

Descriptive geometry

Introduction

Descriptive geometry is one of the fundamental disciplines making up an engineering education. Drawing is a key way to transfer technical thoughts. There are three basic requirements, produced to the images in a drawing: **reversibility, measurability, obviousness**.

It is concerned with setting forth and justifying methods of constructing representations of three-dimensional forms in the plane, as well as methods of solving geometrical problems on the basis of given representations of these forms. As is known, three-dimensional forms can be represented not only in the plane, but on some other surface, for instance, a cylinder or sphere. The latter cases are studied in special branches of descriptive geometry.

The representations constructed according to the rules of descriptive geometry enable us to visualize the shape of objects and their relative positions in space, to determine their dimensions, and to study their geometrical properties.

Descriptive geometry develops the student's three-dimensional imagination by making frequent appeals to it.

Finally, descriptive geometry provides a number of practical means for engineering drawings, ensuring their clarity and accuracy, and, hence, the possibility of manufacturing the represented objects.

The rules for constructing representations, set forth in descriptive geometry are based on the method of projections.

It is standard practice to begin studying the method of projection with the construction of the projections of the point, since the construction of the projections of any three-dimensional form involves considering a number of points belonging to this form.

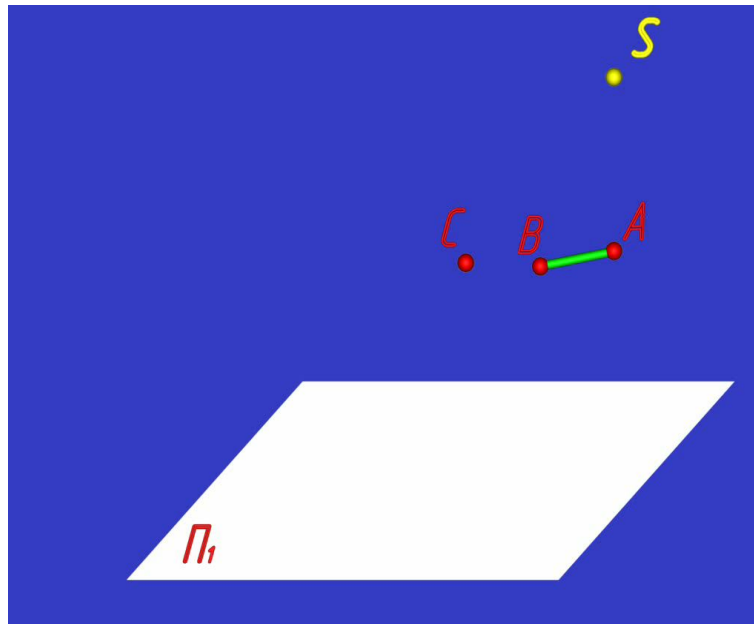
Image is a main component of graphical design documents (drawings). 1

The Method of Projection

In graphic language the shape is described by projection, which is the image of the object, formed by rays of light, taken in some particular direction, from the object into a picture plane, as it appears to an observer stationed at the point, from or towards which the projection is made.

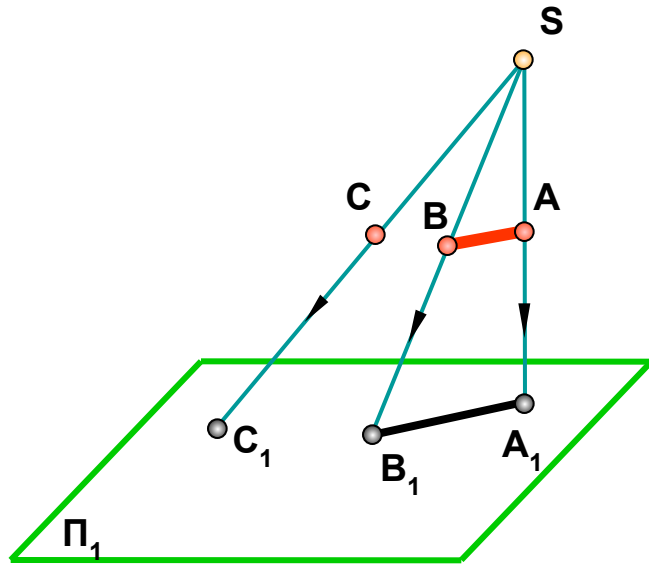
Depending upon the orientation of the object, location of the point of sight, and the direction of lines of sight relative to the picture plane, different types of projections, e.g., central (perspective), parallel, orthographic, axonometric, oblique, etc., can be obtained.

The plane, on which the projection is taken, is called the plane of projection or picture plane. The point, from which the observer is assumed to view the object, is called the station point or the centre of projection.



The Method of Projection

Central Projection



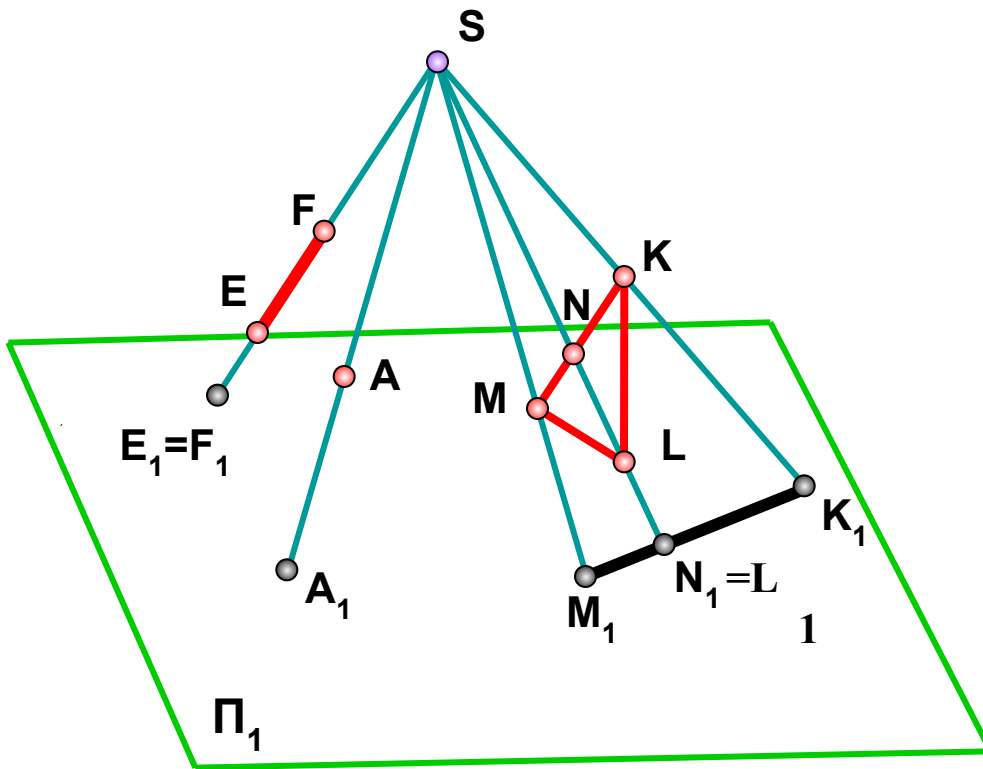
Essentials of projection:

Π_1 – plane of projection (picture plane);
 S – center of projection (station point) ($S \notin \Pi_1$);
 C – a point (original);
 SC – projector (projecting ray);
 C_1 – projection (image) of a point C ;
 AB – straight line segment (original);
 SA, SB – projectors (projecting rays);
 A_1B_1 – central projection of AB line segment onto the plane Π_1 .

Algorithm how to construct central projection of a figure:

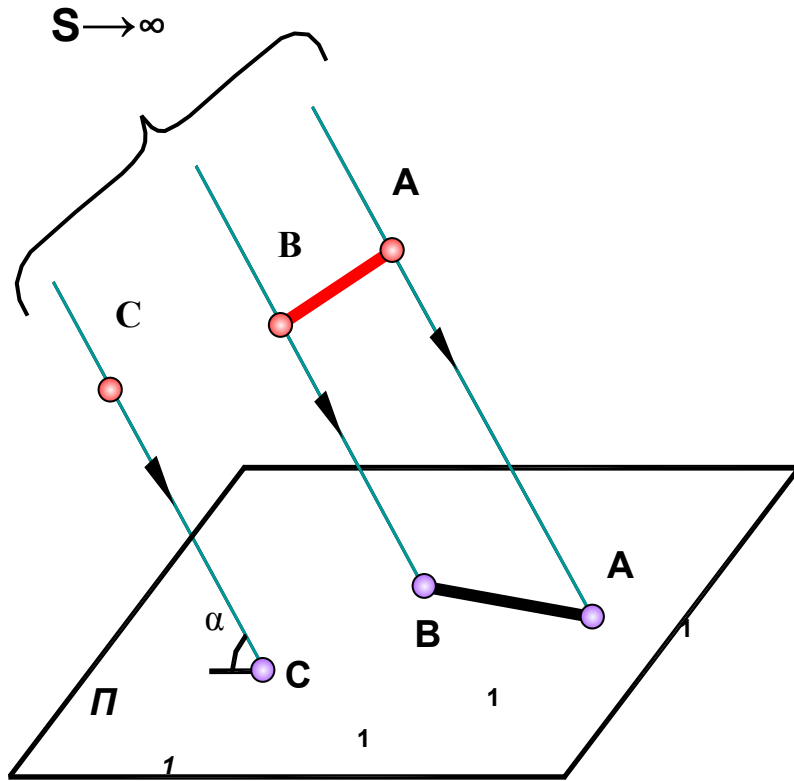
- From the point S draw a projecting ray through the any point of a figure;
- Find piercing point between projector and a plane of projection Π_1 . Obtained point is a projection (image) of an original to the plane of projection;
- Find projections of the points A_1 and B_1 , join them and obtain central projection of AB line segment.

Properties of Projection



1. Point projects to a point ($A \rightarrow A_1$);
2. Straight line projects to a straight line: ($[MK] \rightarrow [M_1K_1]$);
Exception to this property: if a straight line segment belongs to the projecting ray, then it projects to a point: ($[EF] \rightarrow E_1=F_1$);
3. Plane projects to a plane;
Exception to this property: if a plane contains projectors, then it projects to a straight line.
4. If the point belongs to some geometrical object, then projection of that point belongs to the projection of that geometrical object ($N \in [MK] \rightarrow N_1 \in [M_1K_1]$);

Parallel Projection



Parallel nonorthogonal projection.

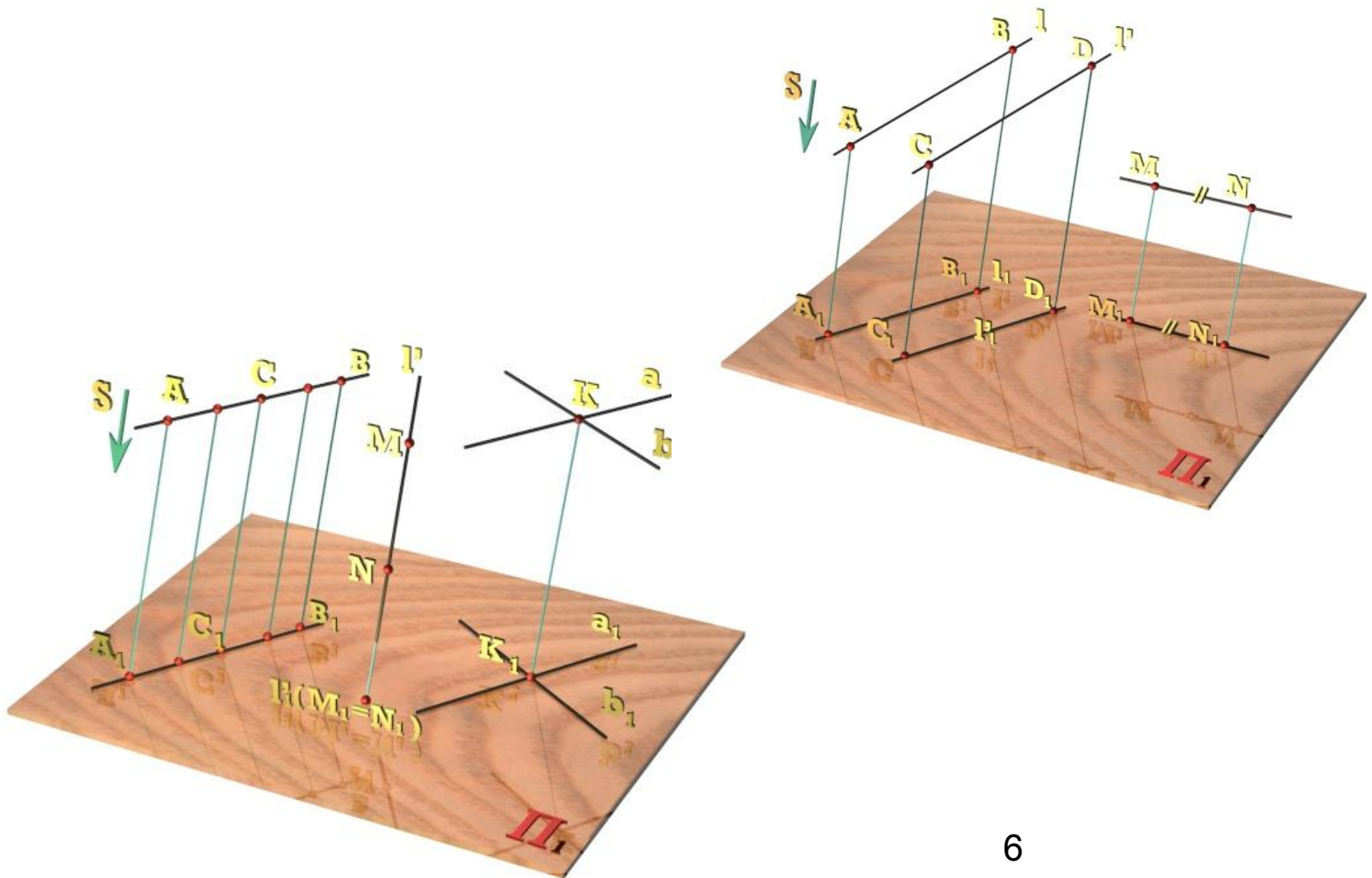
1. When center of projection S is a point at infinity, all projectors (AA1, BB1, CC1) are parallel. Such a projection is called parallel projection.

2. If angle between projectors and a plane of projection is less than 90° $\alpha \neq 90^\circ$, then it is parallel nonorthogonal projection.

All the properties of central projection preserved.

For this type of projection sizes of the image can exceed sizes of the original.

Parallel Projection



Properties of Parallel Projection

If projecting rays are perpendicular to the plane of projection ($\alpha=90^\circ$), then such a projection is called *orthogonal (orthographic) projection*.

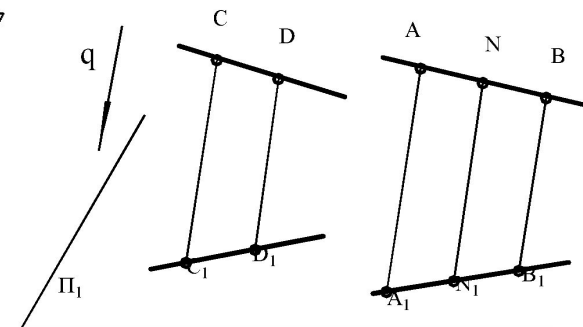
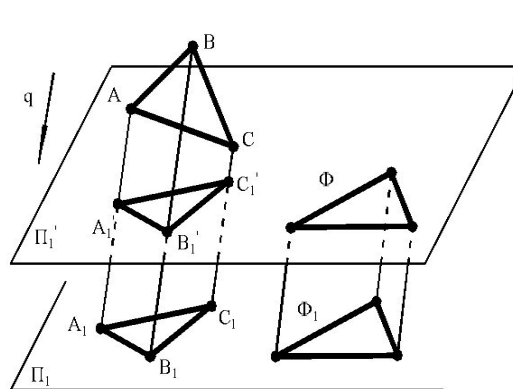
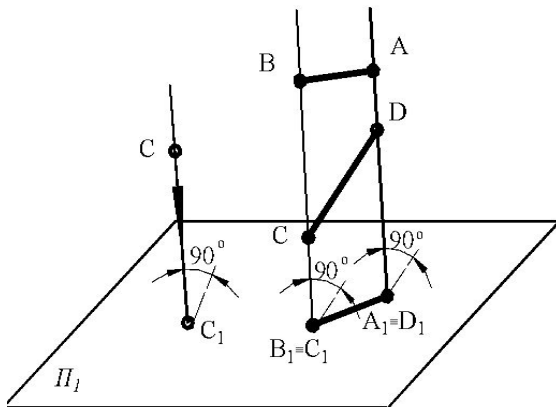
Properties:

1. Parallel lines project to parallel lines ($(CD) \parallel (AB) \rightarrow (C_1D_1) \parallel (A_1B_1)$).

2. Ratio between line segments of the same line or parallel lines preserves $\frac{[AN]}{[NB]} = \frac{[A_1N_1]}{[N_1B_1]}$

3. Planar figure, parallel to the plane of projection, projects to a true size onto that plane.

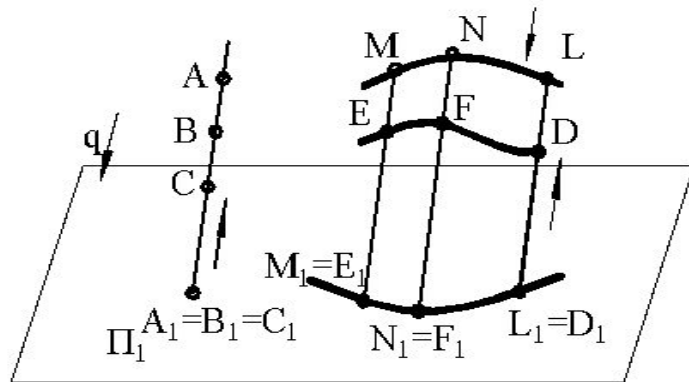
4. For orthographical projection sizes of the image can't exceed sizes of the original. If planar object is parallel to the plane of projection, then it projects to a true size.



Orthographical projection of a point

Images on the drawings should be reversible to imagine real shape of an object.

It's impossible to restore figure shape by only one orthographic projection.



Methods to obtain reversible images.

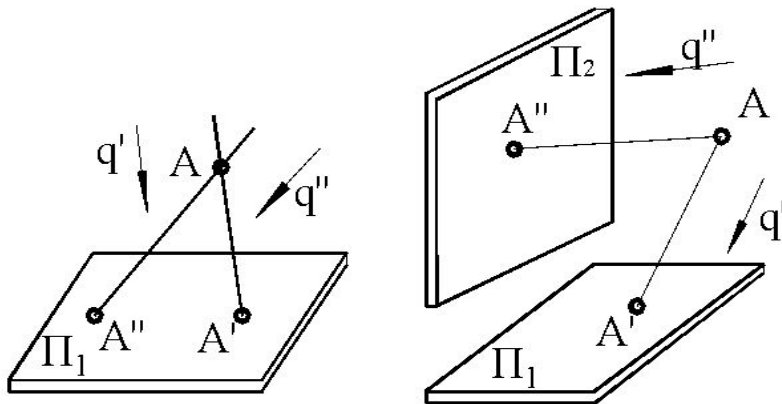
To obtain reversible images some additional terms have to be met. To meet these terms following methods can be used:

1. Orthographical projection on two planes (Monge's method);
2. Axonometric projection method;
3. Method of projection with elevations.
4. Vector constructions.

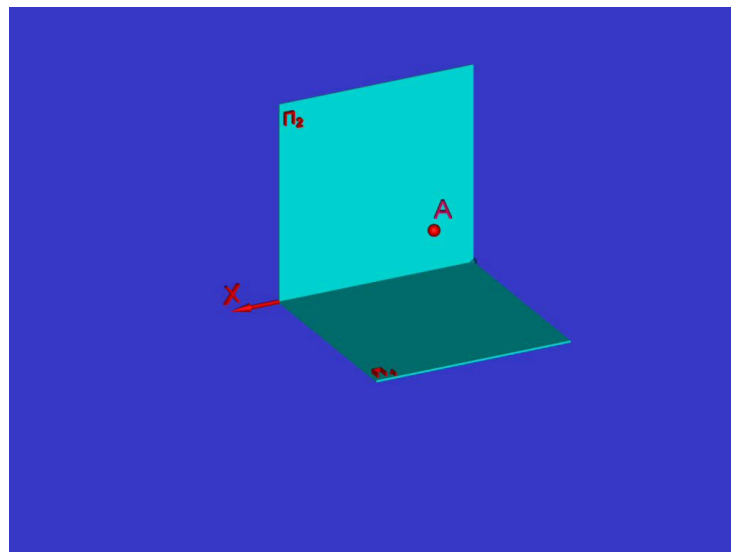
Orthographical projection of a point

Reversibility in projection

A point projects onto two directions or two different planes. Points of original object are obtained from intersection of corresponding projectors.



Monge's method



A point projects onto two mutually perpendicular principal planes (planes of projection). After that two principal planes are superposed by rotating one plane around the axes (which is a line of intersection between two planes of projection) up to coinciding to the other plane (rotating horizontal principal plane Π_1 around x-axis).

