Introduction to Game Theory

Simultaneous-move games with pure strategies (discrete strategies)

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Simultaneous games

Simultaneous

Simultaneous – actions taken at exactly the same time or in isolation – *imperfect information* (*imperfect knowledge*).

Examples:

- Pricing
- Designing new product
- Elections
- Football (goal keeper (goalie) and striker during penalty kick)

Simultaneous games

1. Depicting simultaneous-move games with discrete strategies

- Strategy = action (in pure strategy games)
- Game table, game matrix, payoff table, payoff matrix
- The matrix is called the normal form (strategic form) of the game

Simultaneous games

1. Depicting simultaneous-move games with discrete strategies

		COLUMN		
		Left	Middle	Right
ROW	Top	3/1	2/3	10/2
	High	4/5	3/0	6/4
	Low	2/2	5/4	12/3
	Bottom	5/6	4/5	9/7

Let us have a look at {L, M}

Simultaneous games

3. Dominance

Prisoner's dilemma

		WIFE	
		Confess (Defect)	Deny (Cooperate)
HUSBAND	Confess (Defect)	10yr/10yr	1yr/25yr
	Deny (Cooperate)	25yr/1yr	3yr/3yr

Simultaneous games

3. Dominance

Prisoner's dilemma

- Dominant vs. dominated strategy
- Dominant strategy – one that outperform all other strategies available
- Dominant strategy – my action is done regardless of my belief of the other player's choice

Simultaneous games

A. Both players have dominant strategies

Prisoner's dilemma (PD)

Three essential features:

- each player has two strategies: cooperate and defect from cooperation

- each player has dominant strategy

- dominance solution equilibrium is worse for both players

Simultaneous games

B. Only one player has a dominant strategy

- Analysis of pick a color games
- 2a. Pick a color

		Player 2	
		White	Blue
Player 1	White	50/50	75/25
	Blue	50/50	25/75

Simultaneous games

B. Only one player has a dominant strategy

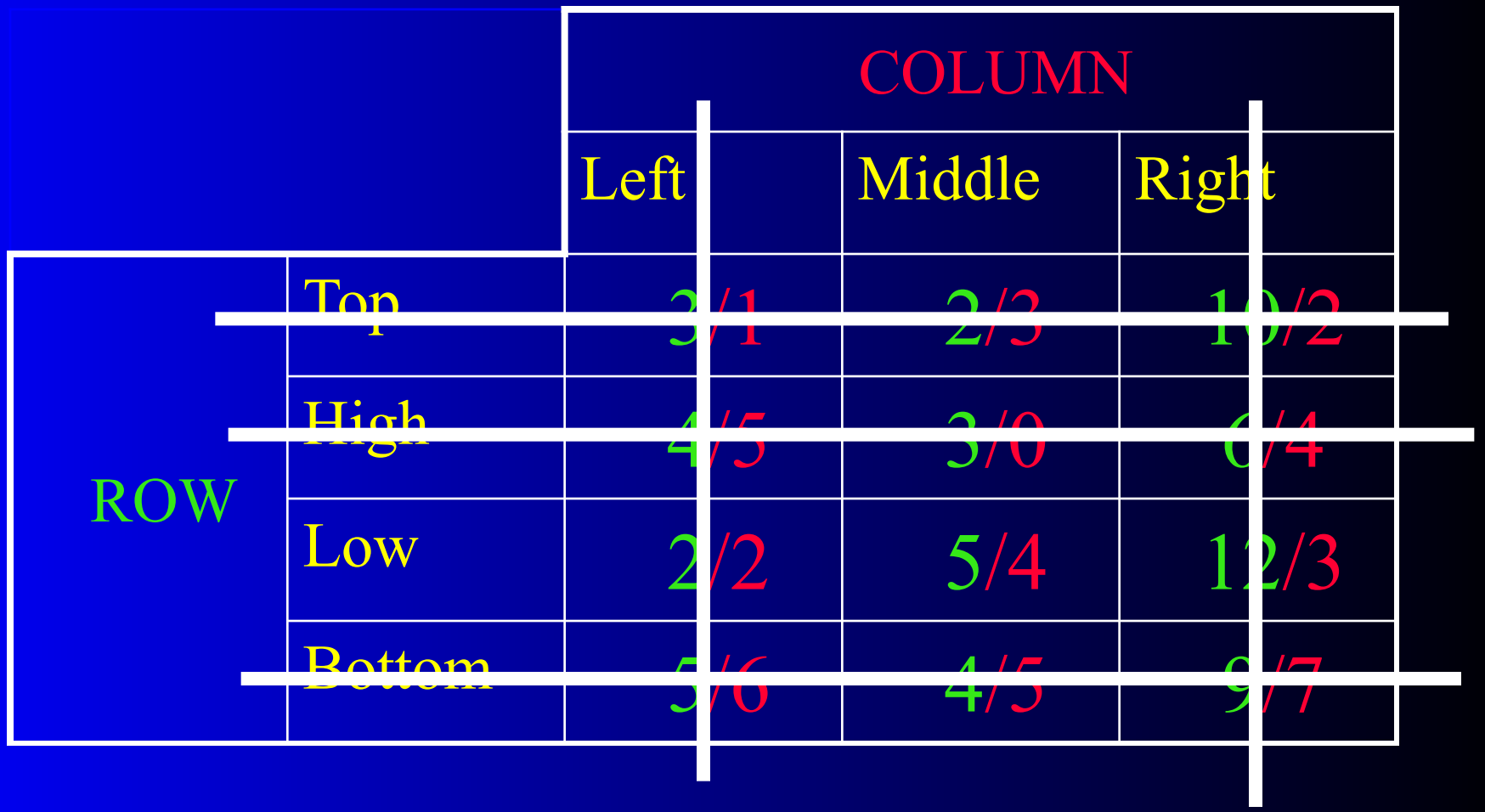
- Analysis of pick a color games
- 2b. Pick a color

