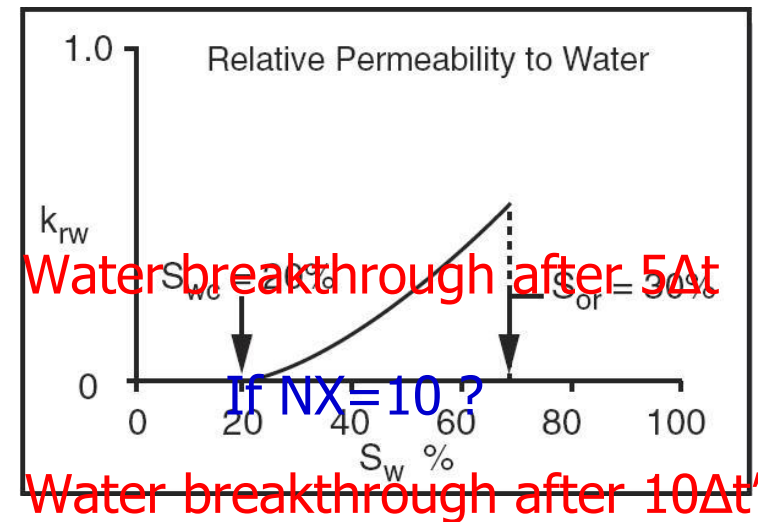
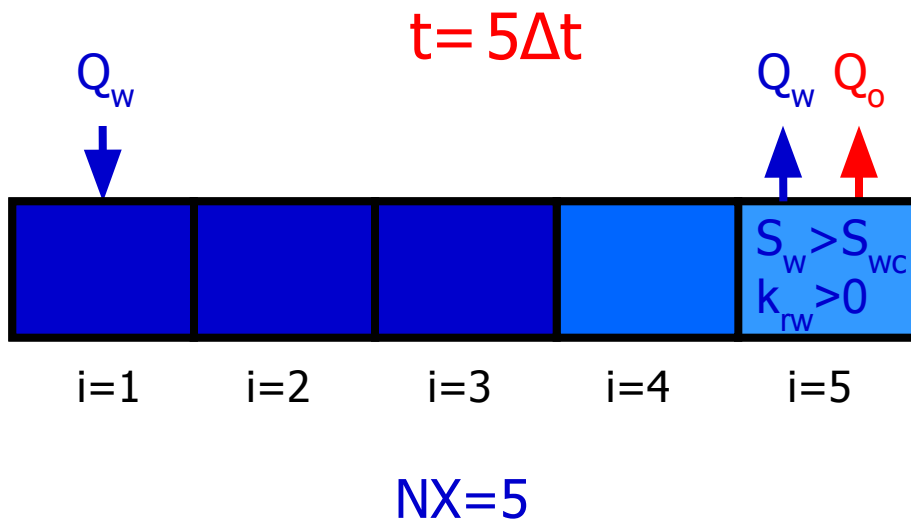




# Gridding in Reservoir Simulation

- Numerical Dispersion**

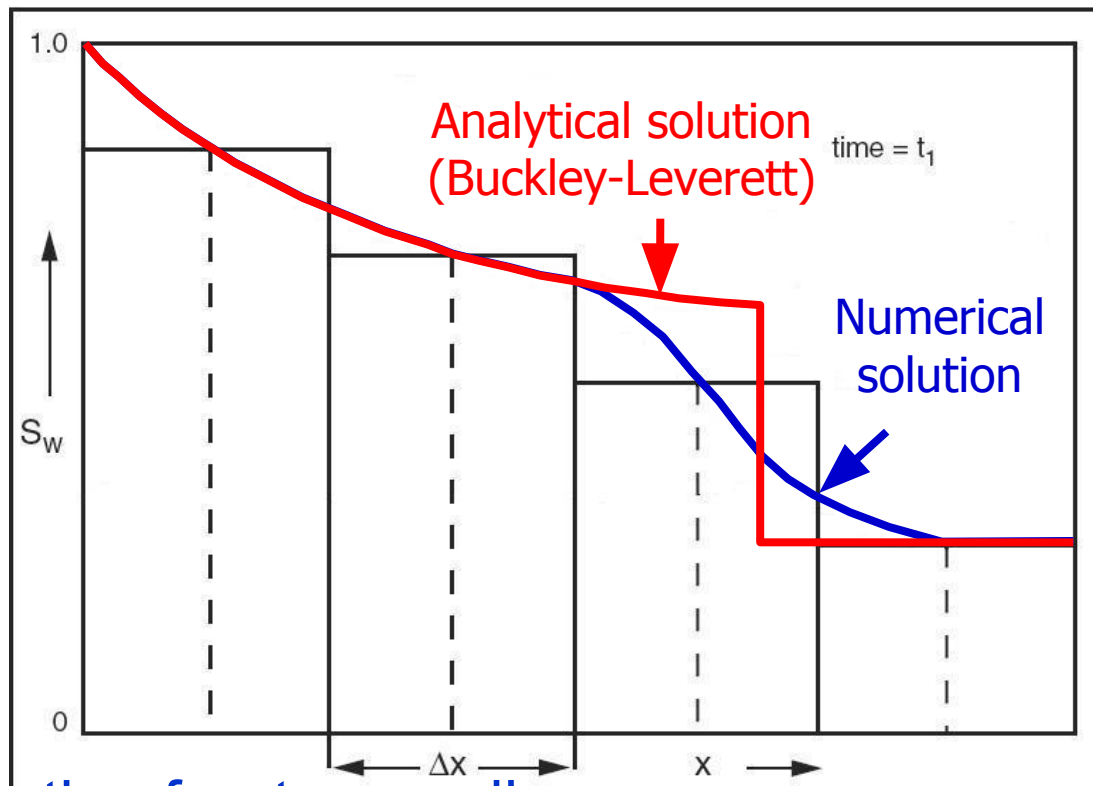
- numerical error due to grid block approximation for solving the flow equations





# Gridding in Reservoir Simulation

- Numerical Dispersion

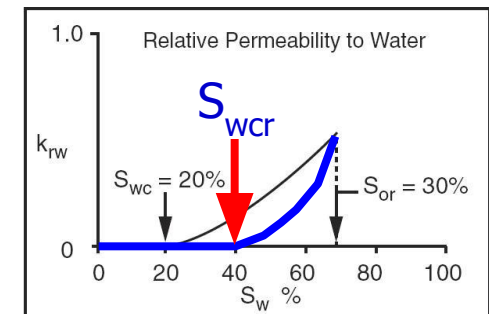


– saturation front spreading



# Gridding in Reservoir Simulation

- Numerical Dispersion
  - Methods of reducing
    - Grid refinement (increase in number of grid blocks)
    - Use of Pseudo-relative permeabilities (2 phase upscaling)
    - Use of alternative simulation scheme (Streamline simulation)

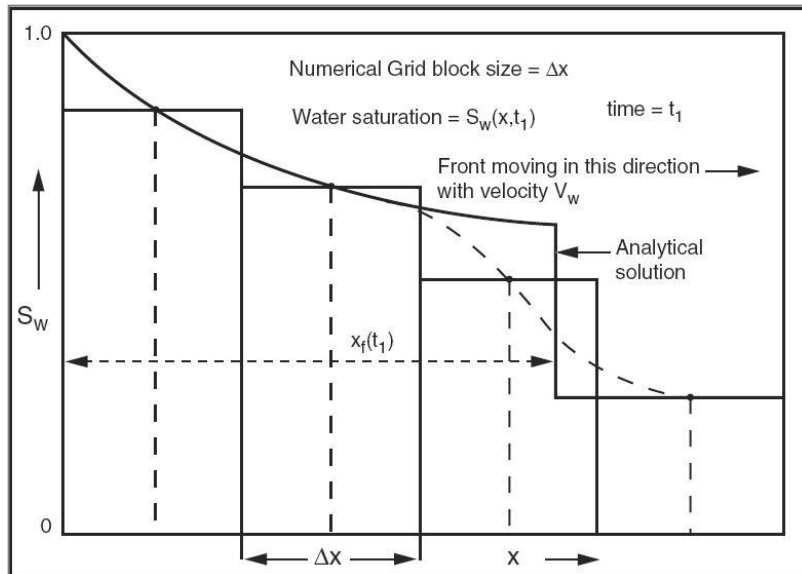




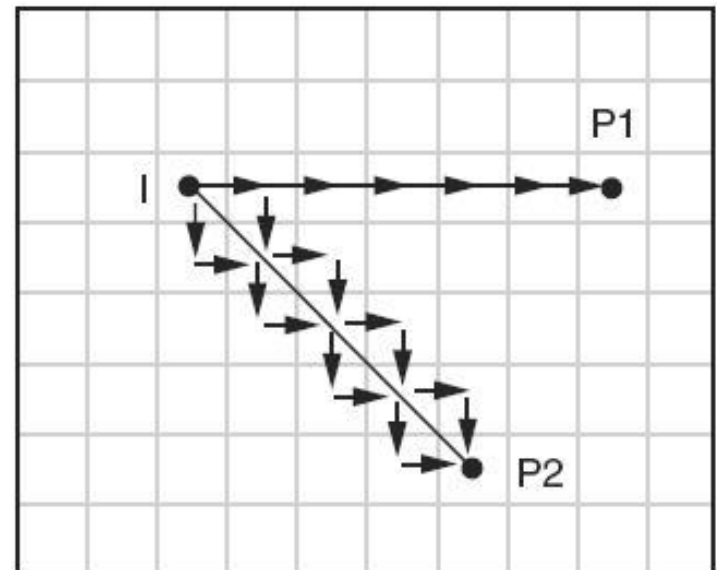
# Gridding in Reservoir Simulation

## Numerical problems

### Numerical dispersion



### Grid orientation

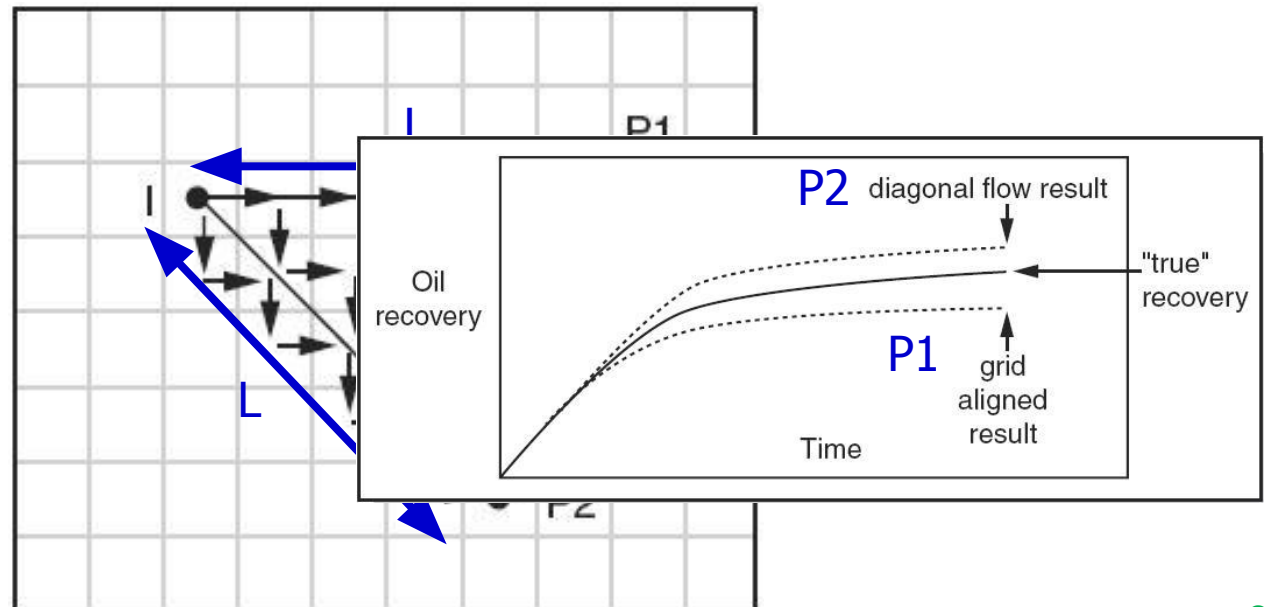






# Gridding in Reservoir Simulation

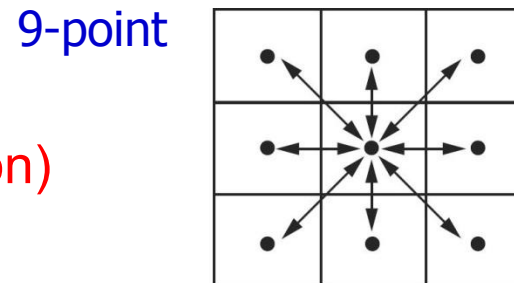
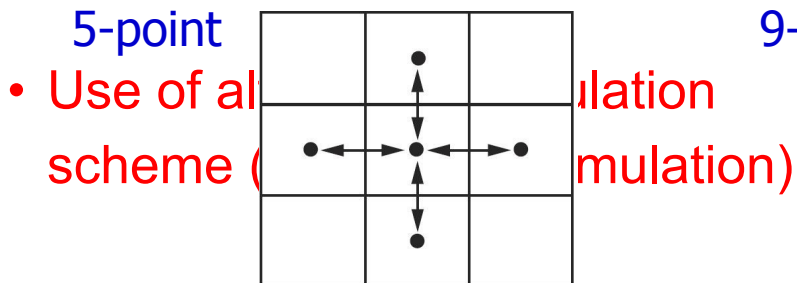
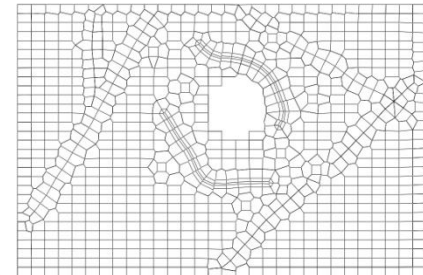
- **Grid Orientation effect**
  - arises when the fluid flow both oriented with the principal and diagonal grid direction





# Gridding in Reservoir Simulation

- **Grid Orientation effect**
  - **Methods of reducing**
    - **Grid refinement (increase in number of grid blocks)**
    - **Use of distorted grids (PEBI)**
    - **Use of improved numerical schemes (9-point)**