Monge's method

Designations

Π_1 – horizontal principal plane (plane of projection) - H;

Π_2 – frontal (vertical) principal plane (plane of projection) - F (V);

A_1 – horizontal projection of a point A;

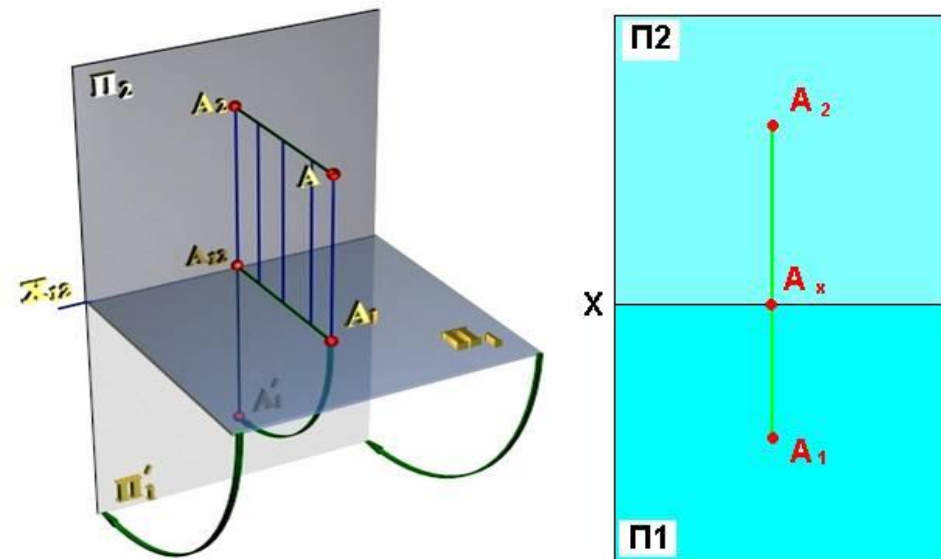
A_2 – frontal projection of a point A;

$A_x A_1$ – depth of A-point (distance from Π_2 -plane);

$A_x A_2$ – height of A-point (distance from Π_1 -plane);

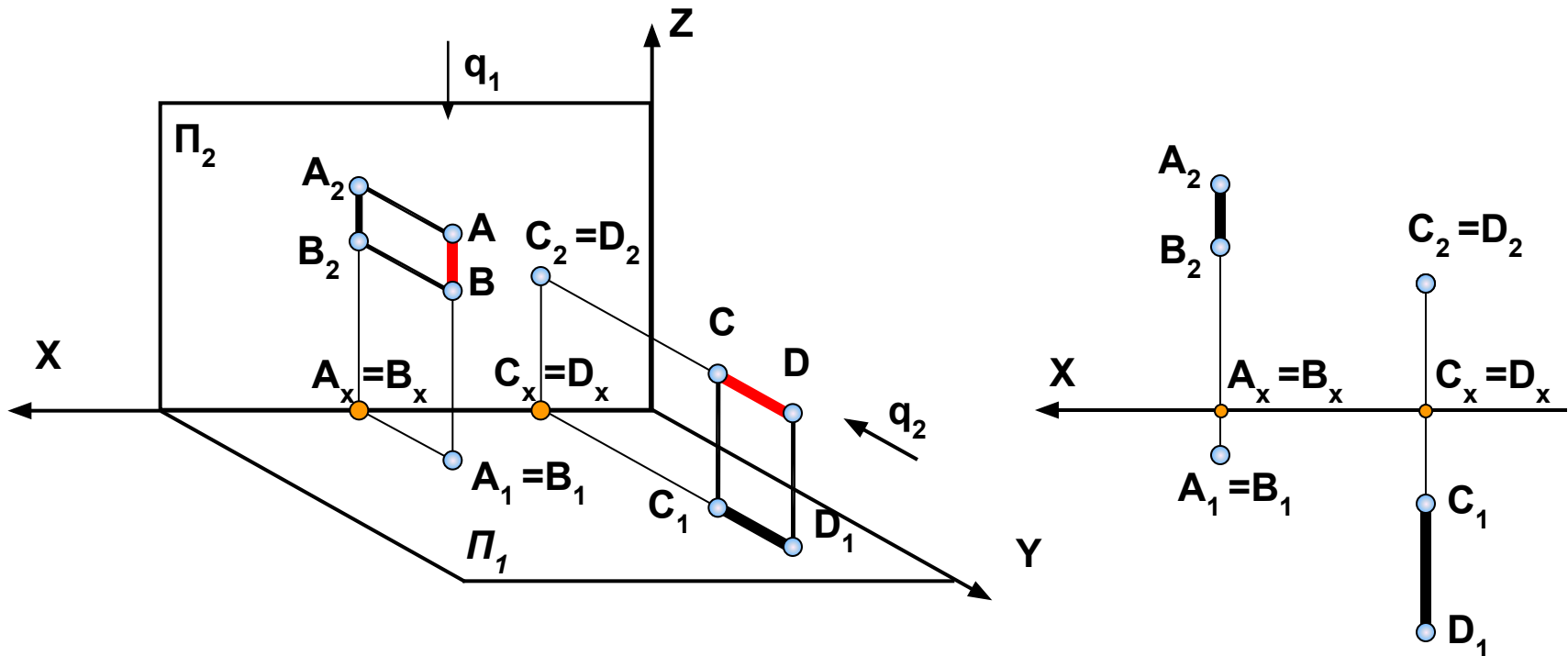
$A_1 A_2$ – projectors (projecting lines).

Horizontal and frontal projections of a point lie on the same vertical projector, perpendicular to x-axis.



Concurrent points

If projection of points are coinciding on the one principal plane, but different on the other plane, these points are called concurrent points.

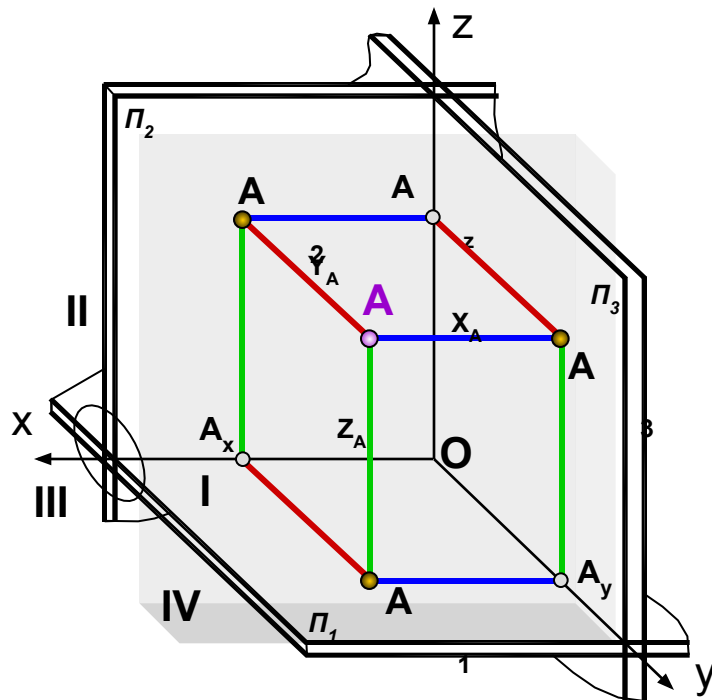


A and B — horizontally concurrent points
C and D — frontally concurrent points

Orthographical projection of a point

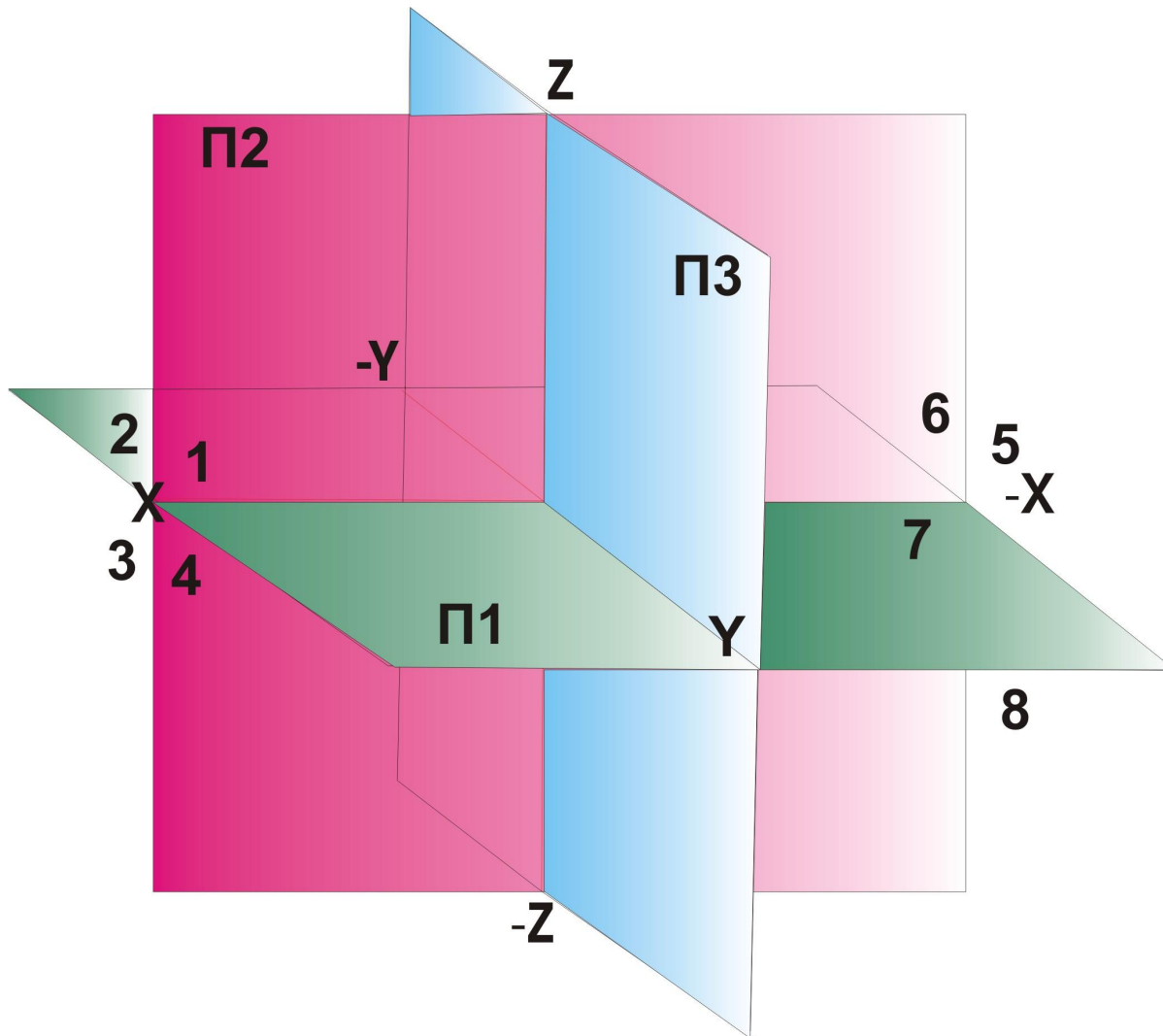
Some problem can be solved easier, if two principle planes of projection would be supplemented by the third plane, perpendicular to them. Such a plane is called profile plane of projection (Π_3). This principal planes is superposed by rotating around the axes (which is a line of intersection between two planes of projection) up to coinciding to the combined planes (rotating profile principal plane Π_1 around z-axis).

Coordinates of the point A: X_A, Y_A, Z_A — numeric values of line segments along coordinate axes.



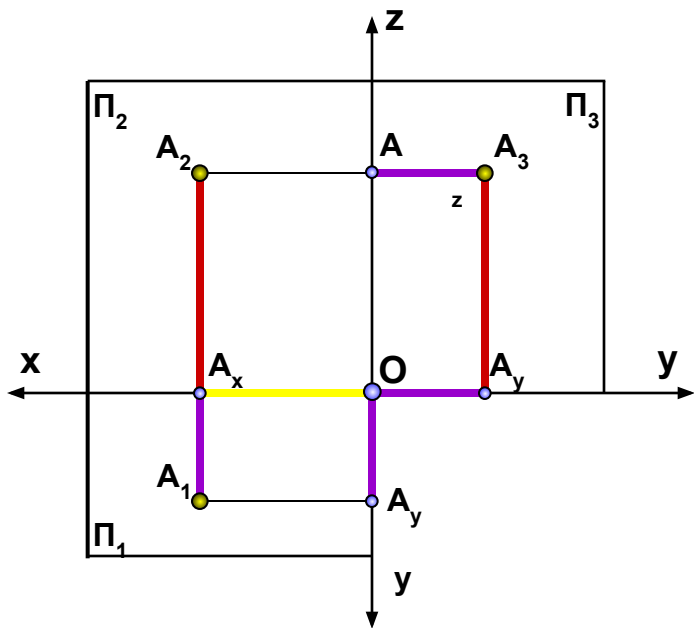
- Π_1 – horizontal principal plane (plane of projection) - H;
- Π_2 – frontal (vertical) principal plane (plane of projection - F (V);
- Π_3 – profile (vertical) principal plane (plane of projection – P(W);
- X, Y, Z – axes;
- A_1, A_2, A_3 – projections of A-point;
- $AA_3=XA; AA_2=YA; AA_1=ZA;$
- $A_x A_1$ – depth;
- $A_x A_2$ – height;
- $O A_x$ – width.

Principal planes of projection



octant	X	Y	Z
1	+	+	+
2	+	-	+
3	+	-	-
4	+	+	-
5	-	+	+
6	-	-	+
7	-	-	-
8	-	+	-

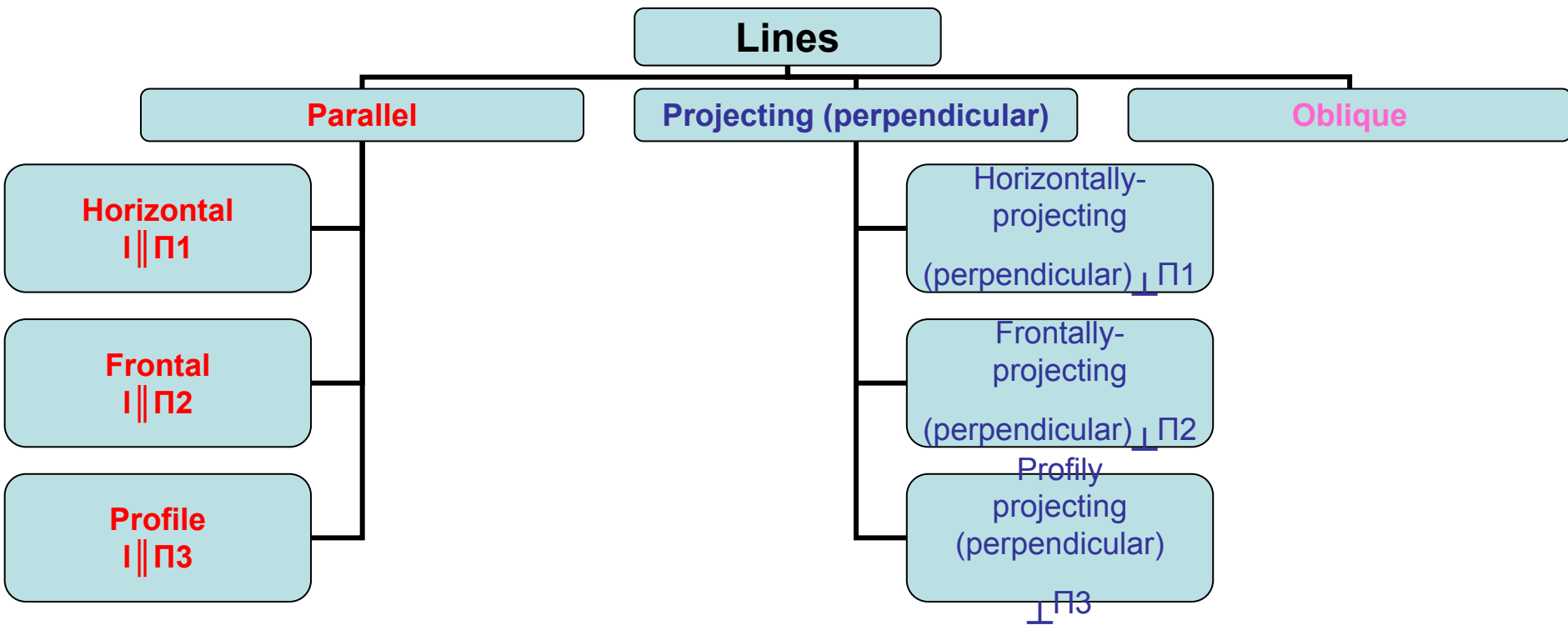
Orthographical projection of a point $A (X_A, Y_A, Z_A)$



1. Drop X_A coordinate (width) from origin point along X -axis, **designated A_x** ($OA_x = X_A$)
2. Construct vertical projector $\perp X$ -axis
3. Construct A_2 protracting Z_A - coordinate (height) along z -axis ($A_xA_2 = Z_A$)
4. Construct A_1 protracting Y_A - coordinate (depth) along y -axis ($A_xA_1 = Y_A$)
5. Construct horizontal projector $\perp Z$ -axis and find A_z .
6. Construct A_3 protracting Y_A - coordinate (depth) along horizontal y -axis ($A_zA_3 = Y_A$).

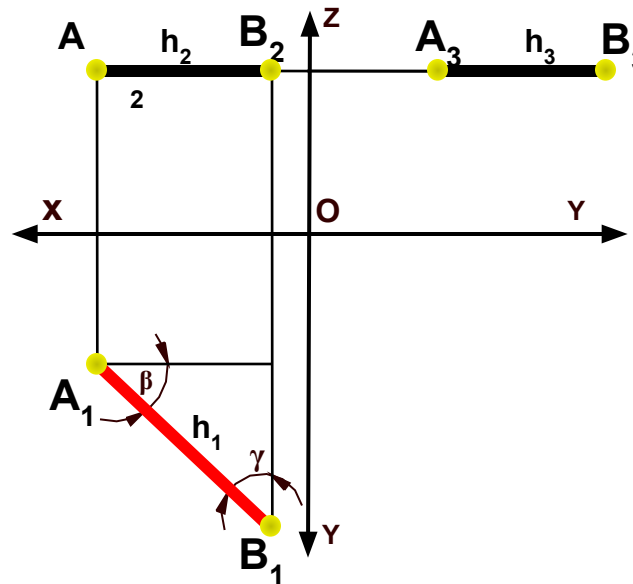
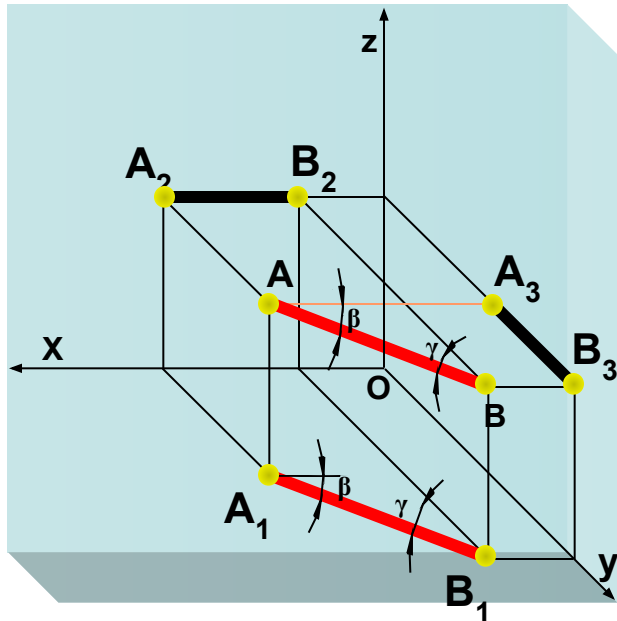
$A_2A_zA_3$ - Horizontal projector
 $A_2A_xA_1$ - Vertical projector

Straight Lines. Classification.



Orthographic projection of a Straight Line. Horizontal line

Horizontal (horizontally parallel) – straight line, parallel to Π_1 .



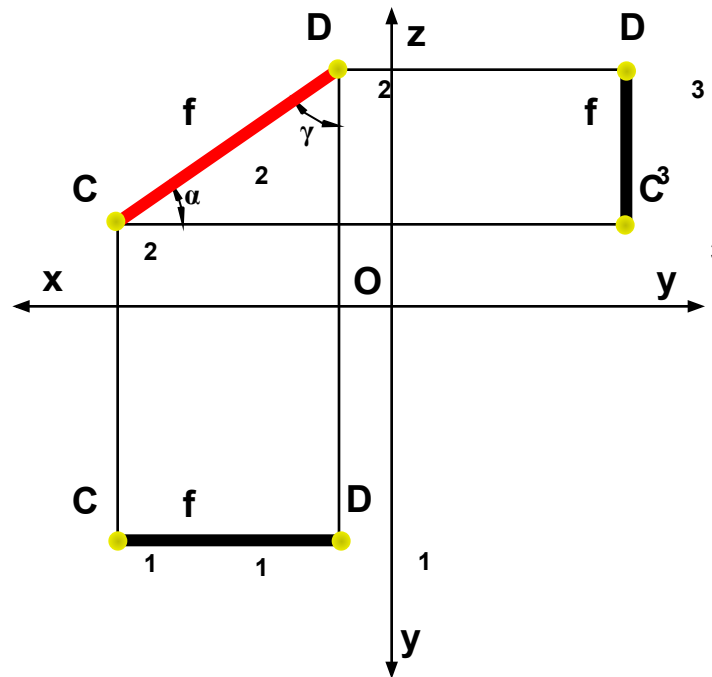
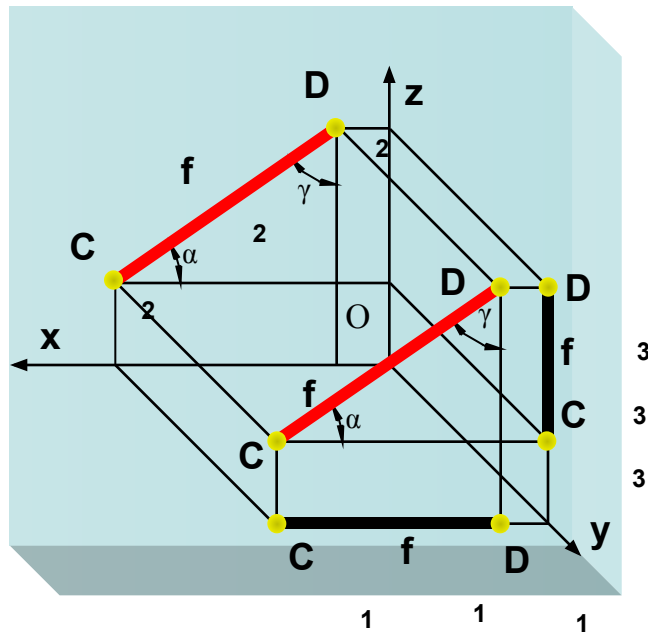
Horizontal line projects onto Π_1 to a True Length.

Angles β and γ on that projection are true angles between a line and principal planes of projection Π_2 and Π_3 .

Horizontal line projects onto Π_2 and Π_3 to horizontal line segments of a length, less than a true size.

Orthographic projection of a Straight Line. Frontal

Frontal (frontally parallel) – straight line, parallel to Π_2 .



