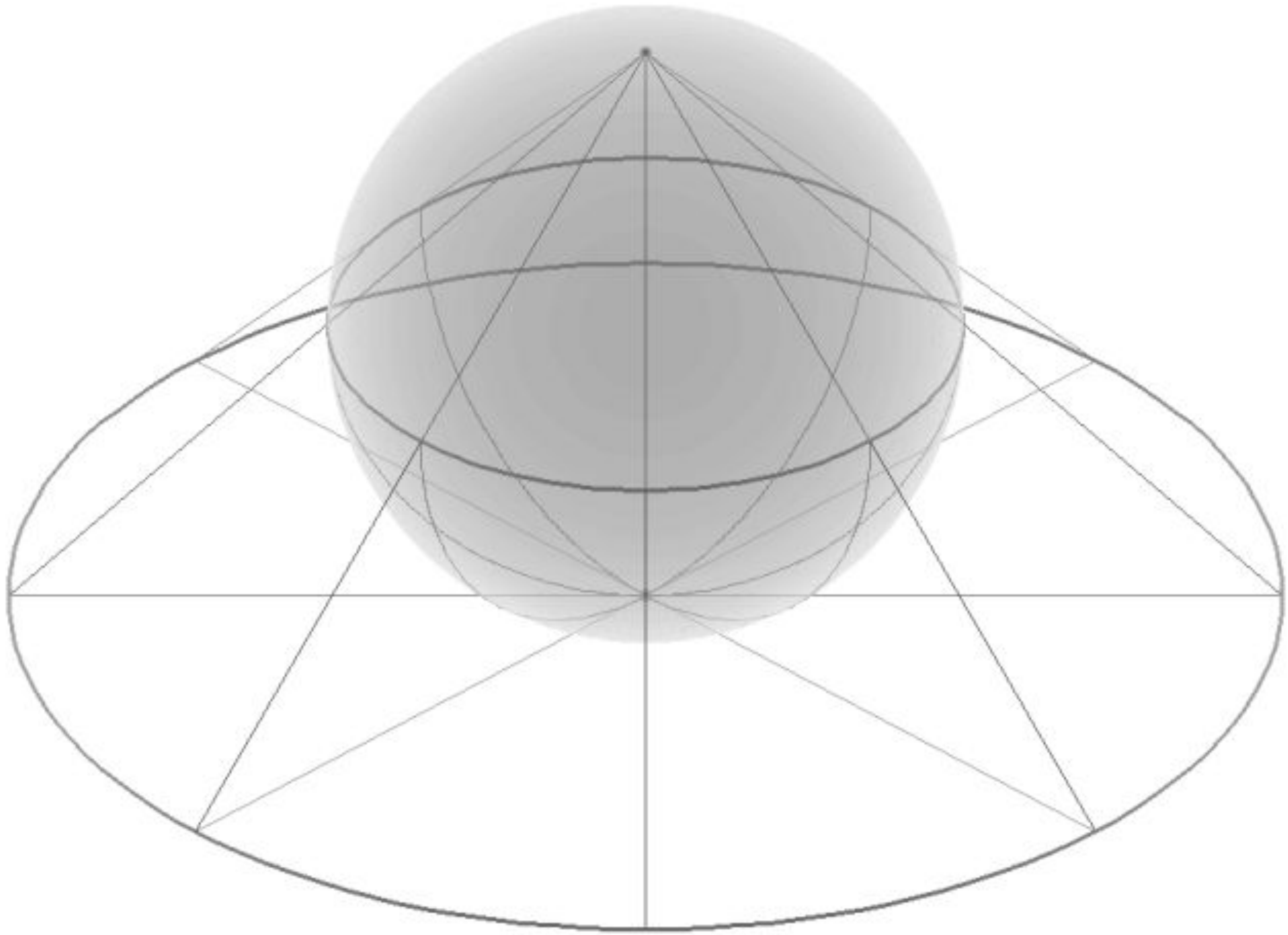
Cyclographic and stereographic projection



What is important in the representation of a crystal ?

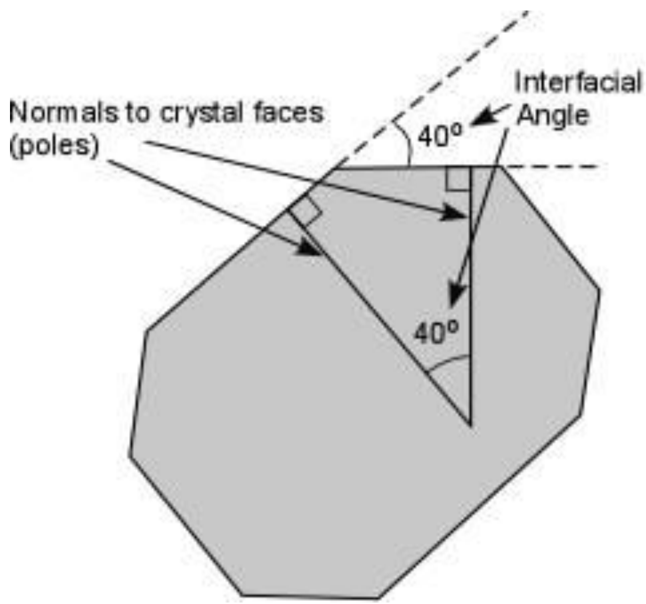
What is important in the representation of a crystal ?

It is not its shape

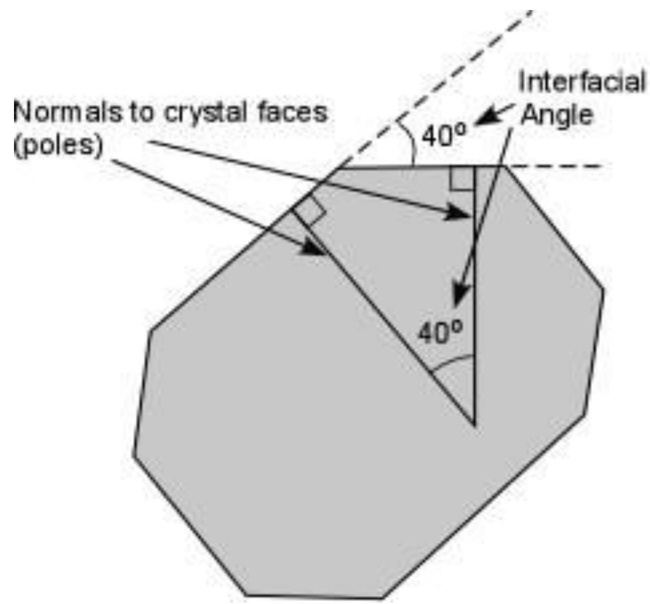
It is not its extension

What is important in the representation of a crystal ?

It is its orientation,
especially with respect to the other faces



What is important in the representation of a crystal ?



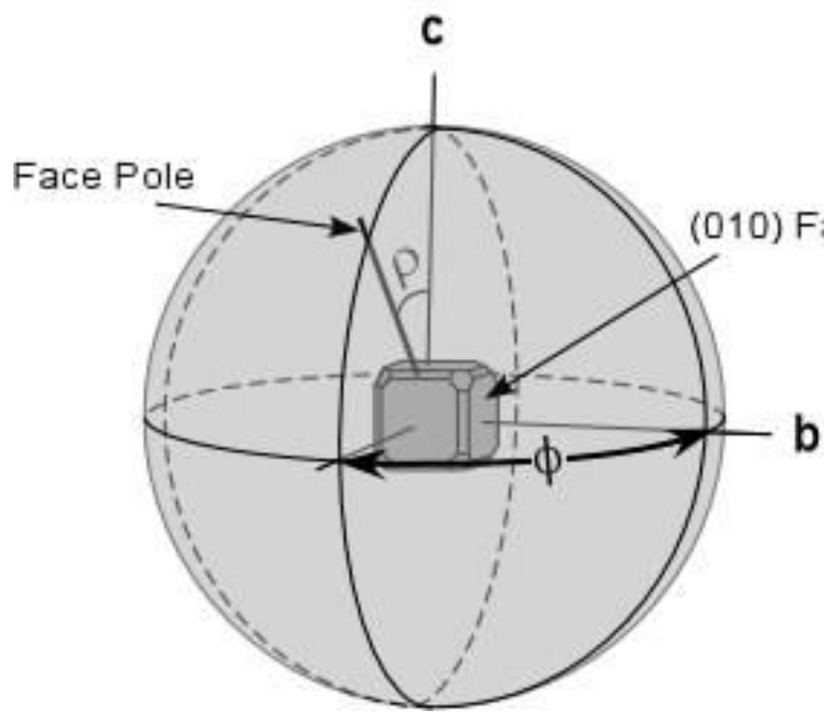
The interfacial angle is measured with the goniometer. It is also available by geometric construction when considering the normals of the two faces.
A **line perpendicular to a face** is called a **pole of the crystal face**. It is named Φ.

How to flatten the Earth ?

Illustration by **Rubens** for
"Opticorum libri sex philosophis juxta ac mathematicis utiles",
by **François d'Aiguillon**.
He demonstrated how the **projection** is computed.

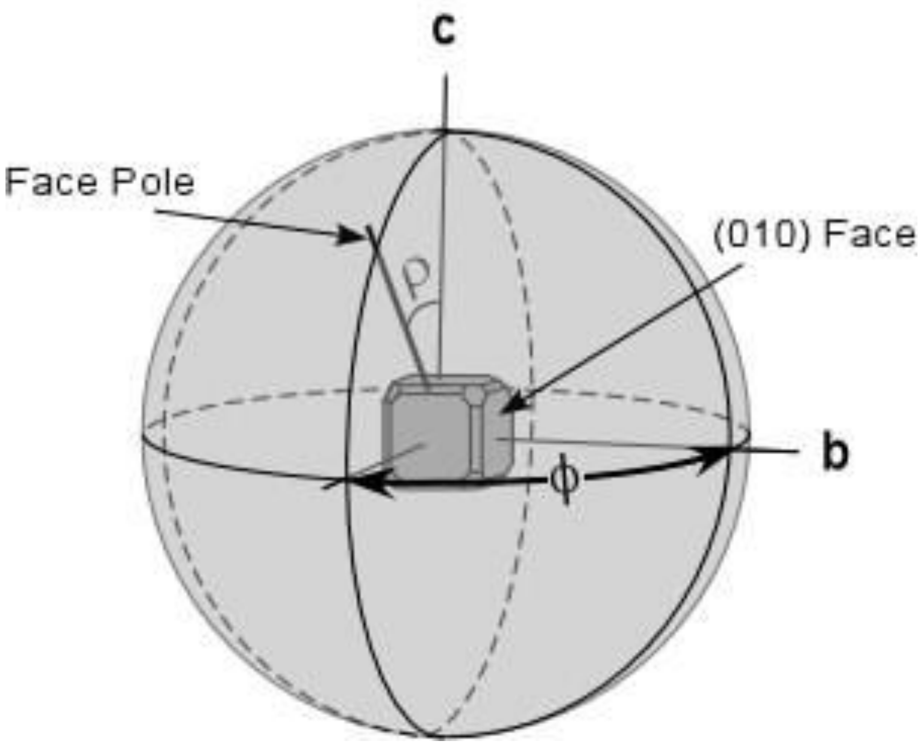
How to proceed ?

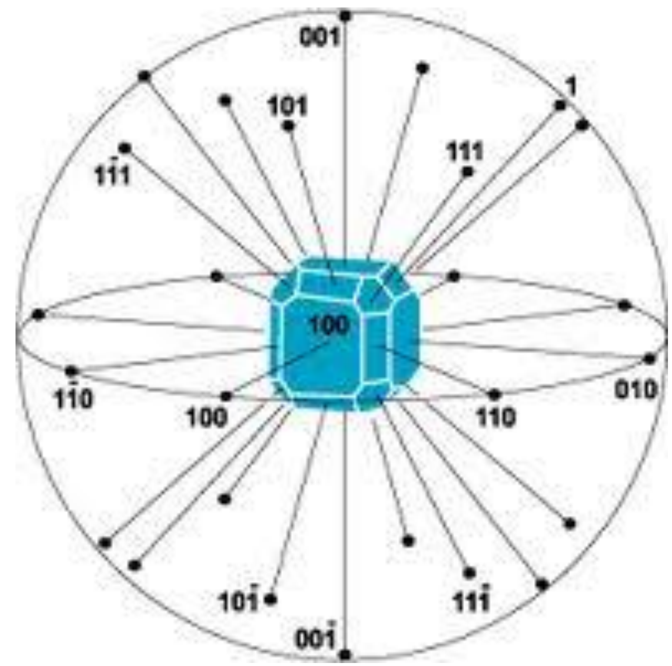
1) The crystal is imagined at the **center of a sphere**
2) **Lines perpendicular to the faces** are drawn.