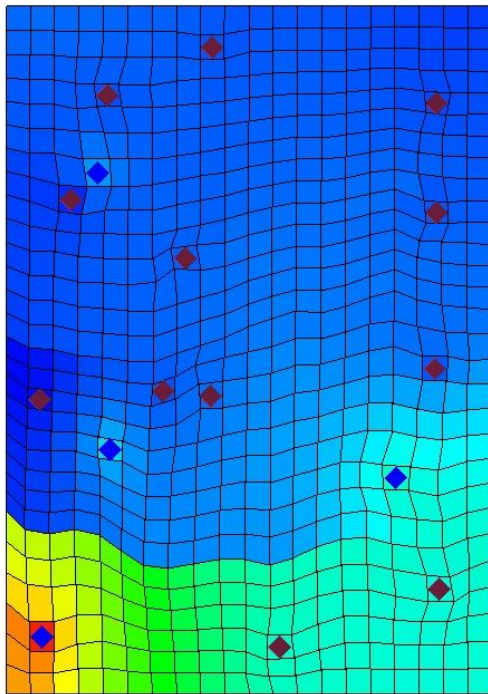




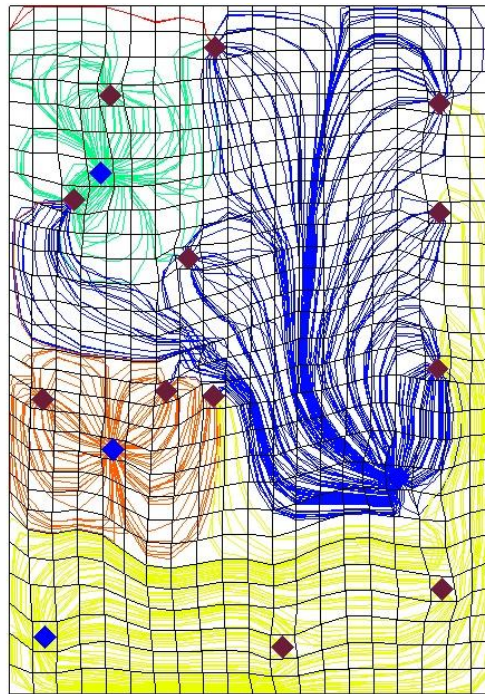
# Gridding in Reservoir Simulation

- **Streamline Simulation**

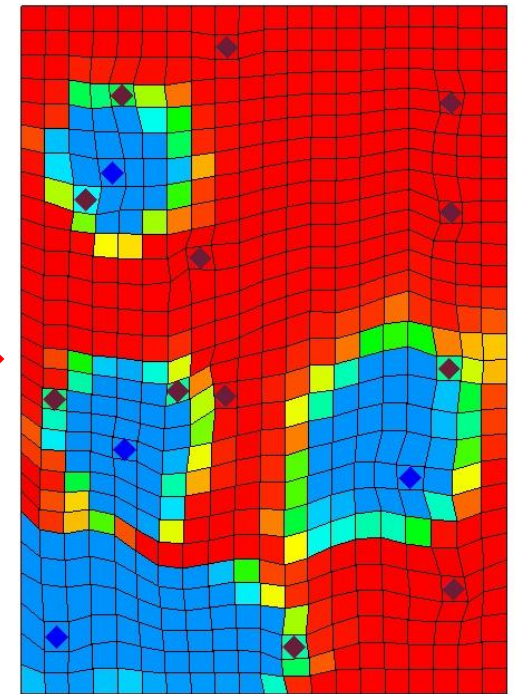
Pressure



Streamlines



Saturation





# Gridding in Reservoir Simulation

- Issues in Choosing Reservoir Simulation Grid

- Grid Dimension

- 1D, 2D, 3D ?

- Grid Geometry/Structure

- Cartesian, Distorted, LGR ?

- Grid Fineness/Coarseness

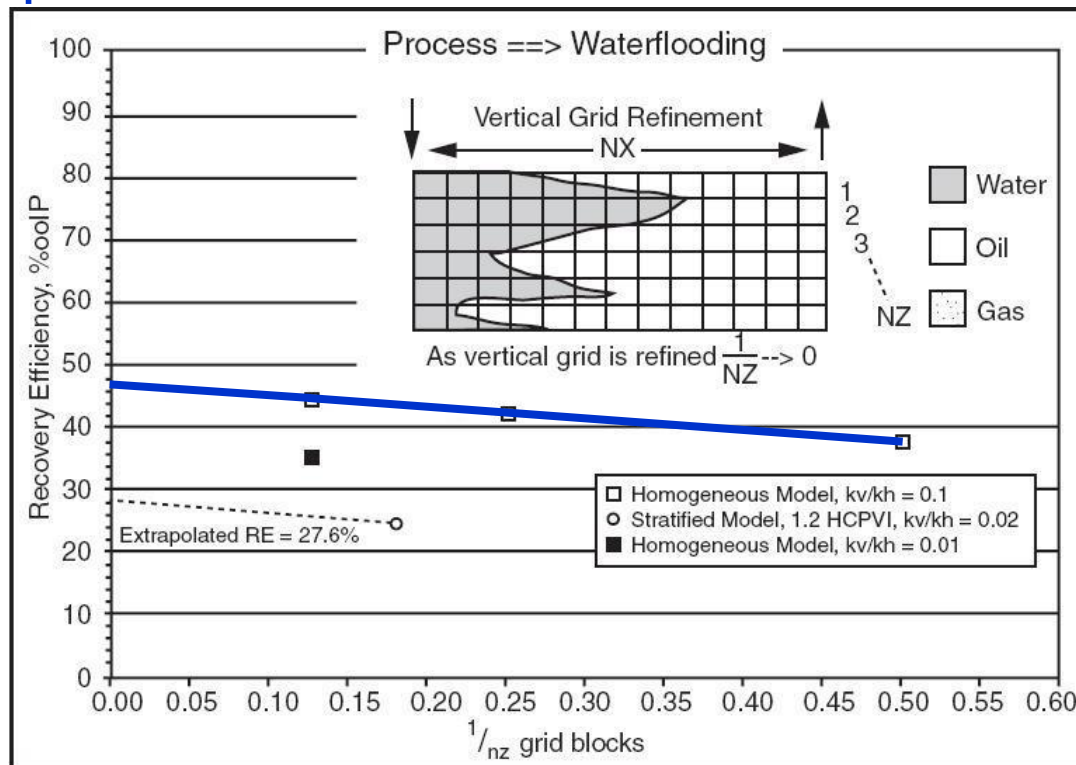
- Grid block size (number of grid blocks) ?

should be related to  
the simplicity/complexity  
of the problem



# Gridding in Reservoir Simulation

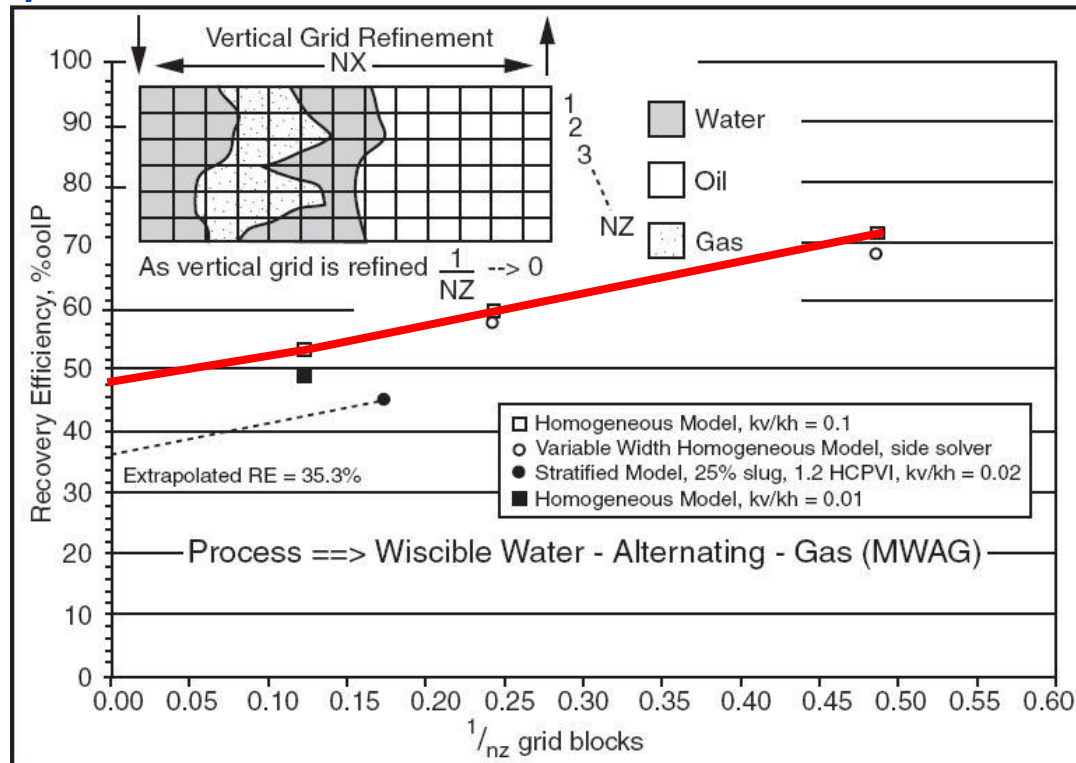
- Issues in Choosing Reservoir Simulation Grid
  - Example 1





# Gridding in Reservoir Simulation

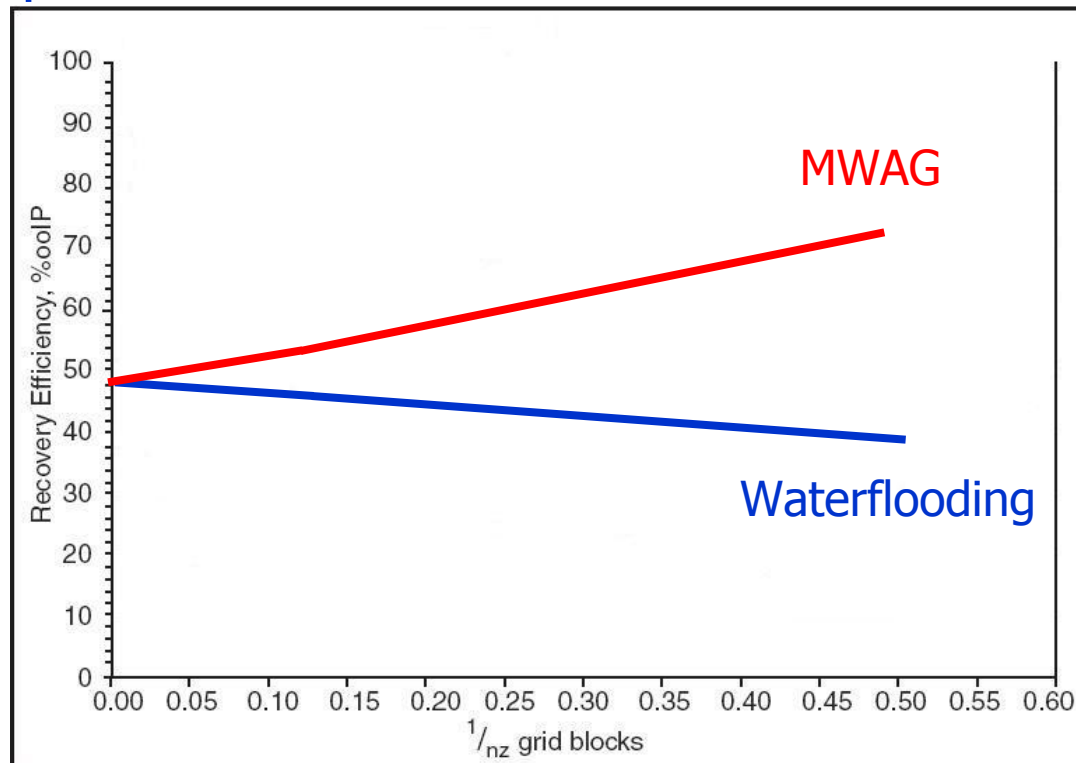
- Issues in Choosing Reservoir Simulation Grid
  - Example 1





# Gridding in Reservoir Simulation

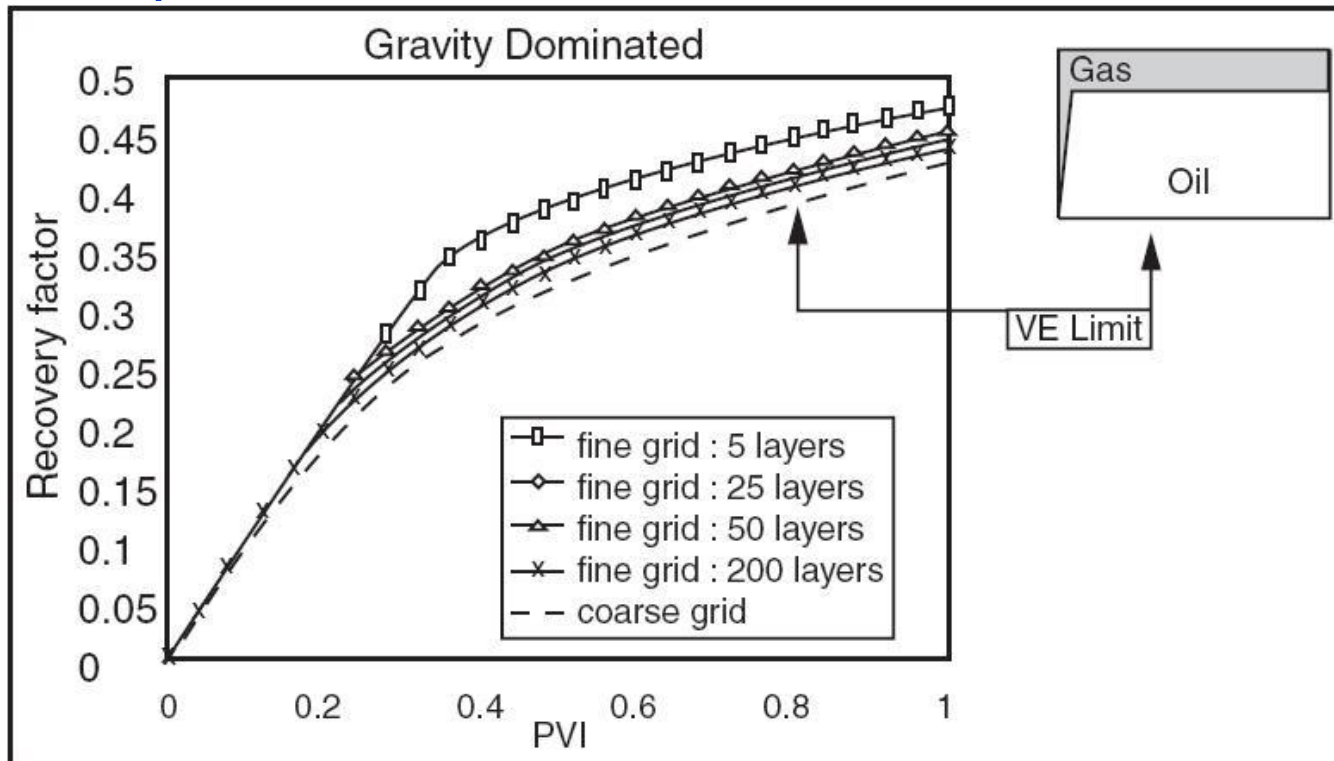
- Issues in Choosing Reservoir Simulation Grid
  - Example 1