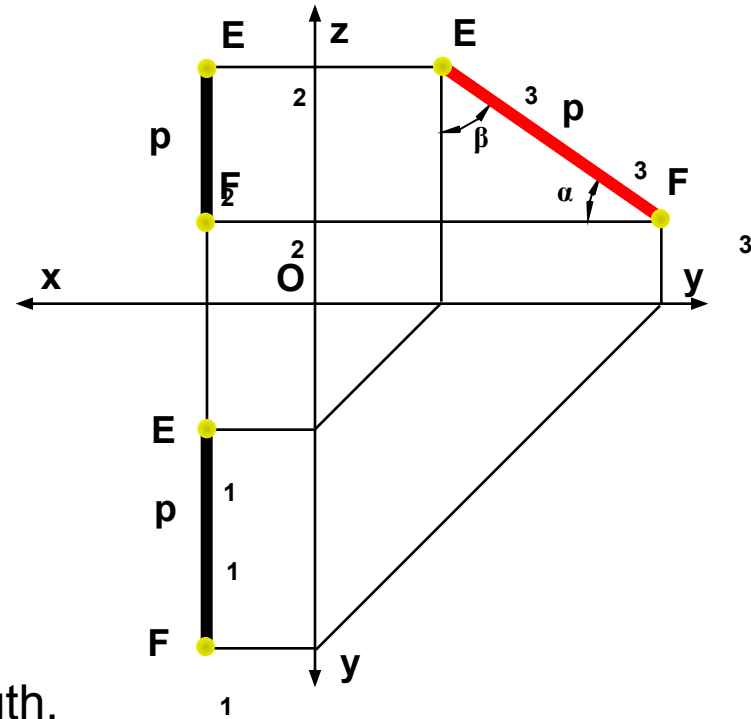
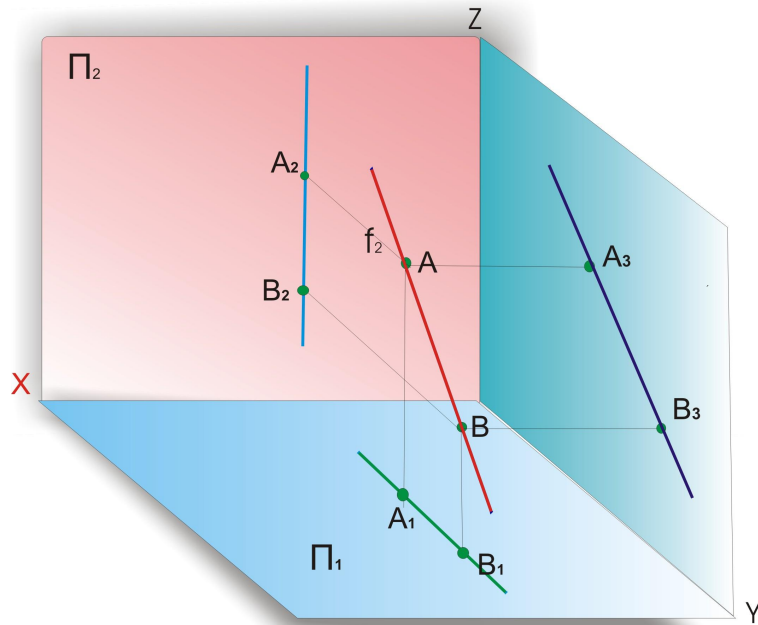
Frontal line projects onto Π_2 to a True Length.

Angles α and γ on that projection are true angles between a line and principal planes of projection Π_1 and Π_3 .

Frontal line projects onto Π_1 and Π_3 to vertical line segments of a length, less than a true size.

Orthographic projection of a Straight Line. Profile line

Profile (profilly parallel) – straight line, parallel to Π_3 .



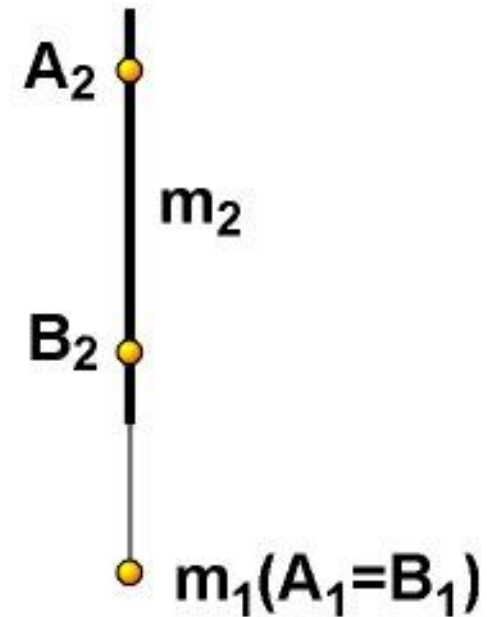
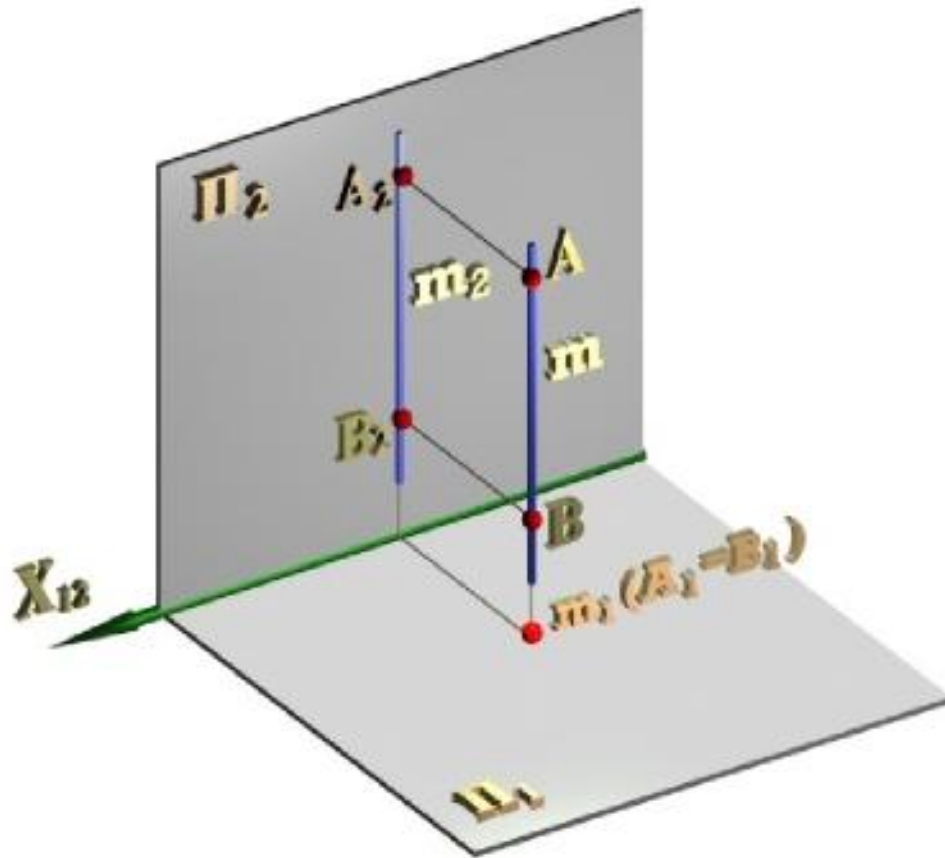
Frontal line projects onto Π_3 to a True Length.

Angles α and β on that projection are true angles between a line and principal planes of projection Π_1 and Π_2 .

Profile line projects onto Π_1 and Π_2 to vertical line segments of a length, less than a true size.

Orthographic projection of a Straight Line. Horizontally projecting (perpendicular) line

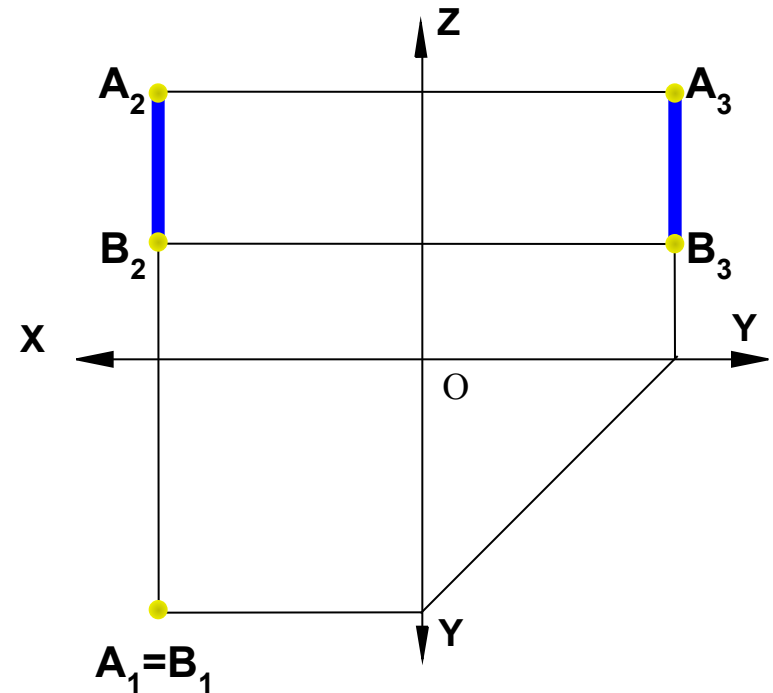
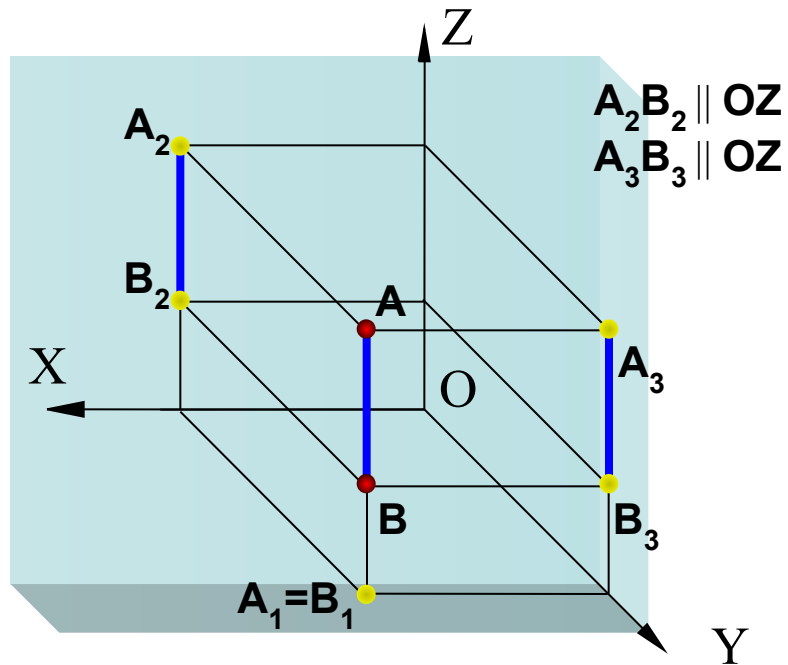
Horizontally-perpendicular line ($AB \perp \Pi_1$).



Line segment onto Π_1 projects to a point.

Orthographic projection of a Straight Line. Horizontally projecting (perpendicular) line

Horizontally-perpendicular line ($AB \perp \Pi_1$).

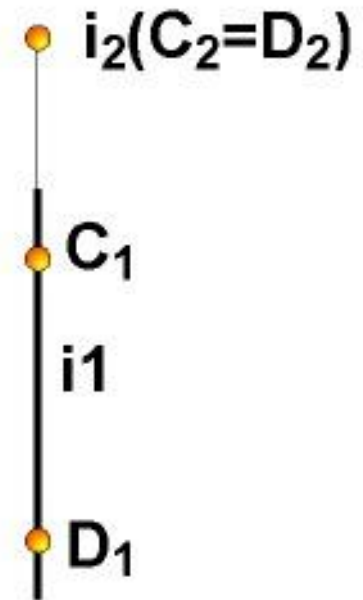
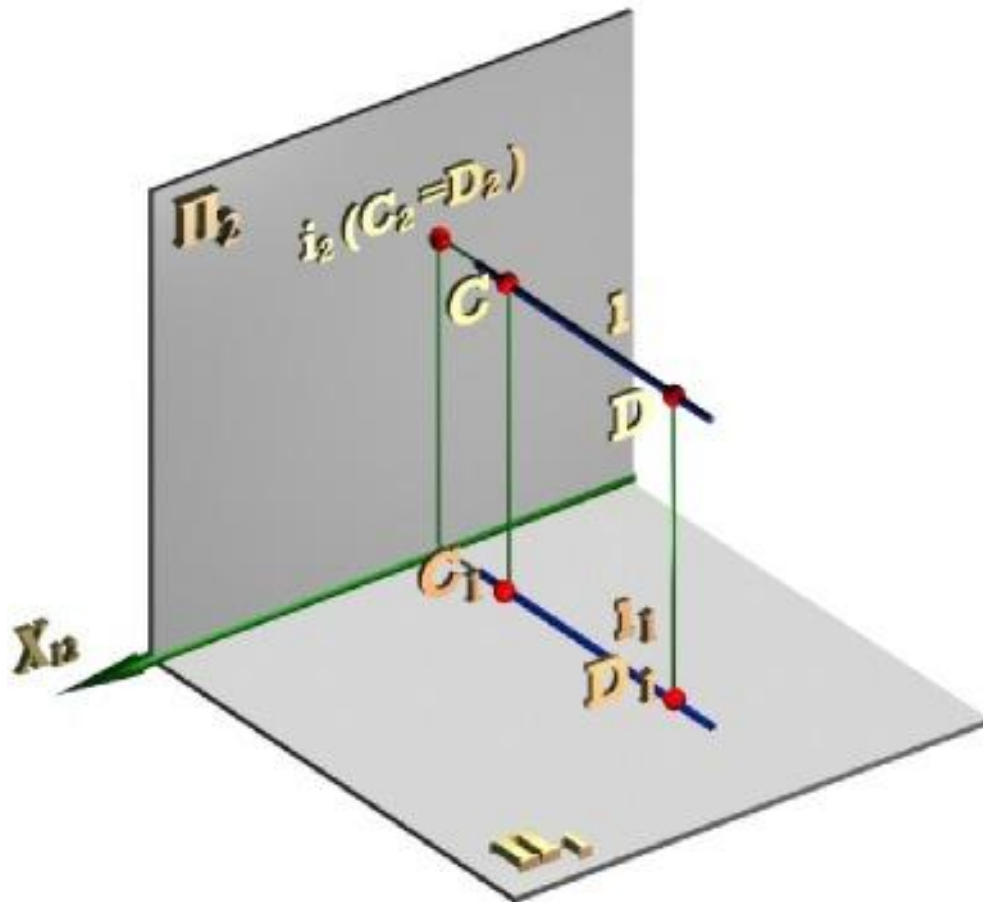


Line segment onto Π_1 projects to a point.

Projections on Π_2 and Π_3 are vertical lines of a true length.

Orthographic projection of a Straight Line. Frontally projecting (perpendicular) line

Frontally-perpendicular line ($CD \perp \Pi_2$).

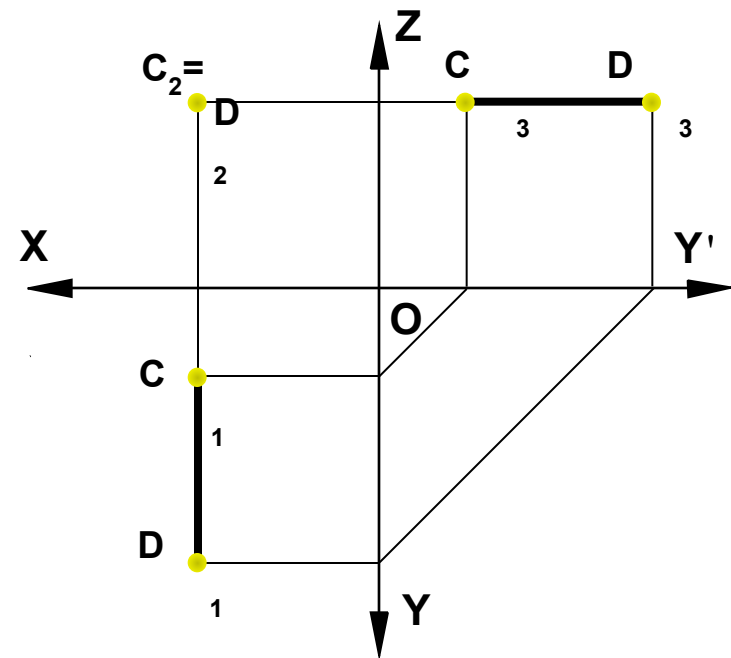
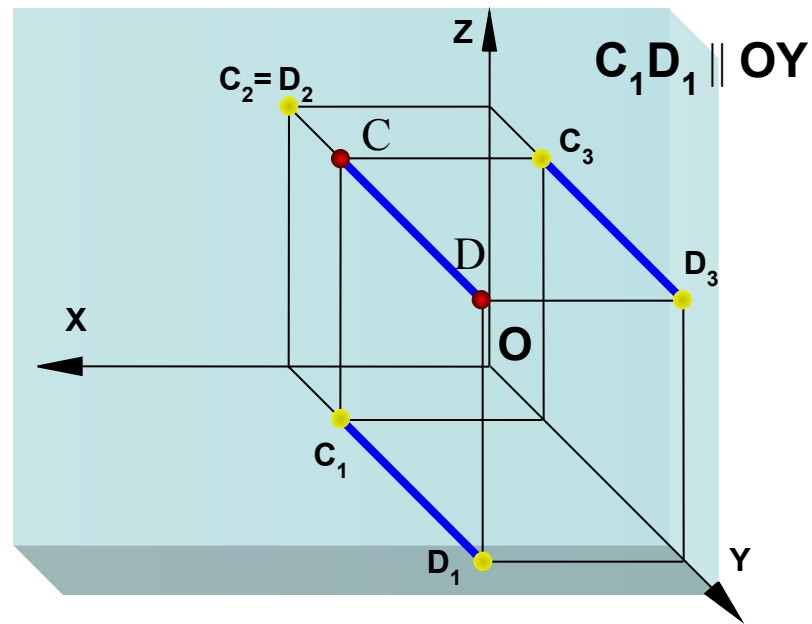


Line segment onto Π_2 projects to a point.

Orthographic projection of a Straight Line.

Frontally projecting (perpendicular) line

Frontally-perpendicular line ($CD \perp \Pi_2$).

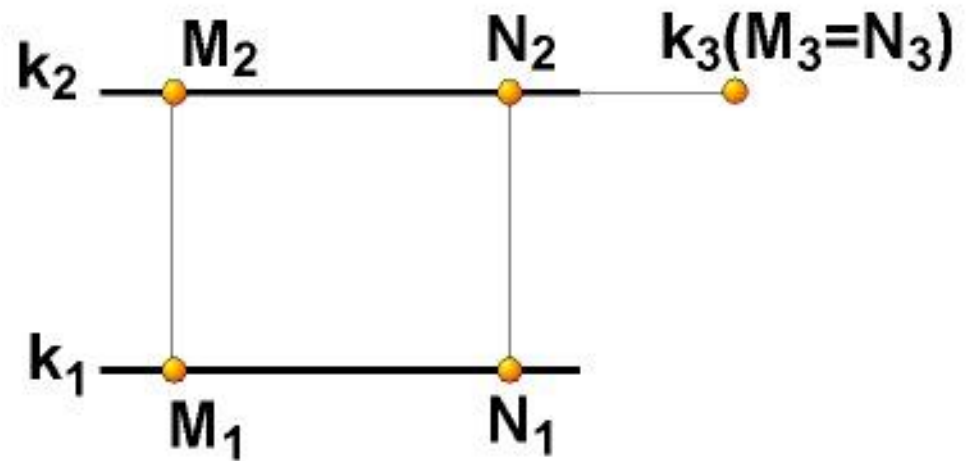
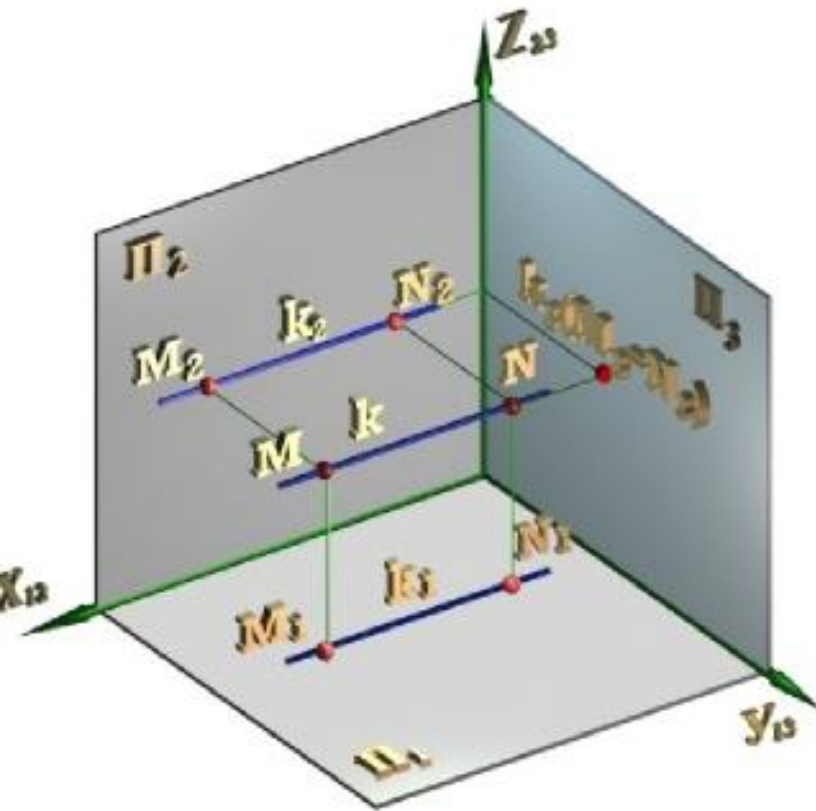


Line segment onto Π_2 projects to a point.
 Projections on Π_1 and Π_3 are lines of a true length.

Orthographic projection of a Straight Line.