Each face is represented by the **intersection** between the **sphere** and the **normal to the face**. It's the **polar projection.**

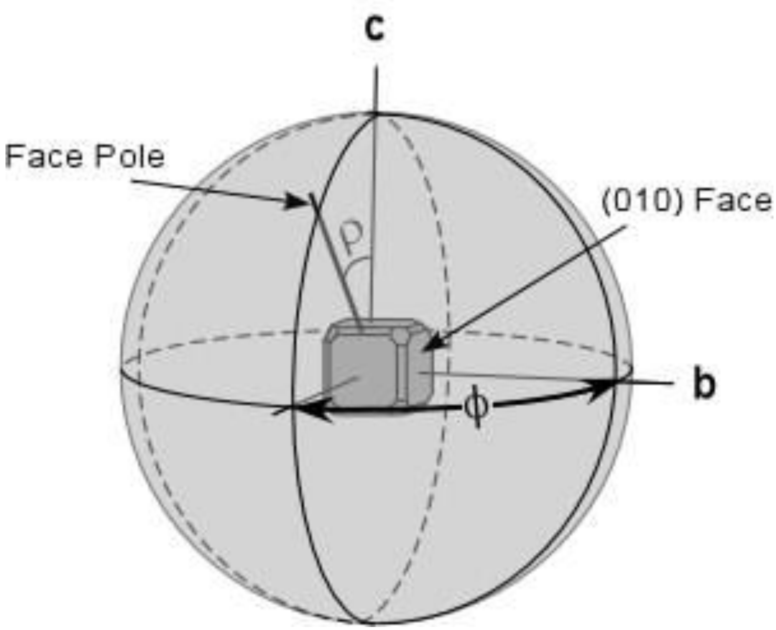
Cyclographic projection





The **angle between the normals** of two faces are found on the **sphere**

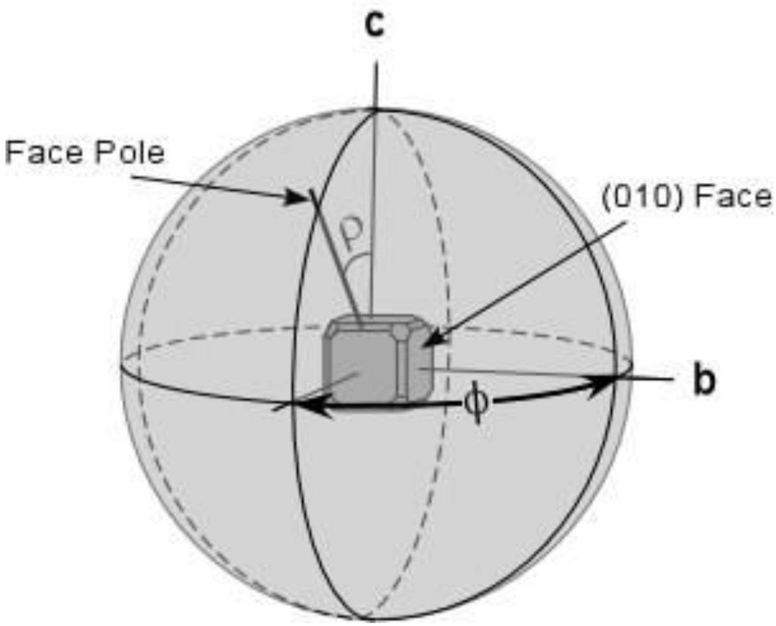
Systematic way to define crystallographic angles



1) The pole of an **hypothetical face** (010) is put **parallel to the b** axis. It impinges inside the sphere at the equator. The Φ angle is equal 0°.
2) For any other face, the angle will be different of 0° (except for // faces.

It will be **measured from the b axis in a clockwise sense in the plane of the equator.**

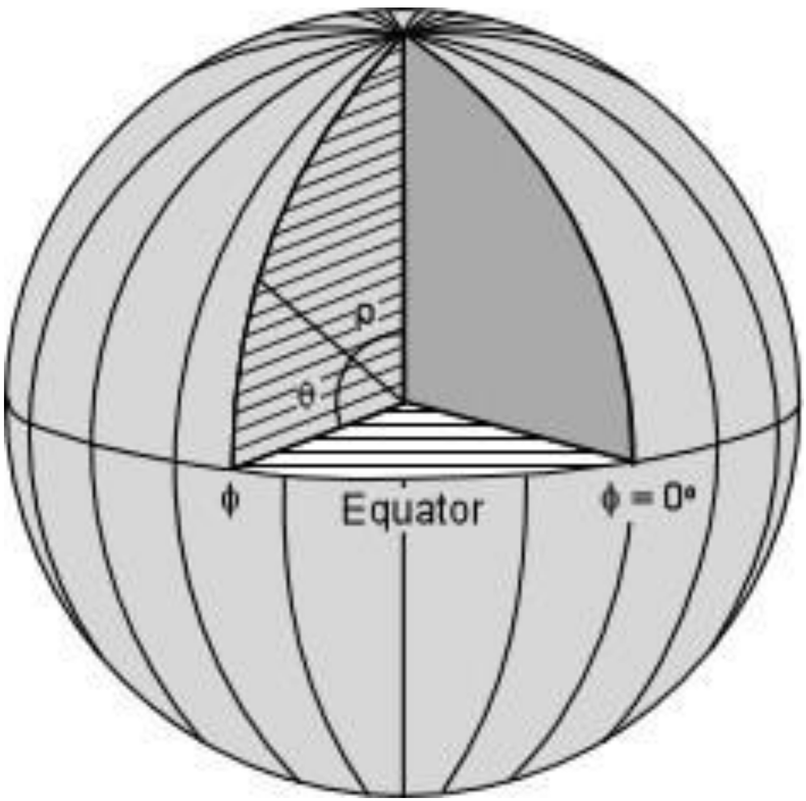
Systematic way to define crystallographic angles



1) **A ρ angle** is defined between the c axis and the pole of the crystal face.
2) This is measured **downward from the North pole** of the sphere

The ρ angle of the face (010) is equal to 90°.
Considering an other face, (101).
The ρ angle is measured in a **vertical plane** containing the **axis c** of the sphere and the **pole** of the plane (101).
The **Φ angle** is equal to 90°. It is measured in the equatorial horizontal plane from b.

Ressemblance with the earth projection system

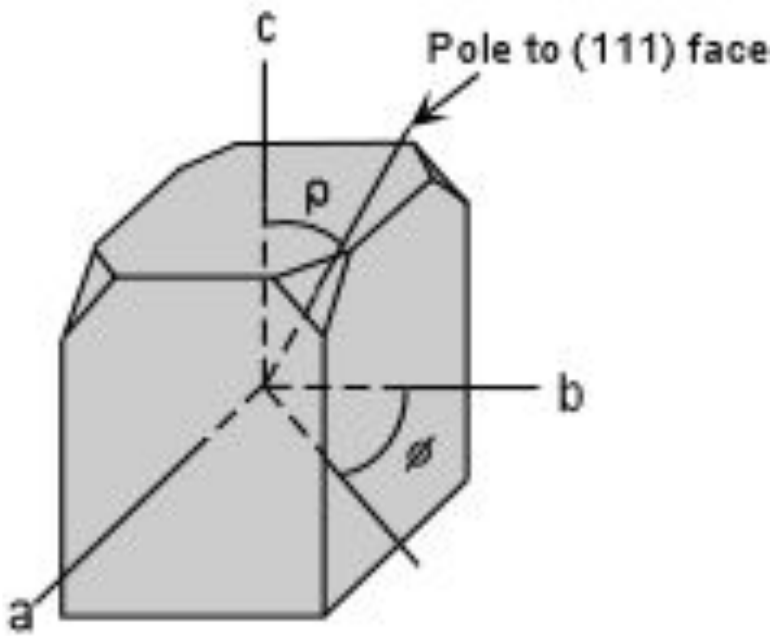


Nelson

1) The **Φ angle** measured in the **equatorial** plane Corresponds to the **longitude**. It is measured from a referential plane, the Greenwich meridian, defined as Φ = 0°.

2) The **ρ angle** is measured in the **vertical** plane as the latitude.
However, the **latitude (Θ)** is measured up from the equator.
So the **ρ angle (90°- Θ)** is called the **colatitude**.

Example for a crystal - face (111)

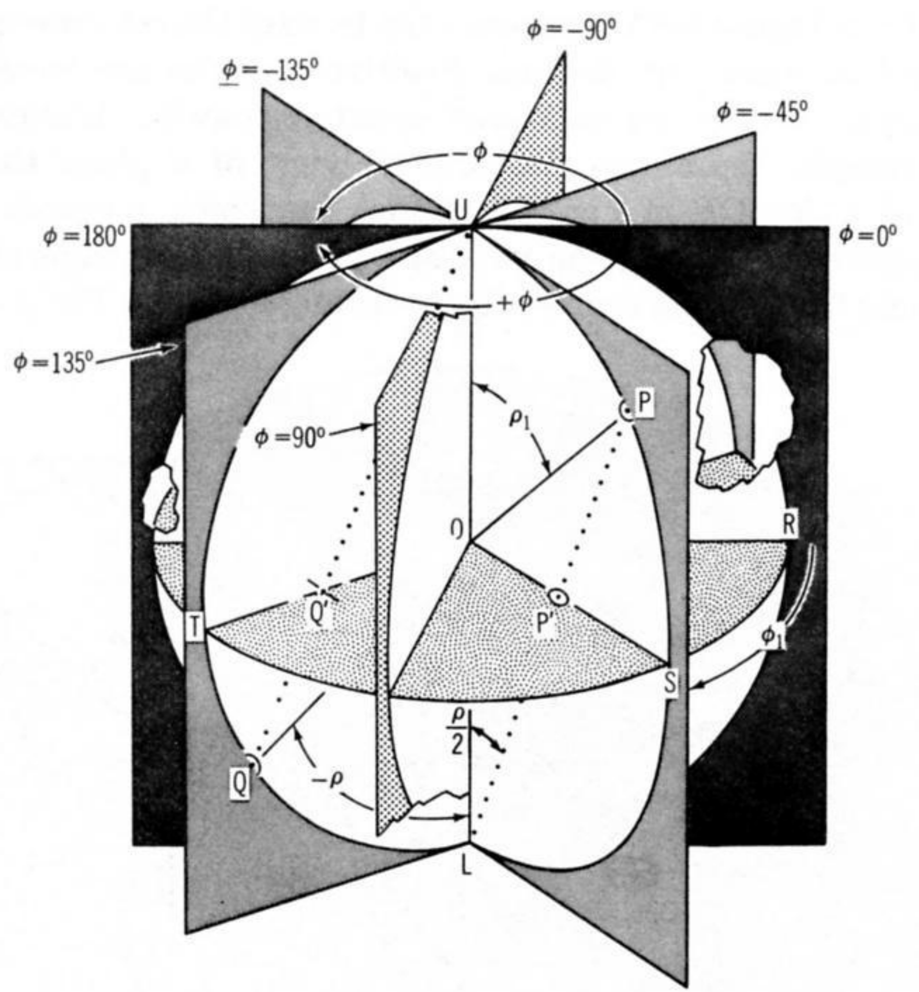


1) The **Φ angle** measured in the **equatorial** plane, clockwise from the b axis.

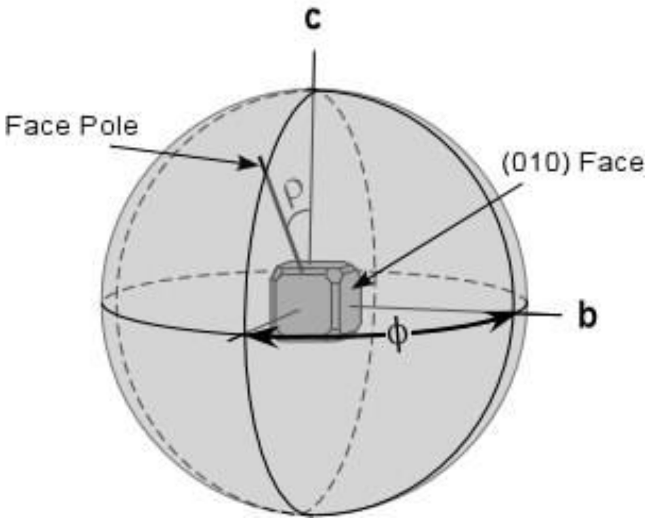
2) The **ρ angle** is measured in the **vertical** plane containing the c axis and the pole of the Face (111).

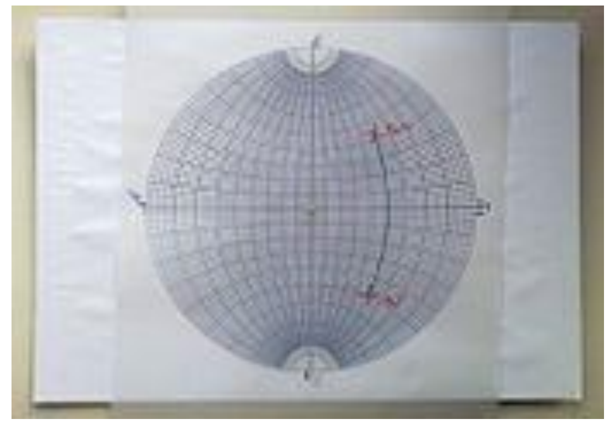
To project a face on a stereographic projection, we need only to know the **Φ angle** and **ρ angle**.
Knowledge of these data for two faces permits to calculate the **interfacial angle** between the two faces, either by trigonometry or by stereography.
And later on, the **mineral symmetry**

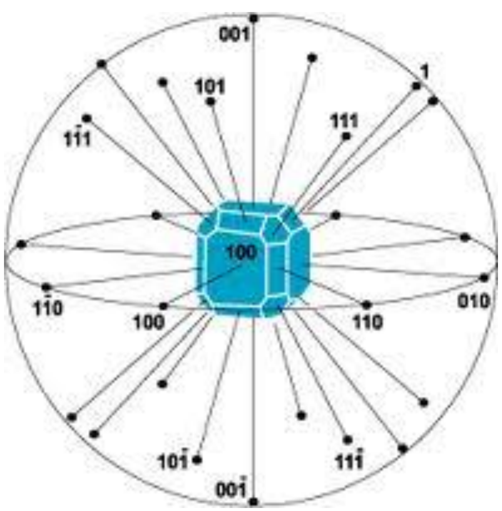
Spherical coordinate system for a sphere



Projection on a plane - Stereographic projection







2) The second step is to imagine a system of projection on a plane of the polar faces.
It's a similar problem as for the projection of the earth. 3 types of projections occur:

- Orthogonal
- gnomic
- stereographic

Why to use a stereographic projection in crystallography?