# Gridding in Reservoir Simulation

- Issues in Choosing Reservoir Simulation Grid
  - Example 2







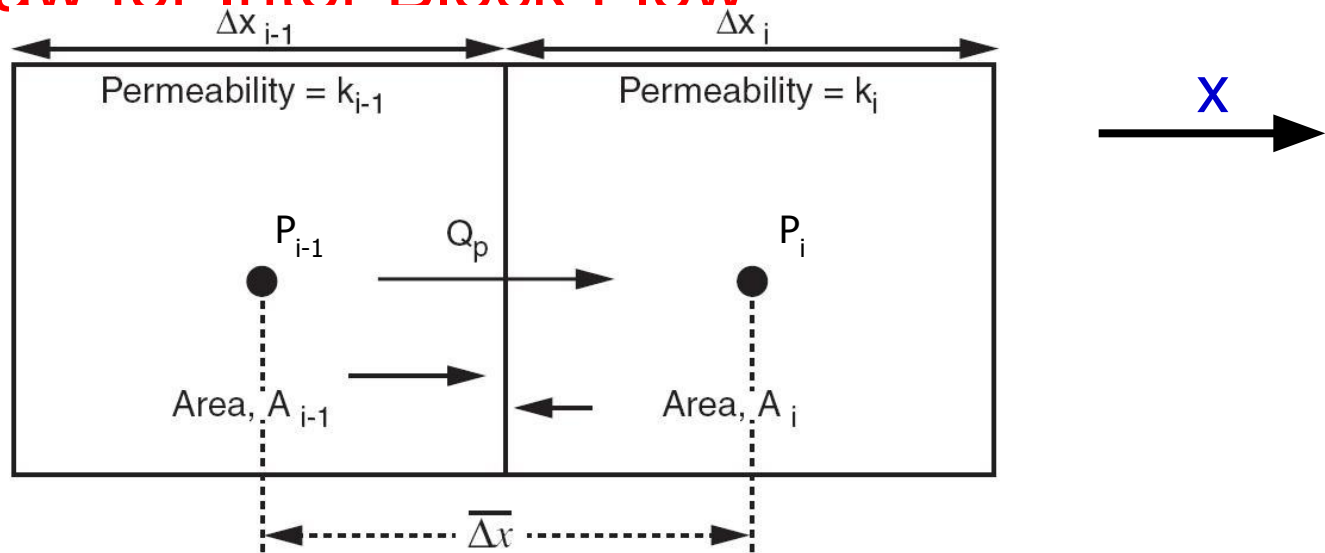
# Outline

- **Gridding in Reservoir Simulation**
- Calculation of Block to Block Flows in Reservoir Simulators
- Wells in Reservoir Simulation



# Calculation of Block to Block Flows in Reservoir Simulators

- Darcy Law for Inter Block Flow

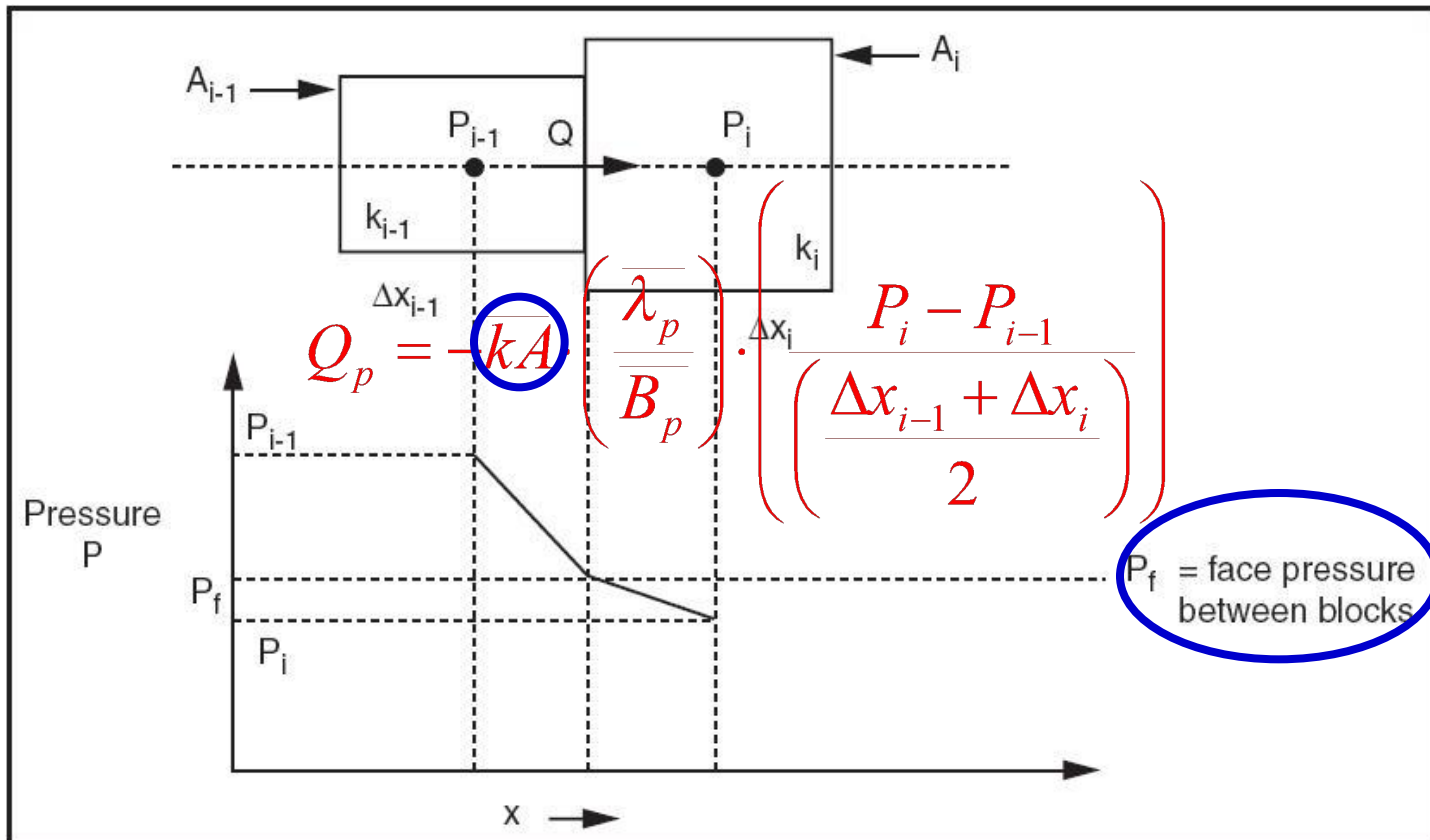


$$Q_p = -\bar{k}A \cdot \left( \frac{\lambda_p}{B_p} \right) \cdot \left( \frac{\Delta P}{\Delta x} \right) = -\bar{k}A \cdot \left( \frac{\lambda_p}{B_p} \right) \cdot \left( \frac{P_i - P_{i-1}}{\left( \frac{\Delta x_{i-1} + \Delta x_i}{2} \right)} \right)$$



# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging k-A Product,  $kA$

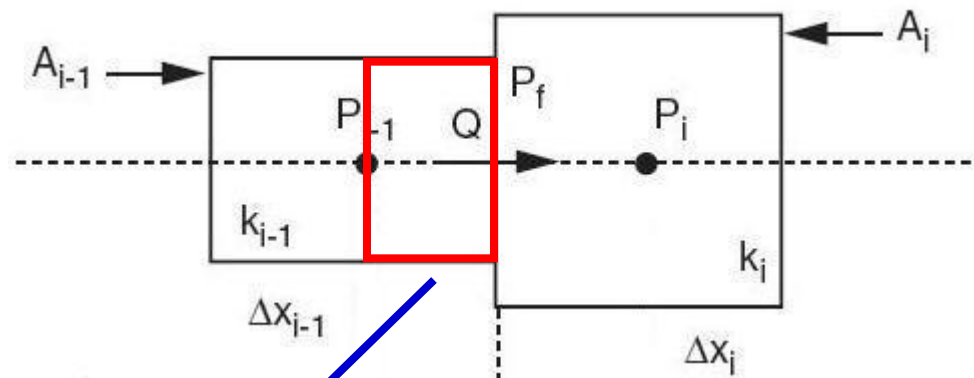




# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging k-A Product,  $kA$

block i-1



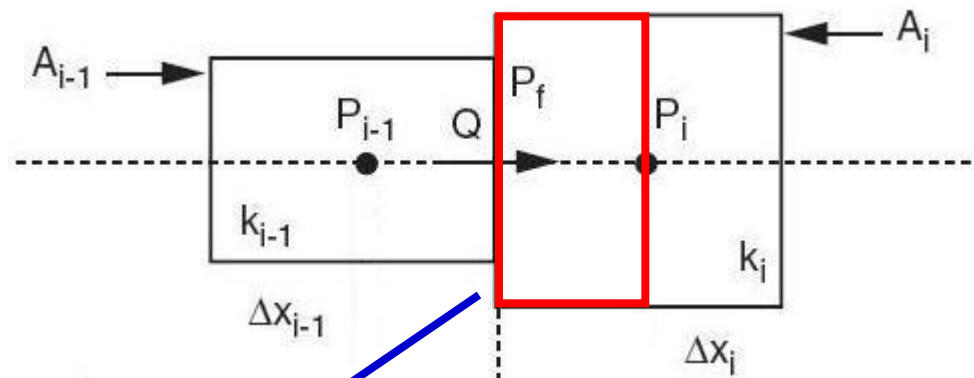
$$Q = -\frac{k_{i-1} \cdot A_{i-1}}{\mu} \cdot \left( \frac{P_f - P_{i-1}}{\frac{\Delta x_{i-1}}{2}} \right) \rightarrow (P_f - P_{i-1}) = -Q\mu \cdot \frac{\Delta x_{i-1}}{2} \cdot \left( \frac{1}{(k \cdot A)_{i-1}} \right)$$



# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging k-A Product,  $kA$

block i

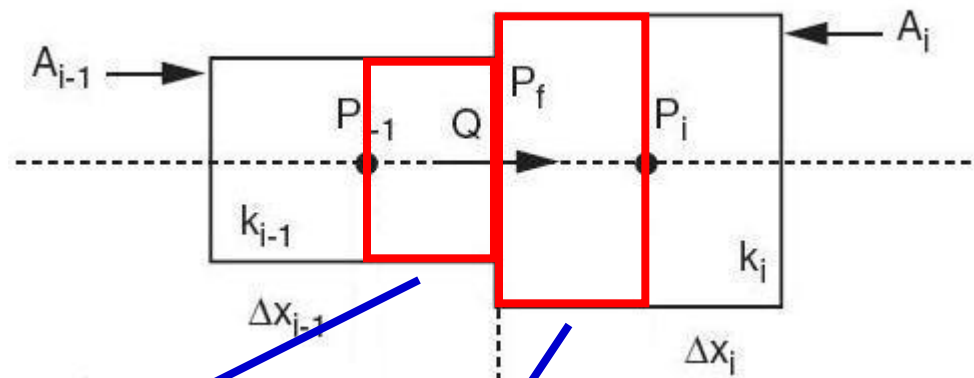


$$Q = -\frac{k_i \cdot A_i}{\mu} \cdot \left( \frac{P_i - P_f}{\frac{\Delta x_i}{2}} \right) \quad \rightarrow \quad (P_i - P_f) = -Q\mu \cdot \frac{\Delta x_i}{2} \cdot \left( \frac{1}{(k \cdot A)_i} \right)$$



# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging k-A Product,  $kA$



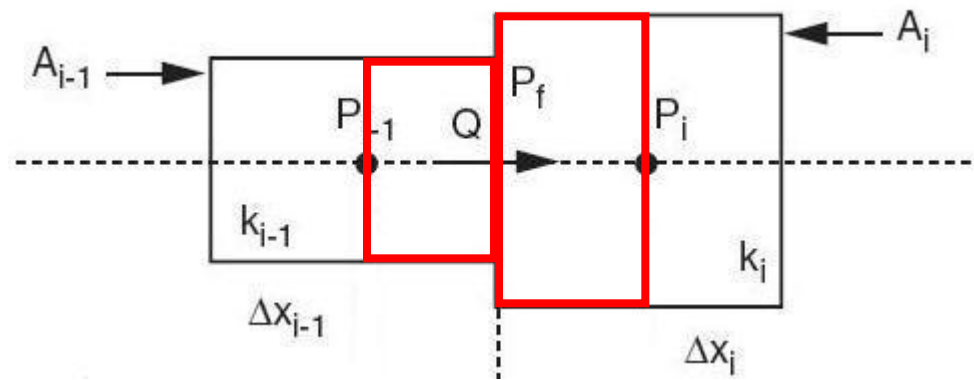
$$(P_f - P_{i-1}) = -Q\mu \cdot \frac{\Delta x_{i-1}}{2} \cdot \left( \frac{1}{(k \cdot A)_{i-1}} \right) + (P_i - P_f) = -Q\mu \cdot \frac{\Delta x_i}{2} \cdot \left( \frac{1}{(k \cdot A)_i} \right)$$

$$(P_i - P_{i-1}) = -\frac{Q\mu}{2} \cdot \left( \frac{\Delta x_{i-1}}{(k \cdot A)_{i-1}} + \frac{\Delta x_i}{(k \cdot A)_i} \right)$$



# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging k-A Product,  $kA$

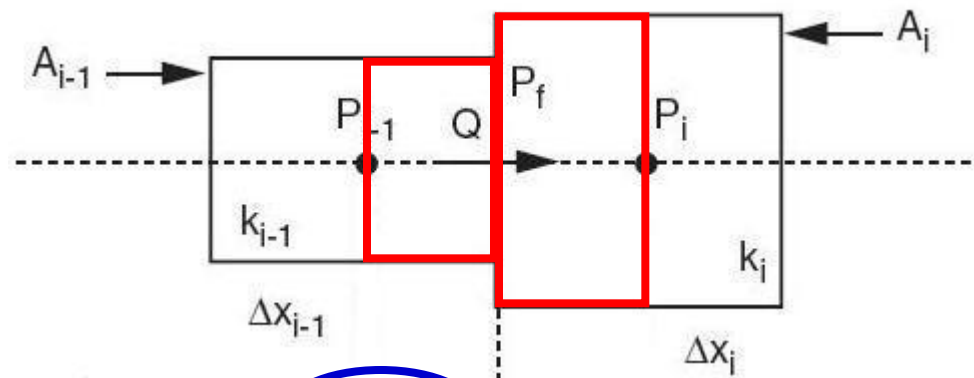


$$Q = -\frac{1}{\mu} \cdot \left( \frac{2(P_i - P_{i-1})}{\frac{\Delta x_{i-1}}{(k \cdot A)_{i-1}} + \frac{\Delta x_i}{(k \cdot A)_i}} \right) = -\frac{Q\mu}{(P_i - P_{i-1})} \cdot \left( \frac{\Delta x_{i-1}}{(k \cdot A)_{i-1}} + \frac{\Delta x_i}{(k \cdot A)_i} \right)$$



# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging k-A Product,  $\bar{kA}$



