Electrostatic Field

- Electric charges, interaction of charges
- Electrostatic field and its characteristics
- Intensity of the electrostatic field. Force lines
- Electric displacement
- Vector E circulation
- Potential of the field
- Superposition principle
- Relation between intensity and potential
- Electric flux. Gauss's theorem
- Application of Gauss's theorem

Fundamental concepts

Electrostatics is a science which studies properties and reciprocal action of electric charges in state of rest with respect to an intertial frame of reference.

There are two classes of electric charges: positive and negative. ***The charges of the same sign (like charges) repel each other, and the charges of opposite sigh (unlike charges) attract each other***.

Electric charge of a body consists of a series of elementary charges. The smallest particle which possesses the elementary negative charge is known as an ***electron***. The smallest particle with positive elementary charge is ***proton***.

Interaction of charges

Coulomb's law: The force **F** of reciprocal electrostatic action between two point electric charges q_1 and q_2 in vacuum, is proportional to the product of magnitudes of the charges and inversly proportional to the square of the distance r between them,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

ε_0 is dialectric constant of vacuum

$$\frac{1}{4\pi\varepsilon_0} = 9\text{x}10^9 \ \text{C}^{-2}\,\text{Nm}^2$$

Interaction between the charges in a medium

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\varepsilon r^2}$$

ε is the ***relative permittivity of the medium***; it expressess how many times the force of interaction between the charges in the medium is less than in vacuum.

Intensity of the electrostatic field

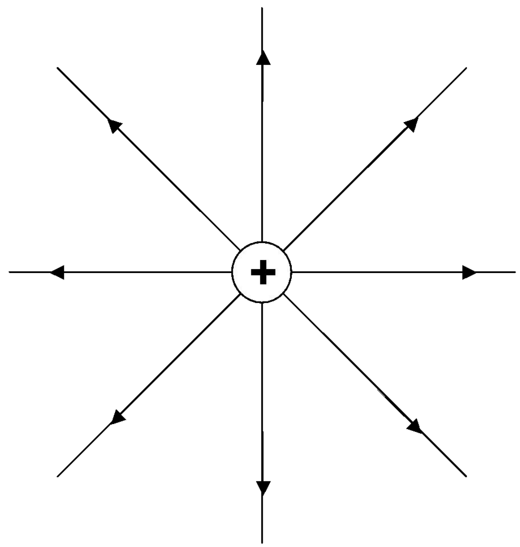
The interaction of charges is realized through electrostatic field created by these charges in the surrounding space.

Intensity of the electrostatic field in a certain point is numerically equal to the force the field acts on a unit positive charge located in this point; its direction coincides with the direction of the force.

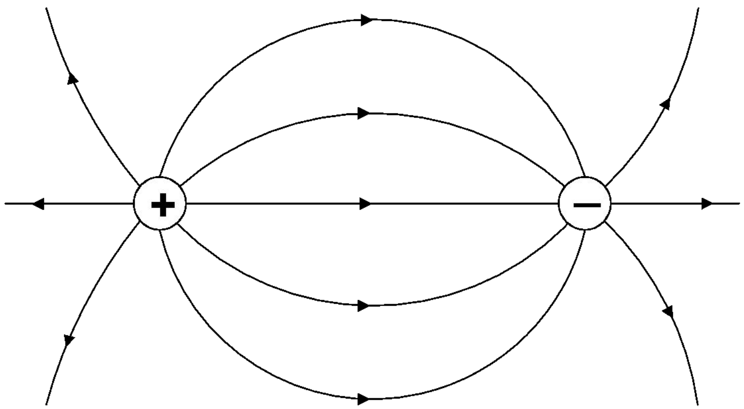
$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

Electrostatic field is said to be ***uniform***, if its intensity E in all the points of the field is the same. Otherwise the field is ***non-uniform***.

Force lines



Carga aislada



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Force line is any line drawn in the region where the field exists in such a way that the vector of field intensity is tangent to the line at any of its points.

Intensity of the electrostatic field of a sole charge

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{q}{4\pi\varepsilon_0 \varepsilon r^2}$$

Electric displacement

$$\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E}$$

Electric displacement (or electric induction) **D** is a vector quantity which characterizes the field and is independent of the medium.

Circulation of E vector of electrostatic field

Total work along a closed path: $\oint (\mathbf{F} \cdot d\mathbf{r}) = 0$

$$\oint dW = 0$$

Work to displace a unit charge along a closed path:

$$\oint (\mathbf{E} \cdot d\mathbf{r}) = 0$$

"Law of vector E circulation": the circulation of intensity E vector of an electrostatic field over a closed path is zero.