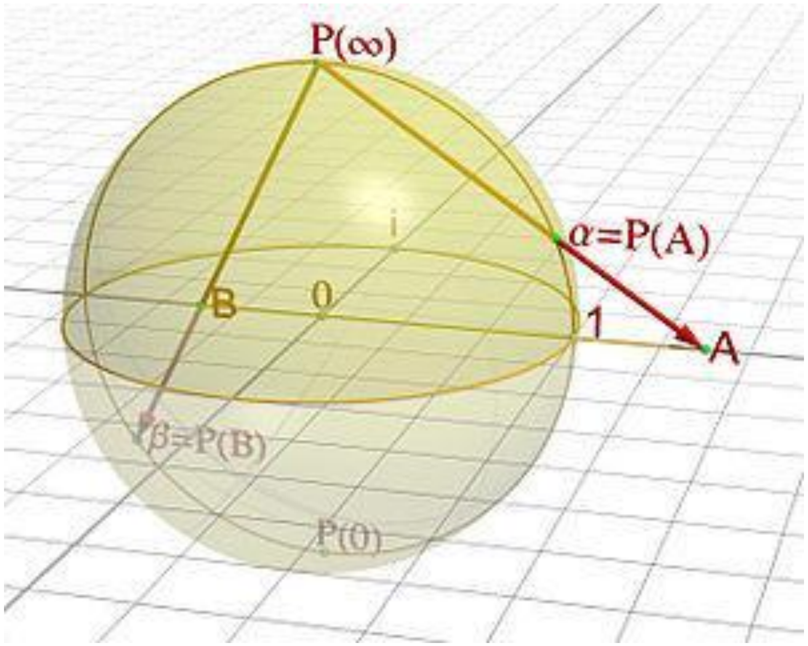
Why to use a stereographic projection in crystallography?

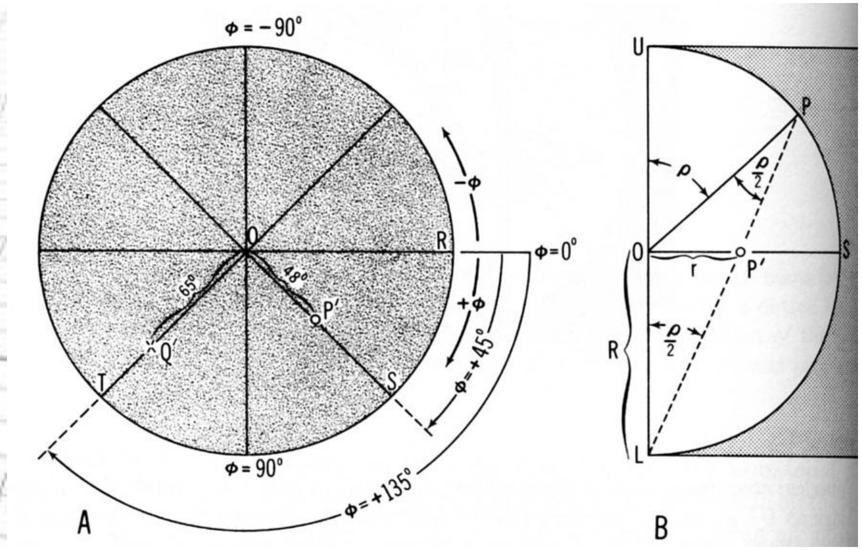
The projection of a circle is maintained as a circle
The **angles between great circles are also maintained**.

Gnomic projection is used when drawing crystals with perspective.

Stereographic projection is used in numerous geologic aspects : structural geology but also applied geology

Stereographic projection

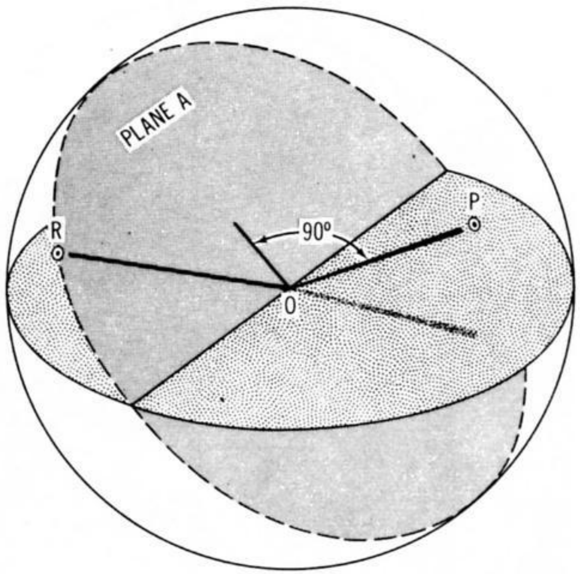


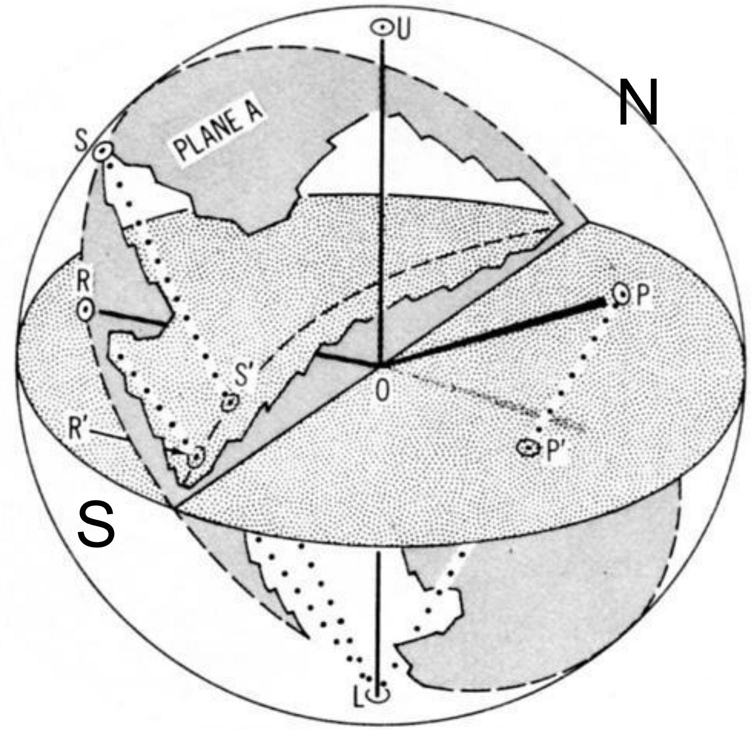


View point

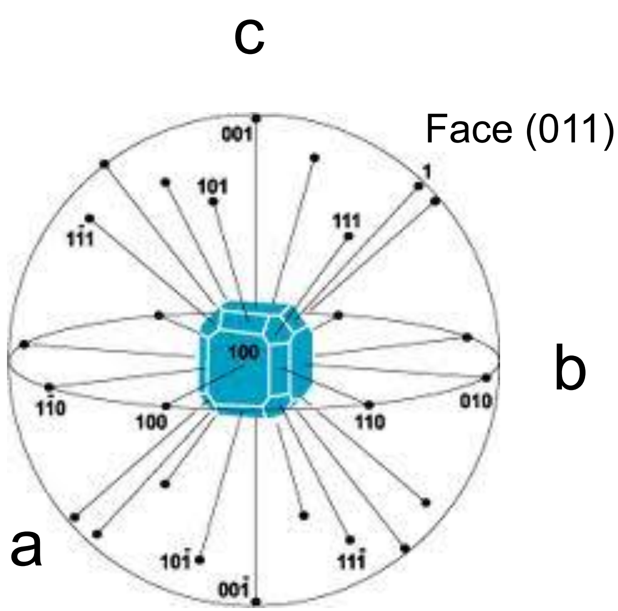
Projections are done on the **equatorial plane** which is also called **projection circle**. Points within the north (upper) hemisphere are projected within the equatorial plane; those from the lower hemisphere are projected outside the equatorial plane.

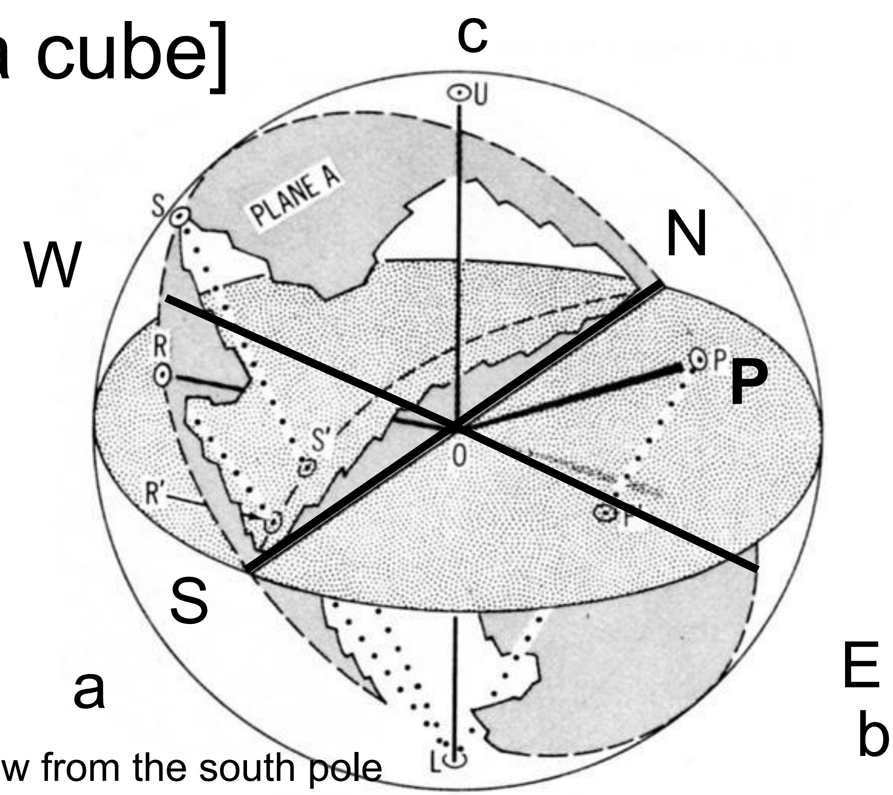
Direct and reciprocal projections





Stereographic projection of crystal faces
[example face (011) of a cube]

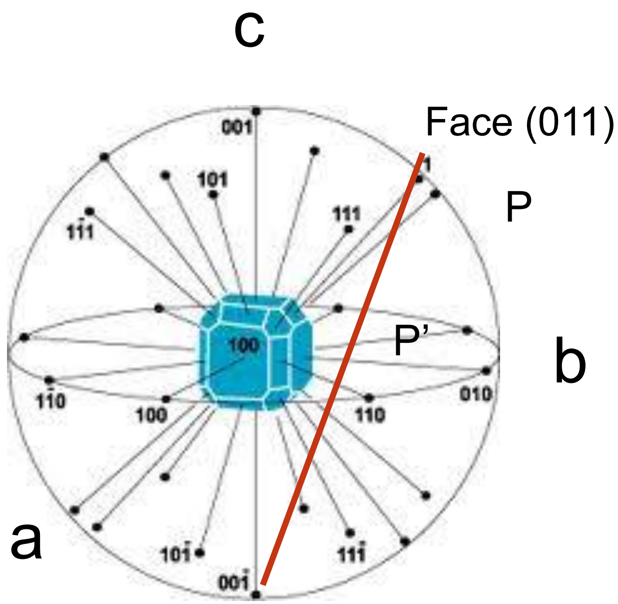


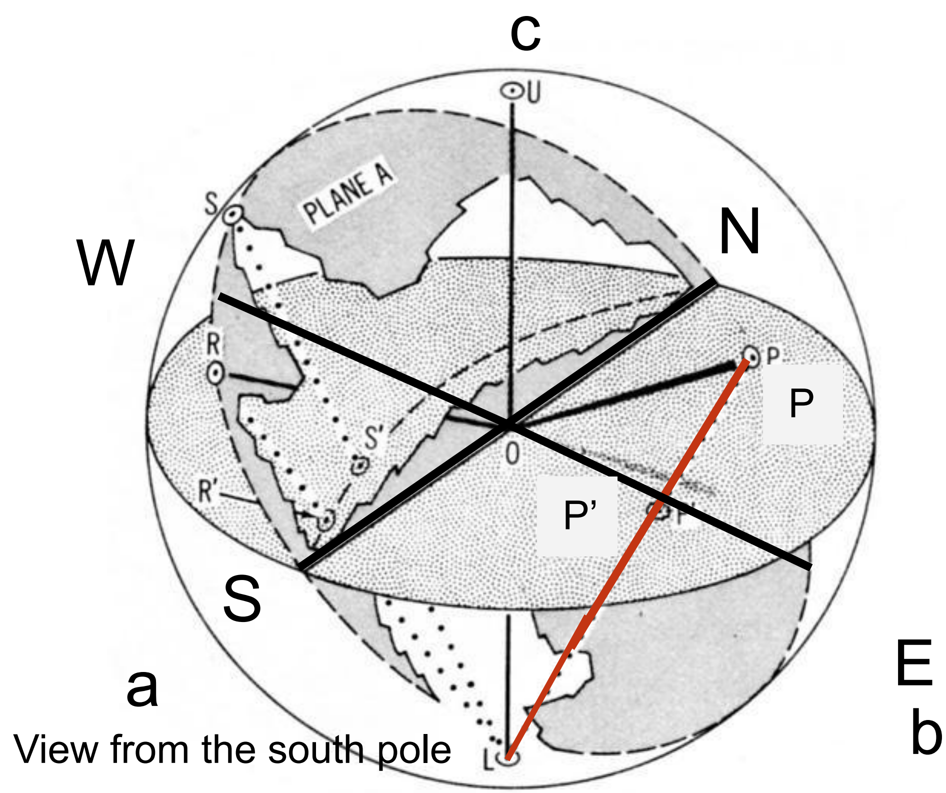


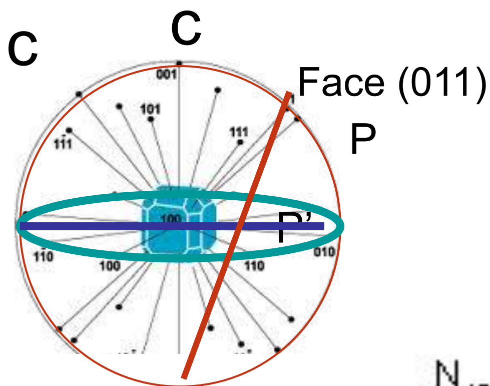
We put the crystal at the center of the sphere, with the (001) axis parallel to c, and the (010) parallel to the b axis.
Face (011) intersects the sphere in P.
We draw a line from P to the south pole of the sphere.
When the line intersects the equatorial plane, we plot the projected point. The stereographic projection is on the equatorial plane.