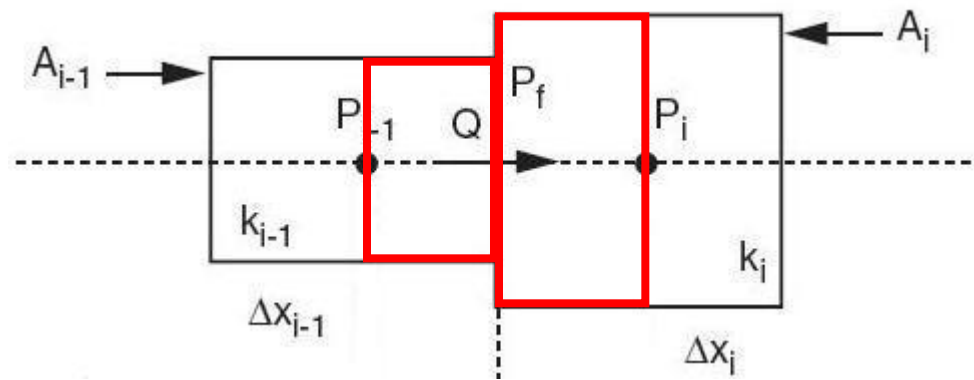
$$Q = - \frac{\bar{kA}}{\mu} \cdot \left( \frac{P_i - P_{i-1}}{\frac{\Delta x_{i-1} + \Delta x_i}{2}} \right) = \frac{1}{\mu} \cdot \left( \frac{\bar{kA}}{\frac{\Delta x_{i-1}}{(k \cdot A)_{i-1}} + \frac{\Delta x_i}{(k \cdot A)_i}} \right) = \left( \frac{2}{\frac{\Delta x_{i-1}}{(k \cdot A)_{i-1}} + \frac{\Delta x_i}{(k \cdot A)_i}} \right) \cdot \frac{\bar{kA}}{2 \cdot (P_i - P_{i-1})}$$





# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging k-A Product,  $kA$



$$\bar{kA} = \frac{\Delta x_{i-1} + \Delta x_i}{2} \left( \frac{k_{i-1} A_{i-1}}{\Delta x_{i-1}} + \frac{k_i A_i}{\Delta x_i} \right)$$

Harmonic average



# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging k-A Product,  $\overline{kA}$

Harmonic  
average

$$Q_p = -\overline{kA} \left( \frac{\overline{\lambda}_p}{B_p} \right) \cdot \left( \frac{P_i - P_{i-1}}{\left( \frac{\Delta x_{i-1} + \Delta x_i}{2} \right)} \right)$$



# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging Two-Phase Mobility,  $\lambda_p$

$$\lambda_p = \frac{k_{rp}}{\mu_p}$$

$$Q_p = -\bar{k}A \cdot \left( \frac{\lambda_p}{B_p} \right) = -\bar{k}A \cdot \left( \frac{P_i - P_{i-1}}{\frac{\Delta x_{i-1} + \Delta x_i}{2}} \right) \cdot \left( \frac{P_i - P_{i-1}}{\frac{\Delta x_{i-1} + \Delta x_i}{2}} \right)$$

$$\bar{\mu}_p = \frac{\mu_{i-1} + \mu_i}{2}$$

$$\bar{B}_p = \frac{B_{i-1} + B_i}{2}$$

Arithmetic average



# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging Two-Phase Mobility,  $\lambda_p$

Harmonic average

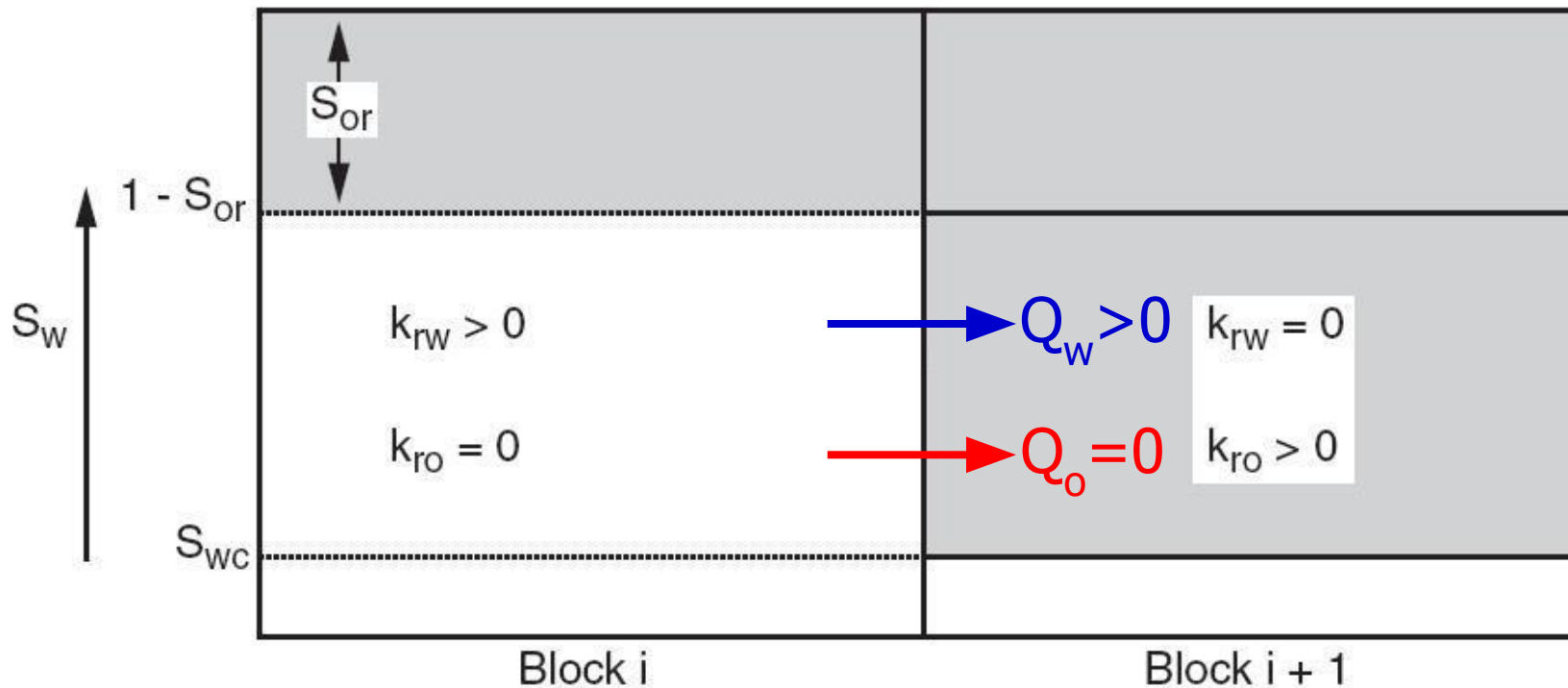
$$Q_p = -\overline{kA} \cdot \left( \frac{\overline{k_{rr}}}{\overline{\mu_p} \cdot \overline{B_r}} \right) \cdot \left( \frac{P_i - P_{i-1}}{\left( \frac{\Delta x_{i-1} + \Delta x_i}{2} \right)} \right)$$

Arithmetic average



# Calculation of Block to Block Flows in Reservoir Simulators

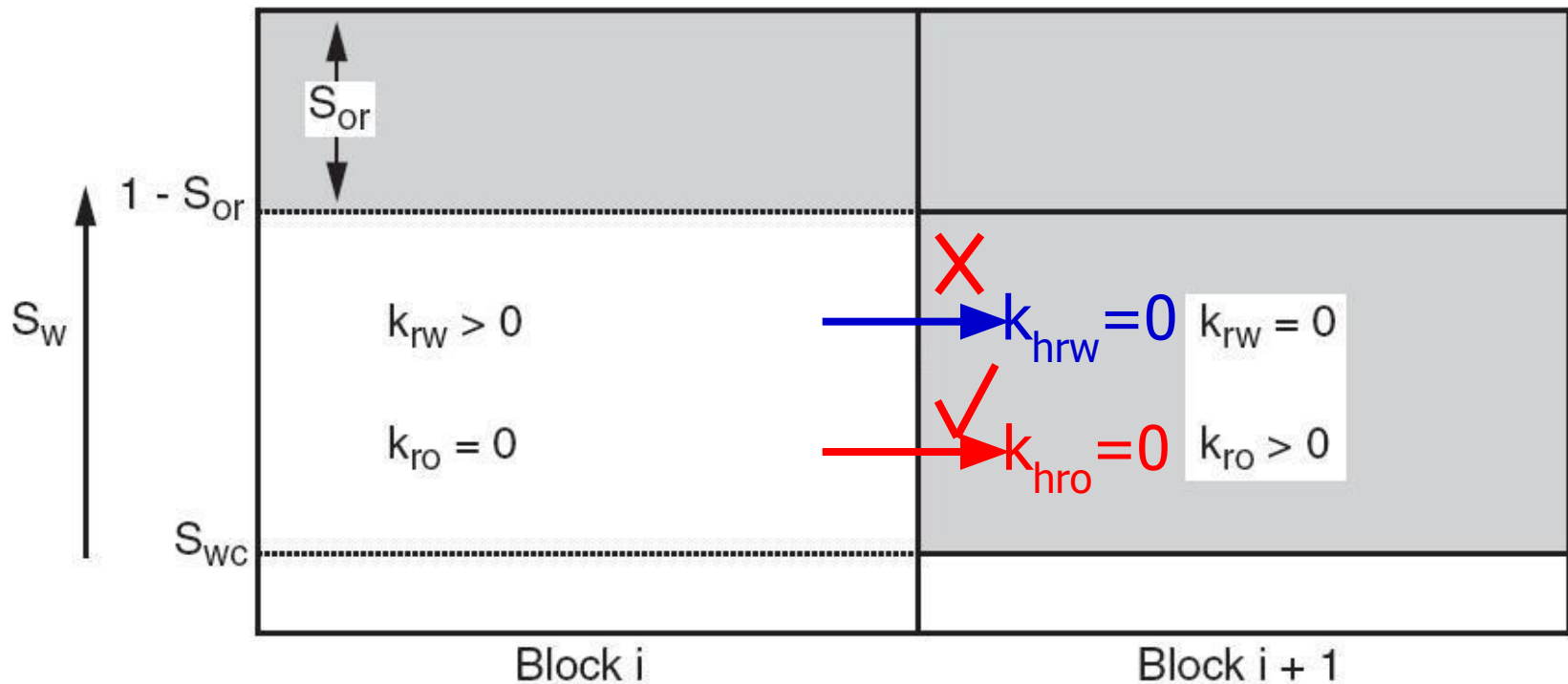
- Averaging Two-Phase Mobility,  $\lambda_p$ 
  - Physically





# Calculation of Block to Block Flows in Reservoir Simulators

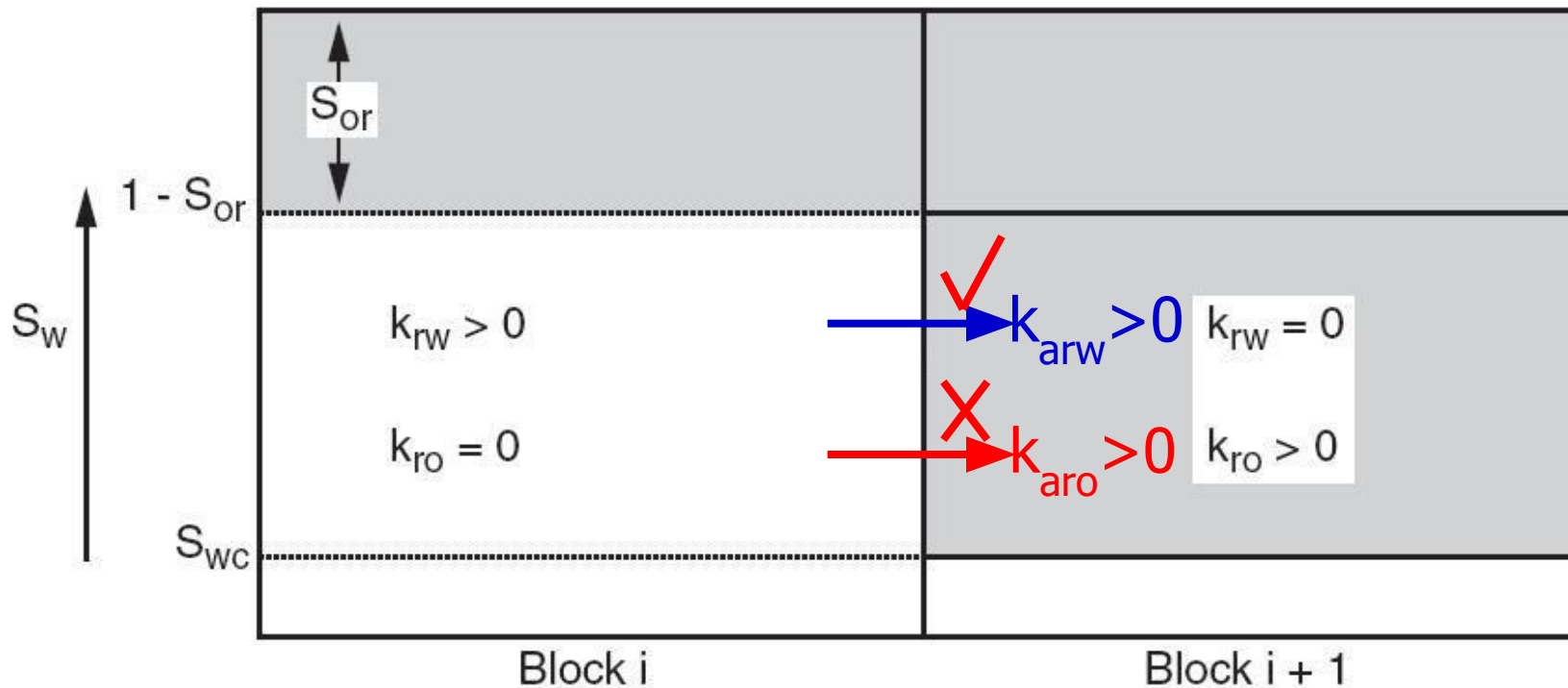
- Averaging Two-Phase Mobility,  $\lambda_p$
- ~~Harmonic average ( $k_h$ ) ?~~