Potential of the field

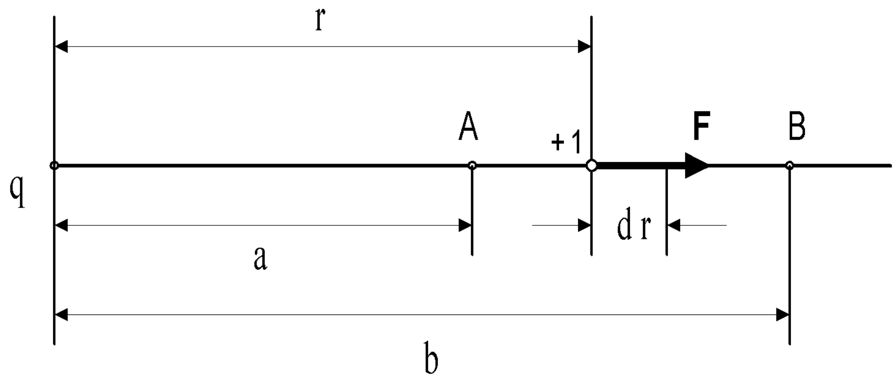
Potential at a point of the field is the work to be done to displace without acceleration a unit positive charge from infinity to this point of the field.

$$\varphi_A = \frac{W_{\infty,A}}{q}$$

$$\varphi_A = \frac{q}{4\pi\varepsilon_0 a}$$

Potential difference

$$\Delta\varphi = \frac{W_{A,B}}{q}$$



Potential difference (cont.)

The work done by the force **F** over the short distance:

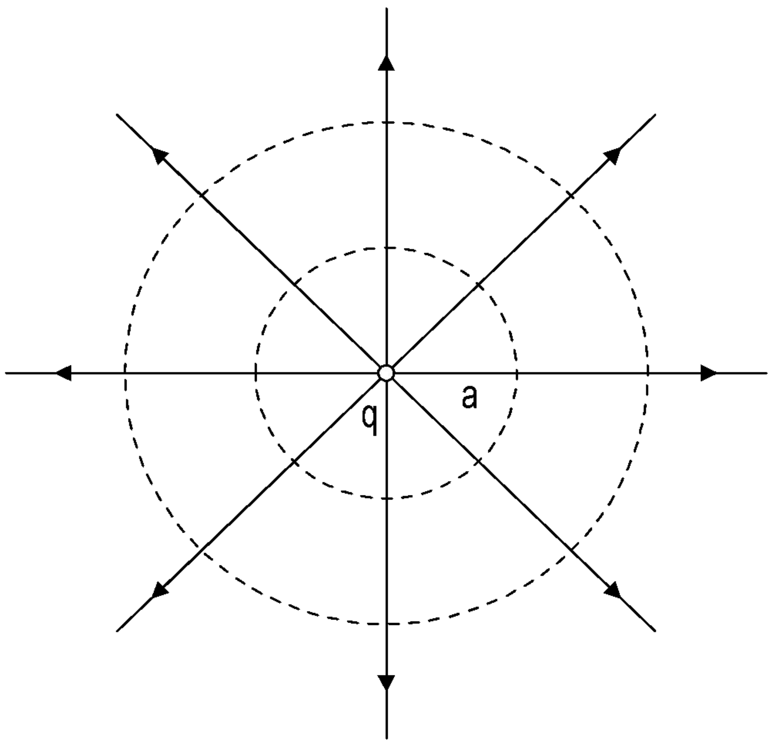
$$dW = \mathbf{F}d\mathbf{r}$$

Over the whole distance from A to B the work done by the force **F** on the unit charge is:

$$\int_A^B dW = \int_{r=a}^{r=b} \mathbf{F}d\mathbf{r} = \int_a^b \frac{q}{4\pi\varepsilon_0 r^2} dr = -\frac{q}{4\pi\varepsilon_0 r} \Big|_a^b = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$V_{AB} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Equipotential surface



Equipotential surface is a geometrical place of points of an electrostatic field with the same potential.

At any point of the equipotential surface, the vector of the electrostatic field intensity is perpendicular to this surface and is directed as potential diminishes.