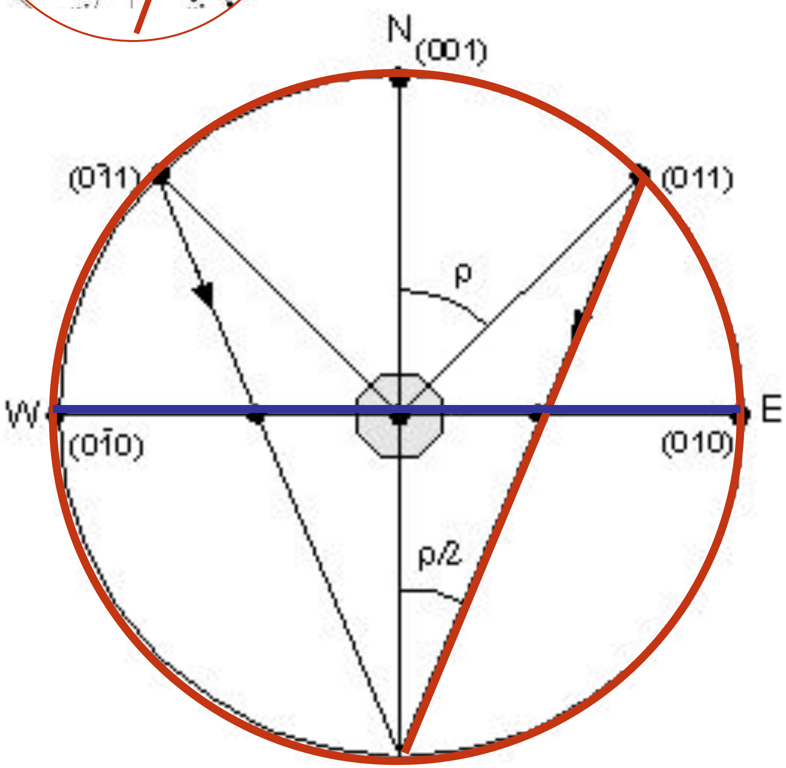
Stereographic projection of crystal faces

[example face (011) of a cube]

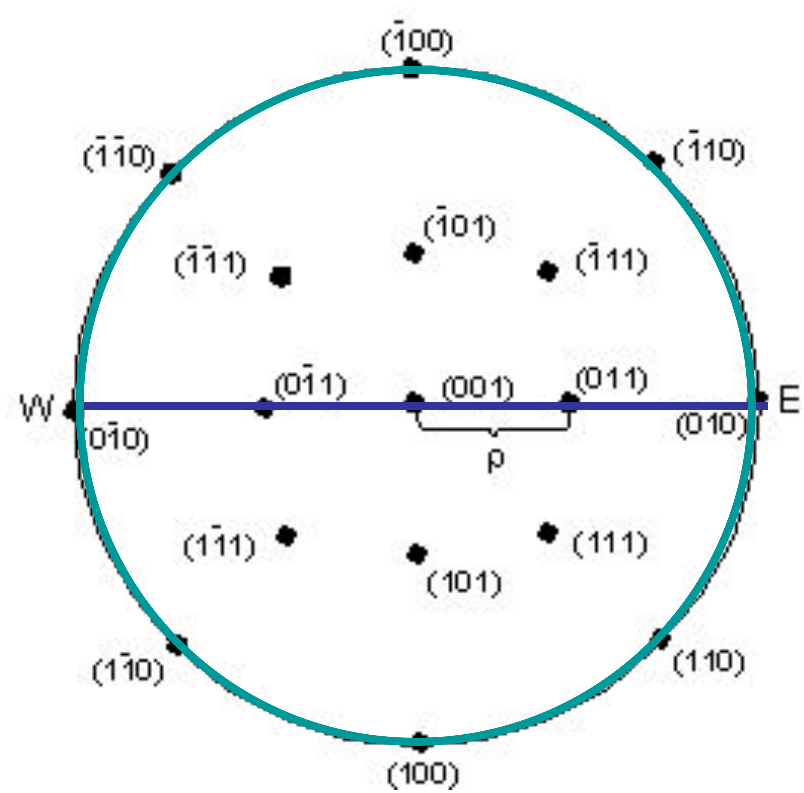




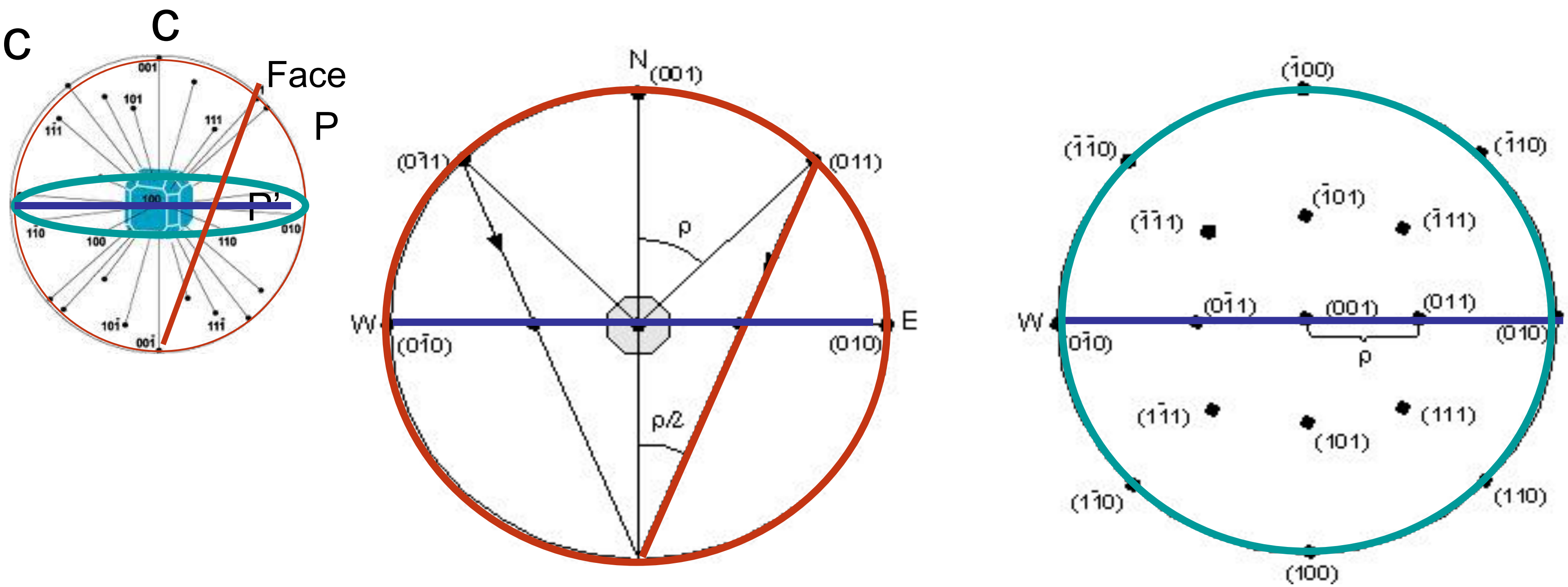




Projection in a vertical plane



Projection in an horizontal plane : the equatorial plane



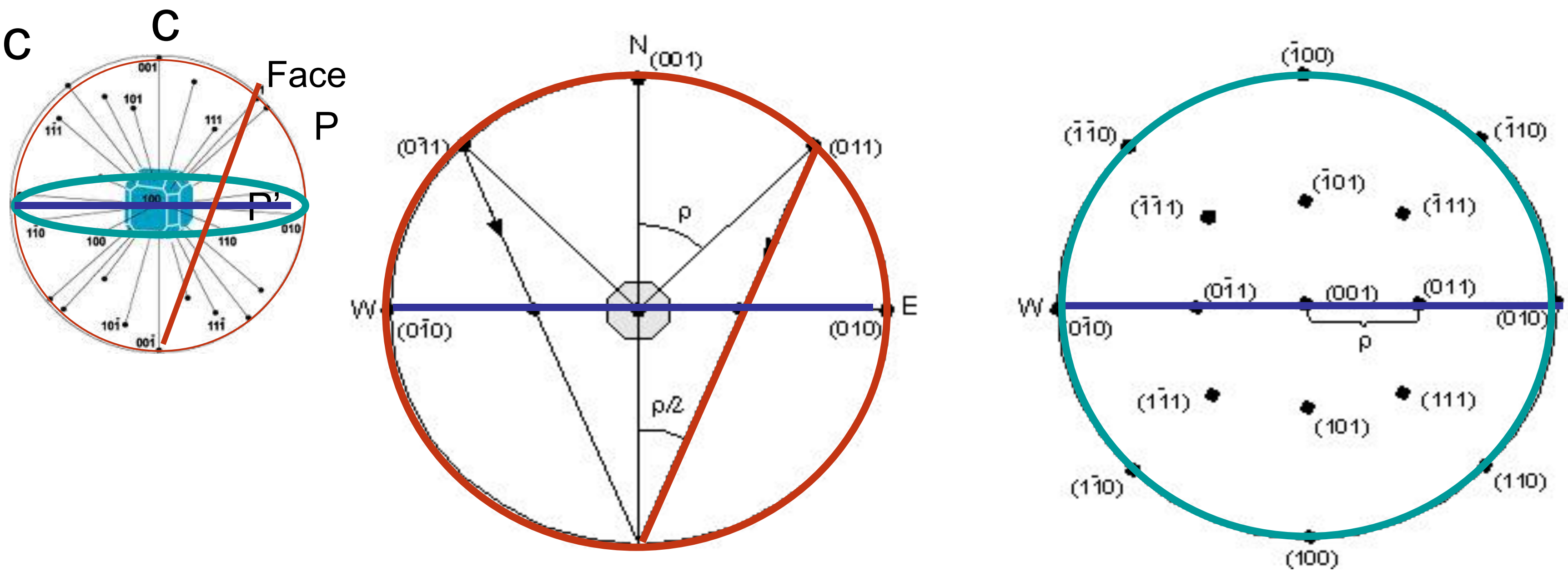
Note : Face (001) horizontal ---> angle ρ = 0°

Face (010) vertical ---> angle ρ = 90°

Face (011) same intercept on b and c axes

---> angle ρ = 45°(see vertical section)

On **equatorial projection**, project point P’ is in intermediate position between the center and the great circle, along the E-W diameter. The angle ρ is measured at this distance **between the center and the projected point P’**.



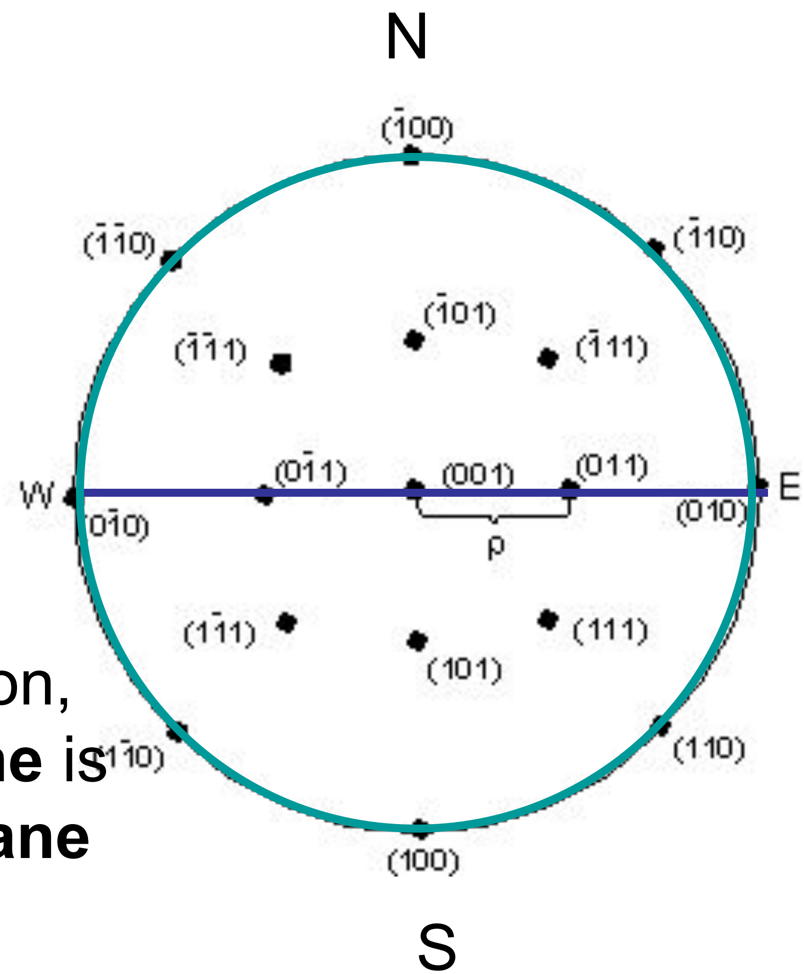
The angle **Φ** is measured **around the circumference** of the circle on the stereographic projection, in a clockwise direction from the East position, which coincide with the b axis or the pole of the face (010).

What we called view from the North and South poles are visible on the vertical projection. The north and south pole are respectively above and below the equatorial plane corresponding to the stereographic projection of crystals.