# Calculation of Block to Block Flows in Reservoir Simulators

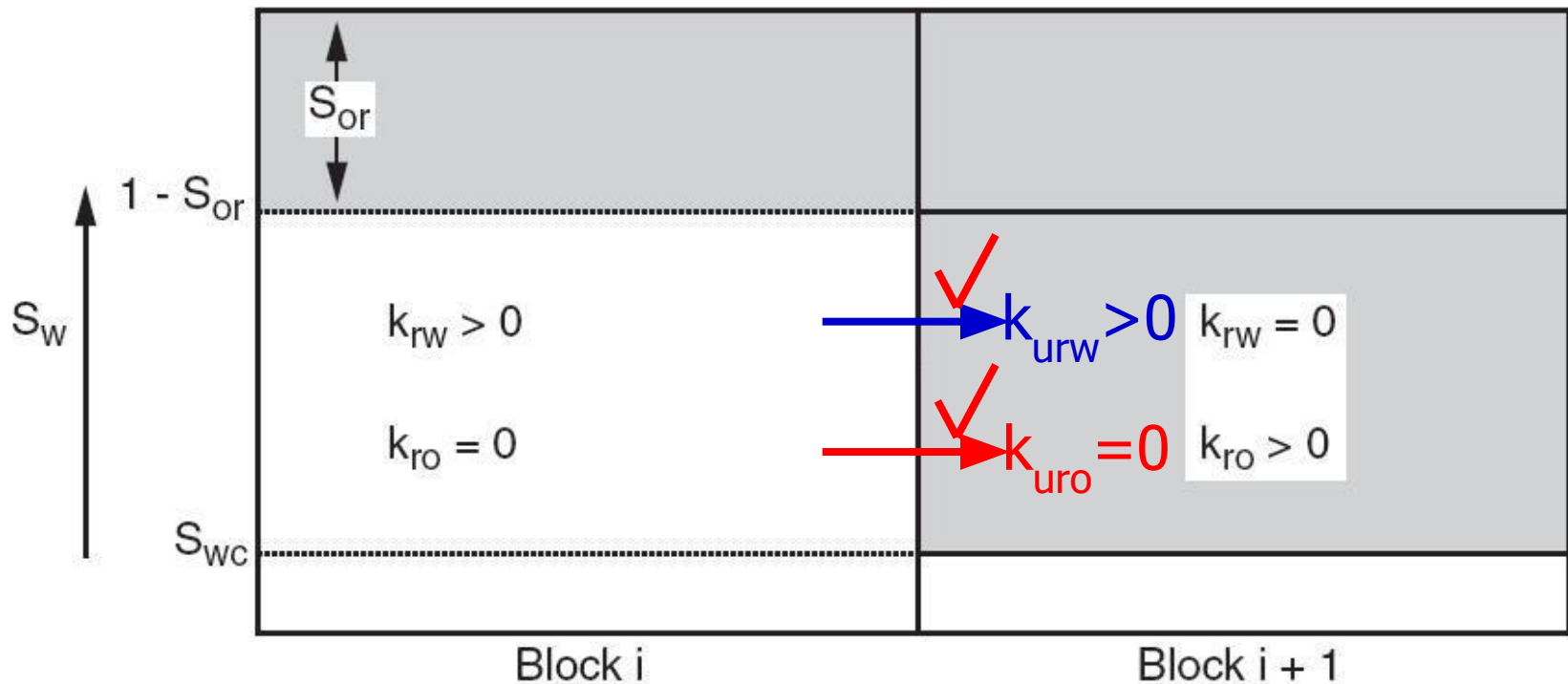
- Averaging Two-Phase Mobility,  $\lambda_p$   
~~Arithmetic average ( $k_a$ ) ?~~





# Calculation of Block to Block Flows in Reservoir Simulators

- Averaging Two-Phase Mobility,  $\lambda_p$ 
  - Upstream value ?

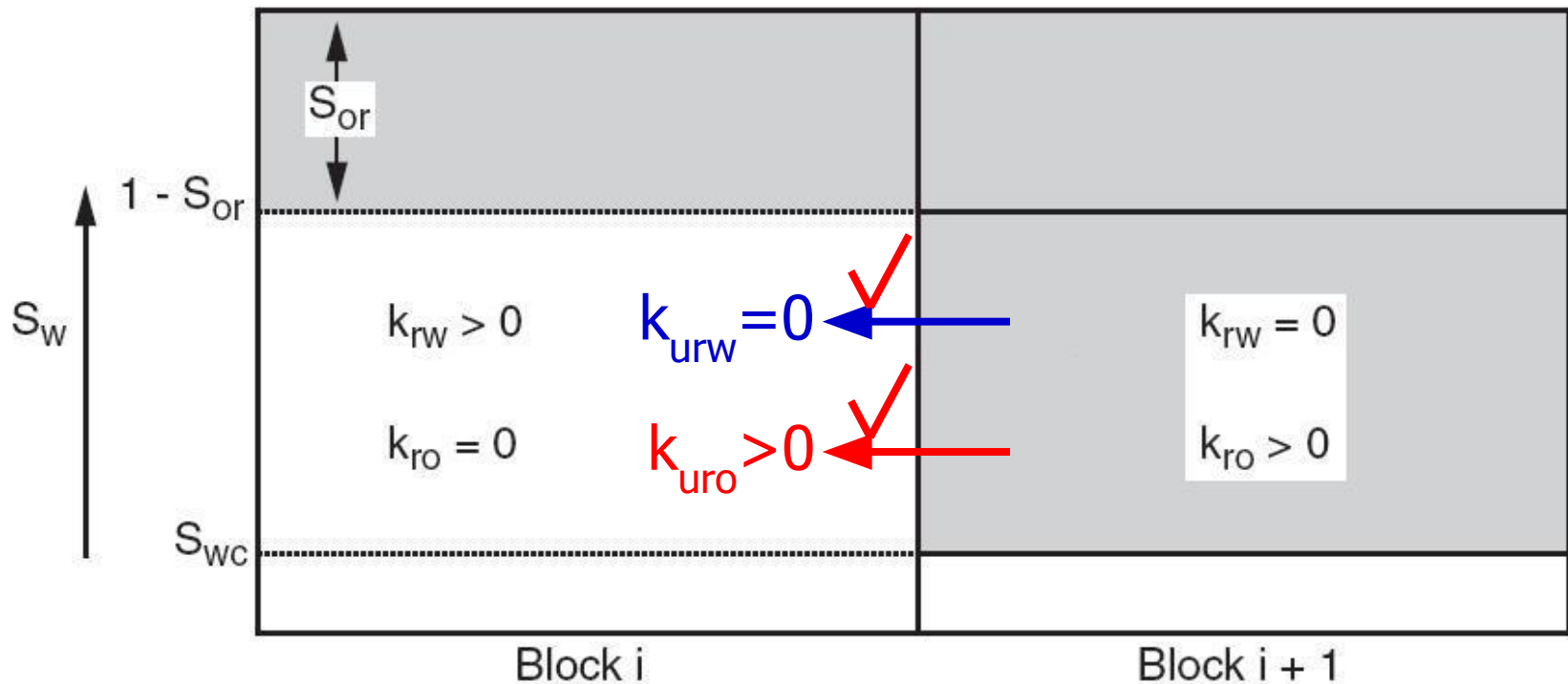






# Calculation of Block to Block Flows in Reservoir Simulators

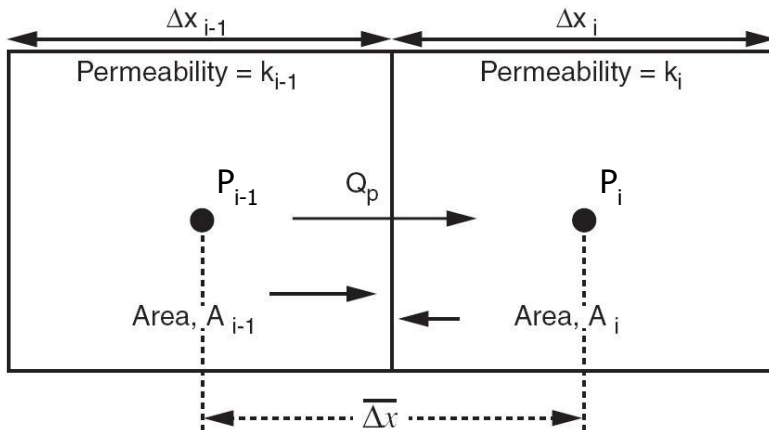
- Averaging Two-Phase Mobility,  $\lambda_p$ 
  - Upstream value ? ✓





# Calculation of Block to Block Flows in Reservoir Simulators

- Summary



Harmonic average

Upstream value

$$Q_p = -kA \left( \frac{k_{rr}}{\mu_r B_r} \right) \left( \frac{\Delta P}{\Delta x} \right)$$

Arithmetic average



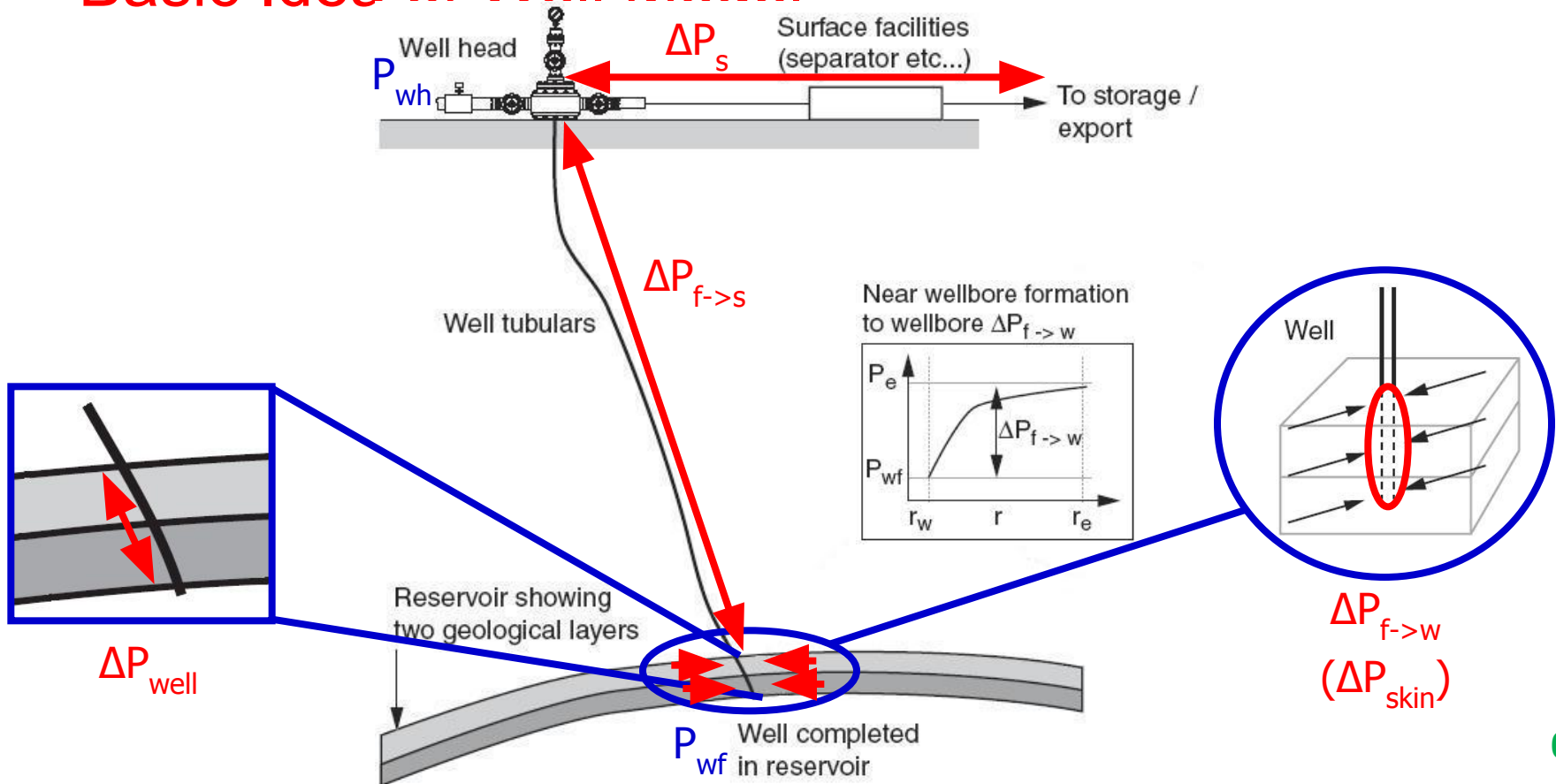
# Outline

- Gridding in Reservoir Simulation
- **Calculation of Block to Block Flows in Reservoir Simulators**
- Wells in Reservoir Simulation



# Wells in Reservoir Simulation

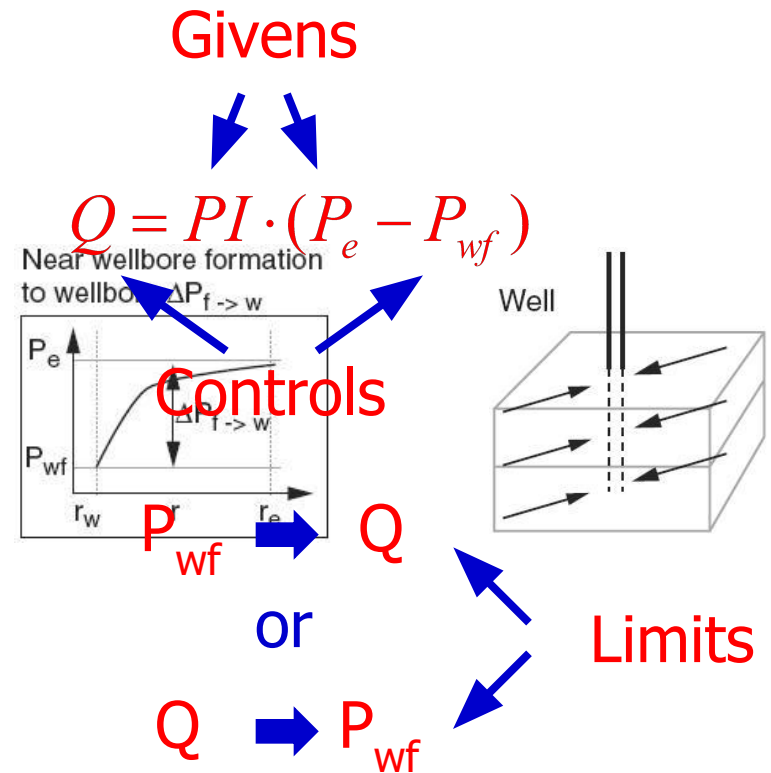
## • Basic Idea of Well Model





# Wells in Reservoir Simulation

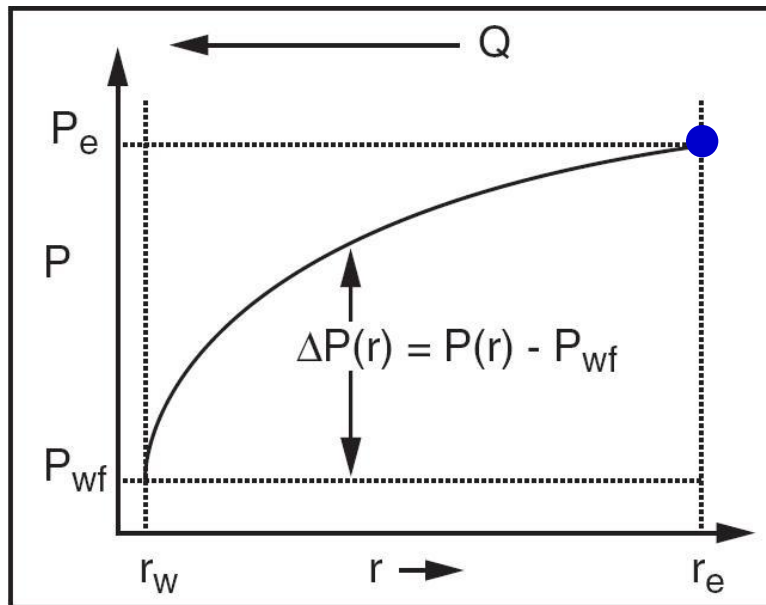
- Well Models for Single and Two Phase Flow