Profile projecting (perpendicular) line

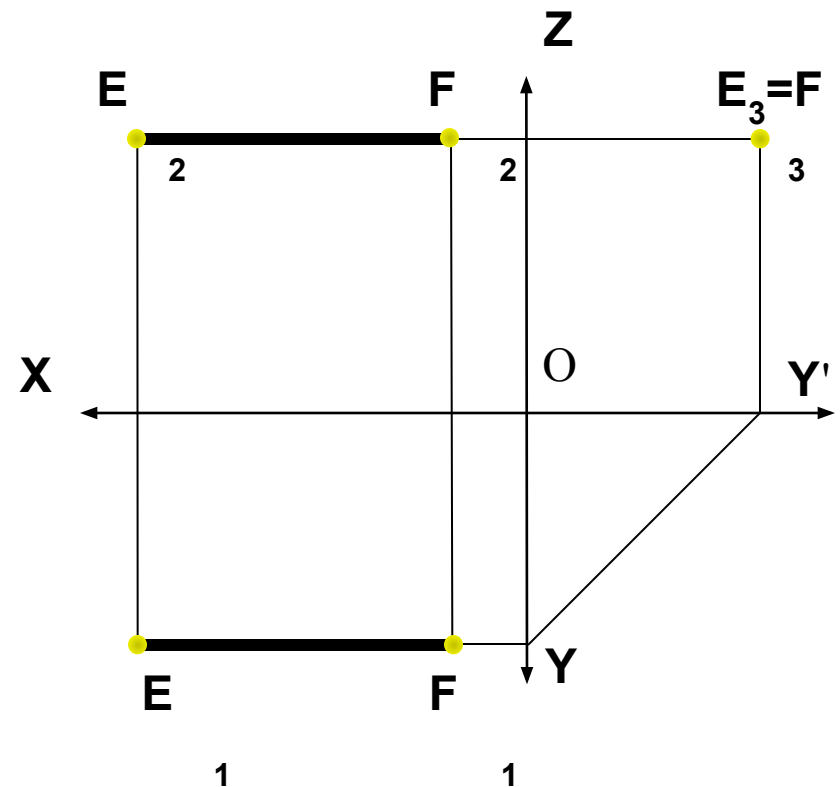
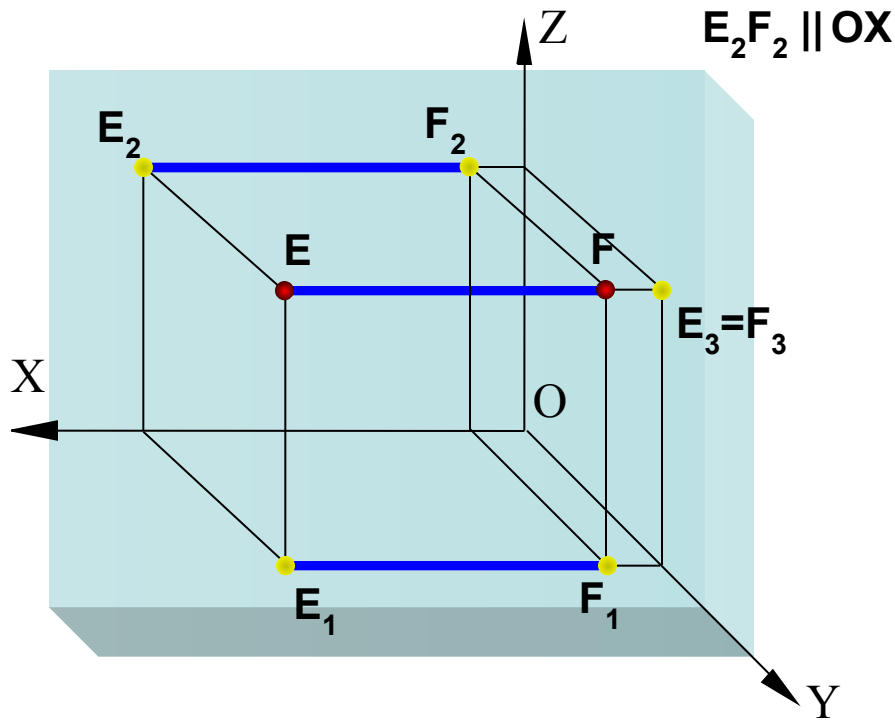
Profile-perpendicular line ($MN \perp \Pi_3$).



Line segment onto Π_3 projects to a point.

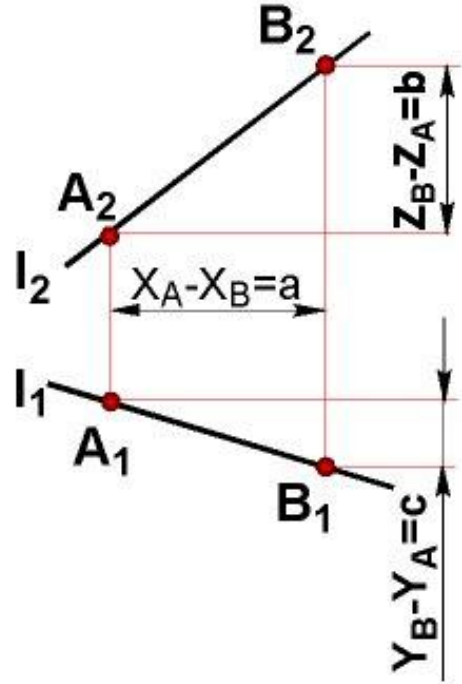
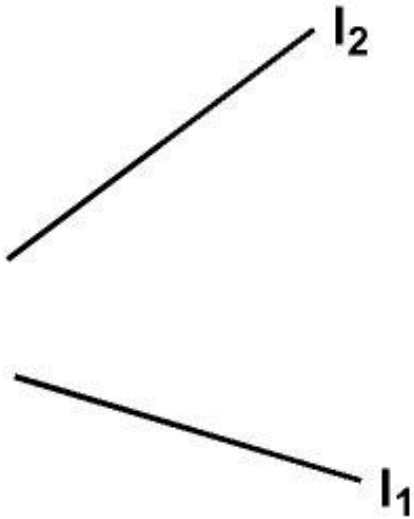
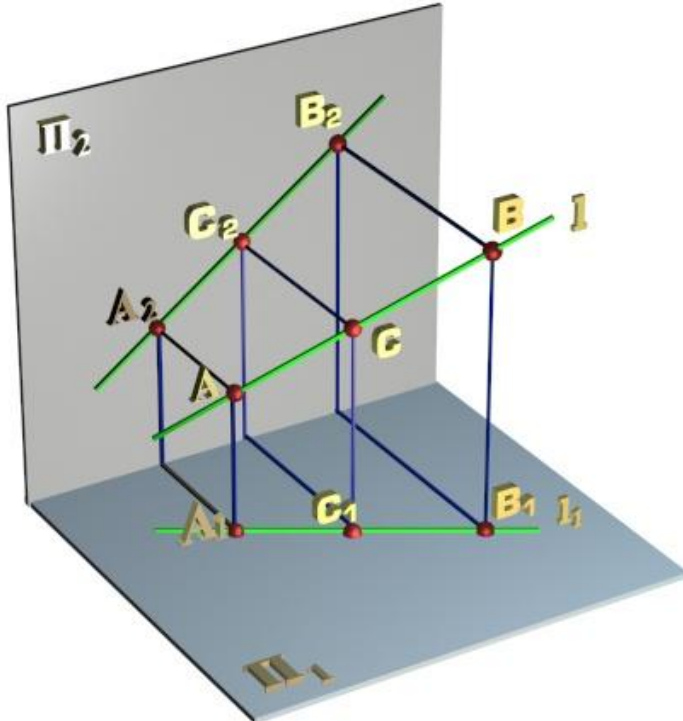
Orthographic projection of a Straight Line. Profile projecting (perpendicular) line

Profile-perpendicular line ($MN \perp \Pi_3$).

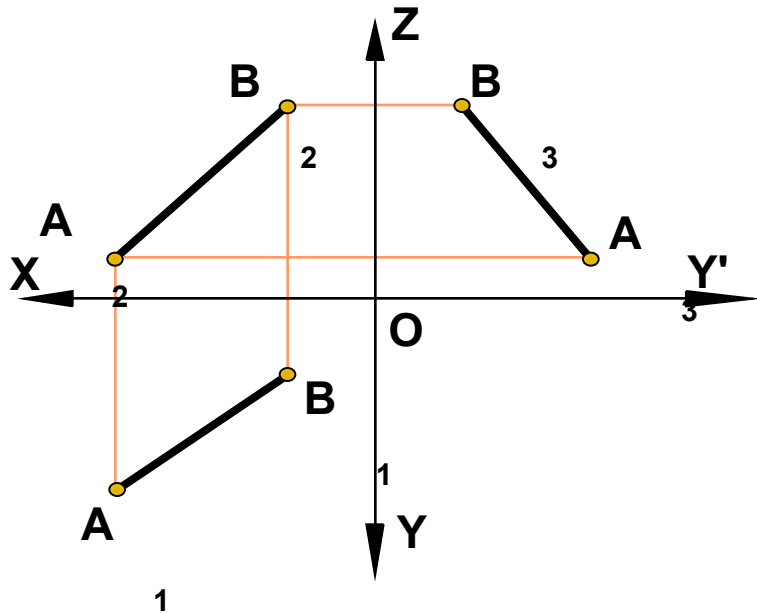


Line segment onto Π_3 projects to a point.
Projections on Π_2 and Π_3 are lines of a true length.

Oblique Line



Oblique Line



Oblique Line neither parallel nor perpendicular to any principal plane of projection.

Oblique line projects onto principal planes Π_1 , Π_2 and Π_3 to line segments of a length, **less** than a true size.



Relative Position of a Line and a Plane

Relative position of a Line and a Plane

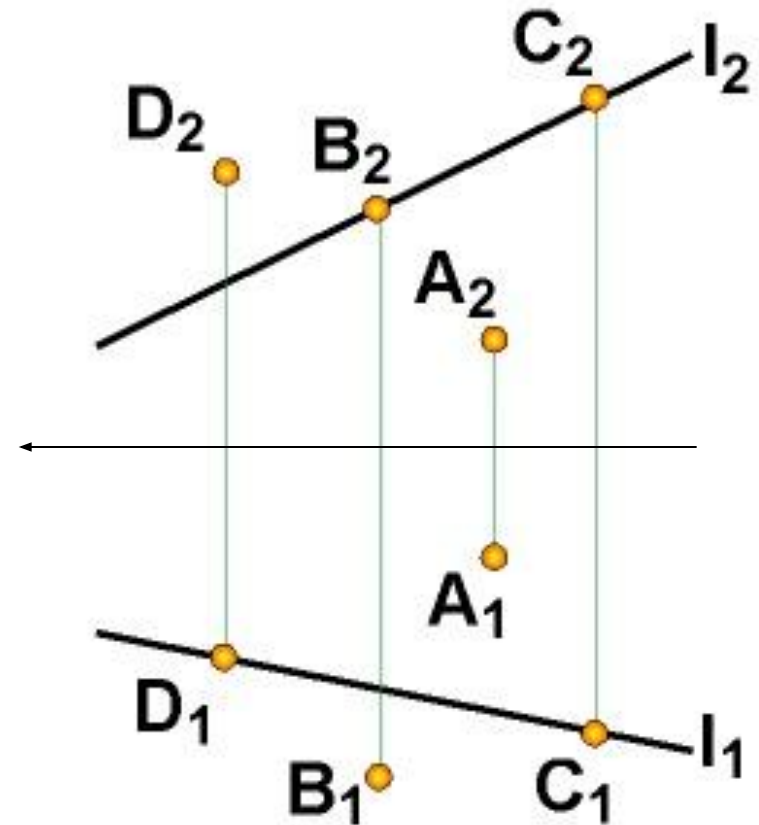
Two variants:

1) Point belongs to a Line.

From the property of a projection it is known: if a point belongs to a line, then projection of that point would belong to the projection of that line.

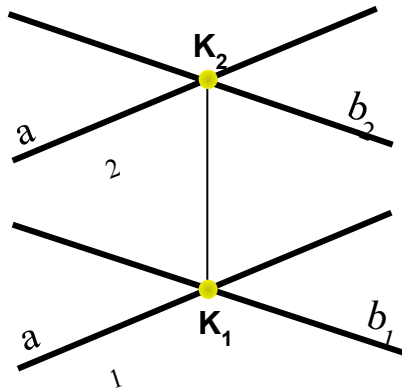
2) Point does not belong to a Line.

In this case relating to the planes of projection a point can be: **above**, **below**, **in front of**, **behind**.



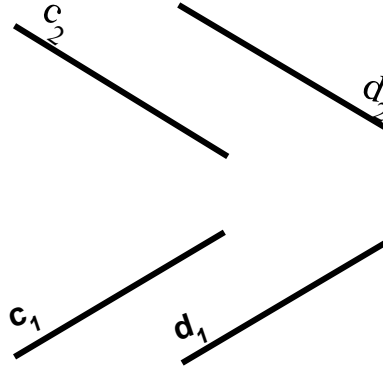
Relative Position of Lines

Intersecting



K_1 and K_2 are on the same projector

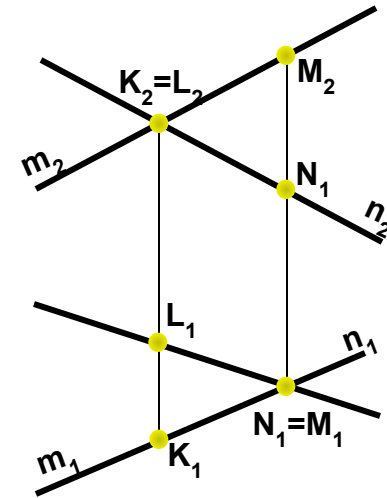
Parallel



Proper projections are parallel.

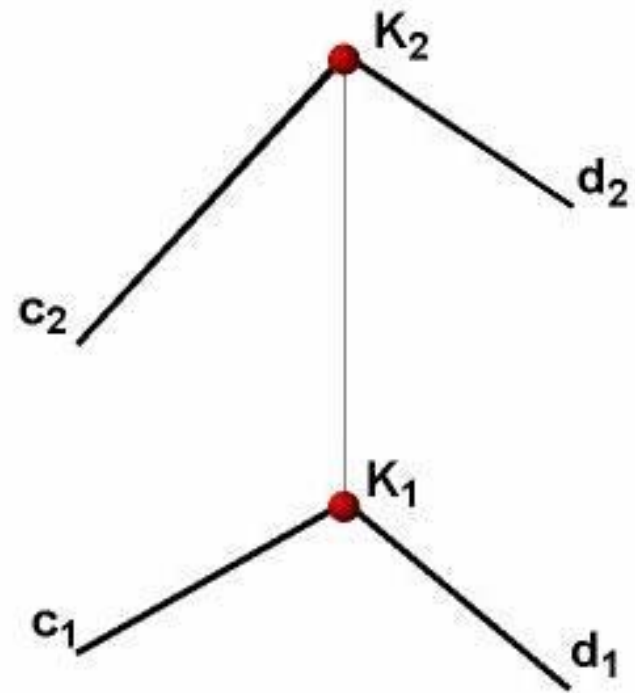
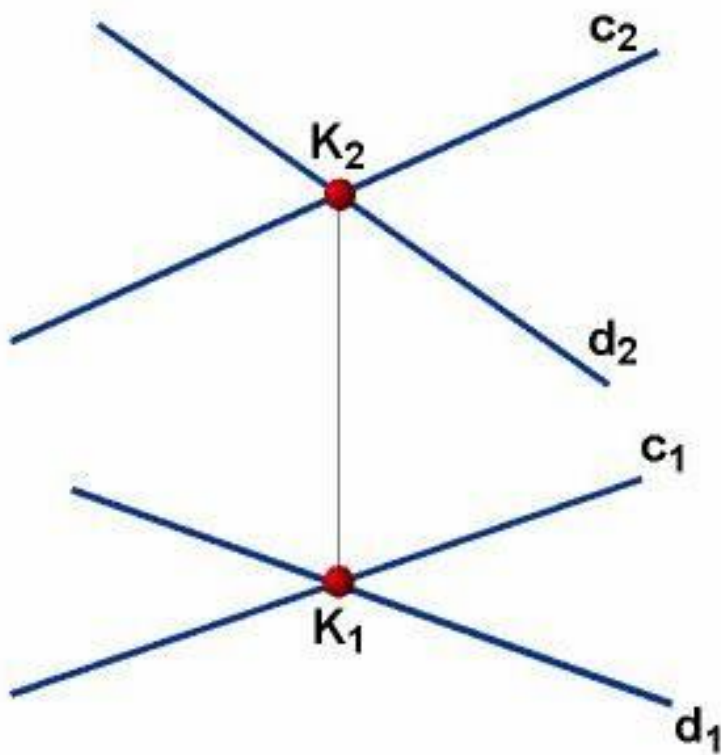
Lines are intersecting in a point at infinity

Skew (Crossing)

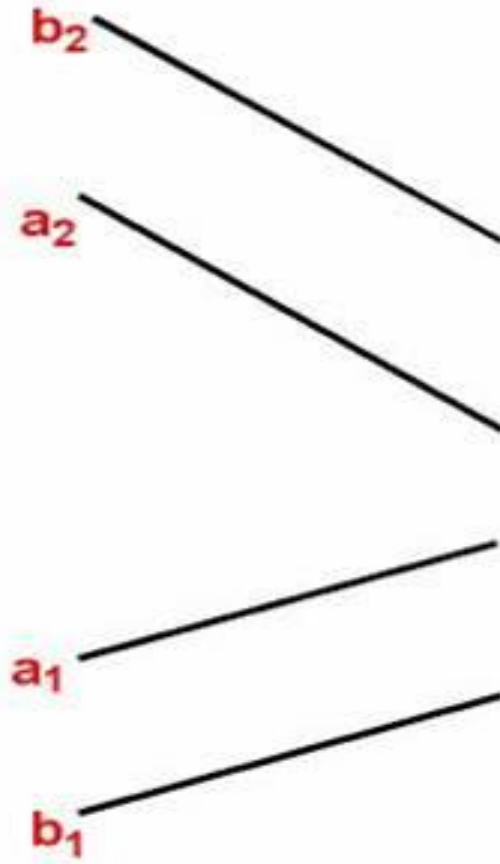


K and L , M and N – concurrent points

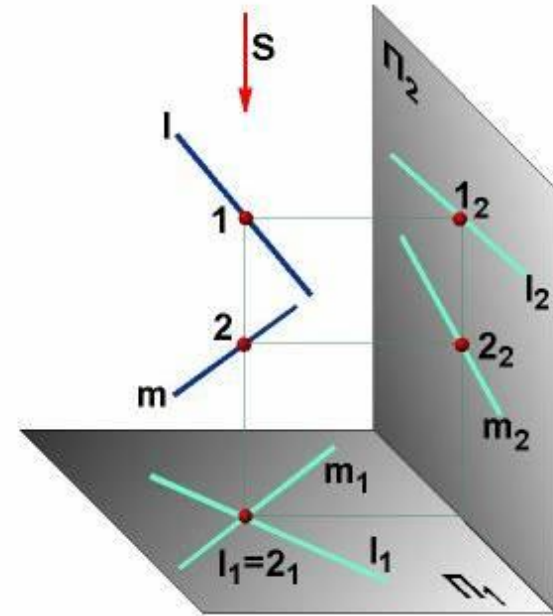
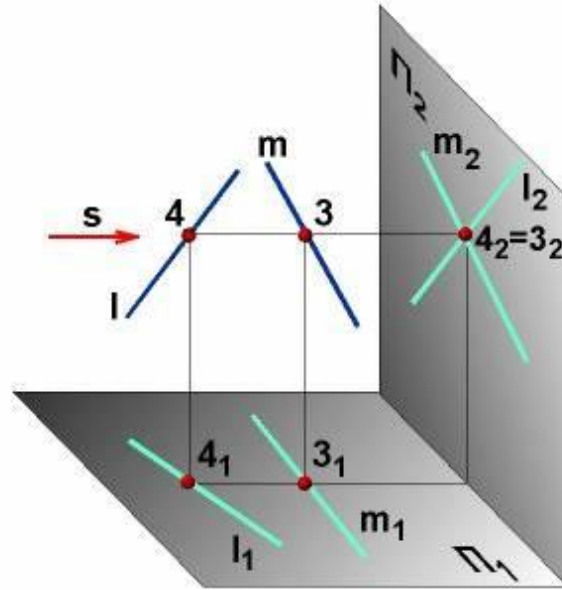
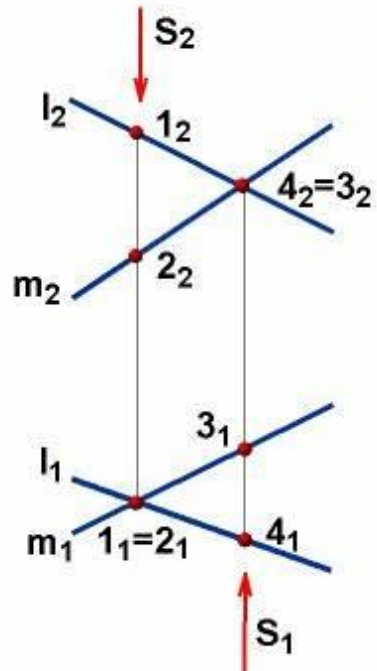
Intersecting $c \cap d = K$



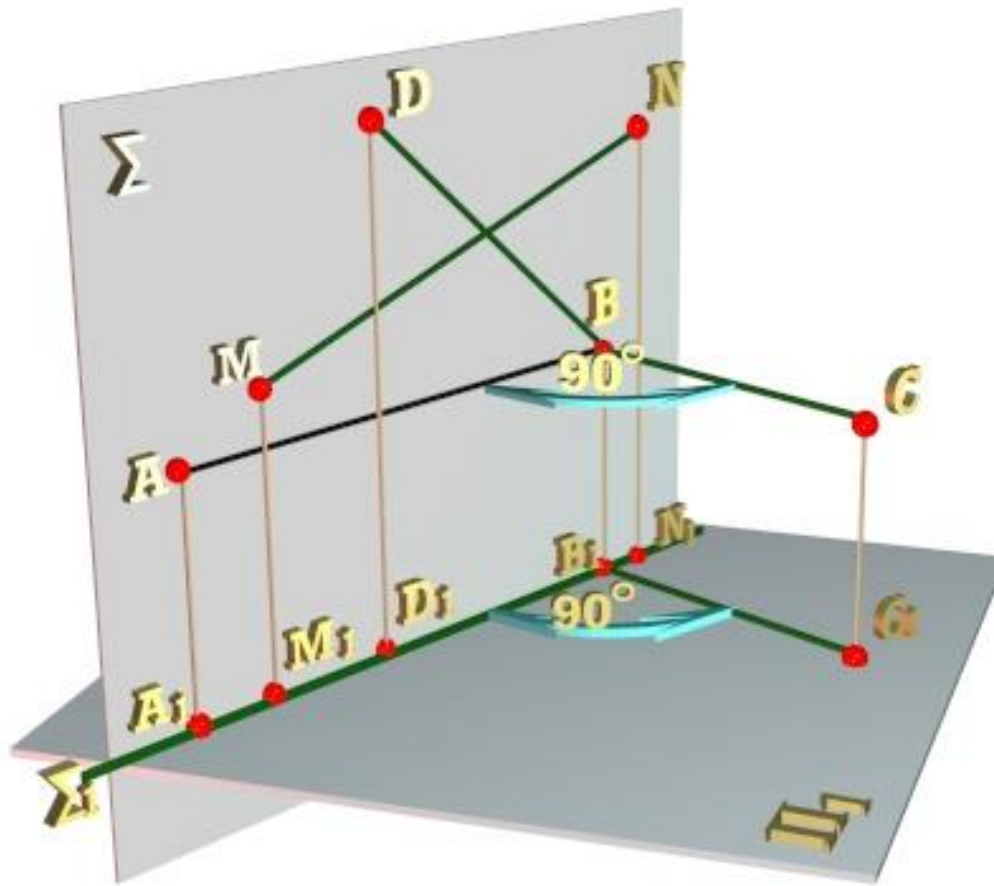
Parallel $a \parallel b$



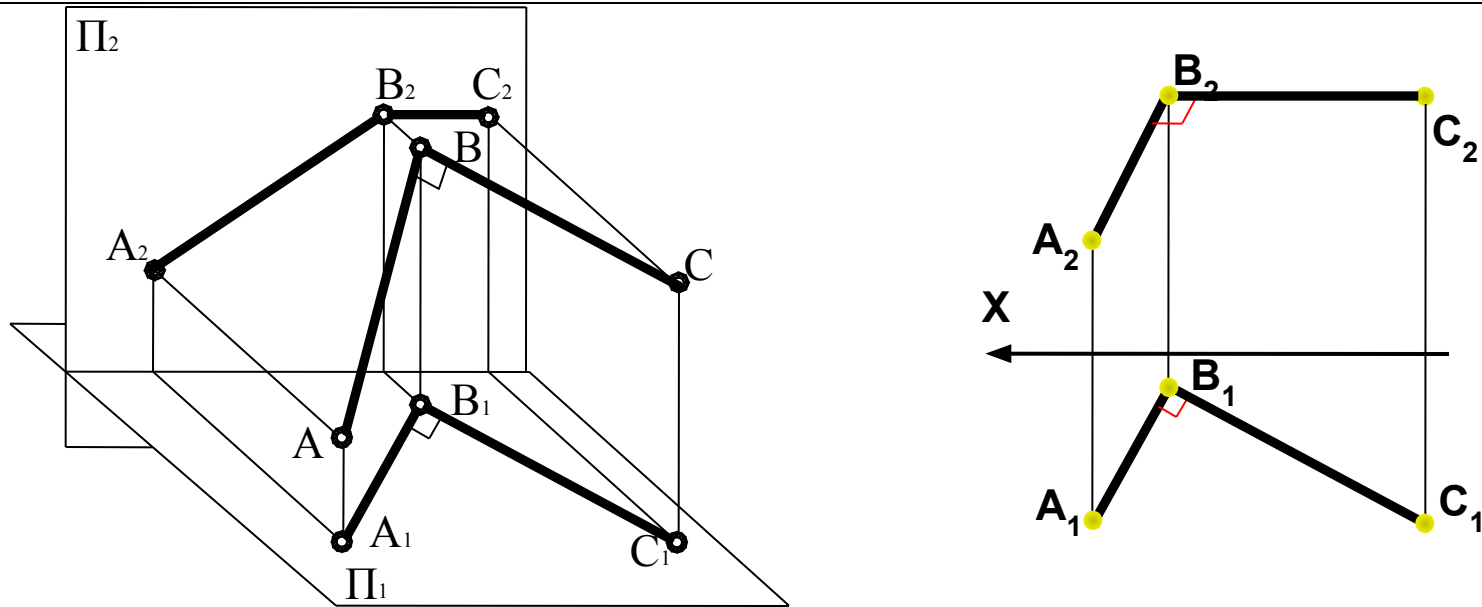
Skew (Crossing) \circ



Projecting of a Right Angle - Theorem.