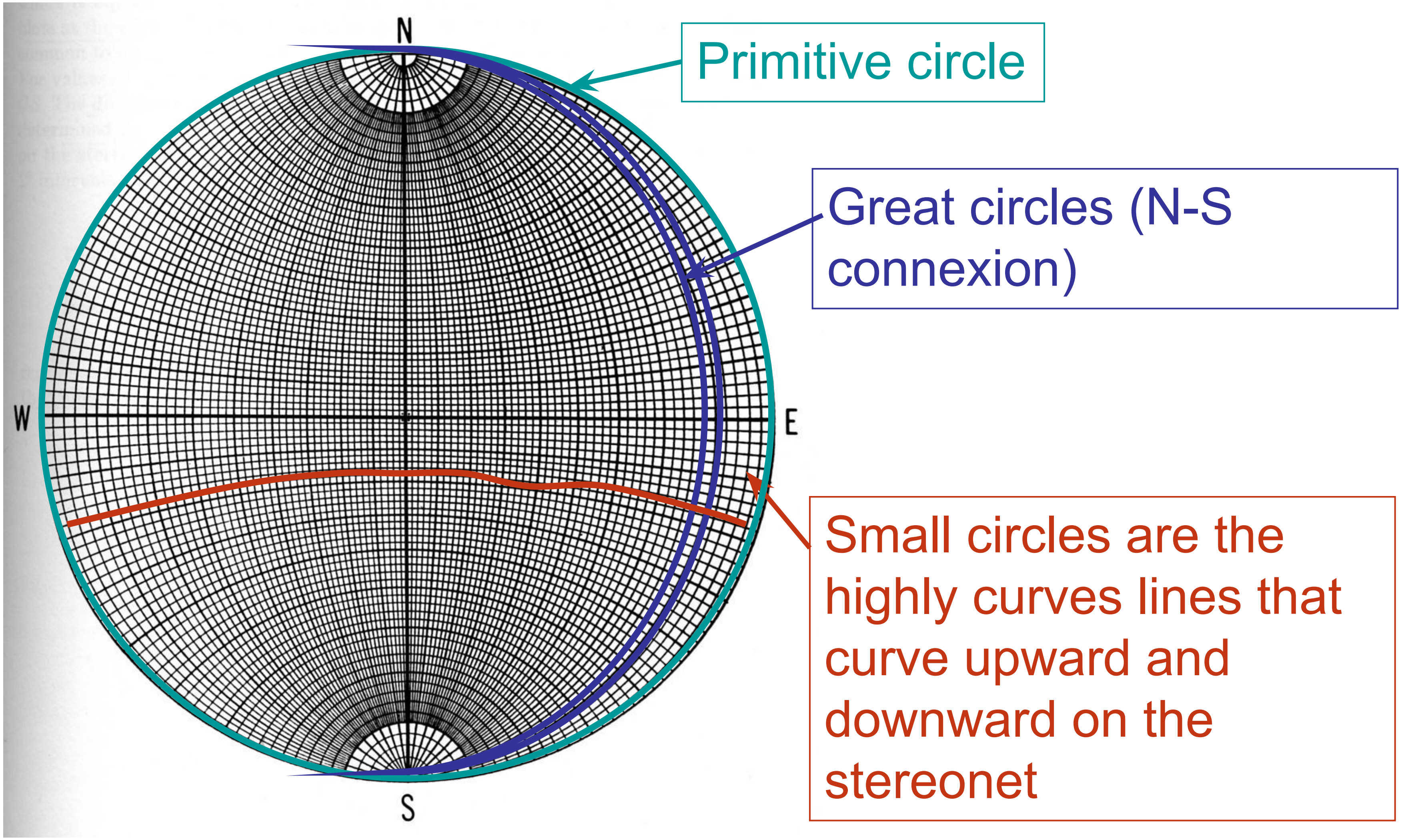
Correspondance with the stereographic projection of the earth and that of tectonic planes

The equatorial projection is orientated as shown on the figure.
The **longitude** is read on the circumference (angle **Φ)** and the **latitude** on the E-W diameter (angle 90-ρ).

When used in structural projection, the **azimut (direction) of a plane** is (angle Φ) and the **dip of the plane** is angle ρ.

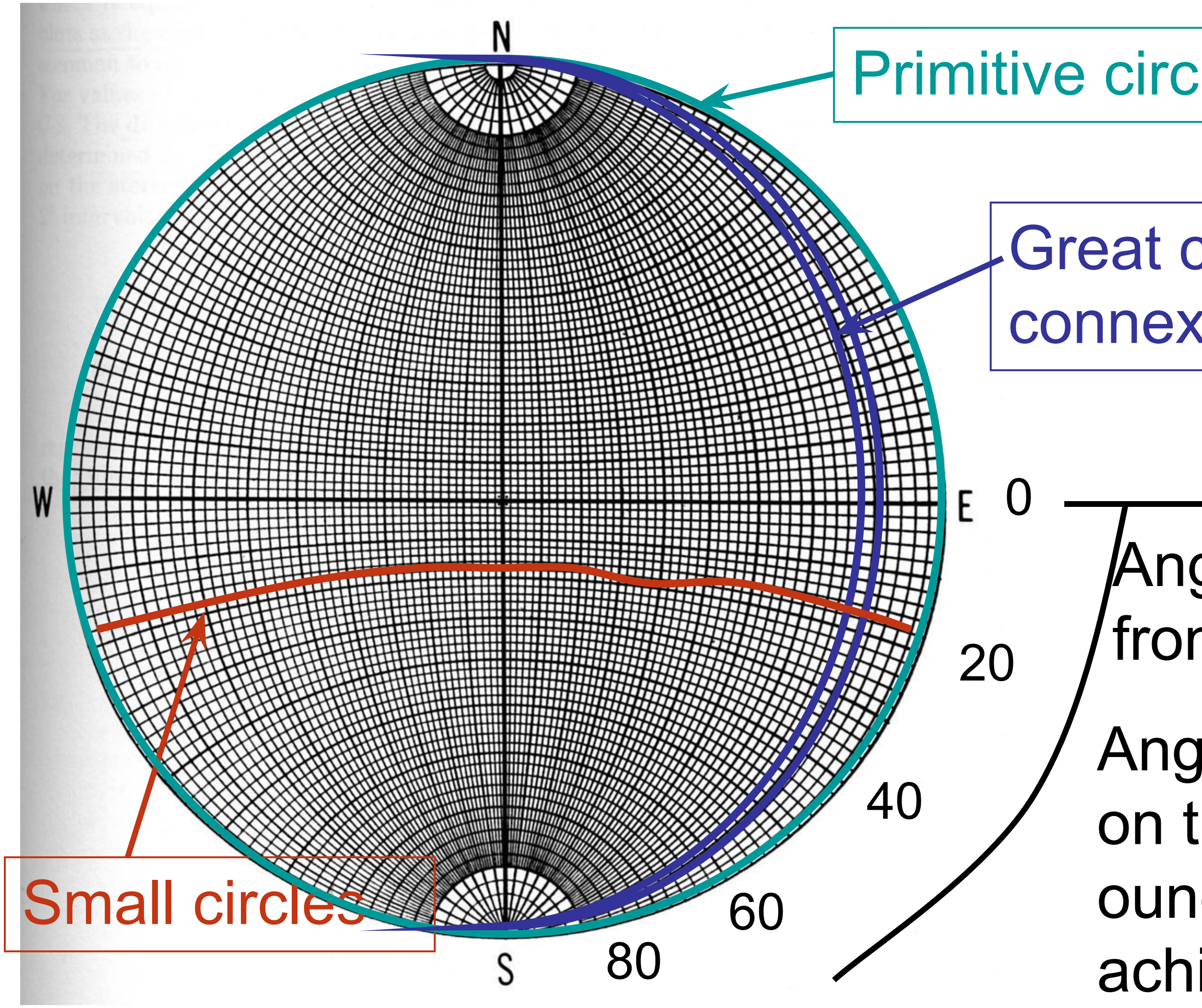


The stereonet - Wulf net
- Components of the net (drawn with 2 intervals)



Note that the primitive circle, the N-S and E-W axes are also great circles

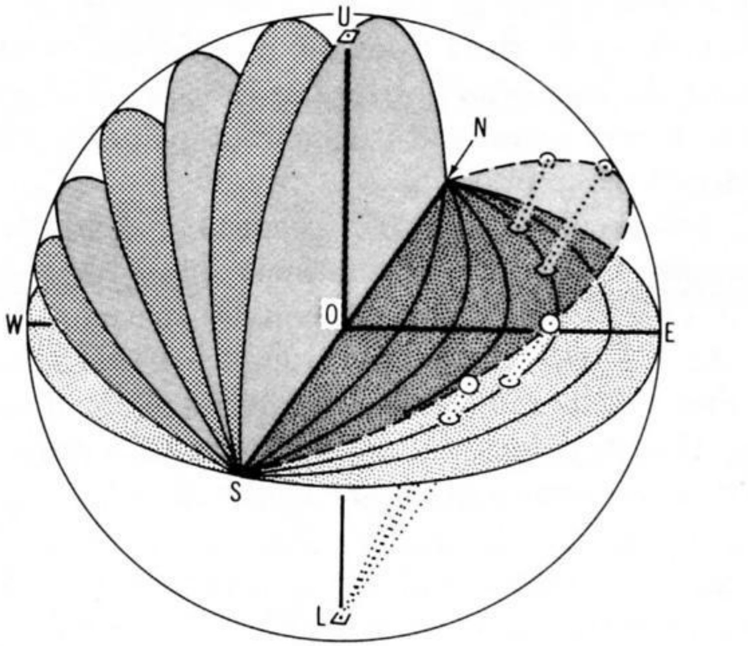
The stereonet - Wulf net
- Components of the net (drawn with 2 intervals)



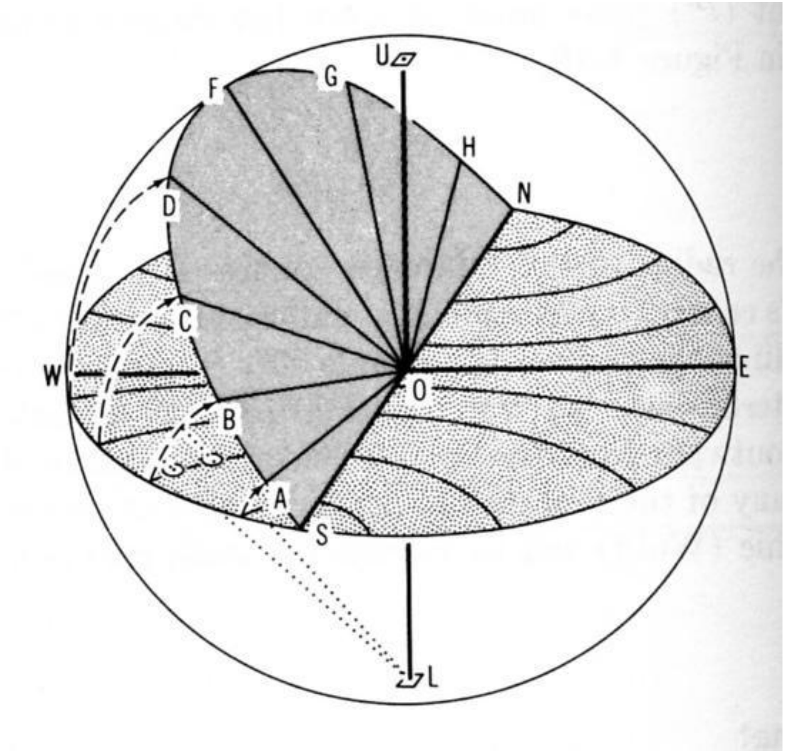
Angle Φ is measured from the great circle

Angle ρ is measured on the E-W line, ounce angle Φ is achieved

Significance of the great and small circles



Great circles



Small circles

Use of the stereonet - (1)

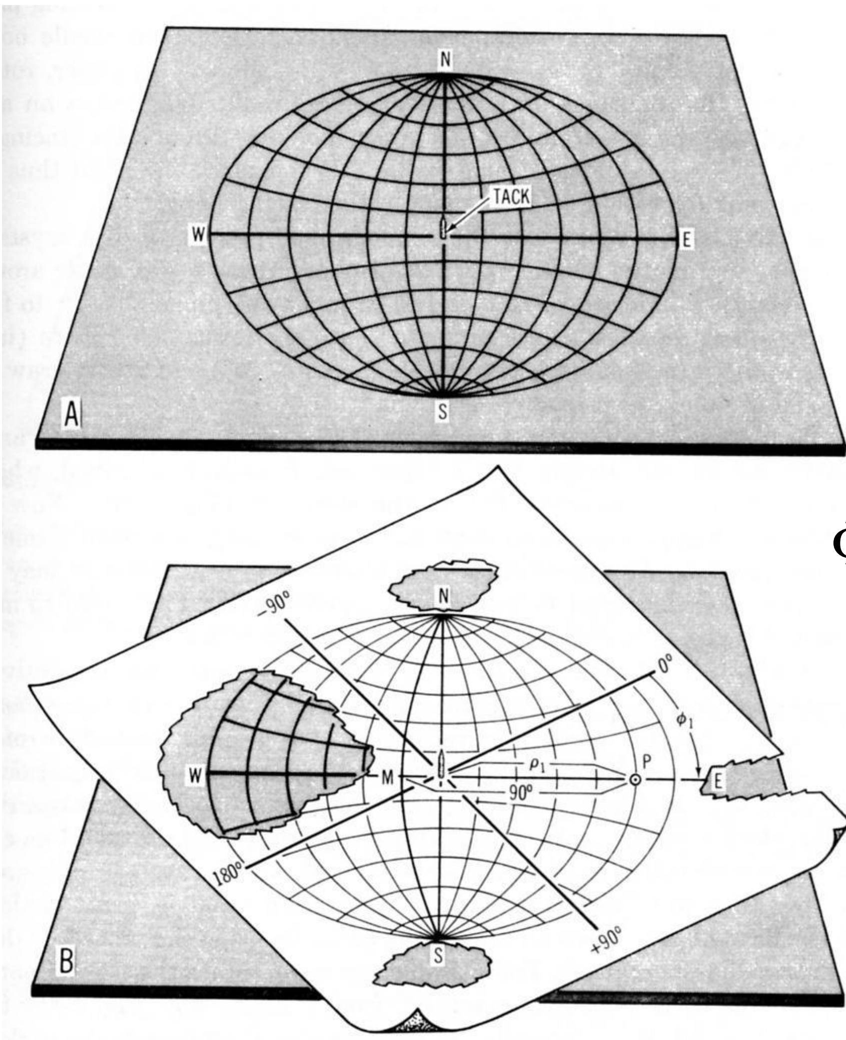
All crystal faces are plotted as **poles** (lines perpendicular to the crystal face.
----> angles between crystal faces are angles between poles

The **b crystallographic axis** is taken as the starting point. This axis is perpendicular to face (010) in any crystal system. The [010] axis (notation as zone symbol) or (010) crystal face will therefore plot at Φ =0° and ρ = 90°.

