CHECKLIST

- Be able to define *angular momentum* in its 3D and 2D form
- Be able to define the *moment of inertia* of a rigid body or system of point particles
- Understand the link between angular momentum, moment of inertia and angular speed
- Be able to define the work done by a torque moving through an angle and the power delivered by this torque
- Know an equation for the kinetic energy of a rotating body moving with angular velocity, ω
- Have a knowledge of the correspondence between linear equations of motion and angular equations of motion
- Be able to perform calculations using any of the equations you have met

Angular momentum. Moment of inertia and more properties of angular motion.

Moment of inertia, I

For linear momentum, $p = mv$

Similarly, for angular momentum,

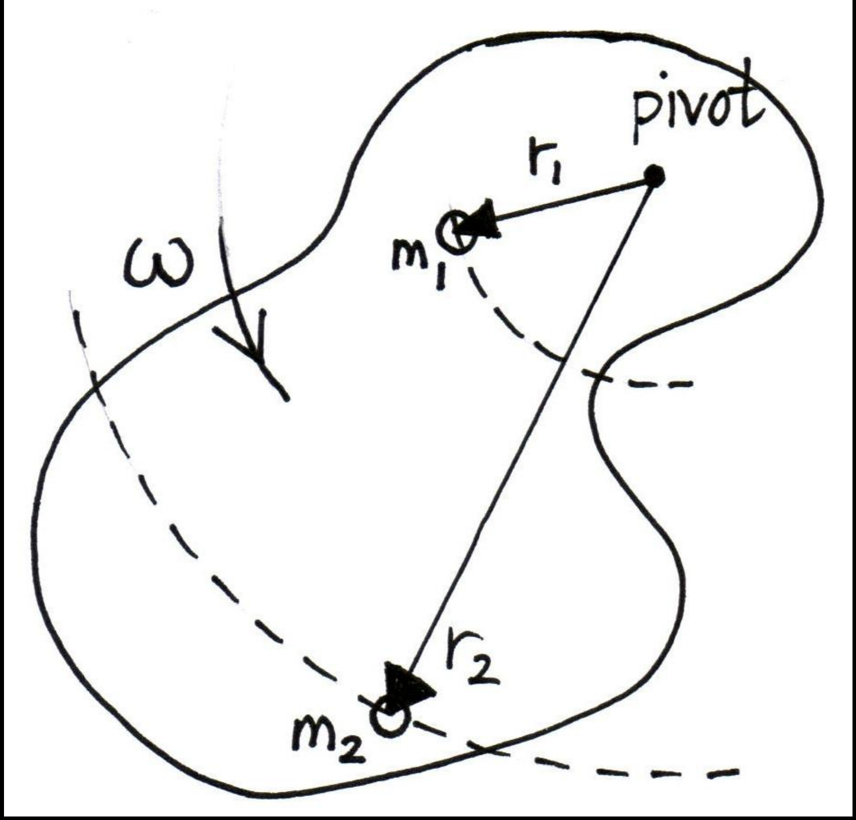
$$L = \text{“something”} \times \omega$$

That is, we seek a property of the body that measures ‘angular inertia’. This is defined as the *moment of inertia*, I.

Thus: $L = I\,\omega$

One way to get a formula for I

Consider a body rotating with angular speed ω. Then consider the body is divided into many small masses, m_1, m_2, etc at distances r_1, r_2, etc, respectively, from a pivot.



Moment of inertia

Moment of inertia

More on I

Note: we know that 1) if no external forces act on the system, linear momentum is conserved.

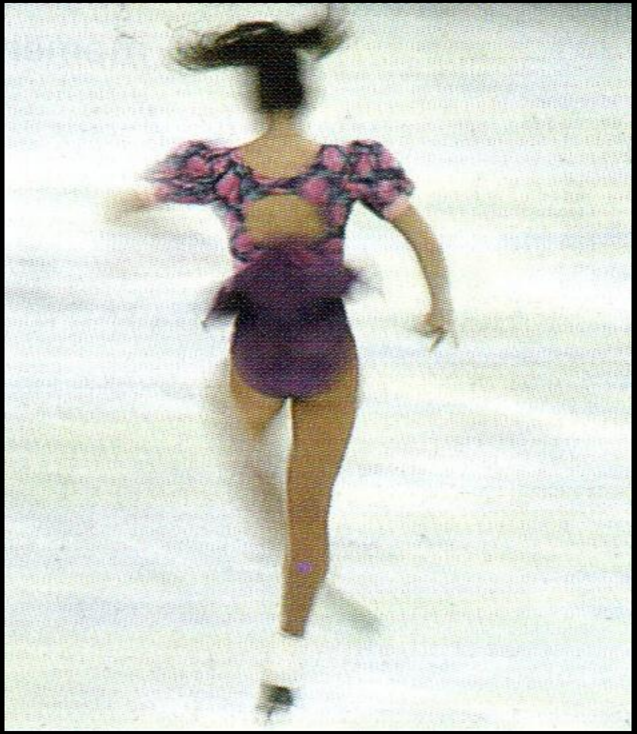
We now know that 2) if no external torques act on the system, angular momentum is conserved. This explains why, e.g., an ice skater rotates faster when she pulls in her arms.

