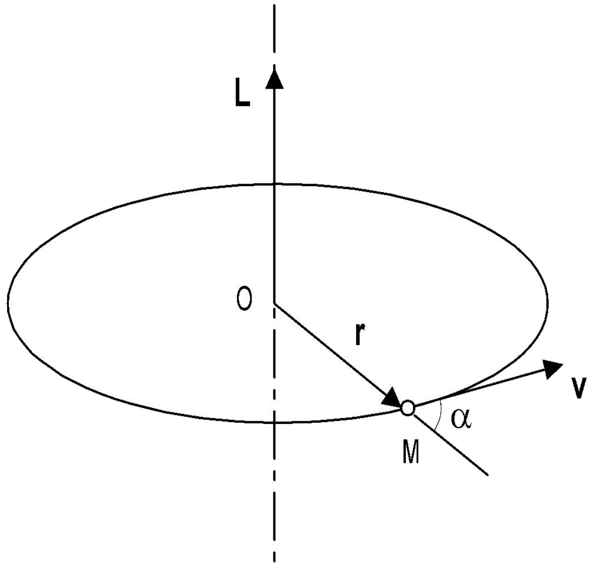
Dynamics of Rotatory Motion

- Angular Momentum
- Torque
- Newton’s Second Law
- Moment of Inertia
- Law of Conservation of Angular Momentum

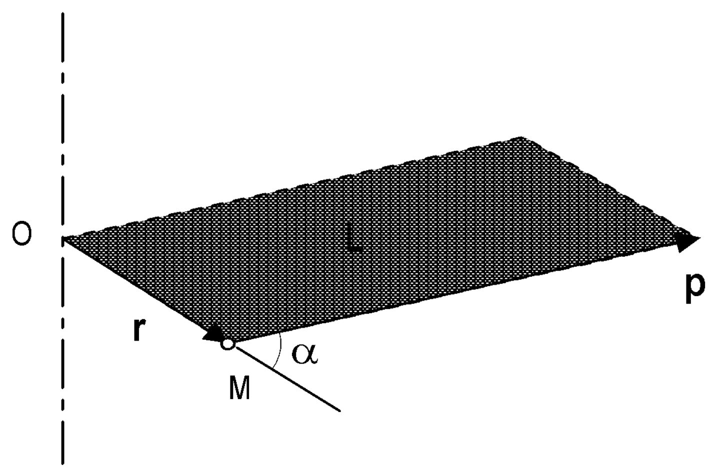
Angular Momentum



The *angular momentum* of this material point is the vector product of position vector and momentum ,

$$\mathbf{L} = [\mathbf{r}, \mathbf{p}]$$

Angular Momentum (cont.)



$$L = r \cdot p \cdot \sin \alpha$$

Torque

$$\mathbf{L} = [\mathbf{r}, \mathbf{p}]$$

$$\frac{d\mathbf{L}}{dt} = [\frac{d\mathbf{r}}{dt}, \mathbf{p}] + [\mathbf{r}, \frac{d\mathbf{p}}{dt}] \qquad \frac{d\mathbf{L}}{dt} = [\mathbf{r}, \frac{d\mathbf{p}}{dt}] = [\mathbf{r}, \mathbf{F}]$$

We shall name the vector product of position vector and force as a moment of force about , or *torque, ,*

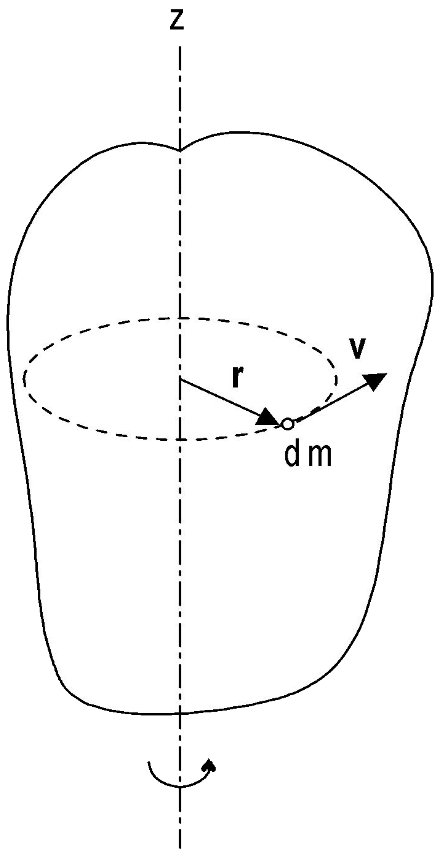
$$\mathbf{M} = [\mathbf{r}, \mathbf{F}]$$

Newton’s Second Law

$$\frac{d\mathbf{L}}{dt} = \mathbf{M}_{ext}$$

The angular momentum of a system of particles will change only if a torque exists

Moment of Inertia



$$L_z = \int r \cdot v \cdot dm = \omega \int r^2 \cdot dm$$

$$\int r^2 \cdot dm = I_z$$

$$\mathbf{L} = I\boldsymbol{\omega}$$

The angular momentum of an object is equal to the product of the moment of inertia of this object and its angular velocity

Newton's Second Law (cont.)

$$\boldsymbol{\varepsilon} = \frac{\mathbf{M}_{ext}}{I}$$

The angular acceleration of an object is directly proportional to the resulting moment of all external forces acting on this object and inversely proportional to the moment of inertia of the object

Law of Conservation of Angular Momentum

If there are no external torques acting on the object with its axis of rotation, then the angular momentum of this object will be not changed:

$$\mathbf{L} = \sum \mathbf{L_i} = const$$

$$\sum_i I_i \boldsymbol{\omega}_i = const$$