Positive Φ angles will be measured clockwise and negative Φ angles, counter- clockwise on the stereonet.

Crystal faces on top of the crystal (ρ < 90°) are plotted as open circles and crystal faces on the bottom (ρ > 90°) as « + » signs.

Use of the stereonet - practical uses (2)



Place a sheet of tracing paper on the stereonet and trace the outermost great circle (primitive circle). Make a reference mark on the right side (east position)

To plot a face:

First measure the value of the Φ angle on the primitive circle.

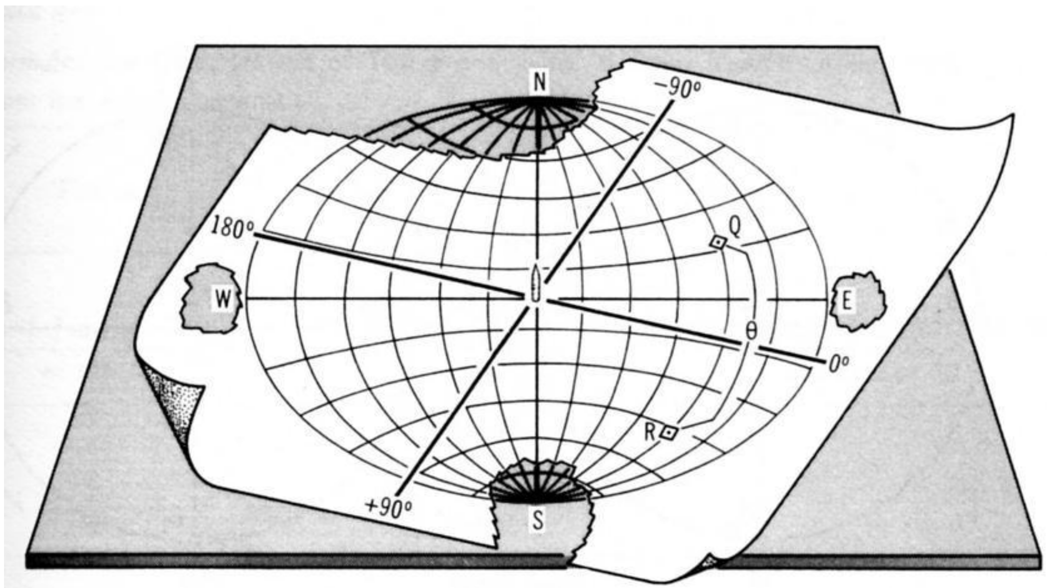
Write a mark

Rotate the sheet of paper to lie the mark on the E position.

Measure the ρ angle out from the center on the E-W axis of the Stereonet.

Note: angles can only be measured along great circles

Use of the stereonet (2)



Standard coordinates for the crystal axes in the different crystal systems

Isometric

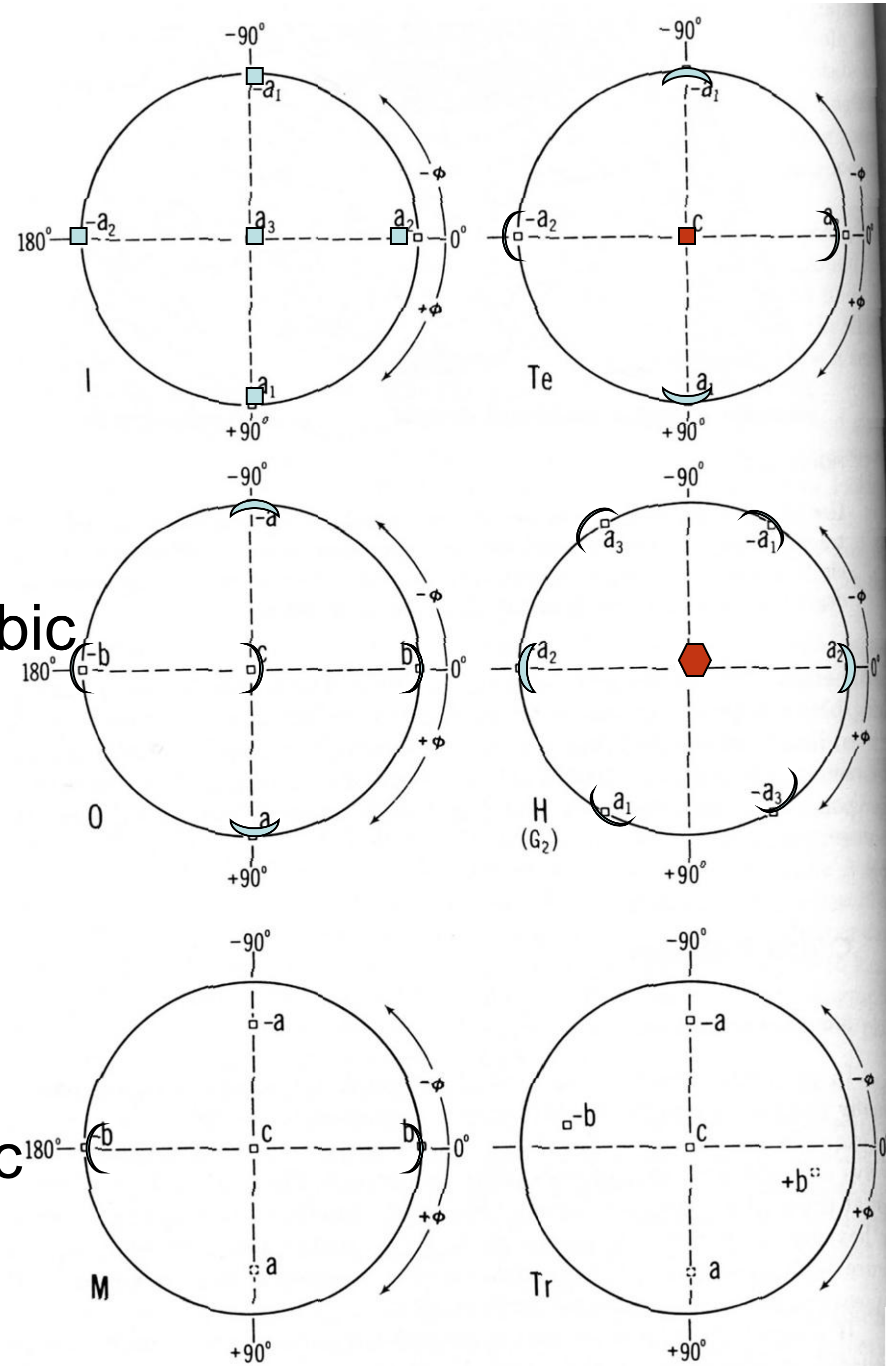
Tetragonal

Orthorhombic

Hexagonal

Monoclinic

Triclinic

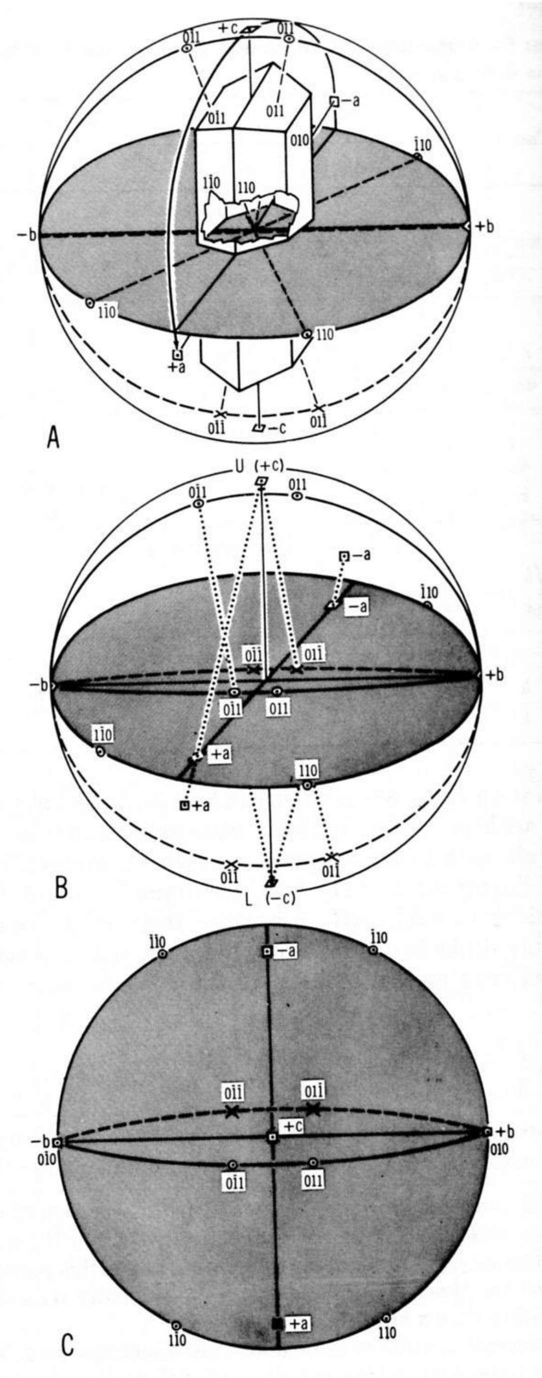


Cyclographic and stereographic projection

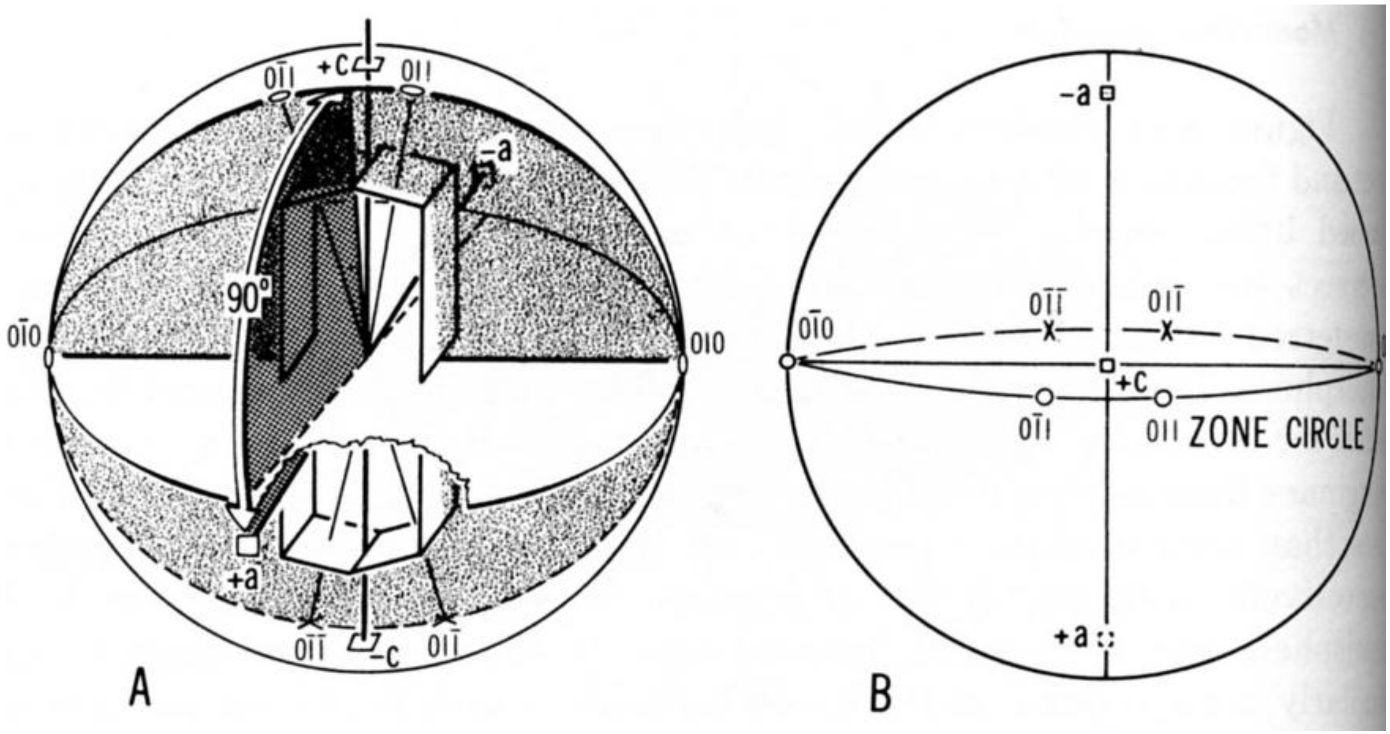
Formulas for Computation of Tan ϕ and ρ for a Face from Its Miller Indices (hkl) and from the Axial Elements

Tan ϕ	Tan ρ	
	$k \neq 0$	$k = 0$
	Monoclinic	
$\dfrac{hc/a + l \sin(\beta - 90)}{kc \sin \beta}$	$\dfrac{kc}{l \cos \phi}$	$\dfrac{hc/a + l \sin(\beta - 90)}{l \sin \phi \cos(\beta - 90)}$
	Orthorhombic	
$\dfrac{h}{ak}$	$\dfrac{kc}{l \cos \phi}$	$\dfrac{hc}{la \sin \phi}$
	Tetragonal	
$\dfrac{h}{k}$	$\dfrac{c}{l}\sqrt{h^2 + k^2}$	
	Hexagonal (G_2)	
$1.1547\left(\dfrac{h}{k} + 0.5\right)$	$\dfrac{kc}{l \cos \phi}$	$\dfrac{hc}{l \sin(\phi - 30)}$
	Isometric	
$\dfrac{h}{k}$	$\sqrt{\dfrac{h^2 + k^2}{l^2}}$	