Projecting of a Right Angle - Theorem.



Projections of angles depend on their location concerning the planes of projection. According to the properties of parallel projection their types and sizes are not preserved.

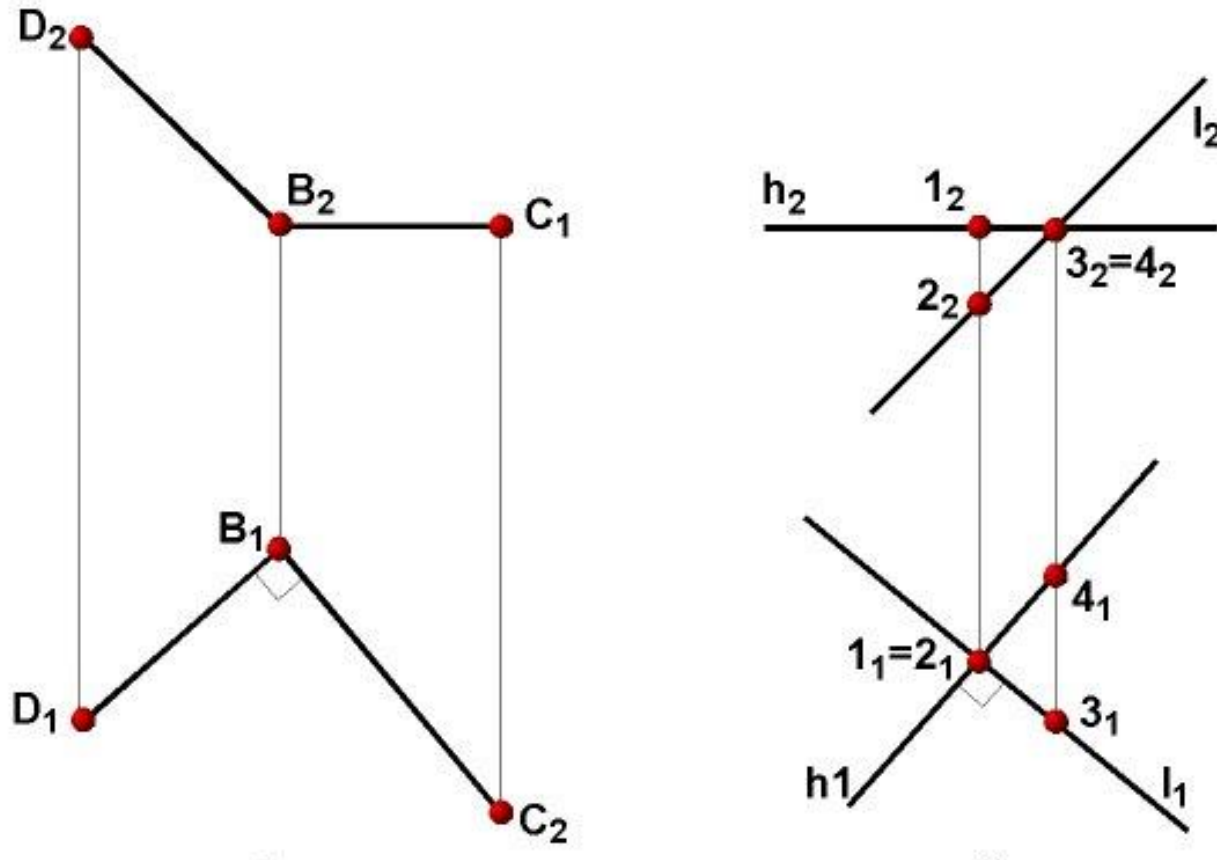
Exception: If one side of an angle is parallel to a plane and the other is not perpendicular to that plane, then acute angle projects to acute angle, obtuse angle projects to obtuse angle and right angle projects to a right angle onto that plane.

The last statement is known as a *theorem about projecting of a right angle*.

It can be formulated this way:

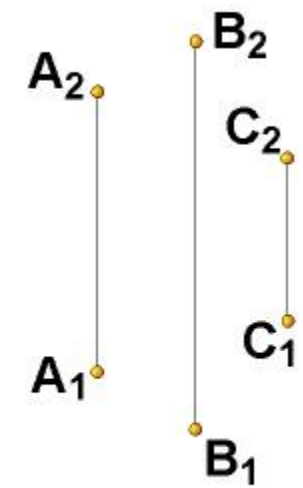
If one side of a right angle is parallel to a plane and the other is not perpendicular to that plane, then right angle projects to a true size (to a right angle) onto that plane.

Projecting of a Right Angle - Theorem.



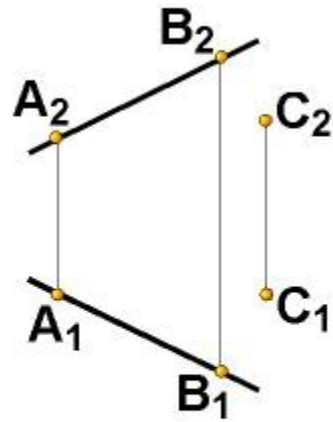
If one side of a right angle is parallel to a plane and the other is not perpendicular to that plane, then right angle projects to a true size (to a right angle) onto that plane.

Orthographic projection of a Plane. Representation of a Plane.



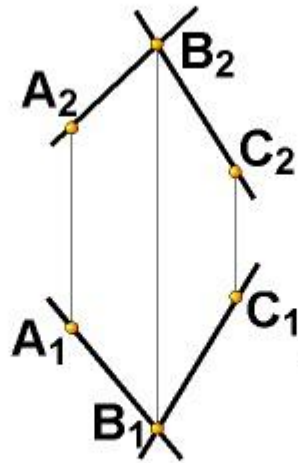
a

Three Points



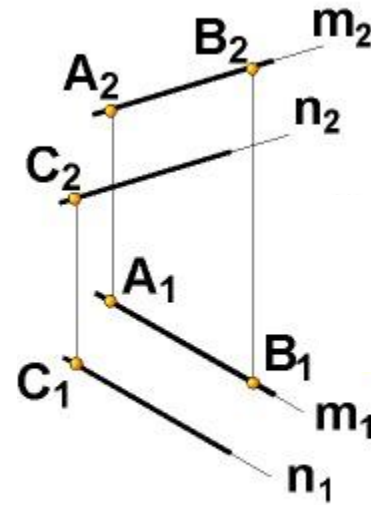
b

A Point and
a Line



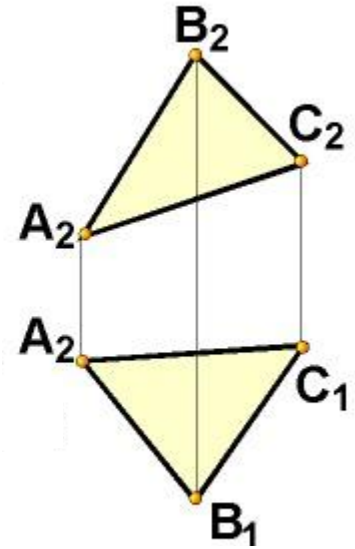
c

Two
Intersecting
Lines



d

Two Parallel Lines



e

Plane Segment



Planes. Classifications.

PLANES

Parallel $\alpha \parallel \Pi_i$

Projecting
(perpendicular) $\alpha \perp \Pi_i$

Oblique

Horizontal $\alpha \parallel \Pi_1$

Frontal $\alpha \parallel \Pi_2$

Profile $\alpha \parallel \Pi_3$

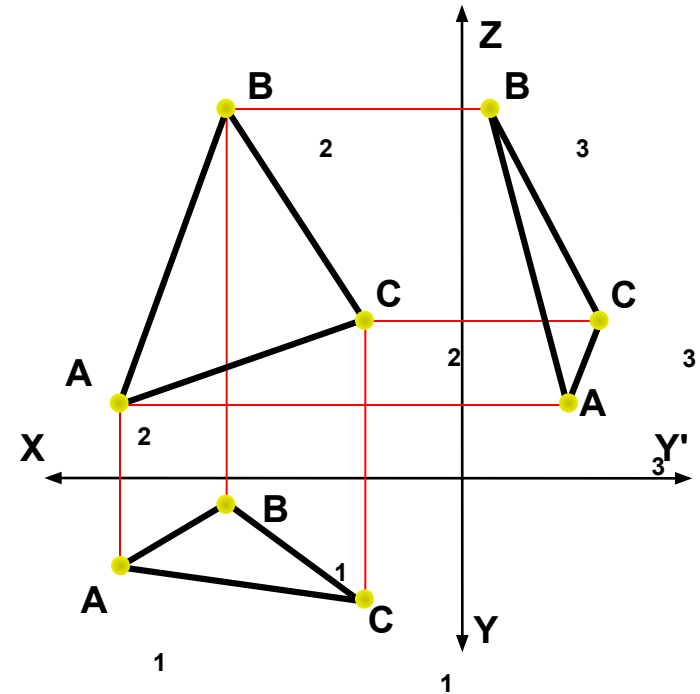
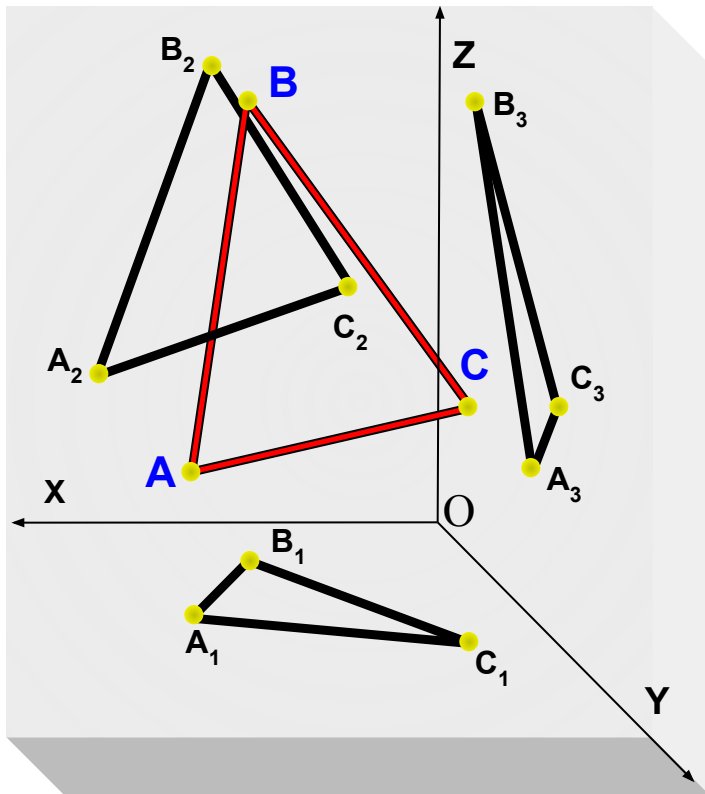
Horizontally-projecting
(perpendicular) $\alpha \perp \Pi_1$

Frontally-projecting
(perpendicular) $\alpha \perp \Pi_2$

Profile-projecting
(perpendicular) $\alpha \perp \Pi_3$

Oblique Plane

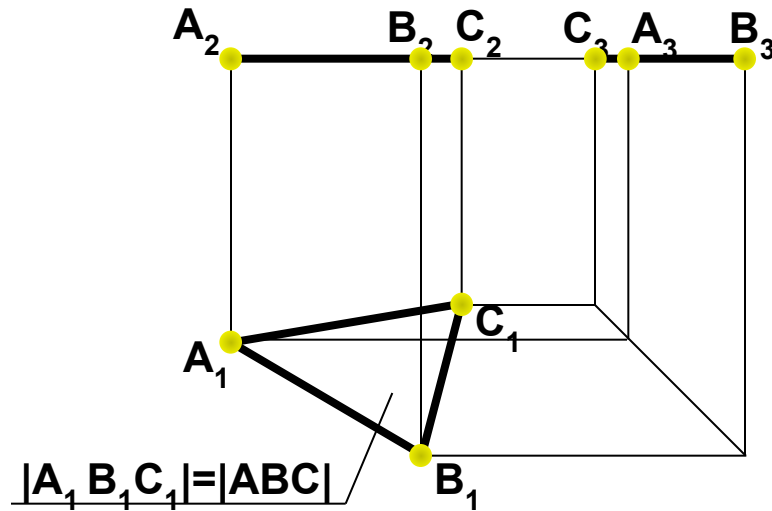
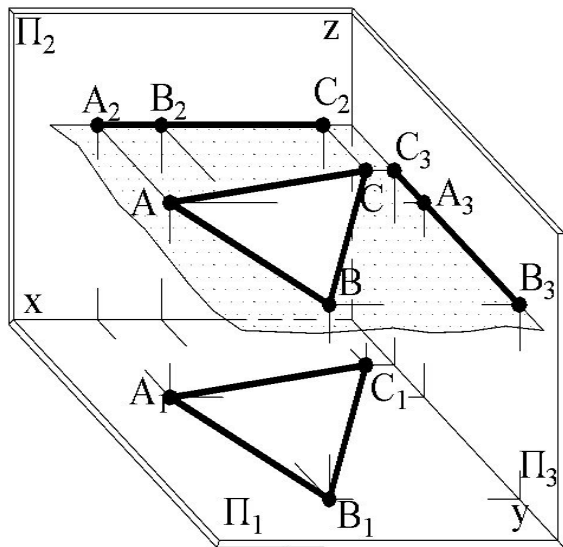
Oblique plane neither parallel nor perpendicular to any principal plane of projection.



Principal Planes.

Horizontal (principal) plane ($(ABC) \parallel \Pi_1$) is a plane, parallel to horizontal principal plane of projection.

Any planar figure, which lies in this plane, projects onto the horizontal principal plane Π_1 to a true size.

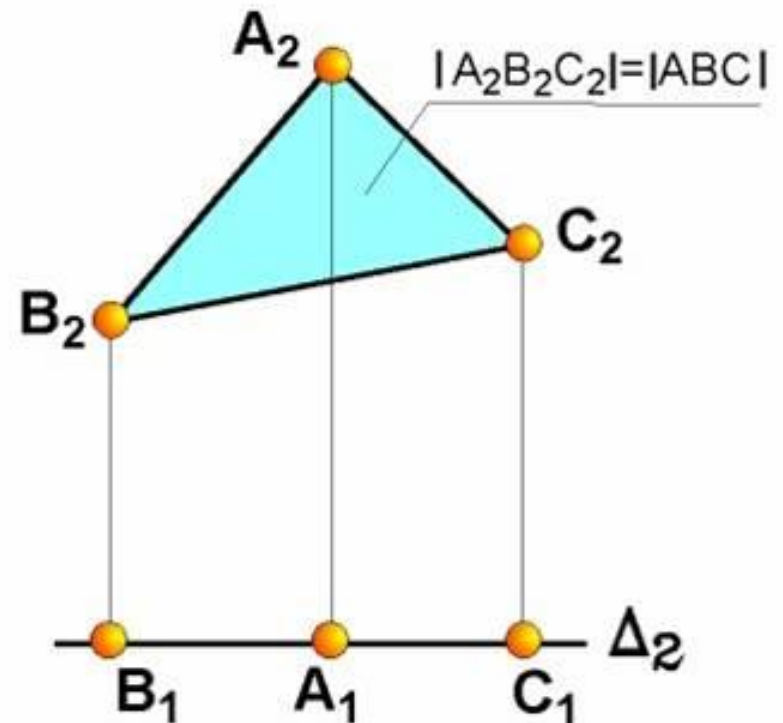
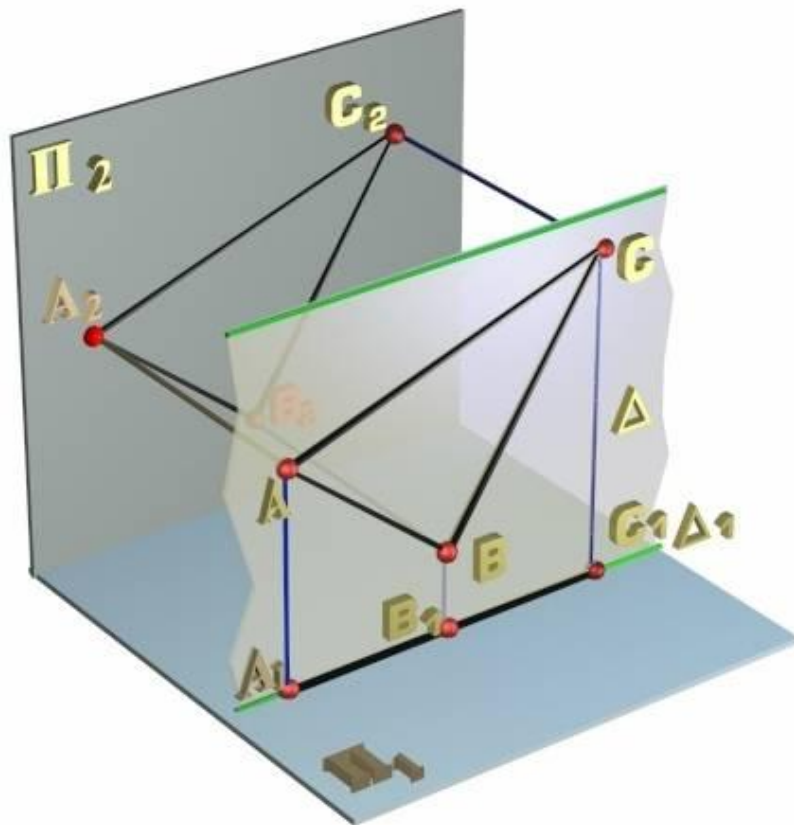


Frontal and Profile planes have similar properties.

Principal Planes.

Frontal (principal) plane ($(ABC) \parallel \Pi_2$) is a plane, parallel to the frontal principal plane of projection.