# Wells in Reservoir Simulation

- Well Models for Single and Two Phase Flow



$$\Delta P(r) = P_e - P_{wf} - \frac{Q\mu}{2\pi k h} \ln\left(\frac{r}{r_w}\right)$$

$P = P_e$  at  $r = r_e$ :

$$P_e - P_{wf} = \frac{Q\mu}{2\pi k h} \ln\left(\frac{r_e}{r_w}\right)$$

$$Q = \frac{2\pi k h}{\mu \cdot \ln\left(\frac{r_e}{r_w}\right)} \cdot (P_e - P_{wf})$$

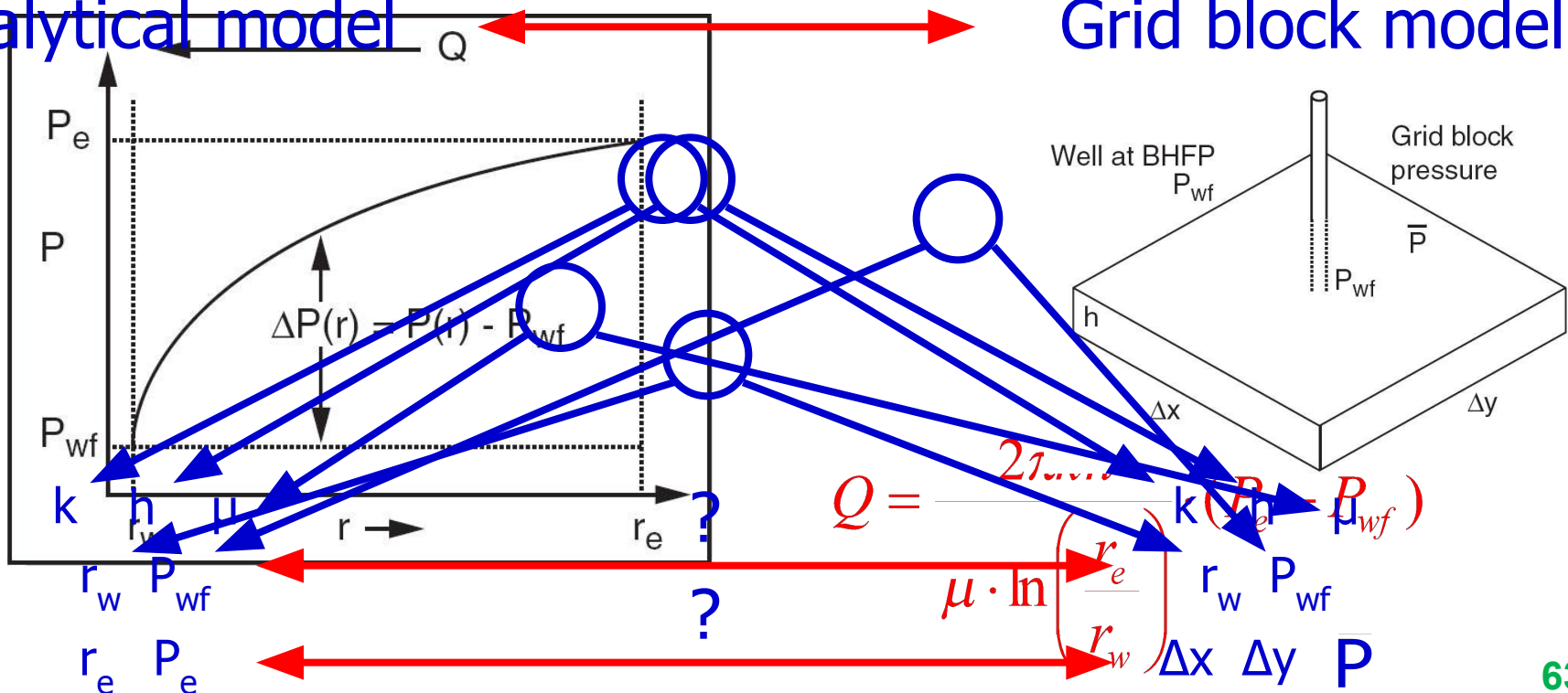


# Wells in Reservoir Simulation

- Well Models for Single and Two Phase Flow

Analytical model

Grid block model





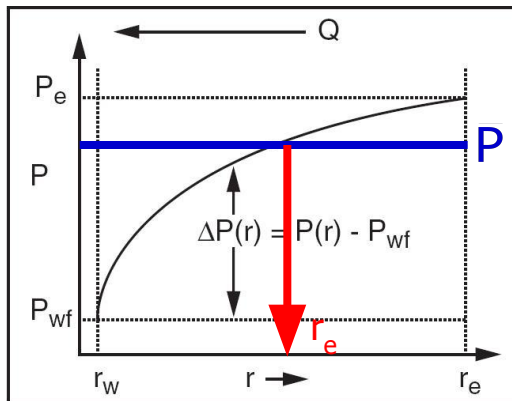
# Wells in Reservoir Simulation

- Well Models for Single and Two Phase Flow

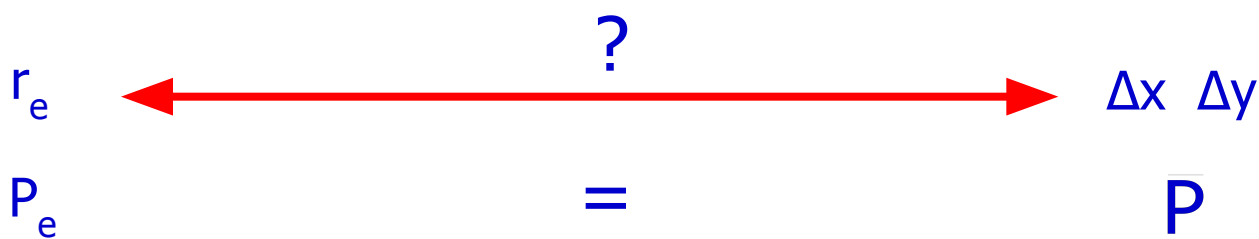
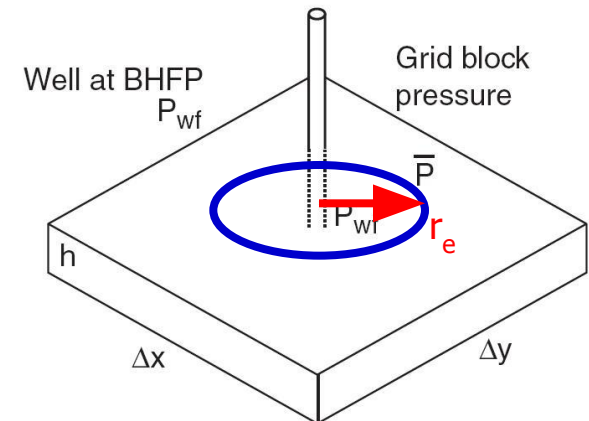
Analytical model



Grid block model



$$Q = \frac{2\pi kh}{\mu \cdot \ln\left(\frac{r_e}{r_w}\right)} \cdot (P_e - P_{wf})$$



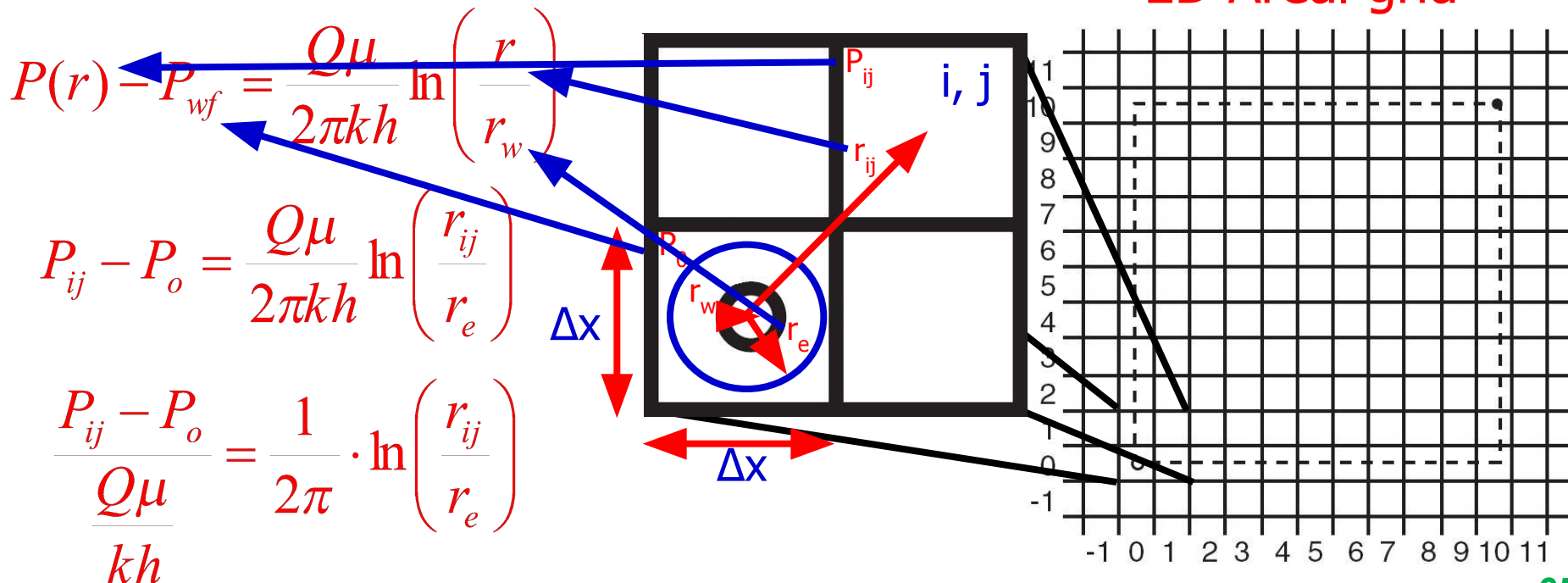




# Wells in Reservoir Simulation

- Well Models for Single and Two Phase Flow
  - Peaceman Equivalent radius,  $r_e$

2D Areal grid





# Wells in Reservoir Simulation

- Well Models for Single and Two Phase Flow
  - Peaceman Equivalent radius,  $r_e$

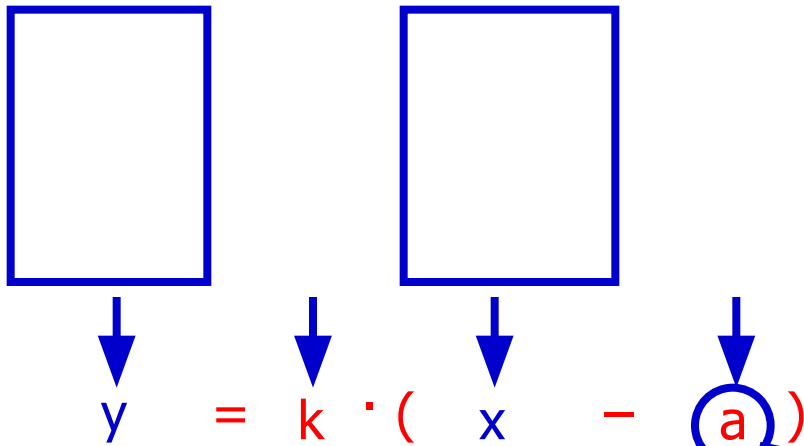
$$\ln\left(\frac{r_{ij}}{r_e}\right) = \ln\left(\frac{r_{ij}}{\Delta x} \cdot \frac{\Delta x}{r_e}\right) = \ln\left(\frac{r_{ij}}{\Delta x}\right) - \ln\left(\frac{r_e}{\Delta x}\right)$$

$$\frac{P_{ij} - P_o}{\frac{Q\mu}{kh}} = \frac{1}{2\pi} \cdot \ln\left(\frac{P_{ij} - P_o}{\frac{r_{ij} Q\mu}{kh}}\right) = \frac{1}{2\pi} \cdot \left[ \ln\left(\frac{r_{ij}}{\Delta x}\right) - \ln\left(\frac{r_e}{\Delta x}\right) \right]$$

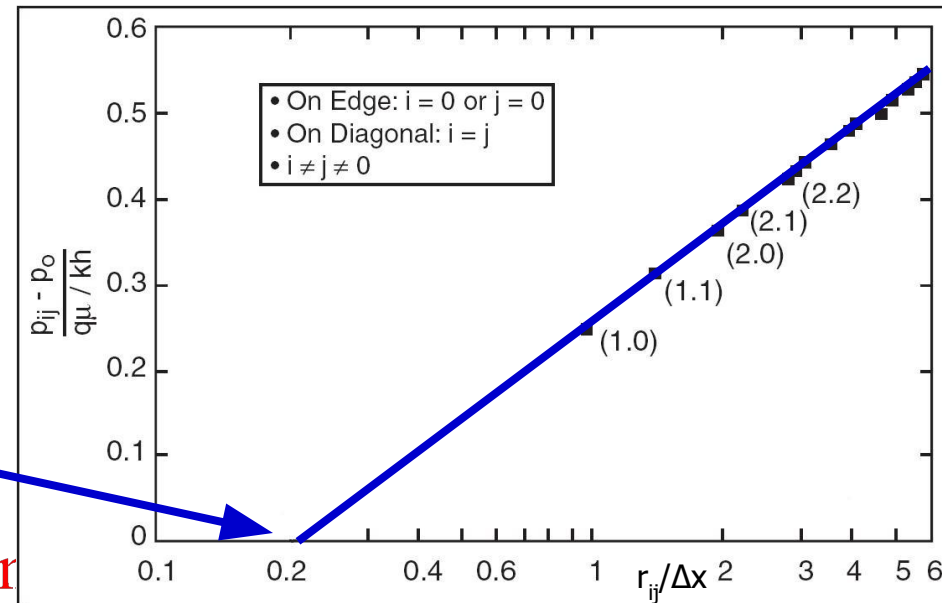


# Wells in Reservoir Simulation

- Well Models for Single and Two Phase Flow
  - Peaceman Equivalent radius,  $r_e$



$$\frac{r_e}{\Delta x} \approx 0.2 \rightarrow \frac{P_{ij} - P_e}{kh} = \frac{1}{2\pi} \left[ \ln \left( \frac{r_{ij}}{\Delta x} \right) + \ln \left( \frac{r_{ij}}{\Delta x} \right) \right]$$