Conservation of angular momentum

Since $L = I\omega$, if no external torques act, L is constant.

So, when the skater moves her arms in, closer to her body,

I decreases and therefore ω increases.

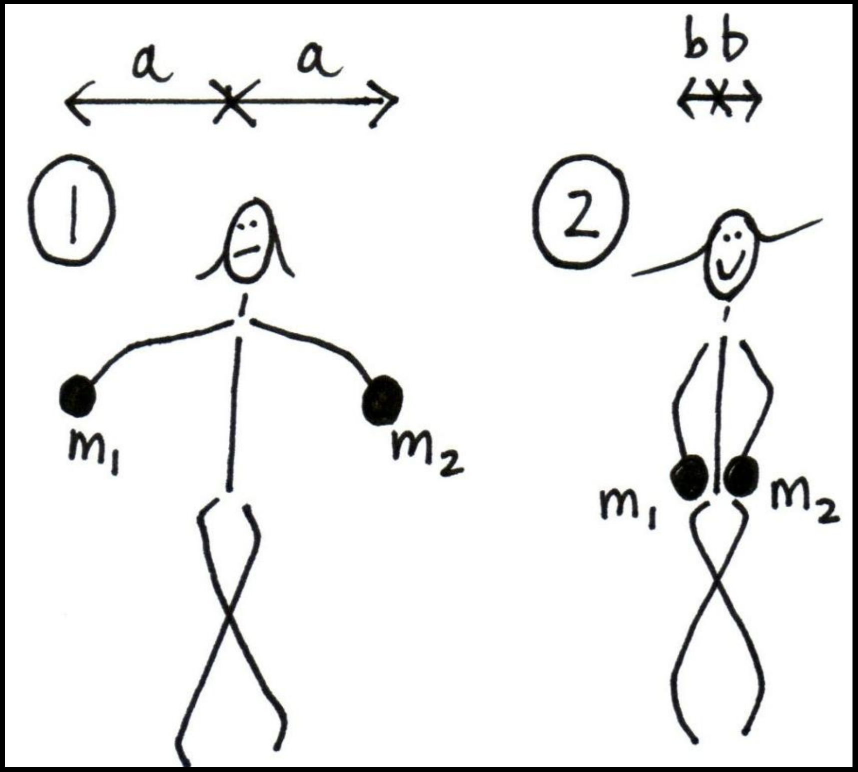


Conservation of angular momentum

In (1) $I = m_1a^2 + m_2a^2$

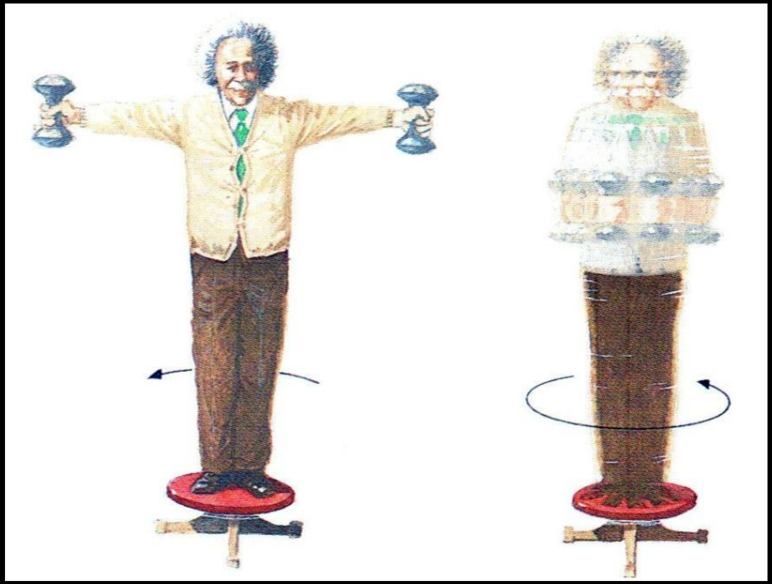
In (2) $I = m_1b^2 + m_2b^2$

For $L = I\,\omega$ to remain constant, the skater rotates faster in (2).