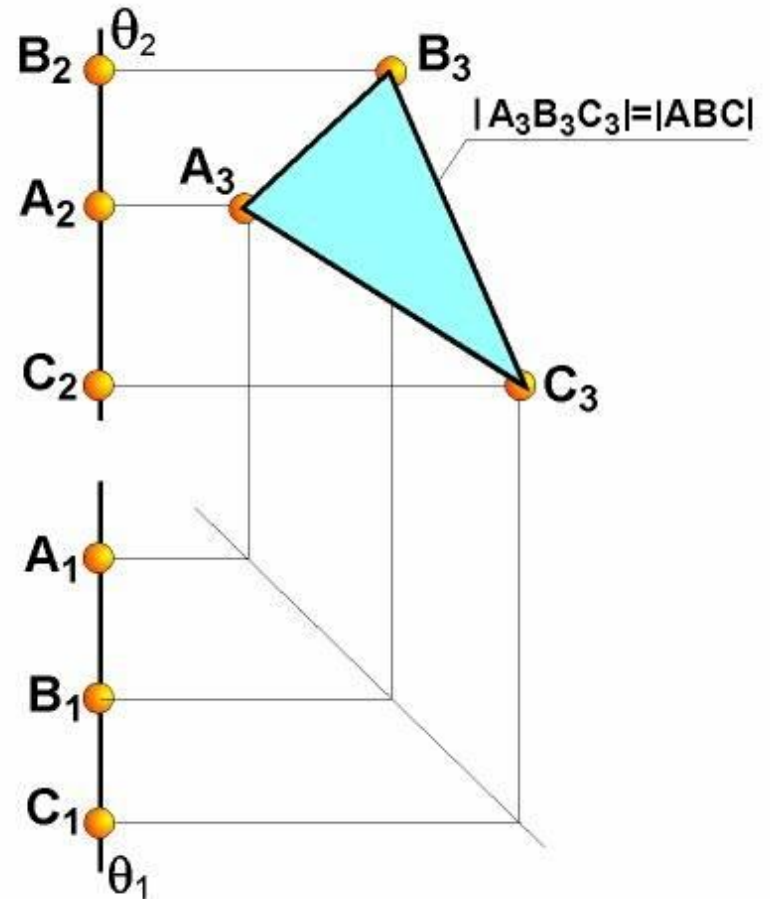
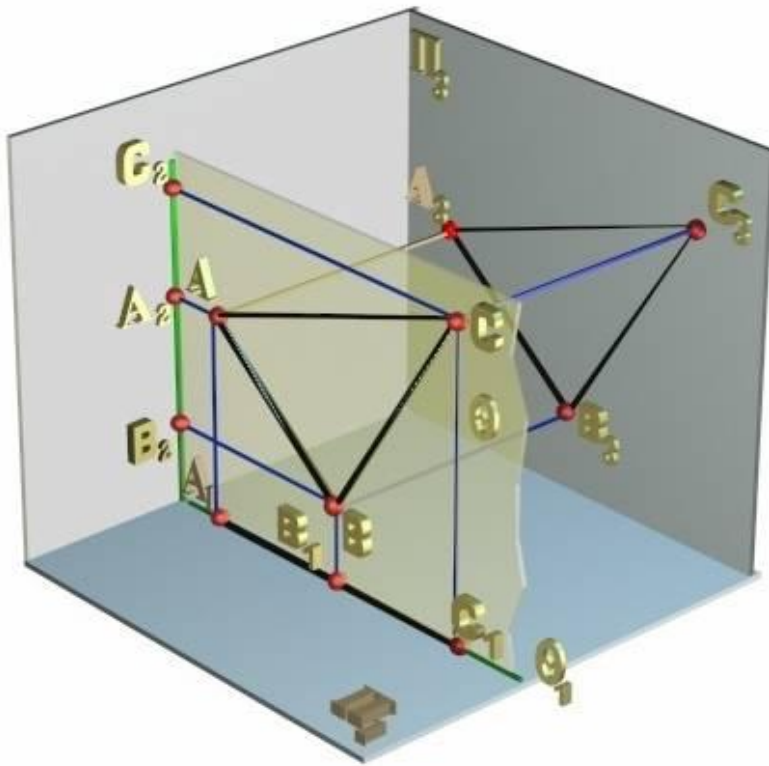
Any planar figure, which lies in this plane, projects onto the frontal principal plane Π_2 to a true size.



Principal Planes.

Profile (principal) plane ($(ABC) \parallel \Pi_3$) is a plane, parallel to the profile principal plane of projection.

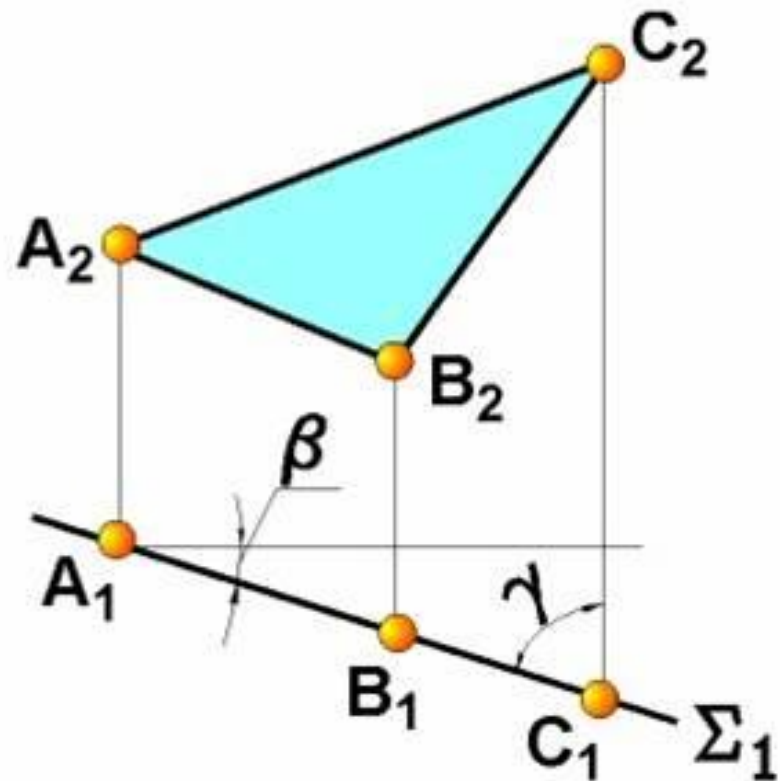
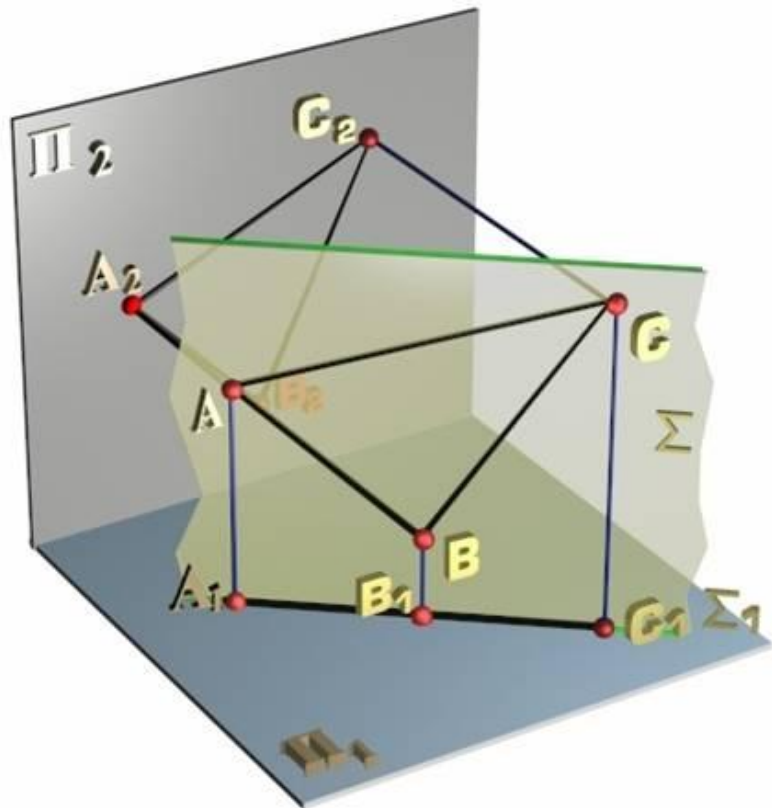
Any planar figure, which lies in this plane, projects onto the profile principal plane Π_3 to a true size.



Planes.

Perpendicular (Projecting) Planes.

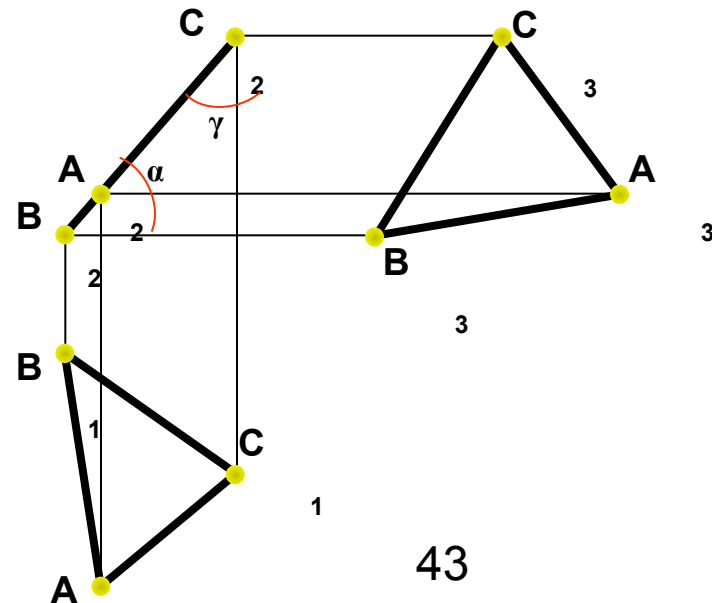
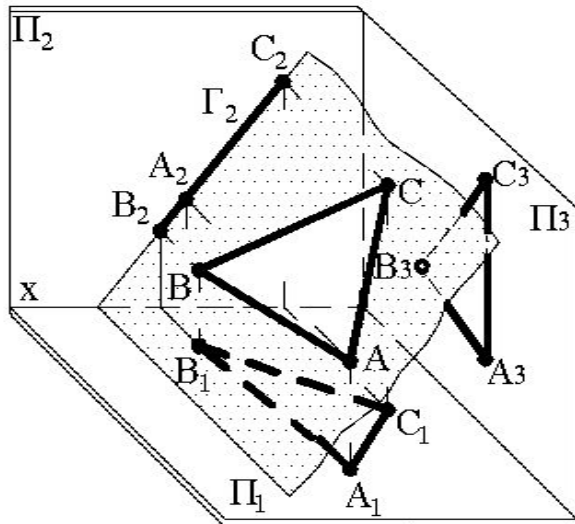
Horizontally-projecting (perpendicular) $((ABC) \perp \Pi_1)$ plane is a plane, perpendicular to the horizontal principal plane of projection. Any planar figure, which lies in this plane, projects onto the horizontal principal plane to a straight line. Horizontally-projecting plane can be represented by its one projection Σ_1 only. Angles β and γ between Σ_1 and orthogonal projectors are true angles between the plane Σ and principal planes Π_2 and Π_3



Planes.

Perpendicular (Projecting) Planes.

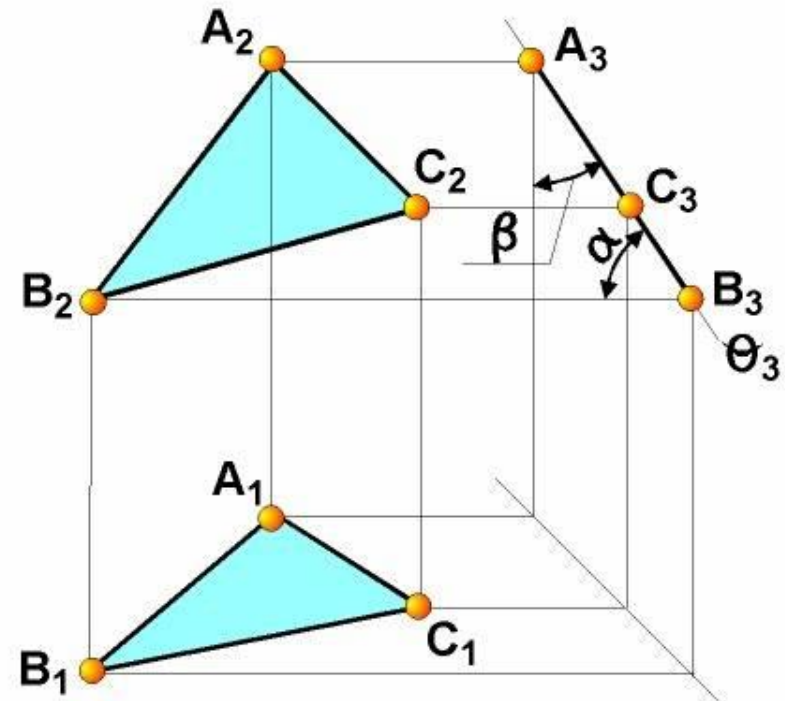
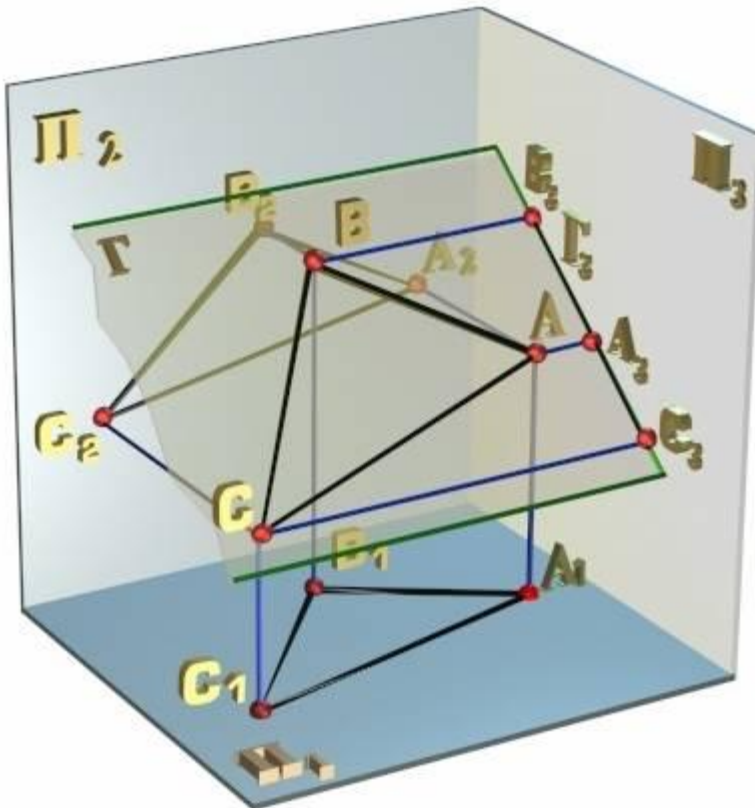
Frontally-projecting (perpendicular) $((ABC) \perp \Pi_2)$ plane is a plane, perpendicular to the frontal principal plane of projection. Any planar figure, which lies in this plane, projects onto the frontal principal plane to a straight line. Frontally-projecting plane can be represented by its one projection Σ_2 only. Angles α and γ between Σ_2 and orthogonal projectors are true angles between the plane Σ and principal planes Π_1 and Π_3 .



Planes.

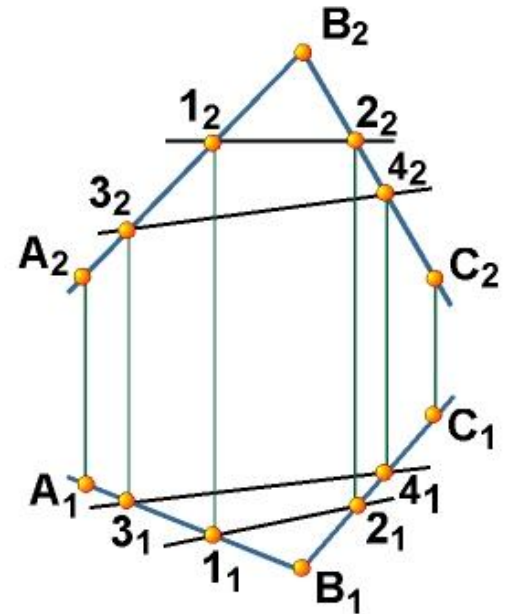
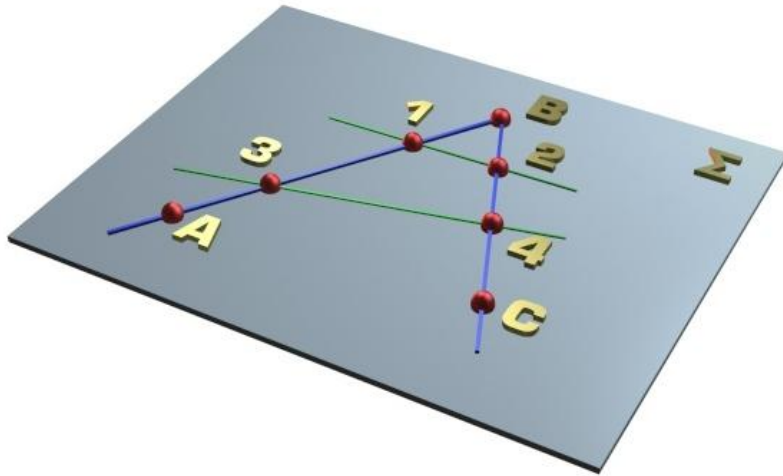
Perpendicular (Projecting) Planes.

Profile-projecting (perpendicular) $((ABC) \perp \Pi_3)$ plane is a plane, perpendicular to the profile principal plane of projection. Any planar figure, which lies in this plane, projects onto the profile principal plane to a straight line. Profile-projecting plane can be represented by its one projection Σ_3 only. Angles α and β between Σ_3 and orthogonal projectors are true angles between the plane Σ and principal planes Π_1 and Π_2 .



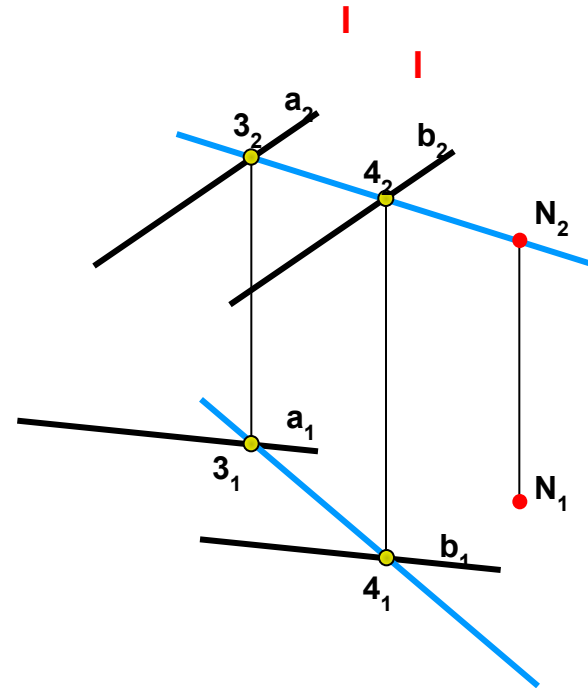
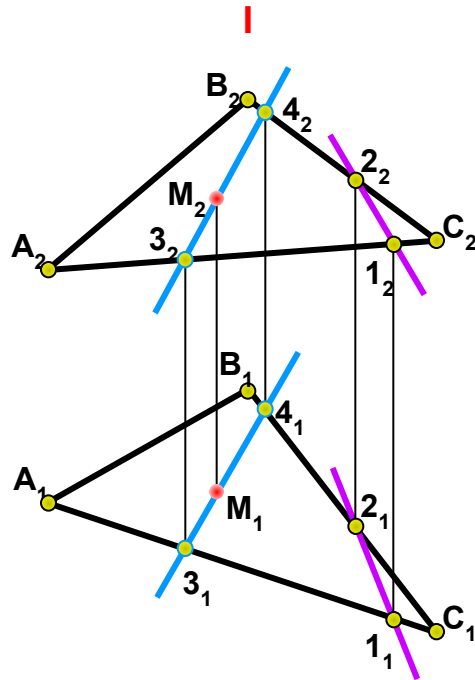
A Line in a Plane

A Straight Line belongs to a Plane, if it passes through two points, which lie in that plane.



A Point in a Plane.

A Straight Line belongs to a Plane, if it passes through two points, which lie in that plane. A Point belongs to a Plane, if it belongs to the Line, which lies in that Plane.

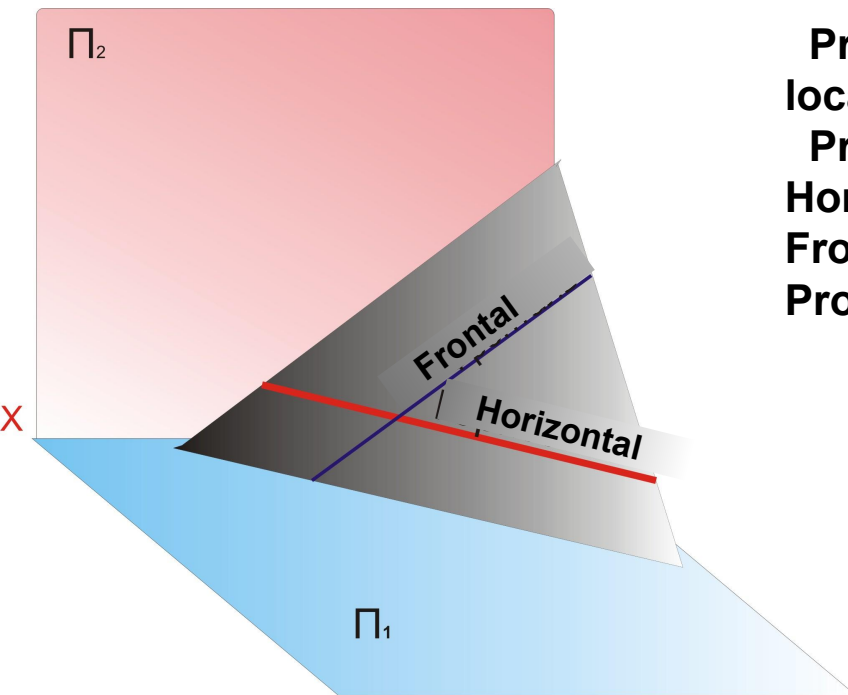


Algorithm:

1. Through the frontal projection of the point $M_2(N_2)$ draw any straight line, which belongs to the plane ((ABC) or (allb)).
 2. Define points 3 and 4, which belong to the plane and the line simultaneously.
 3. Construct horizontal projection of considered line.
 4. On the obtained projection of the line find horizontal projections of the point $M(N)$.
- In the problem II according to the algorithm above, we can see, that point N_1 doesn't belong to the plane.



Principal Lines in a Plane



Principal Lines in a Plane – Lines of a special location, which belong to that plane.