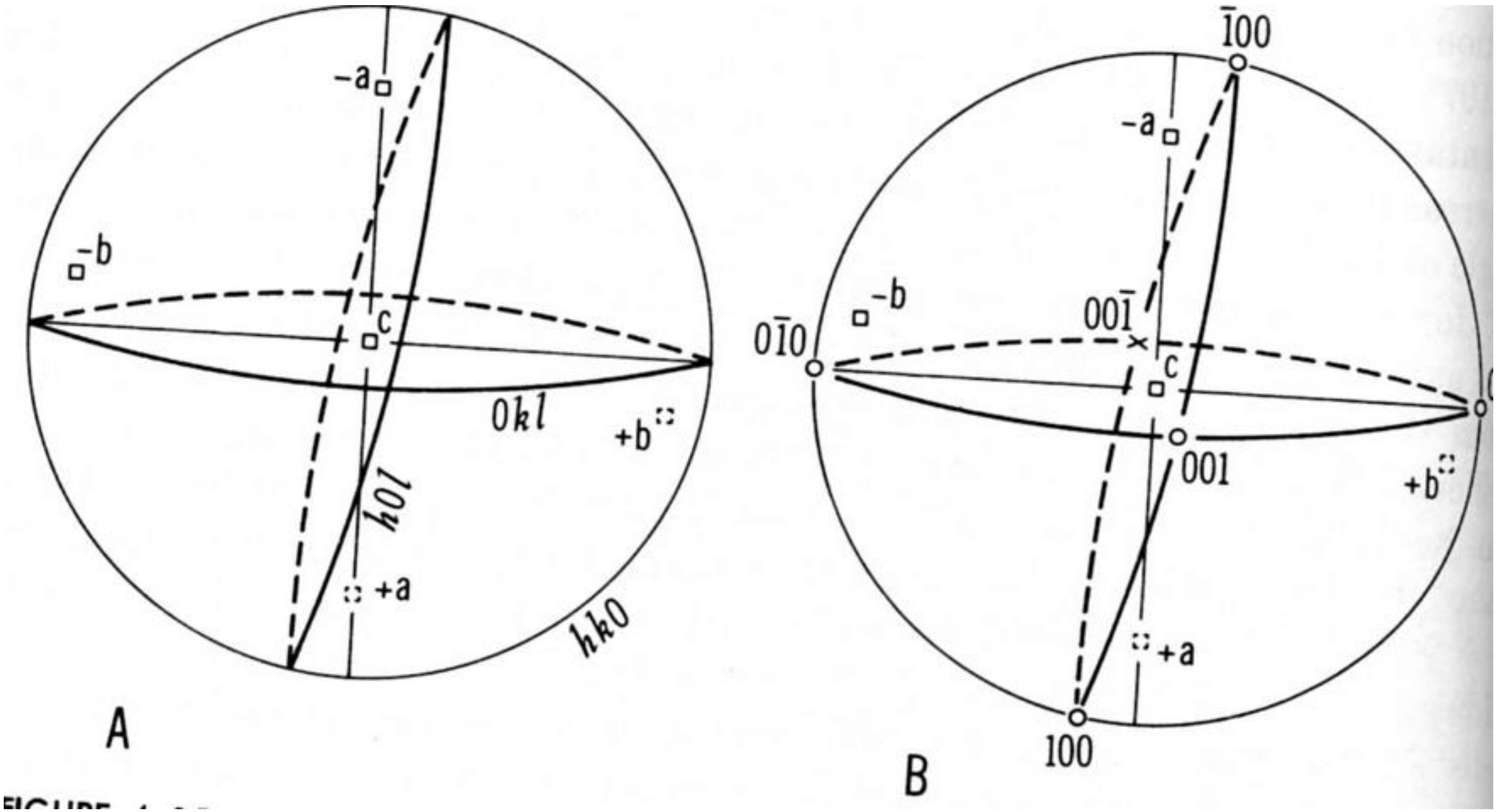
Spherical projection of a monoclinic crystal



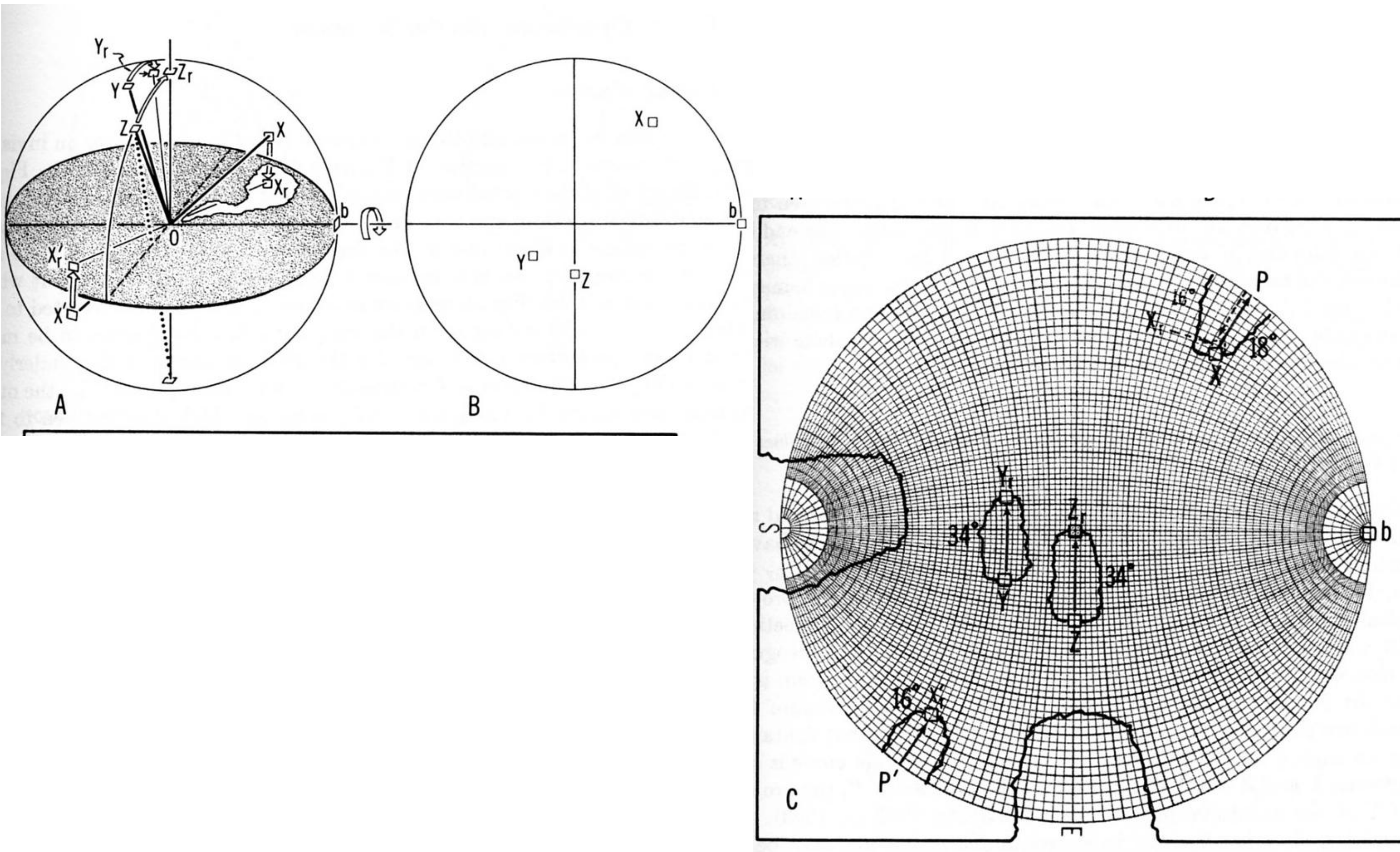
Zone plane - zone circle
Reciprocal projection of the a axe



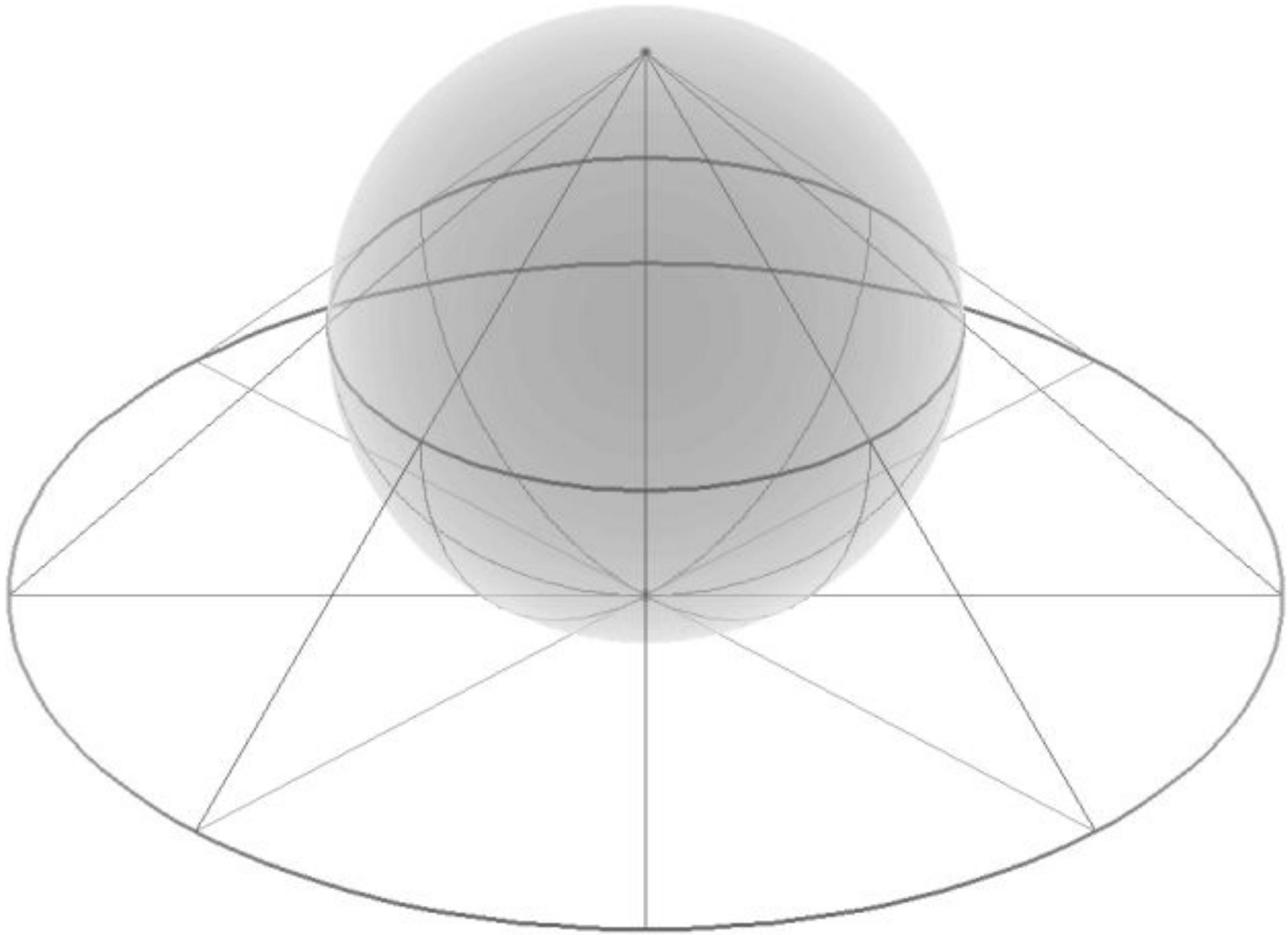
Zone circles of axes a, b and c



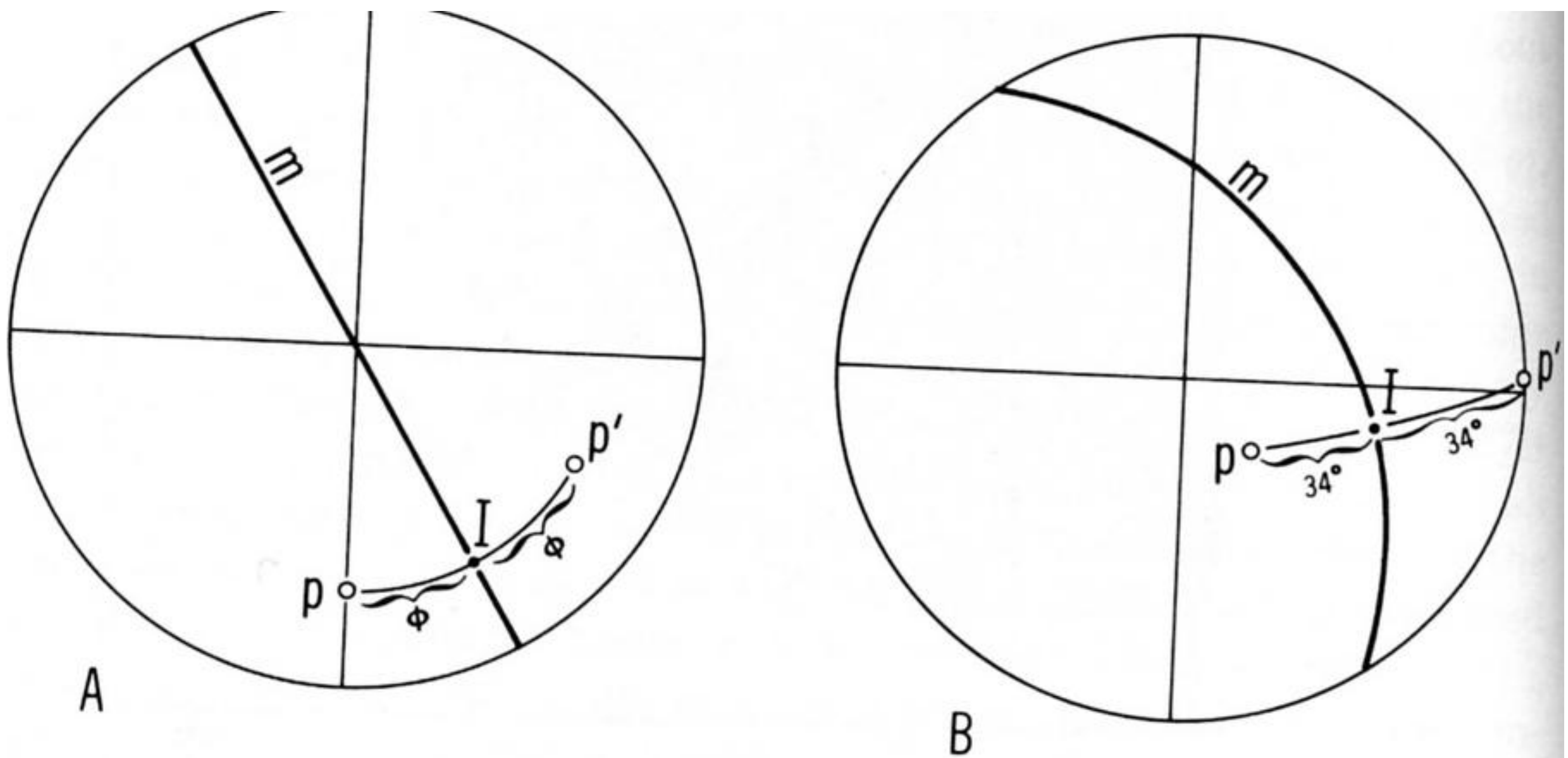
Cyclographic and stereographic projection