Relation between intensity and potential

$$E = V / d$$

$$\mathbf{E} = -\left(\mathbf{i}\frac{\partial \varphi}{\partial x} + \mathbf{j}\frac{\partial \varphi}{\partial y} + \mathbf{k}\frac{\partial \varphi}{\partial z} \right) = -grad\varphi = -\nabla\varphi$$

Intensity at any point of the electrostatic field equals the gradient of potential of the field at this point taken with negative sign. The sign “minus” signifies that the vector **E** is directed in the direction the potential diminishes.

The operator applied to a funtion, indicates what one must do with the function. The Hamilton operator applied to the function, indicates, $\nabla\varphi$ that it is needed to: 1) differenciate the function – find the partial derivatives with respect to *x*, *y*, *z*; 2) multiply these derivatives by respective unit vectors (**i**, **j** o **k**); and 3) add the recieved expressions.

Superposition principle

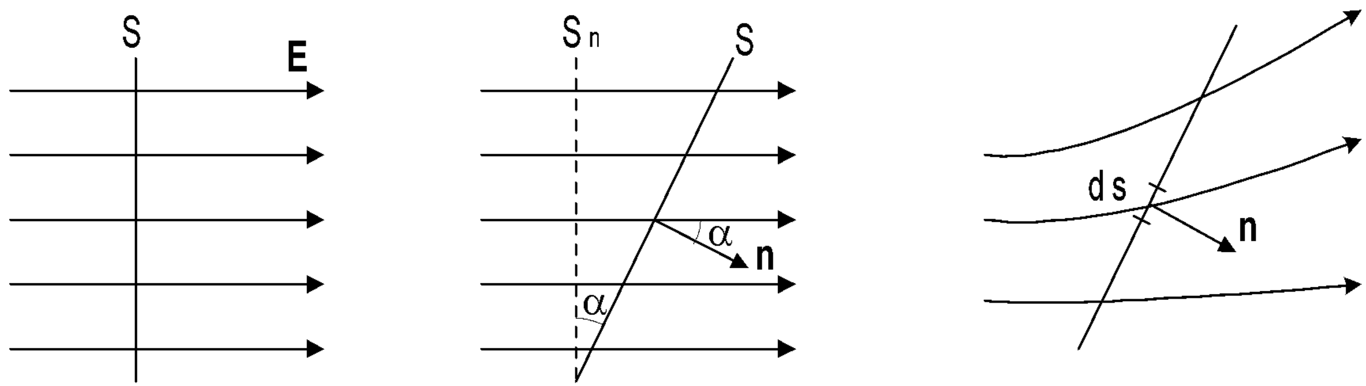
$$\mathbf{E} = \sum_i \mathbf{E}_i$$

The intensity of the electrostatic field of a system of point charges is iqual to the vector sum of the intentisities of the fields created by each of these charges separately.

$$\varphi = \sum_i \varphi_i$$

The potentials are added algebraically.

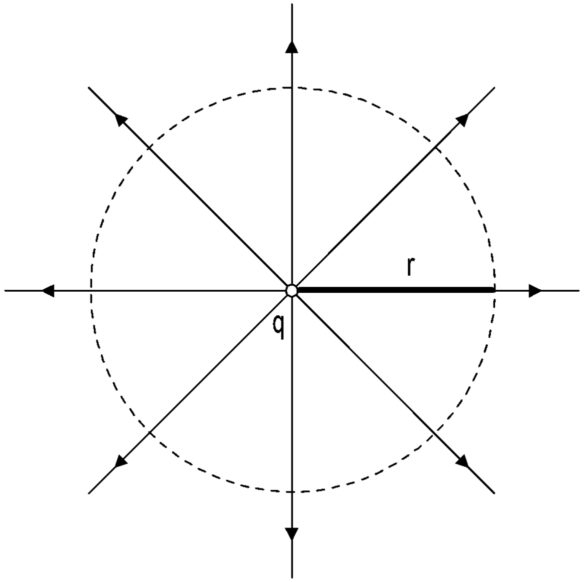
Electric flux



(i) (ii) (iii)

E x $area$ $\quad ES\cos\alpha \quad \int EdS\cos\alpha = \int \mathbf{E}\, d\mathbf{S}$

Gauss's theorem



$$\oint \mathbf{E}\, d\mathbf{S} = E \times 4\pi r^2 = \frac{q}{4\pi\varepsilon_0\varepsilon r^2} \times 4\pi r^2 = \frac{q}{\varepsilon_0\varepsilon}$$