Principal Lines, parallel to the Planes of projection:

Horizontal h – line, parallel to Π_1 ,

Frontal f – line, parallel to Π_2 ,

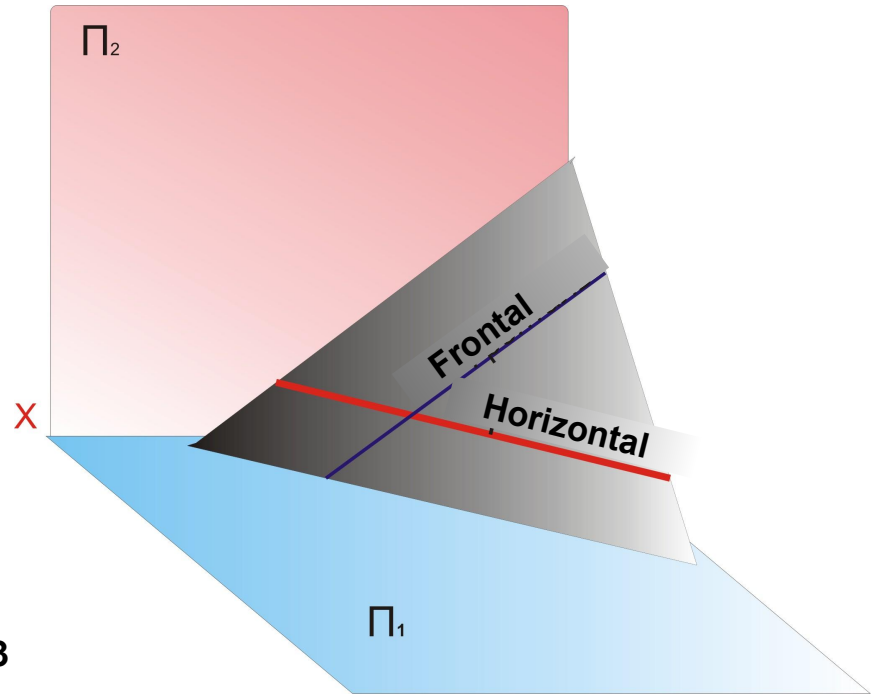
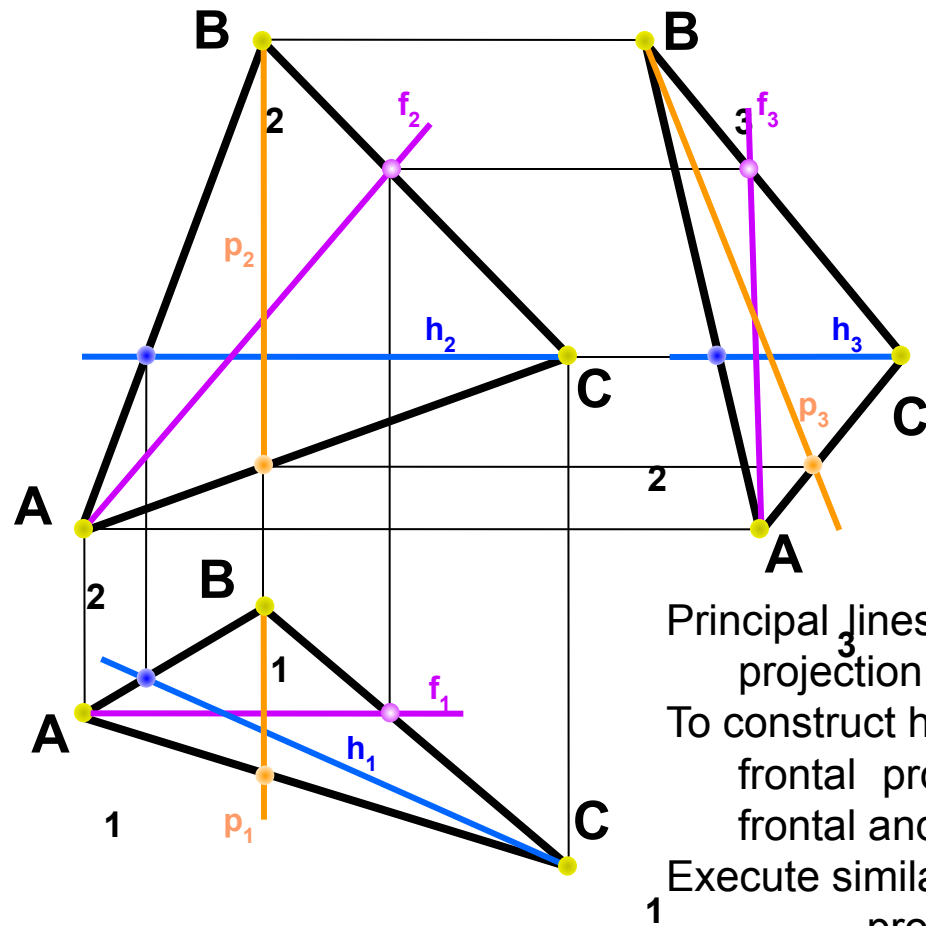
Profile p – line, parallel to Π_3 .

Principal Lines in a Plane

Principal Lines in a Plane – Lines, which belong to that plane and parallel to the Planes of projection. Horizontal h – line, parallel to Π_1 ;

Frontal f – line, parallel to Π_2 ;

Profile p – line, parallel to Π_3 .



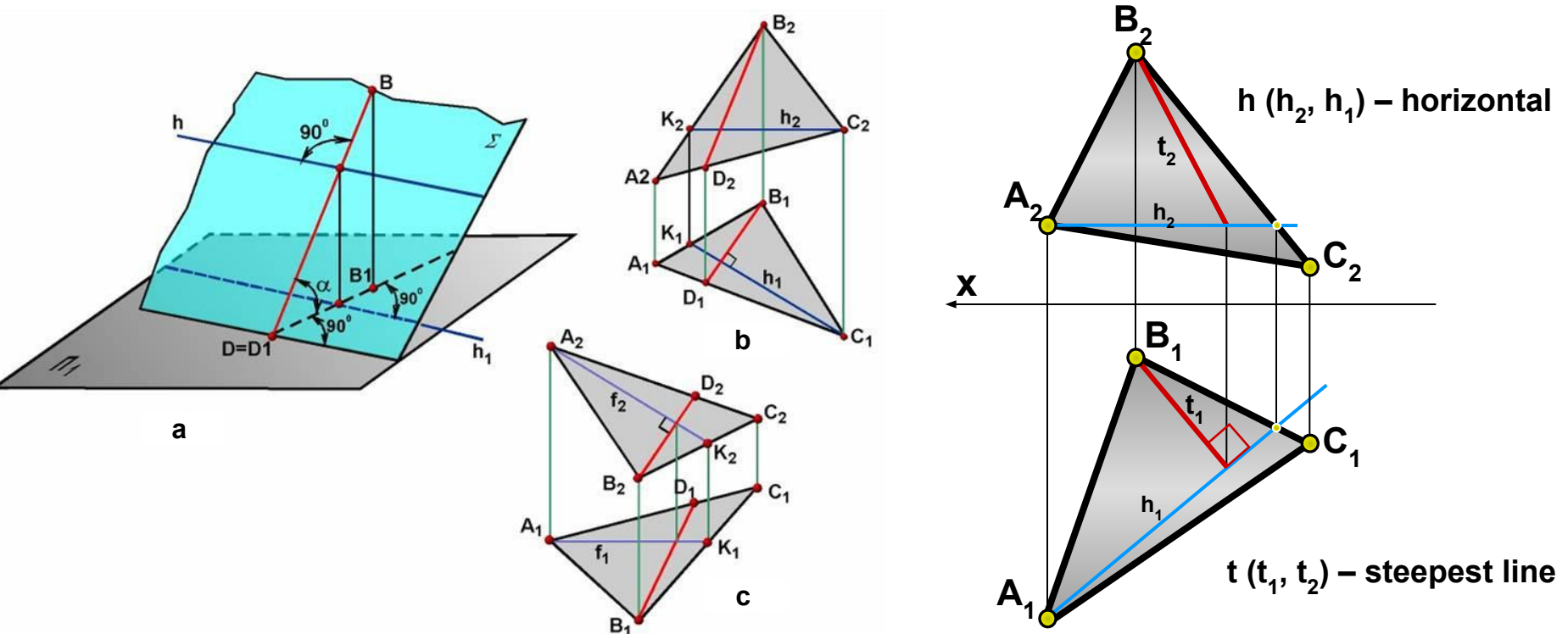
Principal lines in a plane, which are parallel to one plane of projection, parallel to each other.

To construct horizontal principal line of ABC-plane first draw its frontal projection, parallel to x-axis, and then draw its frontal and (if necessary) profile projections.

Execute similar constructions to obtain frontal or profile principal lines in ABC-plane.

Steepest Lines in a Plane

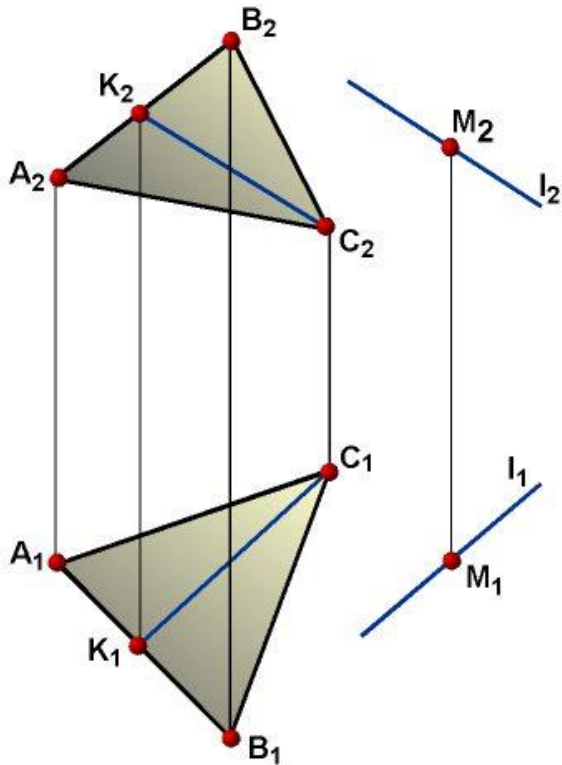
The **Steepest Lines** in a plane are lines, lying in this plane and perpendicular to its **principal lines**.



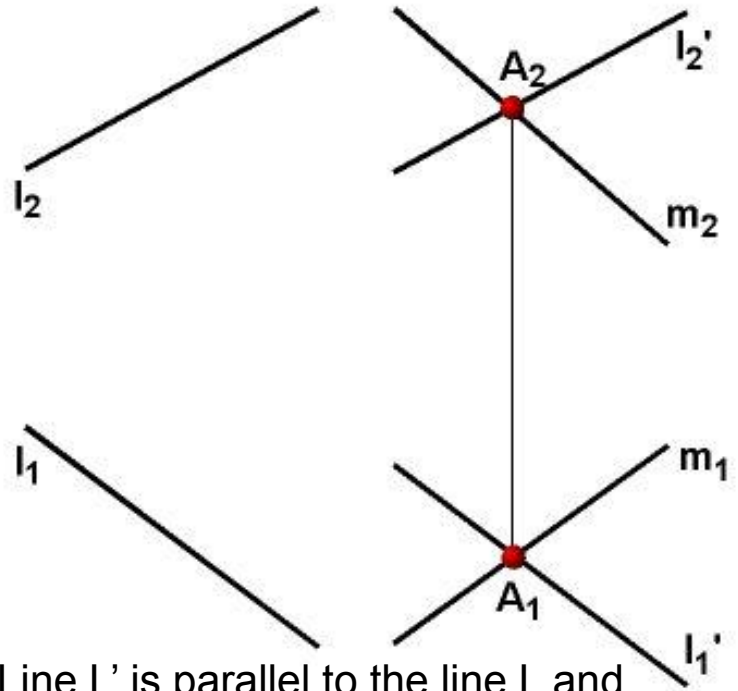
The Steepest line to the plane Π_1 is called Steepest Ascent or Steepest Descent Line. It may serve to determine the angle between the plane (ABC) and horizontal principal plane Π_1 .

Parallelism of a Line and a Plane!

A Line, parallel to a Plane, is a Line, which doesn't belong to the plane and doesn't intersect it. If a Line is parallel to a Plane, then it is parallel at list to the one line, which lies in that plane.



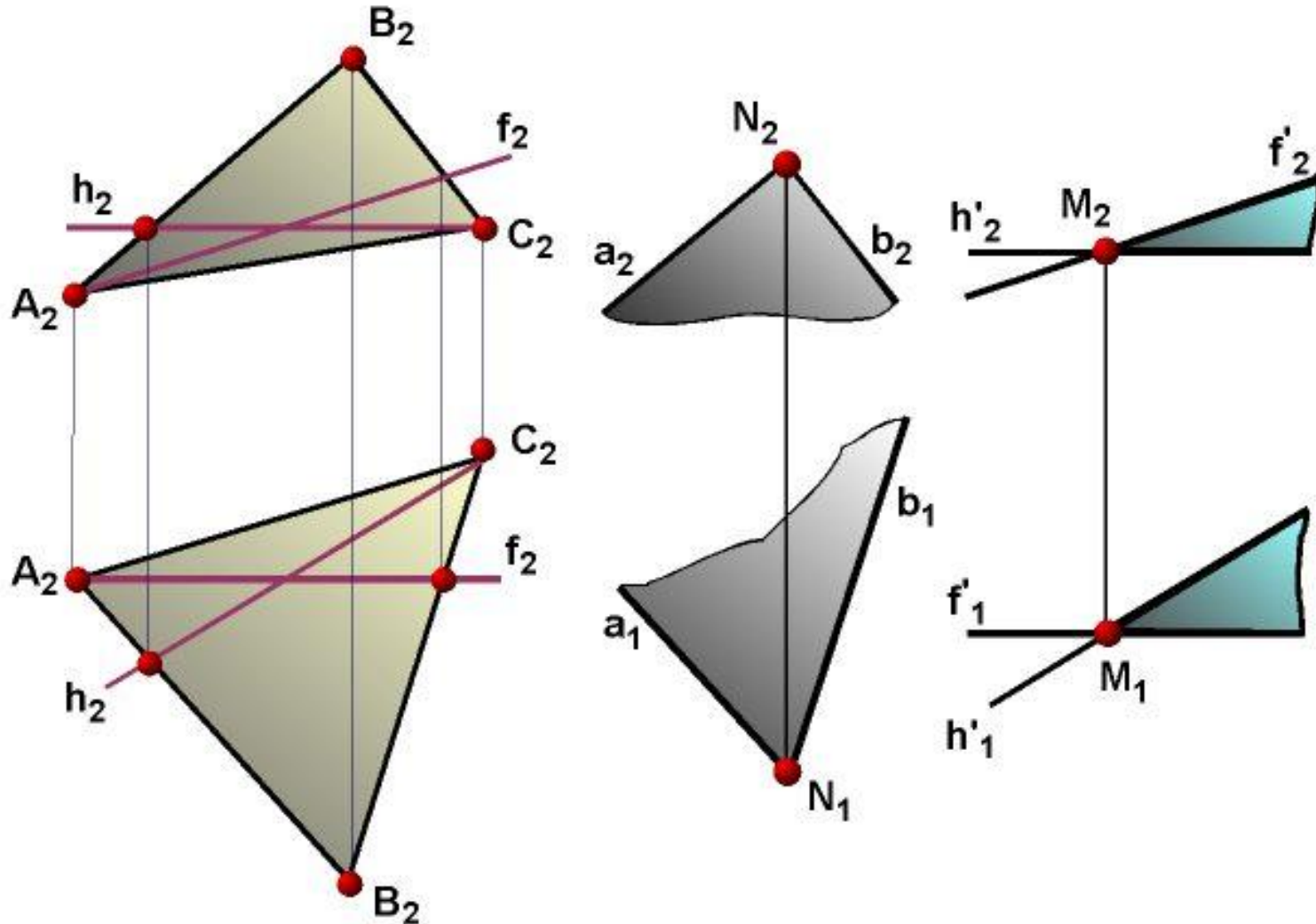
Line L passes through the point M and parallel to ABC-plane.



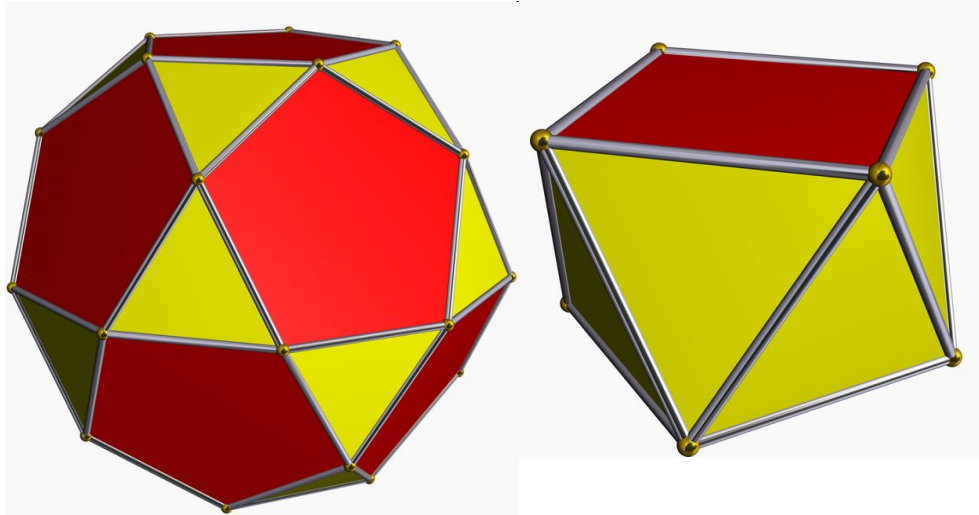
Line L' is parallel to the line L and passes through the point A, which belongs to the line m.

Parallelism of Planes

Two **Planes** are **Parallel**, if two intersecting lines of one plane are **parallel** to two intersecting lines of the other plane.



Polyhedrons. Terms and Definitions

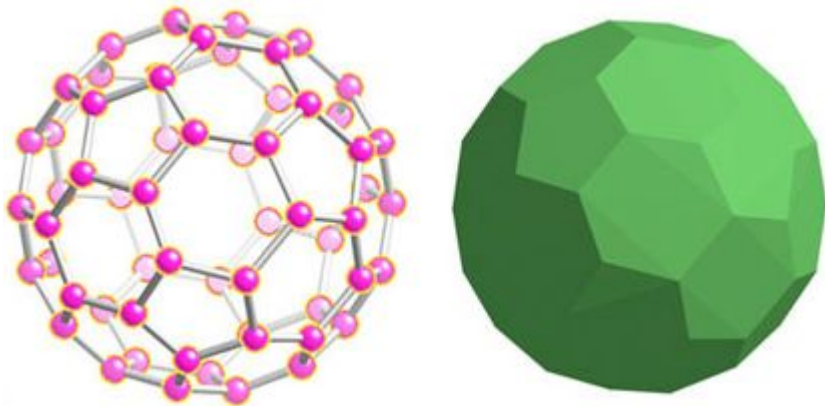


Polyhedron is a solid figure, bounded by plane polygons.

These polygons are called Faces of a polyhedron.

Faces of a polyhedron intersect in straight line segments called Edges.

Edges meet in points called Vertexes (Vertices) of a polyhedron. Multiple of faces form polyhedral surface of a polyhedron.

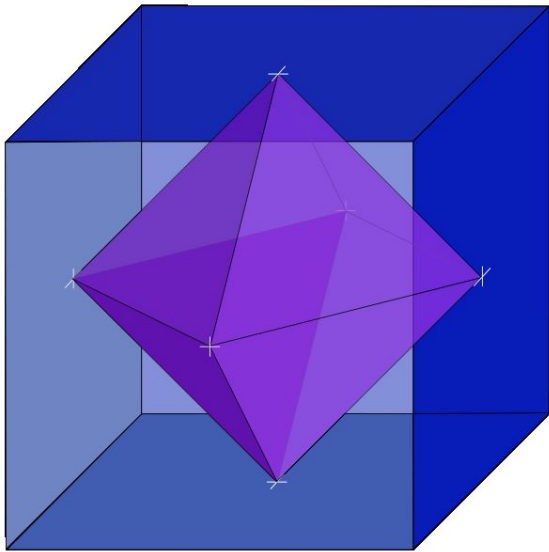


All the vertices and edges of a polyhedron form the mesh of the solid.

To construct orthographic view of a polyhedron means to construct orthographic view of the mesh of the polyhedron.

The Mesh completely defines the polyhedron and is called Determinant of a polyhedron.

Regular Polyhedrons.



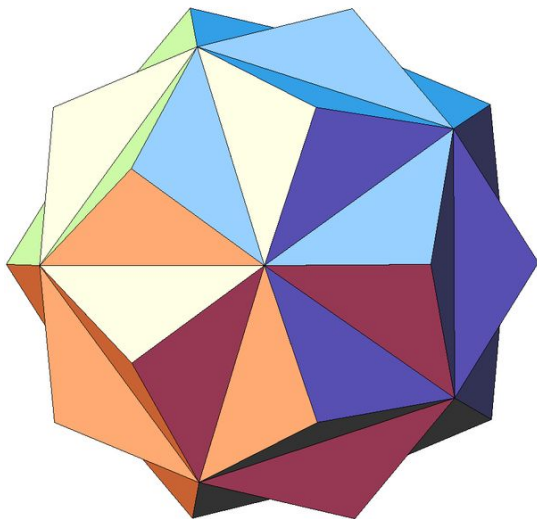
A Regular polyhedron is a polyhedron whose faces are all regular polygons which are identical in both shape and size.

There are two particular spheres associated with any regular polyhedron.

First is the circum-sphere This is the sphere which fits around the outside of the polyhedron so as to touch all its vertices.

Second is the in-sphere This is the sphere which fits inside the polyhedron so as to touch all its faces.

If no face of a polyhedron can cut it on extension, then polyhedron is convex, otherwise it is concave.



