

Definite Integration By Parts

Definite Integration By Parts

- Since there is no change of variable, the limits do not have to be converted.
- Be careful to evaluate both elements of the result of integration by parts between the limits, i.e.:

$$\int_a^b v \frac{du}{dx} dx = [uv]_a^b - \int_a^b u \frac{dv}{dx} dx$$

Example: Evaluate $\int_1^2 x^3 \ln x dx$

• So,

Let $v = \ln x \Rightarrow \frac{dv}{dx} = \frac{1}{x}$

$$\frac{du}{dx} = x^3 \Rightarrow u = \frac{1}{4} x^4$$

Therefore $\int_1^2 x^3 \ln x dx = \left[\frac{x^4 \ln x}{4} \right]_1^2 - \int_1^2 \frac{x^4}{4x} dx$

Example: Evaluate $\int_1^2 x^3 \ln x dx$

$$\text{So } I = \left[\frac{x^4 \ln x}{4} \right]_1^2 - \frac{1}{4} \int_1^2 x^3 dx$$

$$I = \frac{1}{4} [2^4 \ln 2 - 1^4 \ln 1] - \frac{1}{4} \left[\frac{x^4}{4} \right]_1^2$$

$$I = \frac{1}{4} [16 \ln 2 - 0] - \frac{1}{16} [2^4 - 1^4]$$

$$I = 4 \ln 2 - \frac{15}{16} = \ln 16 - \frac{15}{16}$$