Transformation of a Drawing

Some problems can be solved easier if object has a particular location relating to the plane of projection. For instance, if a straight line segment is parallel to the plane of projection, it projects to a true size. Otherwise, to find its true length we need to use method of a right-angular triangle. To achieve this we can apply methods called Transformation of a Drawing.

Three ways how to transform a drawing

- 1. Replacing of planes of projection (projecting to auxiliary planes);
- 2. Revolution of a model about an axes perpendicular to a plane of projection;

3. Planar parallel motion (revolution of a model about an axes without indicating an axes in the drawing).

Basic problems which can be solved by Transformation of a Drawing :

- Oblique line segment transforms to a line segment parallel to the plane of projection;
- Oblique line segment transforms to a line segment perpendicular to the plane of projection;
- Oblique plane segment transforms to a plane segment perpendicular to the plane of projection;
- Oblique plane segment transforms to a plane segment parallel to the plane of projection.

In this method location of an object doesn't change while one of planes of projection is replaced by a new one, perpendicular to the other plane of projection. Properties of transformation:

- 1) New projectors perpendicular to a new axes;
- 2) Distance between a new projection and new axes is equal to a distance between replaced (old) projection and replaced axes.



Projecting of a point on auxiliary plane. Point A is given by its projections in Π_2/Π_1 planes. Construct projections of a point A, if the plane Π_2 is replaced by a new vertical plane Π'_2 .

Horizontal projection A_1 doesn't change because location of Π_1 is not changed. To determine new frontal projection A'_2 draw perpendicular prom A_1 point to intersection with Π'_2 , plane. A'_{2} – new frontal projection of \tilde{A} point.

 $\bar{A'_2}A_{x1} = A_2A_x$. A_2A_x is Z-coordinate of a point A (Z_A). It means that if frontal plane of projection is replaced, Z-coordinates of entire figure doesn't change.



Algorithm:

- 1.
- Draw perpendicular (new projector) from a point A_1 to a new axes of projections X_1 . On a newly obtained projector protract Z_A -coordinate of a point A from the point A_{χ_1} , 2. i.e. $A'_{2}A_{y_{1}} = A_{2}A_{y}$.

Application of the method.

Oblique line segment transforms to a line segment parallel to the plane of projection and in succession perpendicular to the plane of projection.



Algorithm :

1. Implement new frontal plane (primary auxiliary plane) Π'_2 II [AB] to transform A_1B_1 to a line parallel to a principal plane.

- 2. Construct frontal projection of $[A'_2 B'_2]$. Z-coordinates of points A and B doesn't change.
- 3. Implement new horizontal plane (secondary auxiliary plane) $\Pi'_1 \perp [A'_2 B'_2]$.
- 4. Construct horizontal projection of a line $[A'_1 B'_1]$ in a new system Π'_1 .

Construct a true size of a plane segment ABC.



Algorithm :

- 1. Draw horizontal line in the plane (ABC) h_2 .
- 2. Construct horizontal projection of a horizontal line h_1 .
- 3. To implement a primary auxiliary plane Π'_2 draw a new axes χ'_2 perpendicular to h_1 .
- 4. Construct new frontal projection of ABC-plane $A'_2B'_2C'_2$ in a plane Π'_2 .
- 5. To implement a secondary auxiliary plane Π'_1 draw a new axes x''_1 parallel to $A'_2B'_2C'_2$.
- 6. Construct new horizontal projection of ABC-plane $A'_1B'_1C'_1$ in a plane Π'_1 . $A'_1B'_1C'_1$ is a true size of ABC-plane.

Transformation of a Drawing. Revolution about an axes perpendicular to a plane of projection

In this method location of planes of projection doesn't change while an object revolves about an axes, perpendicular to one plane of projection.



All points of a figure are rotated at the same angle α .

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Transformation of a Drawing. Revolution about an axes perpendicular to a plane of projection

Revolution about an axes $\perp \Pi_1$.



Transformation of a Drawing. Revolution about an axes perpendicular to a plane of projection

How to find a True Size of a Surface Segment

Algorithm:

1. Construct horizontal line h in the plane ABC. 2. Revolve horizontal projection of ABC-triangle to locate $h_1 \perp \Pi_2$.