		Player 2	
		Orange	Black
Player 1	Orange	75/25	25/75
	Black	50/50	50/50

Simultaneous games

C. Successive elimination of dominated strategies

- Dominance solvable – solution achieved via successive (iterated) elimination of dominated strategies



		COLUMN		
		Left	Middle	Right
ROW	Top	3/1	2/3	10/2
	High	4/5	3/0	6/4
	Low	2/2	5/4	12/3
	Bottom	5/6	4/5	9/7

Simultaneous games

C. Successive elimination of dominated strategies

- Weak vs. strict dominance
- Using weak dominance we may skip some NE – cell-by-cell check is necessary before elimination

		COLUMN	
		Left	Right
ROW	Up	0/0	1/1
	Down	1/1	1/1

Find NE How many NE? One? NO! Three

Simultaneous games

4. Best-response analysis

If best-response analysis of discrete strategies does not lead to NE then, there are no NE in pure strategies.

		COLUMN		
		Left	Middle	Right
ROW	Top	3/1	2/3	10/2
	High	4/5	3/0	6/4
	Low	2/2	5/4	12/3
	Bottom	5/6	4/5	9/7

Simultaneous games

5. The minimax method for zero-sum games

Logic of strict conflict (zero-sum)

What is good for me is bad for the other player.

		DEFENSE			
		Run	Pass	Blitz	
OFFENSE	Run	2	5	13	min=2
	Short Pass	6	5,6	10,5	min=5,6
	Medium Pass	6	4,5	1	min=1
	Long Pass	10	3	-2	min=-2
		max=10	max=5,6	max=13	

Simultaneous games

Finding NE in pure strategies:

- Cell-by-cell inspection
- Dominant strategies
- Successive elimination of dominated strategies
- Minimax (zero sum games)
- Best-response analysis

Simultaneous games

6. Three players

- Emily's preferences: (others are the same)

6 – don't contribute, Talia & Nina contribute

5 – all contribute

4 – don't contribute, one of the others contributes

3 – contribute, one of the others contributes

2 – don't contribute, Talia & Nina do not

1 - contribute, Talia & Nina do not

Simultaneous games

6. Three players

TALIA chooses

Contribute

		NINA	
		Contri-bute	Don't
EMILY	Contribute	5/5/5	3/6/3
	Don't	6/3/3	4/4/1

Don't Contibute

		NINA	
		Contri-bute	Don't
EMILY	Contribute	3/3/6	1/4/4
	Don't	4/1/4	2/2/2

Simultaneous games

6. Three players

- Searching for NE
  - Check for dominant strategies (check in two pages!)
  - Cell-by-cell inspection NE
  - Best response NE

Simultaneous games

6. Three players- best response

TALIA chooses

Contribute

		NINA	
		Contri-bute	Don't
EMILY	Contribute	5/5/5	3/6/3
	Don't	6/3/3	4/4/1

Don't Contribute

		NINA	
		Contri-bute	Don't
EMILY	Contibute	3/3/6	1/4/4
	Don't	4/1/4	2/2/2

Simultaneous games

7. Multiple equilibria in pure strategies

Pure coordination. „flat tire”

Another example:

		SALLY	
		Starbucks	Local Latte
HARRY	Starbucks	1/1	0/0
	Local Latte	0/0	1/1

Simultaneous games

7.Multiple equilibria in pure strategies

Pure coordination. Players want to coordinate on the same action, no matter what action.

Coordination device - **focal point**

Expectations of the players must converge on focal point.

One must knows, that the other knows, that the first knows, that the other …

Simultaneous games

7.Multiple equilibria in pure strategies

Assurance

		SALLY	
		Starbucks	Local Latte
HARRY	Starbucks	1/1	0/0
	Local Latte	0/0	2/2

Simultaneous games

7.Multiple equilibria in pure strategies

Assurance – multiple equilibria, but players prefer the same equilibrium

Focal point – the obvious equilibrium to choose.

Focal point requires convergence of expectations, otherwise they may fail to coordinate. But assurance (that the other player chooses proper action) is relatively easy to obtain.

Simultaneous games

7.Multiple equilibria in pure strategies

The Battle of Sexes

		SALLY	
		Starbucks	Local Latte
HARRY	Starbucks	2/1	0/0
	Local Latte	0/0	1/2

Simultaneous games

7. Multiple equilibria in pure strategies

The Battle of Sexes – both equilibria are preferred over the other possible outcome of the game, but each player prefers different equilibrium.

If both players are nice – they end up in bad place.

Strategy is needed.

In repeated games – coordination may be negotiated and maintained.

Simultaneous games

7.Multiple equilibria in pure strategies

Chicken game

		DEAN	
		Swerve (Chicken)	Straight (Tough)
JAMES	Swerve (Chicken)	0/0	-1/1
	Straight (Tough)	1/-1	-2/-2

Simultaneous games

7.Multiple equilibria in pure strategies

Chicken game – multiple equilibria, but each player prefers different equilibrium and they want to avoid choosing the same action.

Features:

- two strategies: “tough” and “weak”
- two pure strategy NE
- each player prefers strictly, the other plays “weak”
- payoff for “tough” strategies played by both is very bad for them

Simultaneous games

7. Multiple equilibria in pure strategies

Small summary

In coordination games – there is uncertainty about the other player strategy.

There may be NE in mixed strategies.