Introduction to Game Theory

Simultaneous-move games with pure strategies (discrete strategies)

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Simultaneous

Simultaneous – actions taken at exactly the same time or in isolation – *imperfect information* (*imperfect knowledge*).

- **Examples:**
- Pricing
- Designing new product
- Elections

Football (goal keeper (goalie) and striker during penalty kick)

Depicting simultaneous-move games with discrete strategies

- Strategy = action (in pure strategy games)
- Game table, game matrix, payoff table, payoff matrix
- The matrix is called the normal form (strategic form) of the game

1. Depicting simultaneous-move games with discrete strategies

		COLUMN			
		Left	Middle	Right	
	Тор	3/1	2/3	10/2	
ROW	High	4/5	3/0	6/4	
	Low	2/2	5/4) 12/ <mark>3</mark>	
	Bottom	5/6	4/5	9/7	

Let us have a look at {L, M}

3. Dominance Prisoner's dilemma



3. Dominance

- Prisoner's dilemma
- Dominant vs. dominated strategy
- Dominant strategy one that outperform all other strategies available
- Dominant strategy my action is done regardless of my belief of the other player's choice

- A. Both players have dominant strategies
 - Prisoner's dilemma (PD)
 - **Three essential features:**

 each player has two strategies: cooperate and defect from cooperation

each player has dominant strategy

 dominance solution equilibrium is worse for both players

Simultaneous games B. Only one player has a dominant strategy Analysis of pick a color games 2a. Pick a color

		Player 2	
		White	Blue
Dlovor 1	White	50/50	75/25
rlayer 1	Blue	50/ <mark>50</mark>	25/75

Simultaneous games B. Only one player has a dominant strategy Analysis of pick a color games 2b. Pick a color

		Player 2	
		Orange	Black
Dlovor 1	Orange	75/25	25/75
rlayer 1	Black	50/ <mark>50</mark>	50/ <mark>50</mark>

C. Successive elimination of dominated strategies Dominance solvable – solution achieved via successive (iterated) elimination of dominated strategies



C. Successive elimination of dominated strategies Weak vs. strict dominance

Using weak dominance we may skip some NE – cell-by-cell check is necessary before elimination



Find NE How many NE? One? NO! Three

4. Best-response analysis

If best-response analysis of discrete strategies does not lead to NE then, there are no NE in pure strategies.

		COLUMN			
		Left	Middle	Right	
ROW	Тор	3/1	23	10/2	
	High	4/5	3/0	6/4	
	Low	2/2	$(\underline{34})$	(1)/3	
	Bottom	<u>3/6</u>	4/5	9/7	

5. The minimax method for zero-sum games Logic of strict conflict (zero-sum)

What is good for me is bad for the other player.



max=10 max=5,6

max=13

Finding NE in pure strategies:

- Cell-by-cell inspection
- Dominant strategies
- Successive elimination of dominated strategies
- Minimax (zero sum games)
- Best-response analysis

Simultaneous games 6. Three players

- Emily's preferences: (others are the same)
- 6 don't contribute, Talia & Nina contribute
- 5 all contribute
- 4 don't contribute, one of the others contributes
- 3 contribute, one of the others contributes
- 2 don't contribute, Talia & Nina do not
- 1 contribute, Talia & Nina do not

Simultaneous games6. Three playersTALIA choosesContribute

		NINA				NI	NA
		Contri-b ute	Don't			Contri-b ute	Don't
	Contribute	5/5/5	3/6/3		Contribute	3/3/6	1/4/4
EMILY	Don't	6/3/3	4/4/1	EMILY	Don't	4/1/4	2/2/2

- 6. Three players
- Searching for NE
- Check for dominant strategies (check in two pages!)
- Cell-by-cell inspection NE
- Best response NE

Simultaneous games 6. Three players- best response TALIA chooses

Contribute

Don't Contribute



7.Multiple equilibria in pure strategies Pure coordination. "flat tire"

Another example:

		SALLY		
		Starbucks	Local Latte	
	Starbucks	1/1	0/0	
HAKKY	Local Latte	0/0	1/1	

7.Multiple equilibria in pure strategies

Pure coordination. Players want to coordinate on the same action, no matter what action.

- Coordination device focal point
- Expectations of the players must converge on focal point.
- One must knows, that the other knows, that the first knows, that the other ...

7.Multiple equilibria in pure strategies Assurance

		SALLY		
		Starbucks	Local Latte	
	Starbucks	1/1	0/0	
HAKKY	Local Latte	0/0	2/2	

7. Multiple equilibria in pure strategies

- Assurance multiple equilibria, but players prefer the same equilibrium
- Focal point the obvious equilibrium to choose.
- Focal point requires convergence of expectations, otherwise they may fail to coordinate. But assurance (that the other player chooses proper action) is relatively easy to obtain.

7.Multiple equilibria in pure strategies The Battle of Sexes

		SALLY		
		Starbucks	Local Latte	
	Starbucks	2/1	0/0	
HAKKY	Local Latte	0/0	1/2	

7. Multiple equilibria in pure strategies

- The Battle of Sexes both equilibria are preferred over the other possible outcome of the game, but
- each player prefers different equilibrium.
- If both players are nice they end up in bad place.
- Strategy is needed.
- In repeated games coordination may be negotiated and maintained.

7.Multiple equilibria in pure strategies

Chicken game

		DEAN		
		Swerve (Chicken)	Straight (Tough)	
ТАВЛІТС	Swerve (Chicken)	0/0	-1/1	
JANIES	Straight (Tough)	1/-1	-2/-2	

7.Multiple equilibria in pure strategies

Chicken game – multiple equilibria, but each player prefers different equilibrium and they want to avoid choosing the same action.

Features:

- two strategies: "tough" and "weak"
- two pure strategy NE
- each player prefers strictly, the other plays "weak"
- payoff for "tough" strategies played by both is very bad for them

7. Multiple equilibria in pure strategies

Small summary

In coordination games – there is uncertainty about the other player strategy.

There may be NE in mixed strategies.