Construct frontal view of ABC-triangle. It projects to a line perpendicular to the frontal principal plane.
Revolve obtained view to locate it parallel to horizontal principal plane. A"B"C" || to x-axes

5. Obtained horizontal view is a True Size of a $\triangle ABC$.



Transformation of a Drawing. Revolution about an axes perpendicular to a plane of projection without indicating an axis in a drawing. (Planar Parallel Motion)

How to find a True Size of a Surface Segment



In this method location of planes of projection doesn't change while each point of an object moves in a plane, parallel to one plane of projection.

Algorithm:

- 1. Construct horizontal line h in the plane ABC
- 2. Move horizontal projection of ABC-triangle to locate $h_1 \perp \Pi_2$.
- 3. Construct frontal view of ABC-triangle. It projects to a line perpendicular to the frontal principal plane.
- 4. Move obtained view to locate it parallel to horizontal principal plane. A"B"C" || to x-axes .
- 5. Obtained horizontal view is a True Size of a $\triangle ABC$.

Transformation of a Drawing. Revolution about an axes perpendicular to a plane of projection without indicating an axis in a drawing (Planar Parallel Motion)

Find a True Size of a dihedral angle between plane segments.



Algorithm:

- 1.
- Move ABCD- figure parallel to Π_2 to locate BC-edge parallel to horizontal principal plane $\Pi_1 B'_2C'_2||$ x-axes. Construct horizontal view of A'B'C'D'-figure. Each vertex of a figure moves in a plane, parallel to Π_2 (In Π_1 parallel to 2. x-axes). Vertices A'_1 , B'_1 , C'_1 , D'_1 are placed on intersections between horizontal and vertical projectors. Move obtained A'B'C'D'-figure parallel to Π_1 plane to locate B'C'-edge perpendicular to a frontal principal plane Π_2 -
- 3. $B_{1}^{"}C_{1}^{"} \perp x$ -axes.
- Construct frontal view of A"B"C"D"-figure. Vertices A", B", C", D', are placed on intersections between horizontal 4. and vertical projectors.
- A"B"C" and B"C"D"-planes are perpendicular to the frontal principal plane, because common 5. edge B" C" $\perp \Pi_2$. It enables to find a true size of dihedral angle between them on the frontal view.

Polyhedrons. True size of a planar section.



Algorithm:

- 1. Construct primary auxiliary plane parallel to the section to project section to a true size.
- 2. On projecting a section to primary auxiliary horizontal plane Y-coordinates of a section don't change, and they are protracted from a new X1 axes on the proper projectors, perpendicular to the X1 axes.

True Size of a polyhedron planar section is obtained.

Section can be constructed in a free space of a drawing by placing a new axes parallel to a section plane and protracting Y-coordinates of points 1, 2, 3, 4, 5 and 6 perpendicular to the new axes. 11

Find a distance between two skew lines.



Find a distance between a point and a plane.



Intersection between planes



Intersection between a Line and a Plane

A Line and a Plane have one common Point



Intersection between a Line and a Plane

How to find a piercing point

A Line and a Plane have one common Point



Algorithm.

- 1. Construct cutting plane which would contain the given line.
- 2. Find line of intersection between the given plane and cutting plane.
- 3. Determine point of intersection between obtained line of intersection and given line.

Intersection between two oblique planes. Cutting plane method



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Intersection between two oblique planes. Cutting plane method



Line of intersection between two planes is a multiple of points which simultaneously belong to both planes. To find a line of intersection between two planes it's sufficient to find two points which belong to both planes.

Two Cutting Planes are used to find two common points.

Intersection between two oblique planes. Cutting plane method

Algorithm:

1. Construct two cutting planes intersecting two given oblique planes and perpendicular to some principal plane.

2. Construct lines of intersection between two cutting planes and two oblique planes.

3. Determine points of intersection between obtained lines.

4. Construct line of intersection joining determined points.



The word 'axonometry' (Greek) consists of two words: 'axon'—axis and 'metreo'—I measure, and means 'measuring with the aid of axes', or 'measuring along axes'.