# Wells in Reservoir Simulation

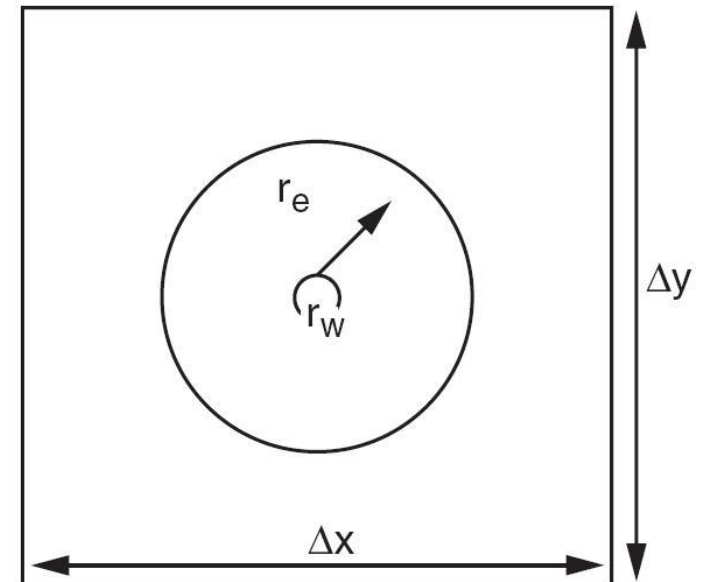
- Well Models for Single and Two Phase Flow
  - Peaceman Equivalent radius,  $r_e$

$$(\Delta x = \Delta y)$$

$$r_e \approx 0.14 \sqrt{\Delta x^2 + \Delta y^2}$$

$$(\Delta x \neq \Delta y)$$

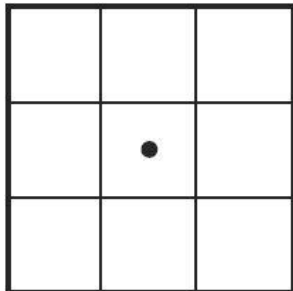
$$r_e \approx 0.2 \Delta x$$



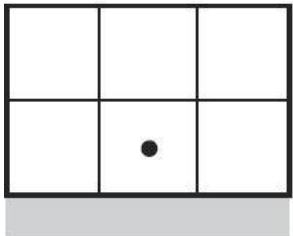


# Wells in Reservoir Simulation

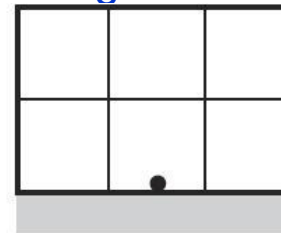
- Well Models for Single and Two Phase Flow
  - Peaceman Equivalent radius,  $r_e$



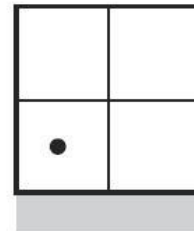
$$r_e \approx 0.2\Delta x$$



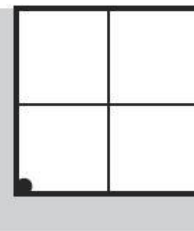
$$r_e \approx 0.196\Delta x$$



$$r_e \approx 0.433\Delta x$$



$$r_e \approx 0.193\Delta x$$



$$r_e \approx 0.72\Delta x$$



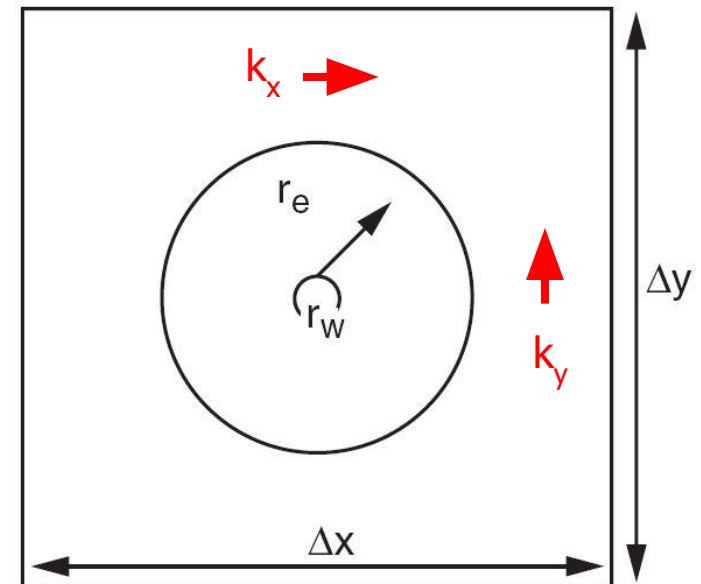
# Wells in Reservoir Simulation

- Well Models for Single and Two Phase Flow
  - Peaceman Equivalent radius,  $r_e$

$(\Delta x \neq \Delta y)$

$(k_x \neq k_y)$

$$r_e \approx 0.28 \frac{\left[ \left( \frac{k_y}{k_x} \right) \cdot \Delta x^2 + \left( \frac{k_x}{k_y} \right) \cdot \Delta y^2 \right]^{\frac{1}{2}}}{\left( \frac{k_y}{k_x} \right)^{\frac{1}{4}} + \left( \frac{k_x}{k_y} \right)^{\frac{1}{4}}}$$





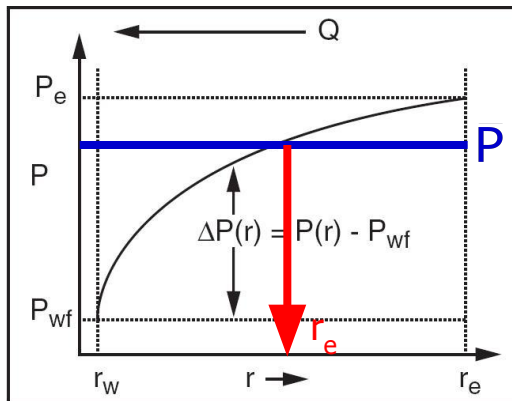
# Wells in Reservoir Simulation

- Well Models for Single and Two Phase Flow

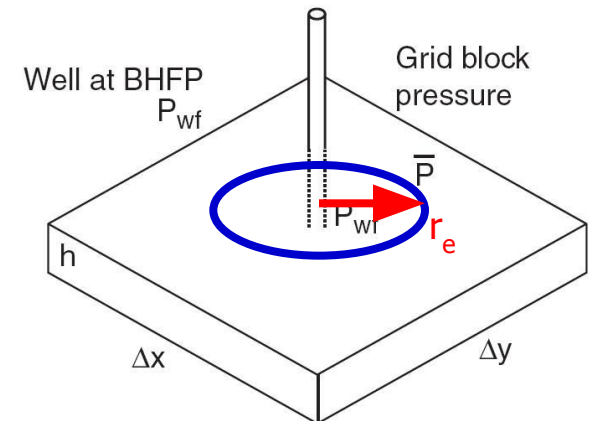
Analytical model



Grid block model



$$Q = \frac{2\pi kh}{\mu \cdot \ln\left(\frac{r_e}{r_w}\right)} \cdot (P_e - P_{wf})$$



$$r_e \quad \leftarrow \quad \stackrel{?}{=} \quad 0.14 \sqrt{\Delta x^2 + \Delta y^2} \quad \rightarrow \quad \Delta x \quad \Delta y$$

$$P_e = P$$



# Wells in Reservoir Simulation

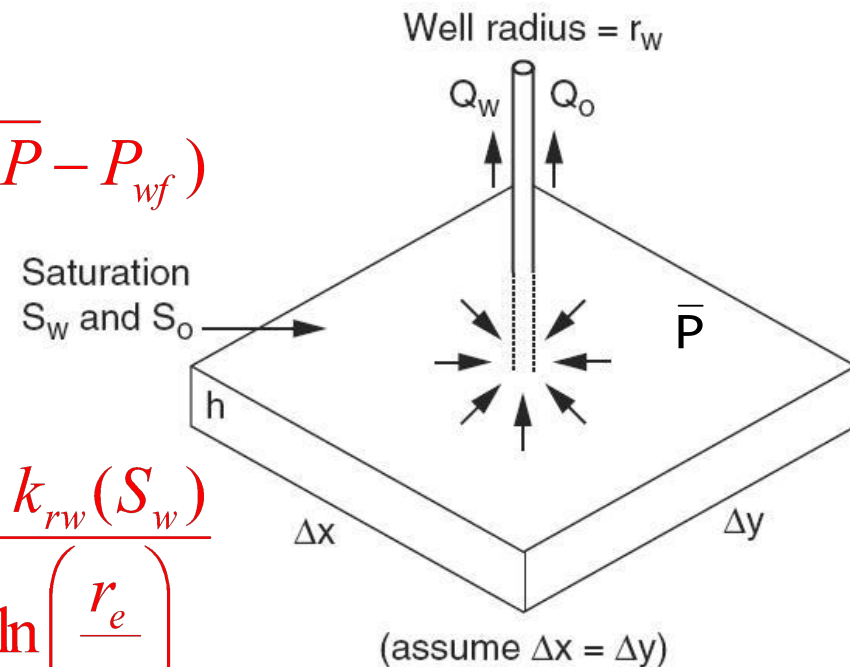
- Well Models for Single and Two Phase Flow
  - Two phase flow

$$Q_o = PI_o \cdot (\bar{P} - P_{wf})$$

$$Q_w = PI_w \cdot (\bar{P} - P_{wf})$$

$$PI_o = \frac{2\pi kh \cdot k_{ro}(S_o)}{\mu_o \cdot \ln\left(\frac{r_e}{r_w}\right)}$$

$$PI_w = \frac{2\pi kh \cdot k_{rw}(S_w)}{\mu_w \cdot \ln\left(\frac{r_e}{r_w}\right)}$$

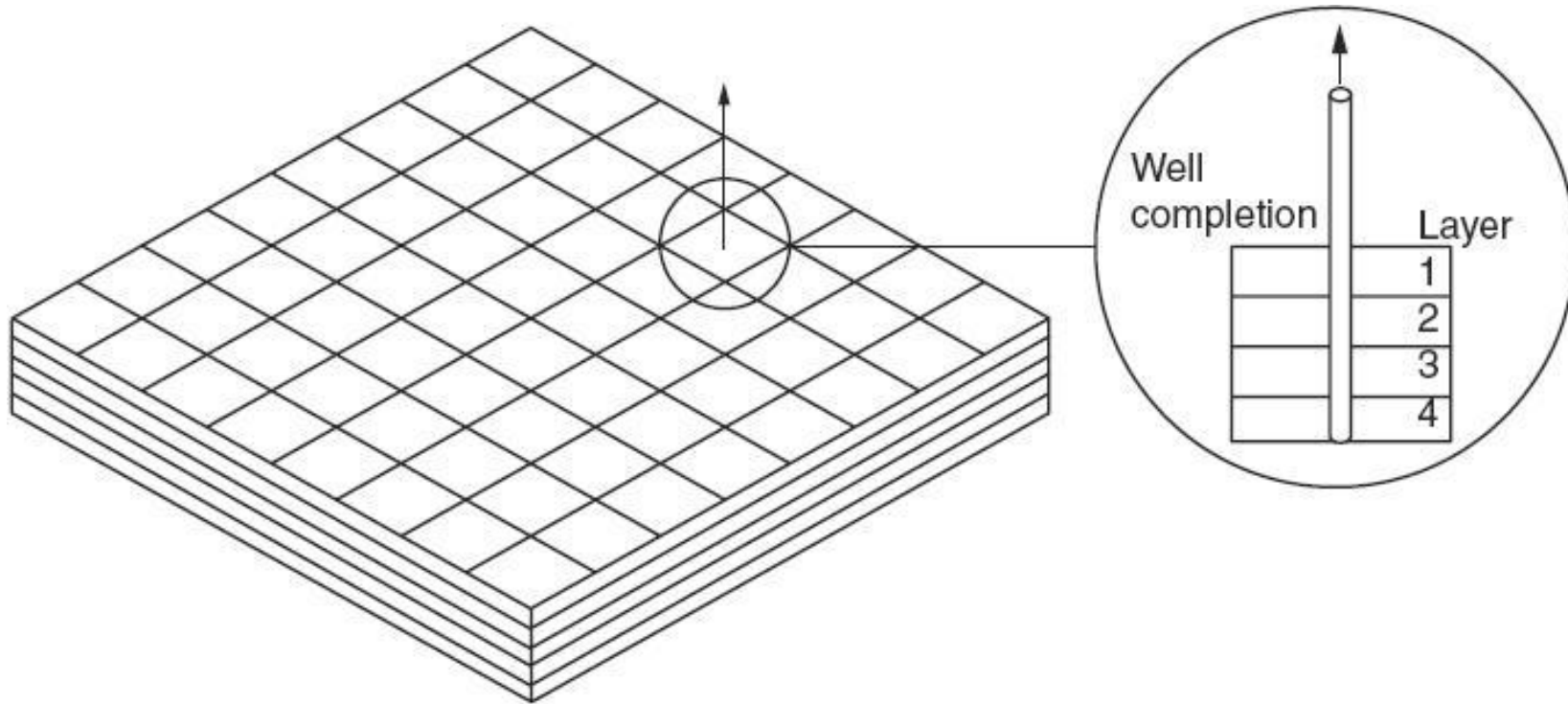






# Wells in Reservoir Simulation

- Well Modelling in a Multi-Layer System





# Wells in Reservoir Simulation

## Well Modelling in a Multi-Layer System

For each layer (k=1..4)

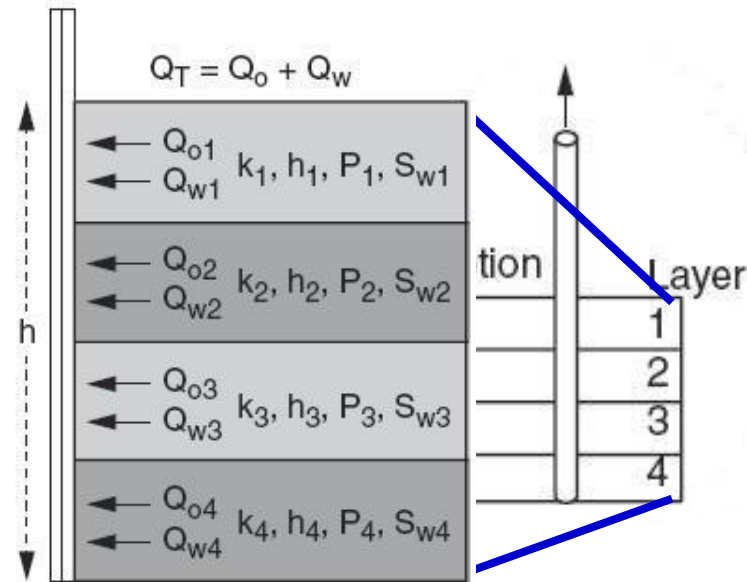
$$Q_{ok} = PI_{ok} \cdot (\bar{P}_k - P_{wfk})$$