The total electric flux passing through any closed surface whatever its shape, is always equal to the total charge enclosed by the surface, divided by $\varepsilon_0\varepsilon$

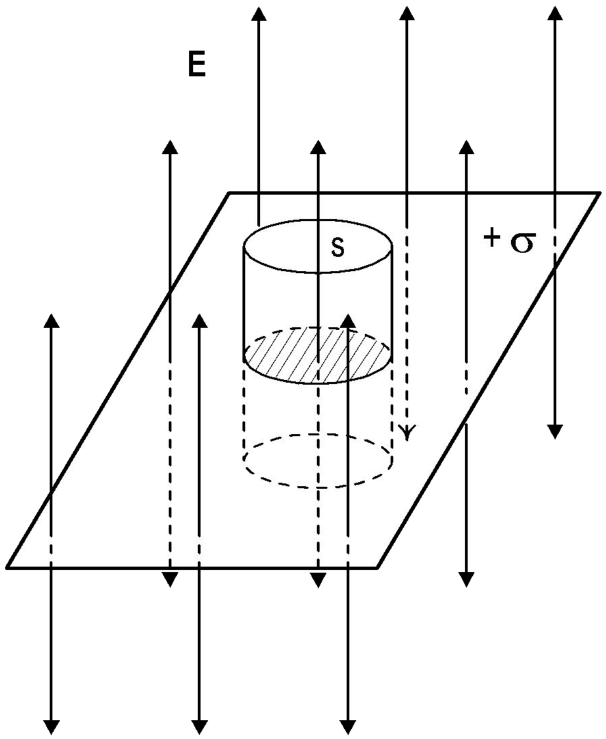
Gauss's theorem (cont.)

$$\oint \mathbf{E}\, d\mathbf{S} = \frac{1}{\varepsilon_0 \varepsilon} \sum_i q_i$$

$$\oint \mathbf{E}\, d\mathbf{S} = \frac{1}{\varepsilon_0 \varepsilon} \int \rho\, dV$$

ρ is the volumetrical density of the charge

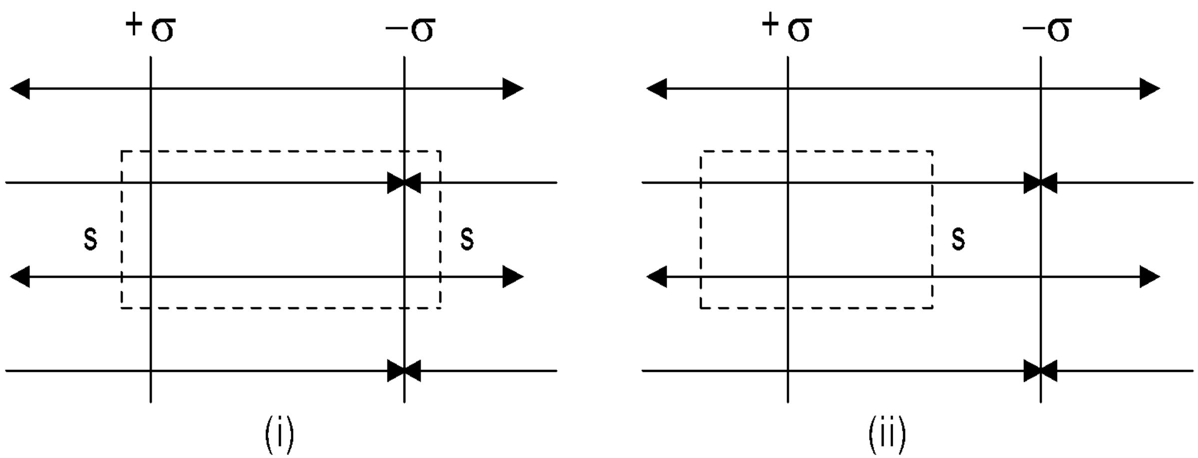
Field Outside a Charged Plane Conductor



$$E\,2S = \frac{\sigma S}{\varepsilon_0 \varepsilon}$$

$$\therefore E = \frac{\sigma}{2\varepsilon_0 \varepsilon}$$

Field Outside and Inside Two Parallel Charged Plane Conductors



$$E2S = \frac{\sigma S + (-\sigma)S}{\varepsilon_0 \varepsilon} = 0$$

$$\therefore E = 0$$

$$ES = \frac{\sigma S}{\varepsilon_0 \varepsilon}$$

$$\therefore E = \frac{\sigma}{\varepsilon_0 \varepsilon}$$