The method of axonometric projection consists in the fact that an object is referred to some coordinate system and then is projected by parallel lines or rays onto a plane together with the system of coordinates.

Axonometric projections differ from orthographic (orthogonal) projections in that in axonometry an object is projected only onto one plane of projection called the **axonometric** (or **pictorial**) plane and is placed in front of the picture plane so as to expose three sides to the viewer.

In mechanical engineering axonometric projections are used as an auxiliary to orthographic projections of a mechanical part when it is the necessary to give a clearer picture of its shapes which are difficult to imagine from the orthographic projections. Without the axonometric picture it is sometimes very difficult to visualize the shape of the object from the three orthographic projections alone.

The basic statement of axonometry" was formulated by K. Polke (in 1851) in the form of the following theorem: any three line segments emanating from a single point in the plane may be taken as a parallel projection of three equal and mutually perpendicular line segments in space. In the sixties of the 19-th century G. Schwartz generalized Polke's theorem. He proved that any complete quadrilateral in the plane may always be regarded as a parallel projection of a tetrahedron similar to any given one.

Therefore, any three non-coincident straight lines passing through a point in the plane may be taken for the axonometric axes with various scales. 20



A line segment **e** is laid off on the axes *x*, *y*, *z*, and is taken as a unit of measurement along these axes (the *true unit*). The line segments e_x , e_y , e_z on the axonometric axes represent the projections of the segment **e**. In the general case they are not equal to /, and are not equal to one another. The segments e_x , e_y , e_z are called the **axonometric units** and are used for measuring along the axonometric axes

The ratios

$$\mathbf{K}_{\mathbf{x}} = \frac{e_x}{e}; \quad \mathbf{K}_{\mathbf{z}} = \frac{e_z}{e}; \quad \mathbf{K}_{\mathbf{y}} = \frac{e_y}{e}$$

are called distortion factors along the axonometric axes. Regarding to distortion factors axonometric views can be:

trimetric – all distortion factors are different $(K_x \neq K_y \neq K_z)$; *dimetric* – any two distortion factors are equal (for example $K_x = K_y$); *isometric* – all distortion factors are equal to each other($K_x = K_y = K_z$).







Algorithm how to construct axonometry by the orthographic view



- 1. From the point O' along O'X' axes protract distance $X_A = |OA_x|$.
- 2. From the obtained point A'x draw a straight line, parallel to O'Y'.
- 3. On the obtained straight line protract distance $Y_A = |A_xA_1|$ and obtain A'_1 point.
- 4. From the point A'₁ draw a straight line, parallel to O'Z' axes. On the obtained straight line protract distance Z = |A A_|.
- distance Z_A= |A_xA₂|. 5. Point A' is a projection of a point A in axonometric coordinate system.



A=1.22d – major axes CD=0.7d – minor axes d – diameter of a circle Major axes of an oval is always perpendicular to the axonometric axes, which doesn't belong to the plane of a circle



How to construct axonometric projection of polyhedron



Algorithm:

- 1. In axonometric coordinate system X', Y', Z' protract points 1, 8, 2, 3.
- 2. Draw line segments |45|, |67| || to X' axes.
- 3. Join points 1, 4, 3, 5, 8, 7, 2, 6 and obtain axonometry of the base.
- Construct lateral edges from vertices of the base draw straight line segments || to Z-axes at a distance of polyhedron height ().
- 5. Join end points of obtained straight line segments.
- 6. Construct (if necessary) axonometric projection of M point, which belongs to a lateral face of a polyhedron.

Algorithm how to construct orthographic view by the axonometry



- Given data: cone of revolution in axonometric projection. Point K belongs to a surface af the cone.
- 1. Through a point K draw a straight line generatrix of a cone.
- 2. Draw its projection on xy-plane.
- 3. Construct point K" on the obtained projection.
- 4. Find coordinates X_{K} and Y_{K} of K''-point.
- 5. Draw obtained coordinates on the orthographical view.

Orthogonal frontal isometric projection



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Orthogonal frontal dimetric projection



Nonorthogonal frontal isometric projection – cavalier (military) pictorial view



Nonorthogonal horizontal isometric projection





Nonorthogonal frontal dimetric projection – cabinet pictorial view