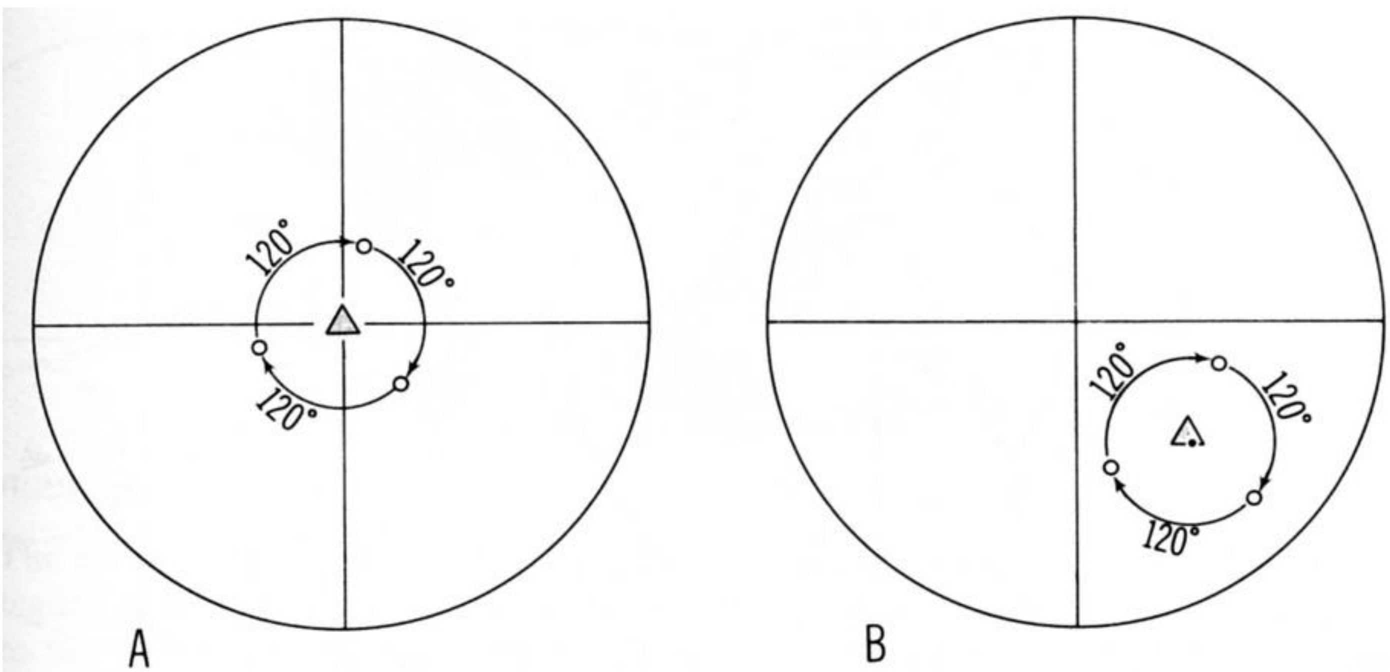
Cyclographic and stereographic projection



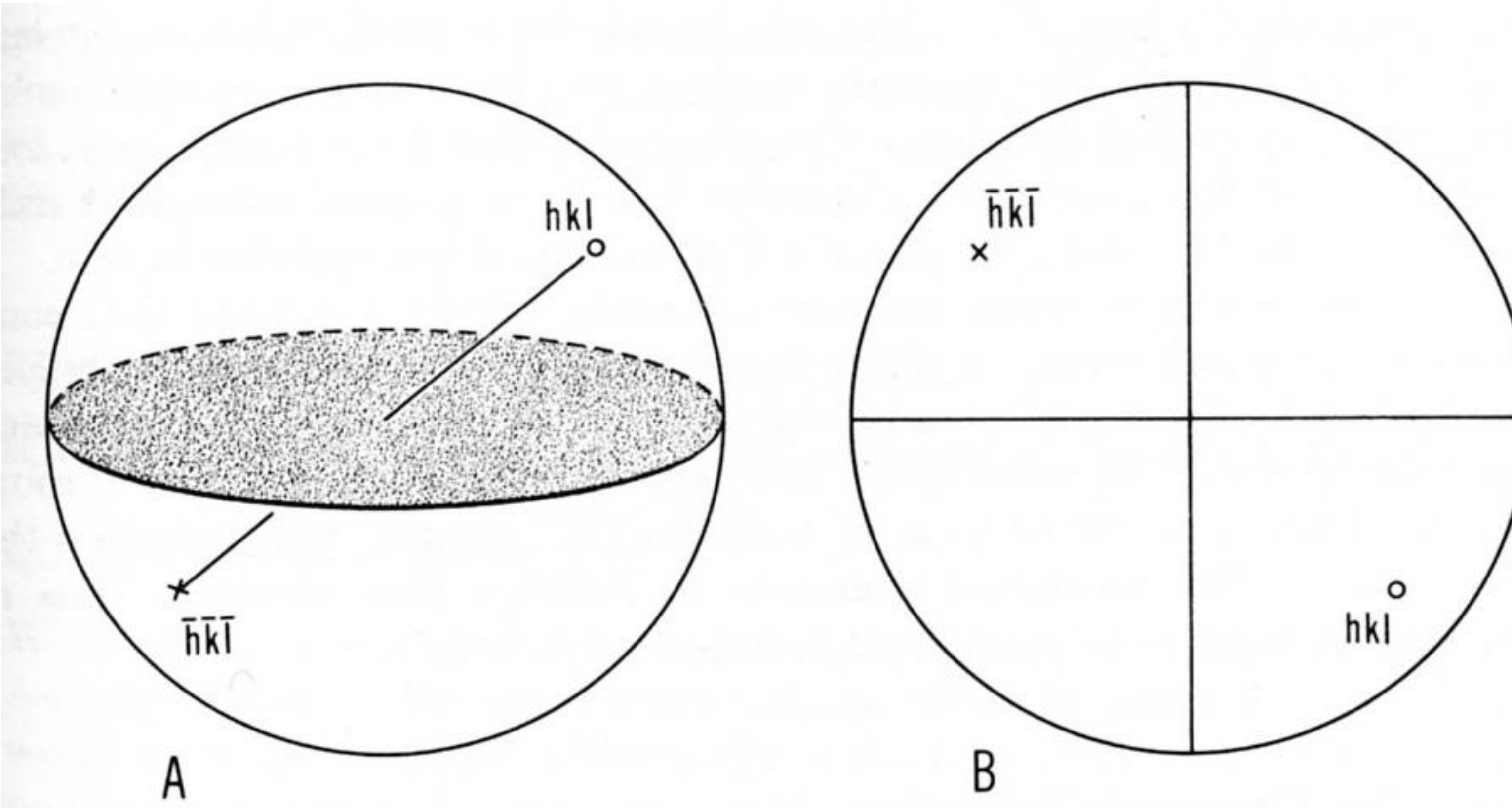
Symmetry related pole faces
1 - Plane of symmetry



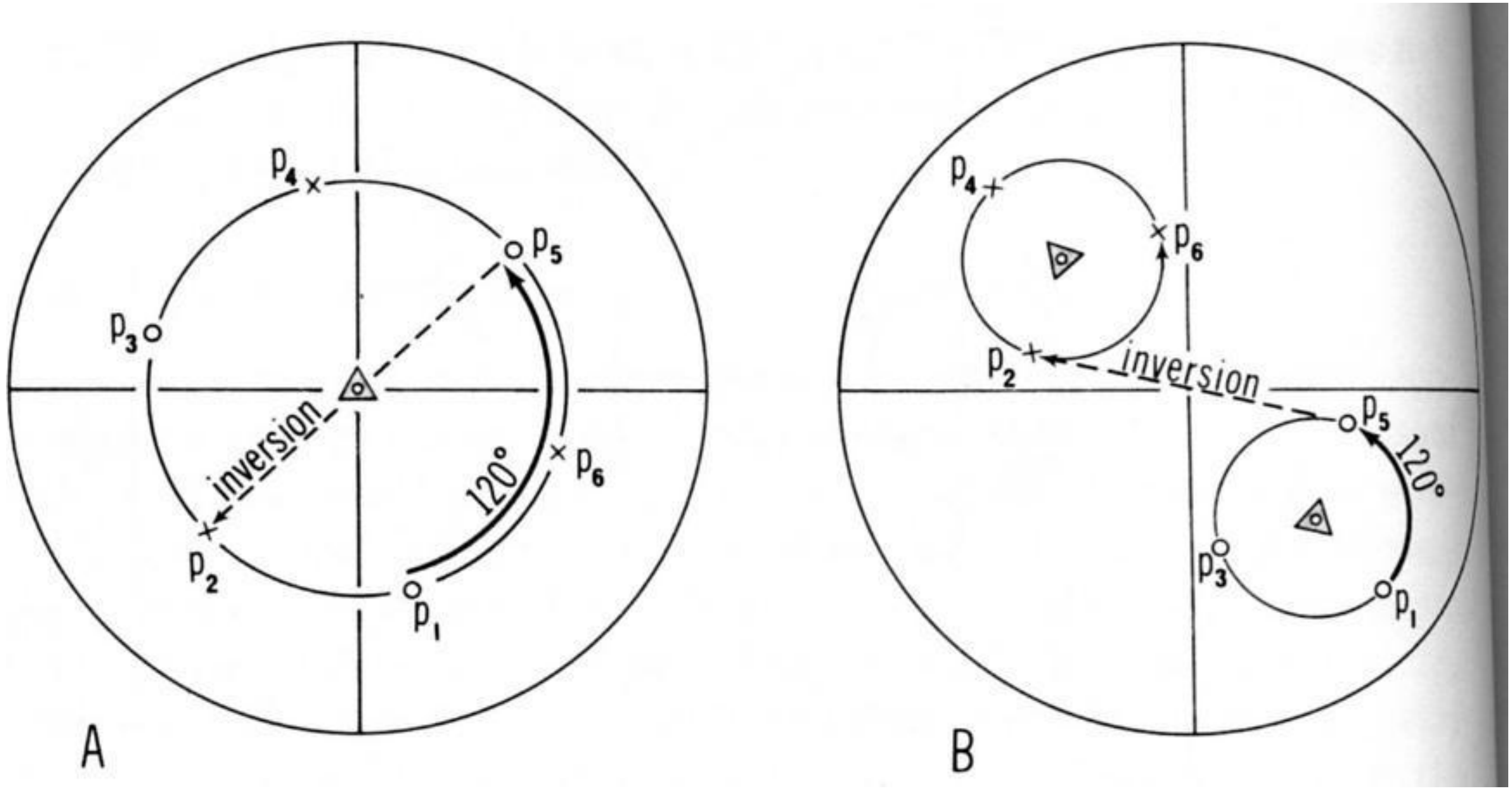
Symmetry related pole faces
2 - Rotation axe:
Exp.: 3-fold rotational axe



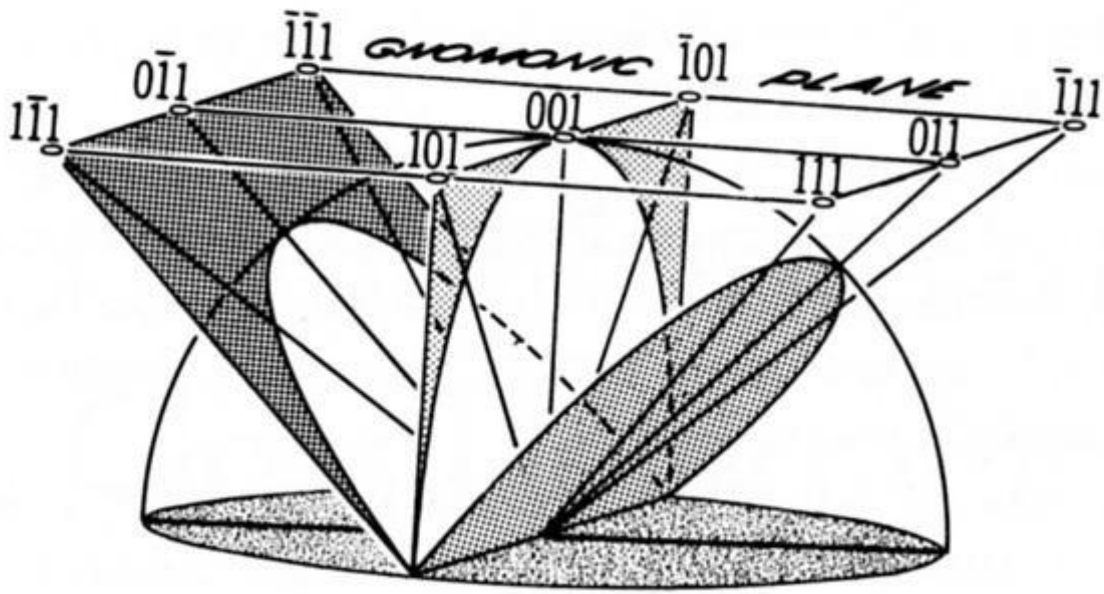
Symmetry related pole faces
3 - Center of Symmetry



Inversion axes



Gnomonic projection



Cyclographic and stereographic projection