+

$$Q_{wk} = PI_{wk} \cdot (\bar{P}_k - P_{wfk})$$



$$Q_{Tk} = \frac{PI_{ok} \cdot 2\pi(kh \cdot k_o) (\bar{P}_k - P_{wfk}) + PI_{wk} \cdot 2\pi(kh \cdot k_w) (\bar{P}_k - P_{wfk})}{\mu_o \cdot \ln\left(\frac{r_{ek}}{r_w}\right)} = \frac{2\pi(kh \cdot k_{rw} (S_w))_k}{\mu_w \cdot \ln\left(\frac{r_{ek}}{r_w}\right)}$$





# Wells in Reservoir Simulation

- Well Modelling in a Multi-Layer System

For each layer (k=1..4)

$$Q_{ok} = PI_{ok} \cdot (\bar{P}_k - P_{wfk})$$

$$Q_{wk} = PI_{wk} \cdot (\bar{P}_k - P_{wfk})$$

$$Q_{Tk} = (PI_{ok} + PI_{wk}) \cdot (\bar{P}_k - P_{wfk})$$

For simplicity

Total

$$Q_o = \sum_{k=1}^4 PI_{ok} \cdot (\bar{P}_k - P_{wfk})$$

$$Q_w = \sum_{k=1}^4 PI_{wk} \cdot (\bar{P}_k - P_{wfk})$$

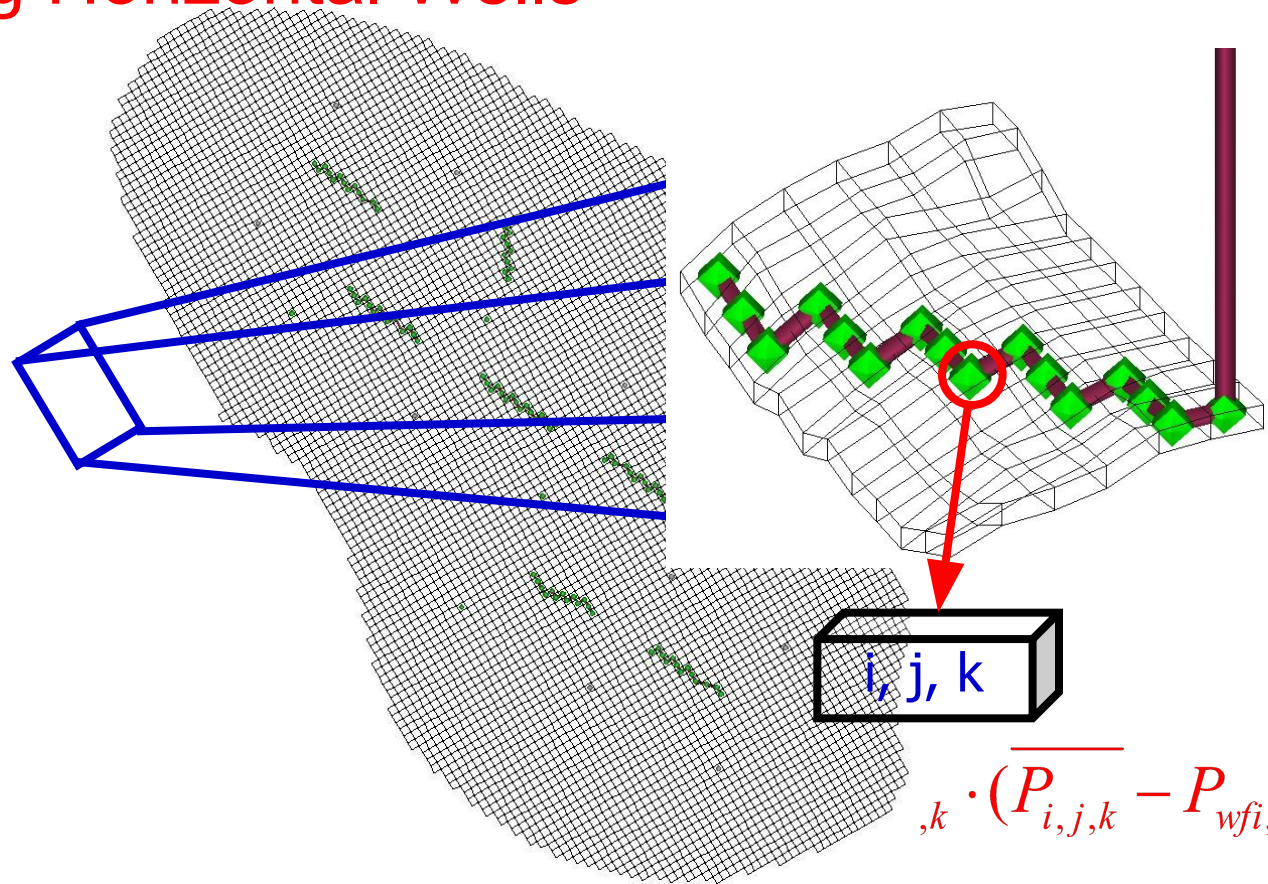
$$Q_T = \sum_{k=1}^4 (PI_{ok} + PI_{wk}) \cdot (\bar{P}_k - P_{wfk})$$

$$P_{wf1} = P_{wf2} = P_{wf3} = P_{wf4}$$



# Wells in Reservoir Simulation

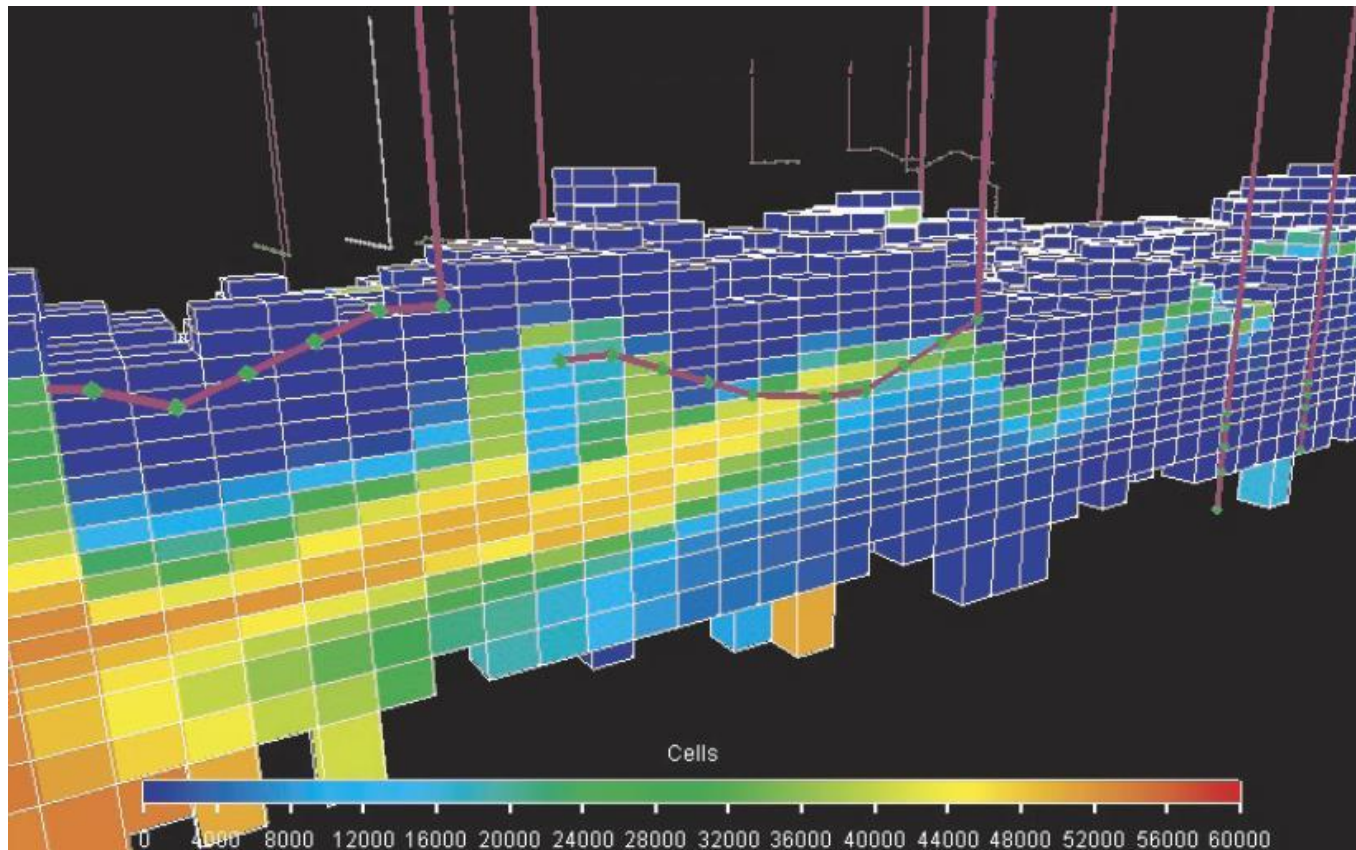
- Modelling Horizontal Wells





# Wells in Reservoir Simulation

- Modelling Horizontal Wells

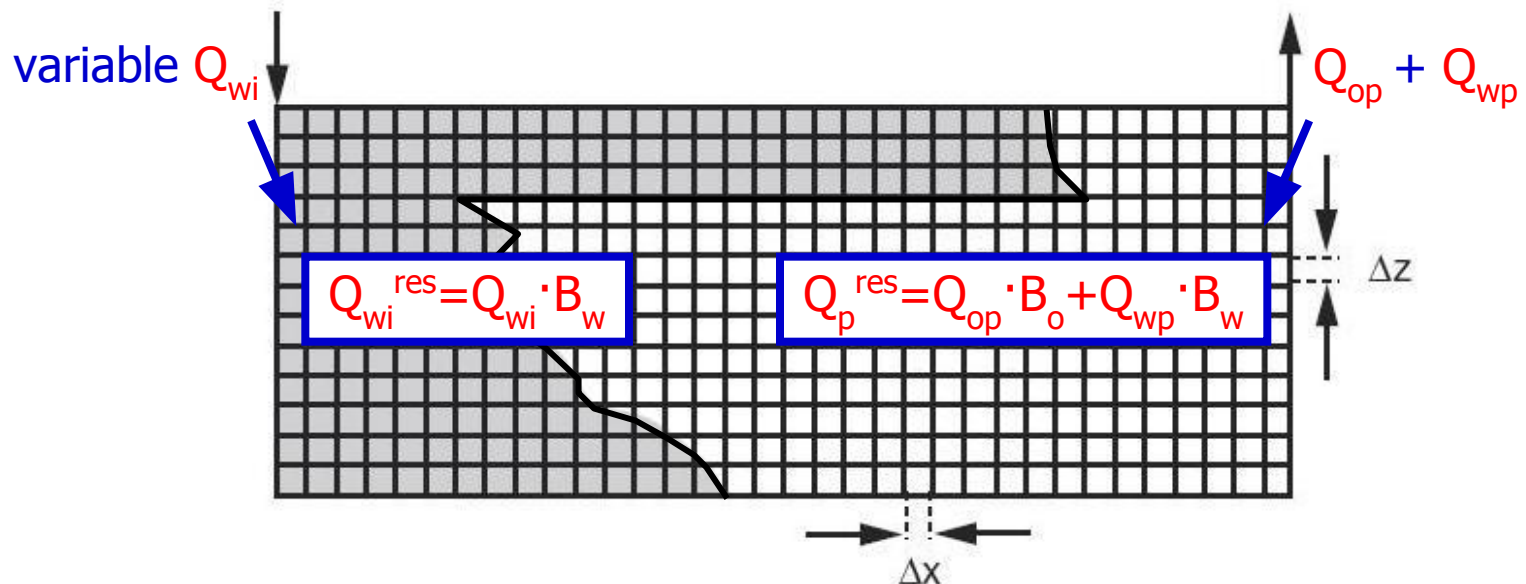






# Wells in Reservoir Simulation

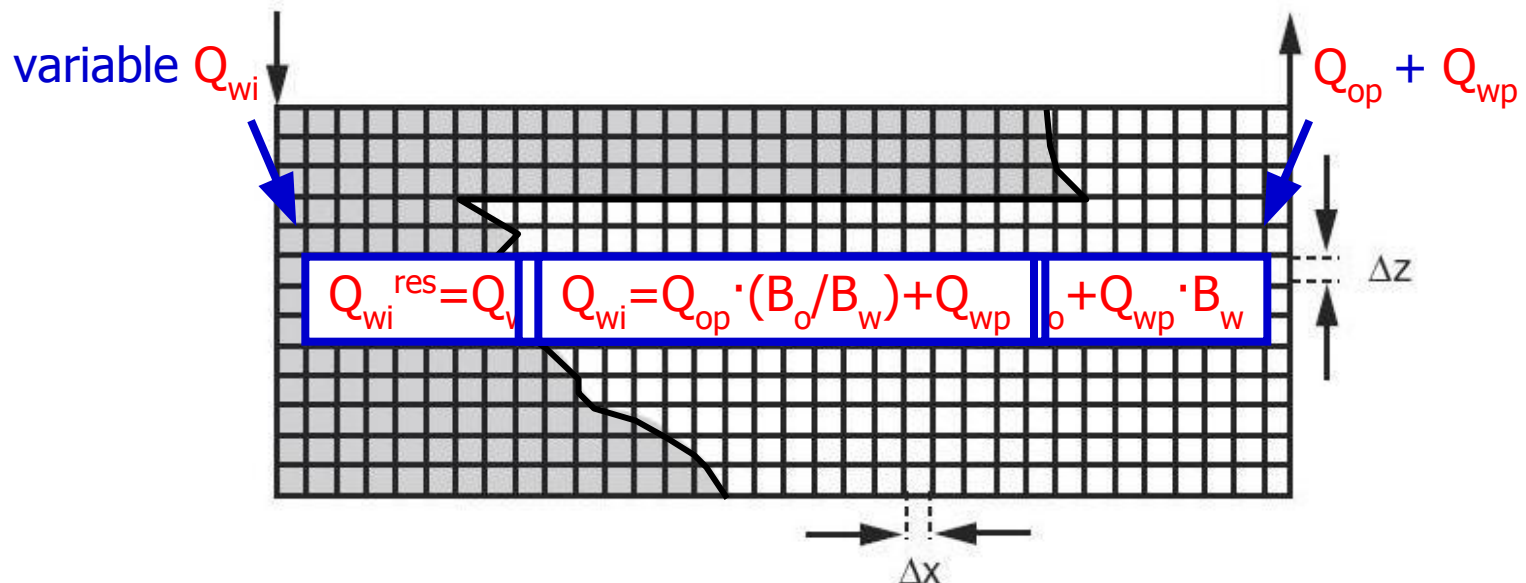
- Hierarchies of Wells and Well Controls
  - Well Controls
    - Rate or Pressure constrained production
    - Voidage replacement (injection)





# Wells in Reservoir Simulation

- Hierarchies of Wells and Well Controls
  - Well Controls
    - Rate or Pressure constrained production
    - Voidage replacement (injection)







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# Thank you for attention!

Reservoir Simulation  
**Gridding and Well Modelling**

Sergey Kurelenkov

2011