Definite Integration By Parts

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- Since there is no change of variable, the limits do not have to be converted.

- Be careful to evaluate both elements of the result of integration by parts between the limits, i.e.:

$$\int_a^b v\frac{du}{dx}\,dx = \left[uv\right]_a^b - \int_a^b u\frac{dv}{dx}\,dx$$

Example: Evaluate $\int_1^2 x^3 \ln x dx$

- So,

$$Let \qquad v = \ln x \Rightarrow \qquad \frac{dv}{dx} = \frac{1}{x}$$

$$\frac{du}{dx} = x^3 \qquad \Rightarrow \qquad u = \frac{1}{4}x^4$$

$$Therefore \qquad \int_1^2 x^3 \ln x dx = \left[\frac{x^4 \ln x}{4}\right]_1^2 - \int_1^2 \frac{x^4}{4x} dx$$

Example: Evaluate $\int_1^2 x^3 \ln x\,dx$

$$So \qquad I = \left[\frac{x^4 \ln x}{4}\right]_1^2 - \frac{1}{4}\int_1^2 x^3\,dx$$

$$I = \frac{1}{4}\left[2^4 \ln 2 - 1^4 \ln 1\right] - \frac{1}{4}\left[\frac{x^4}{4}\right]_1^2$$

$$I = \frac{1}{4}\left[16 \ln 2 - 0\right] - \frac{1}{16}\left[2^4 - 1^4\right]$$

$$I = 4 \ln 2 - \frac{15}{16} = \ln 16 - \frac{15}{16}$$