For every convex polyhedron ratio between faces, edges and vertexes can be defined by Euler formula

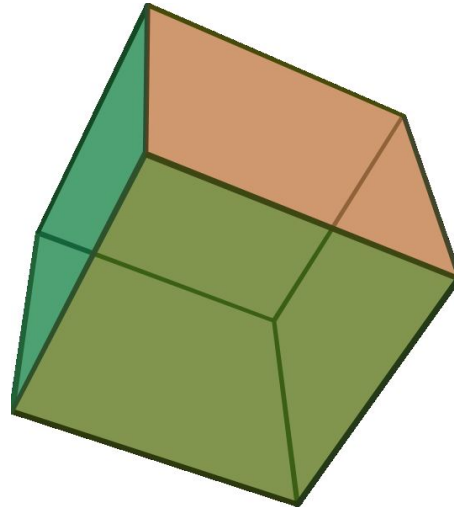
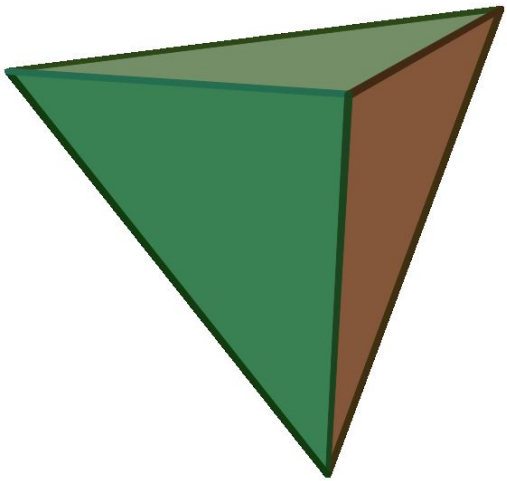
$$F - E + V = 2$$

F - number of faces

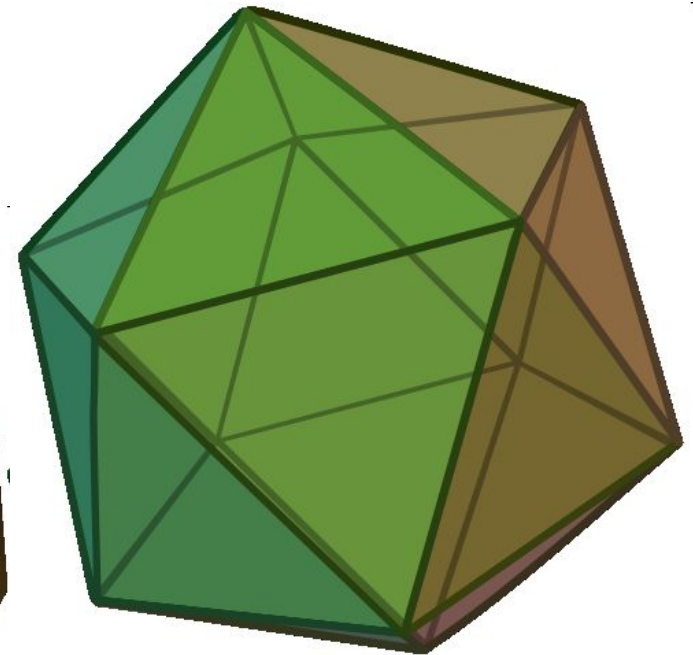
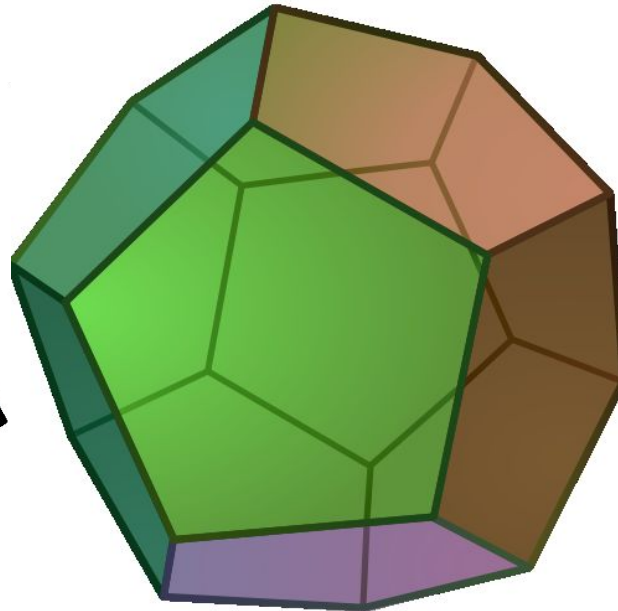
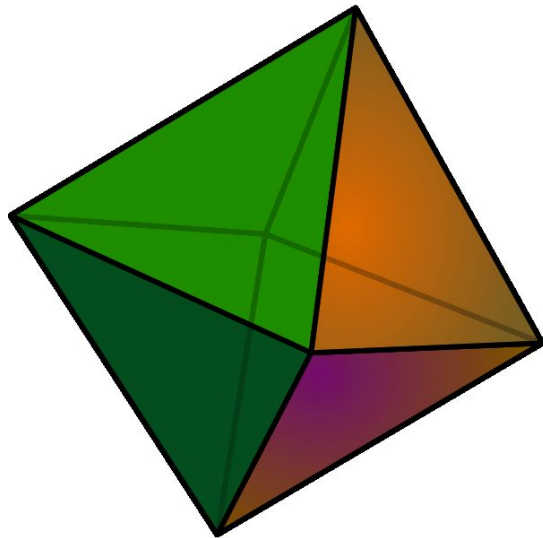
E - number of edges

V - number of vertexes

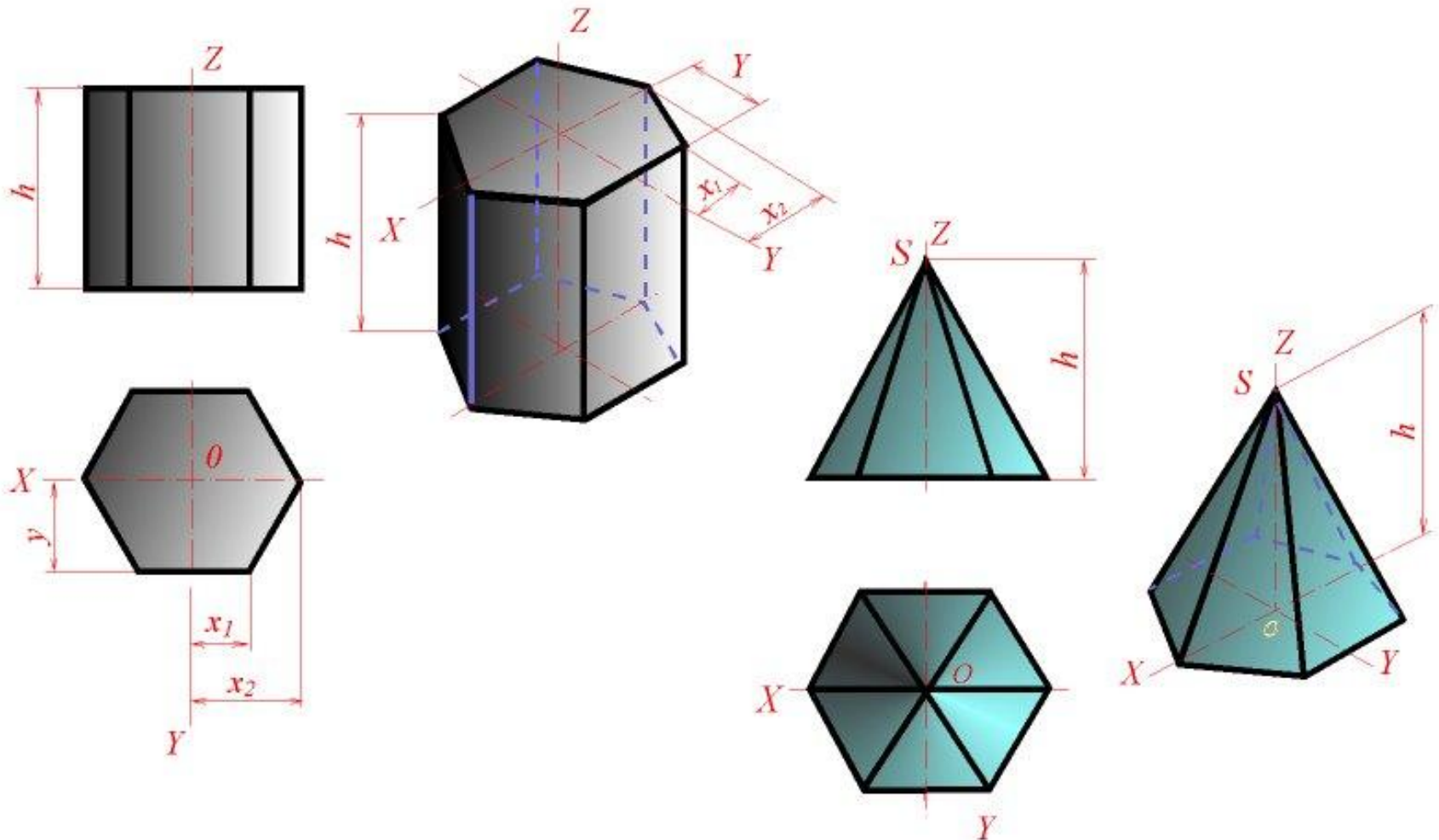
Regular Polyhedrons.



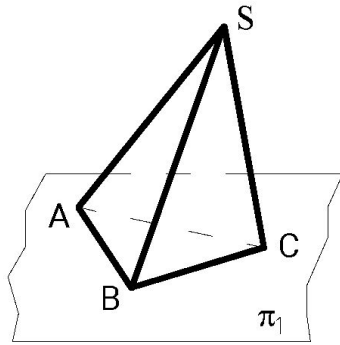
tetrahedron 4 triangles
cube 6 squares
octahedron 8 triangles
dodecahedron 12 pentagons
icosahedron 20 triangles



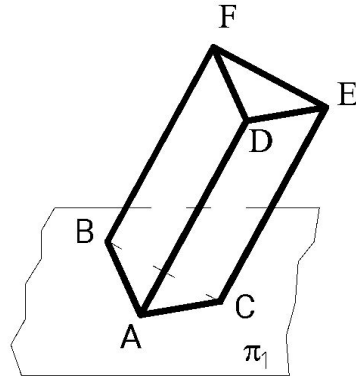
Prism and Pyramid



Pyramid and Prism.

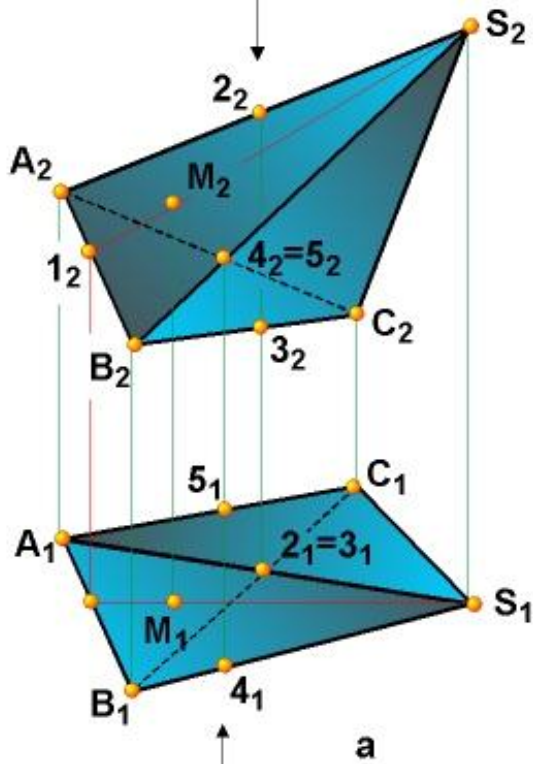


Pyramid

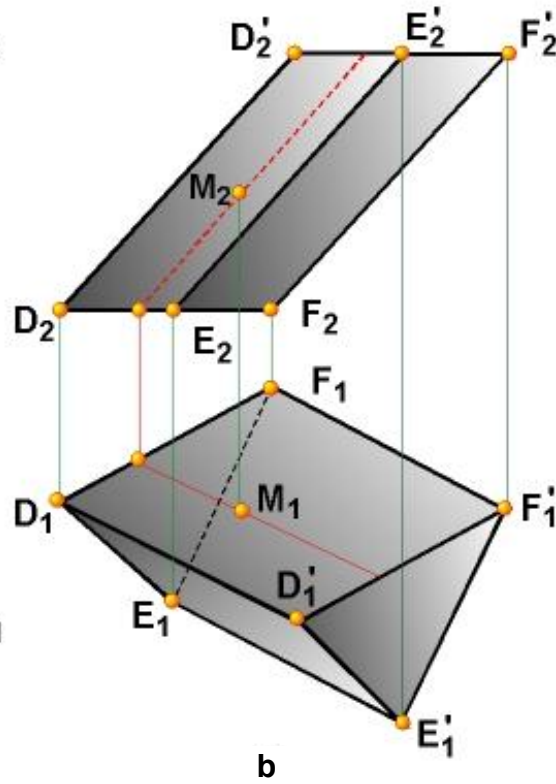


Prism

Pyramid is a polyhedron formed by connecting a polygonal base and a point, which doesn't lie in the base plane. This point is called the apex of a pyramid. Each base edge and apex form a triangle, which called lateral face.



a

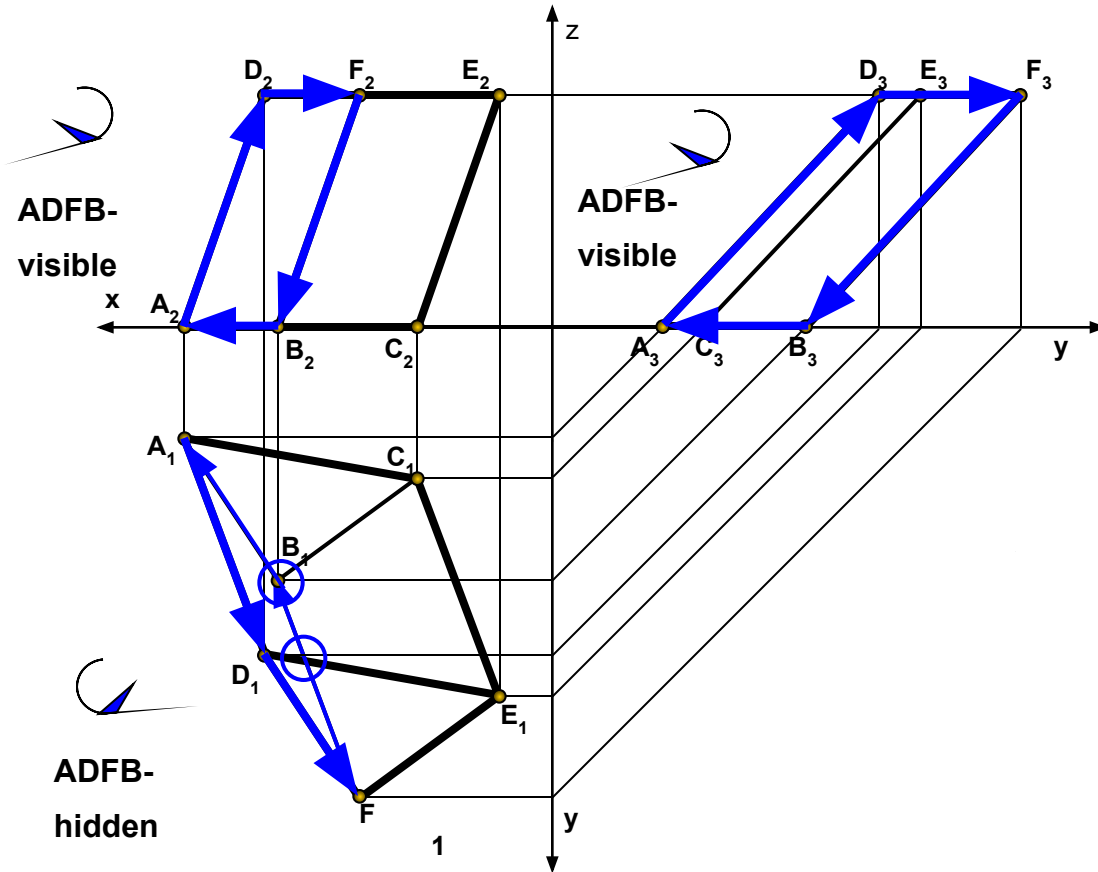


b

Prism is a polyhedron made of polygonal bases and lateral faces, joining corresponding sides. Thus these lateral faces are parallelograms. All cross-sections parallel to the base faces are of the same shape and size.

Polyhedrons. Visibility definition

Prism (Lateral edges intersect in ∞).



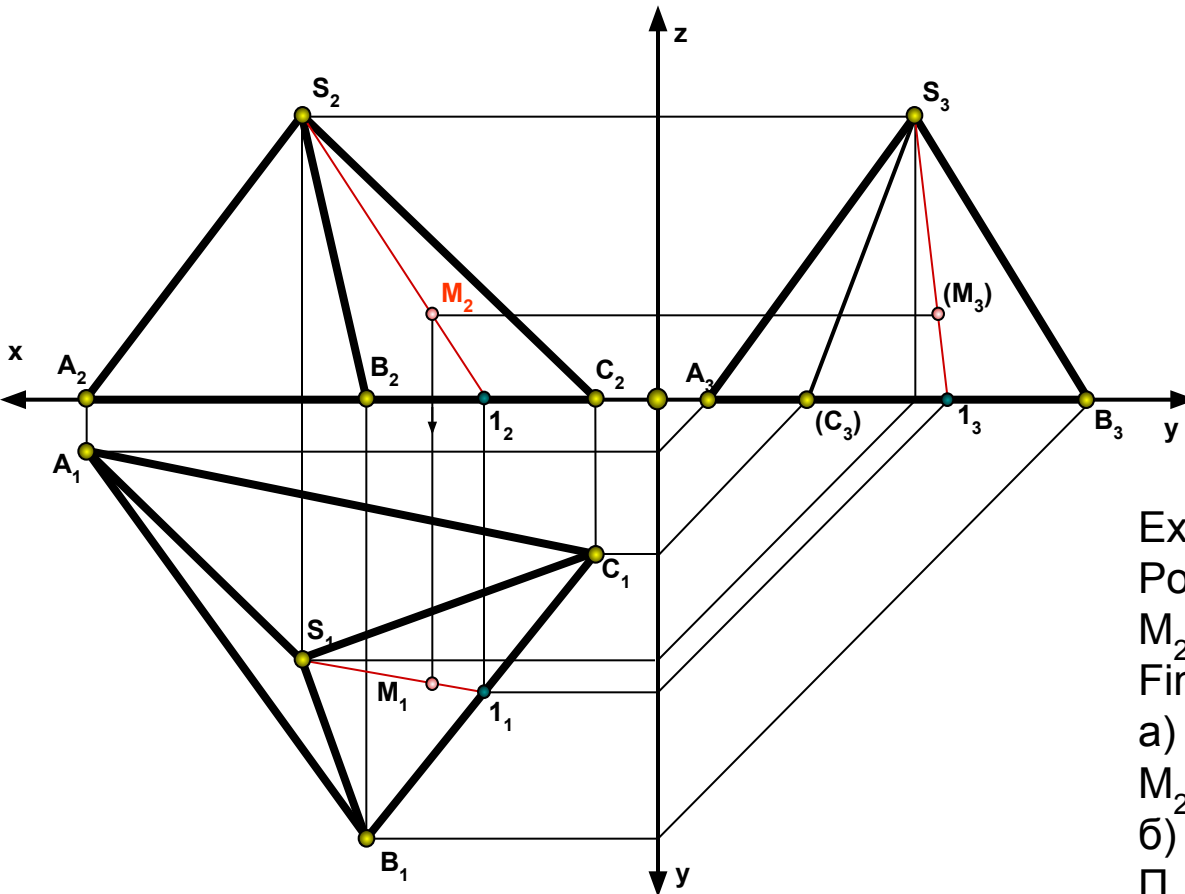
How to define Visibility of edges and faces in orthographic views of a polyhedron.

Four rules for visibility definition in polyhedrons.

1. Outline is always visible.
2. If two edges intersect inside outline, then one is visible, and the other is invisible (hidden).
3. If three edges meet in one point (vertex) inside the contour, then they are all visible or they are all hidden.
4. If sequence of vertexes for the polyhedron's face is the same on orthographic views, then visibility is the same, otherwise it is different.

Polyhedrons. Pyramid

Pyramid - Lateral edges meet in one common point (Apex).



Criteria of representation.
If it's possible to complete projection of the point, which belongs to polyhedral surface, by its one projection, then polyhedron is represented on the orthographic drawing.

Example:

Point M belongs to SBC -face $M \in (SBC)$.
 M_2 is given.

Find M_1 .

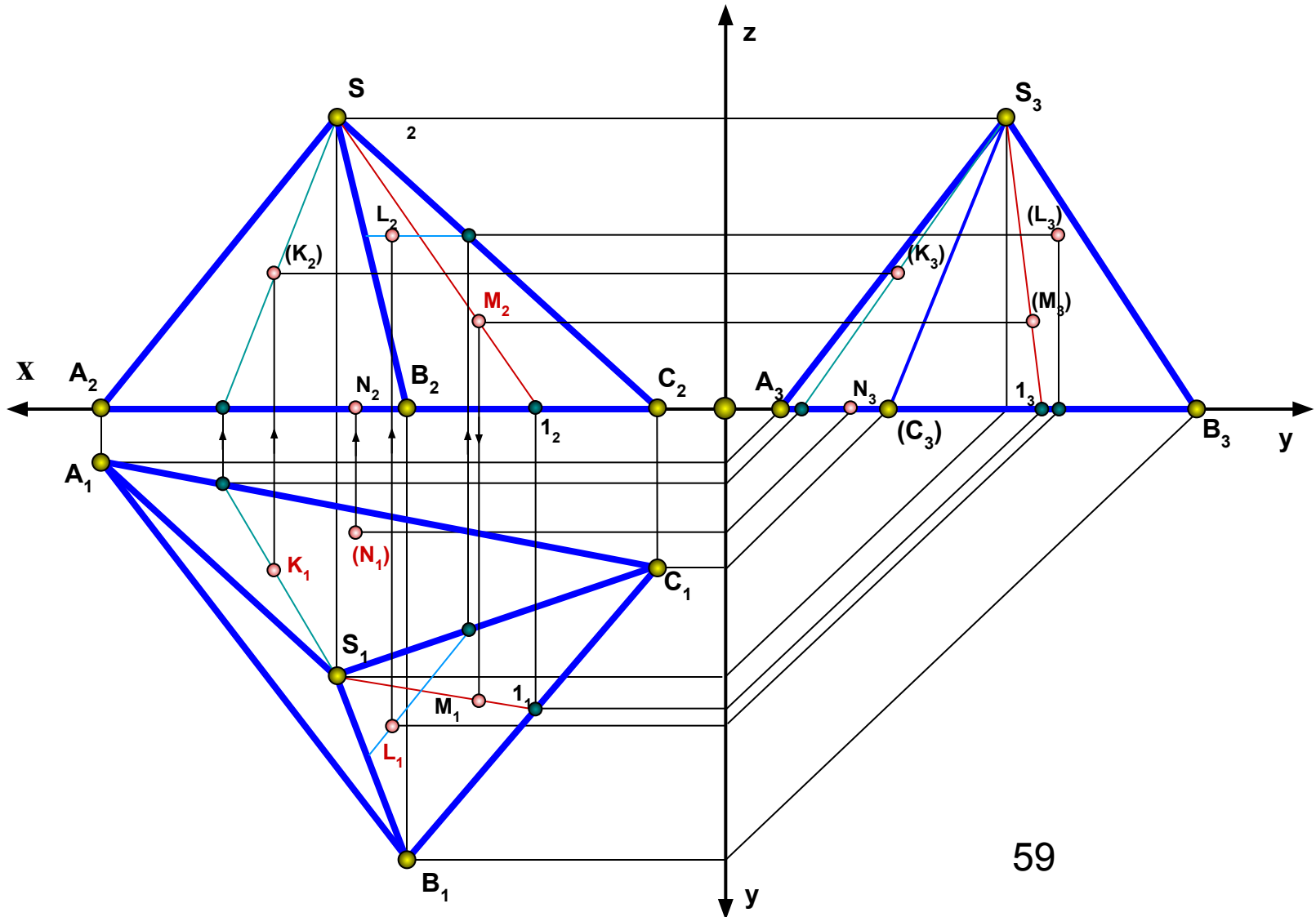
a) Through the projection of the point M_2 draw straight line segment $S_2 1_2$;

б) Find its projection $S_1 1_1$ on the plane Π_1 ;

в) Protract vertical projector from the M_2 and find projection M_1 .

Polyhedrons. Pyramid

Pyramid - Lateral edges meet in one common point (Apex).



Polyhedrons. Prism

Prism (Lateral edges intersect in ∞).