Please don't try this!

A crazy physics professor stands on a swivel chair with weights in his outstretched arms. A student causes him to rotate.

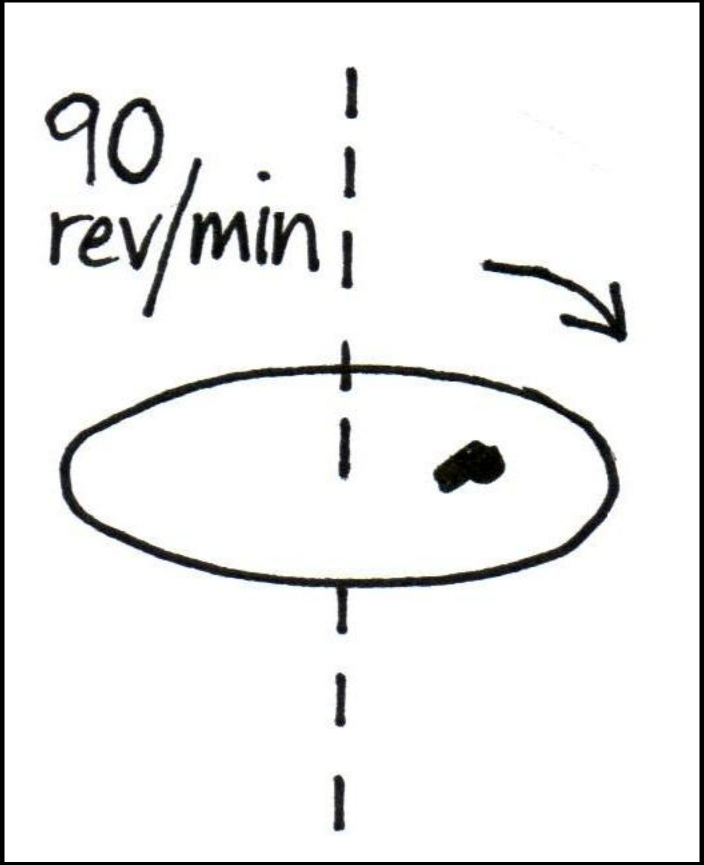


The professor brings in his arms and…

Comparison of equations for linear and angular motion

Example 1. Conservation of angular momentum

A horizontal disc rotates around a vertical axis at 90 rev/min. A small piece of chewing gum of 20 g falls vertically onto the disc and sticks to it at a distance of 5.0 cm from the axis. If the number of revs/min is reduced to 80, find the moment of inertia, I, of the disc.



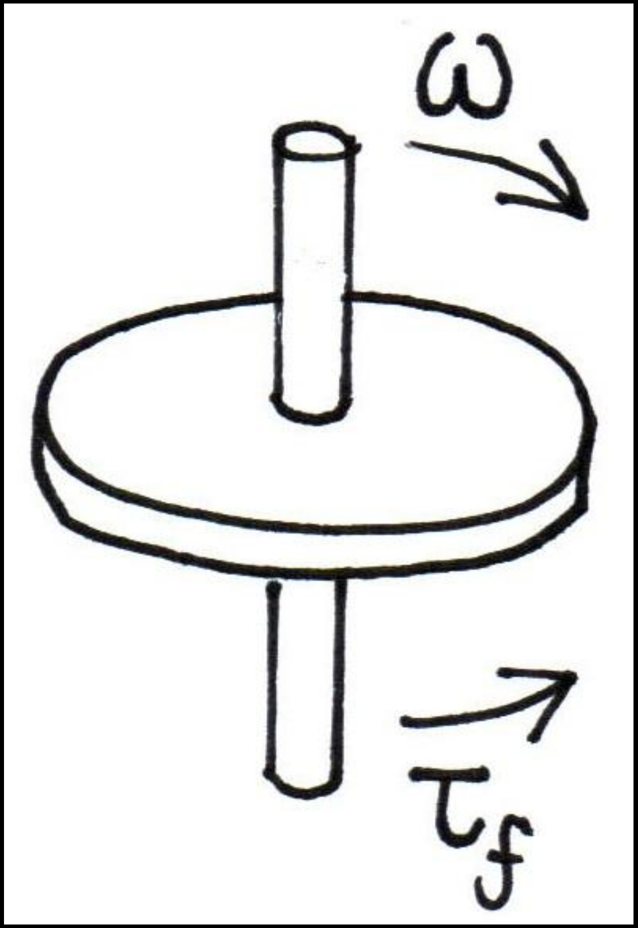
Work done by torque

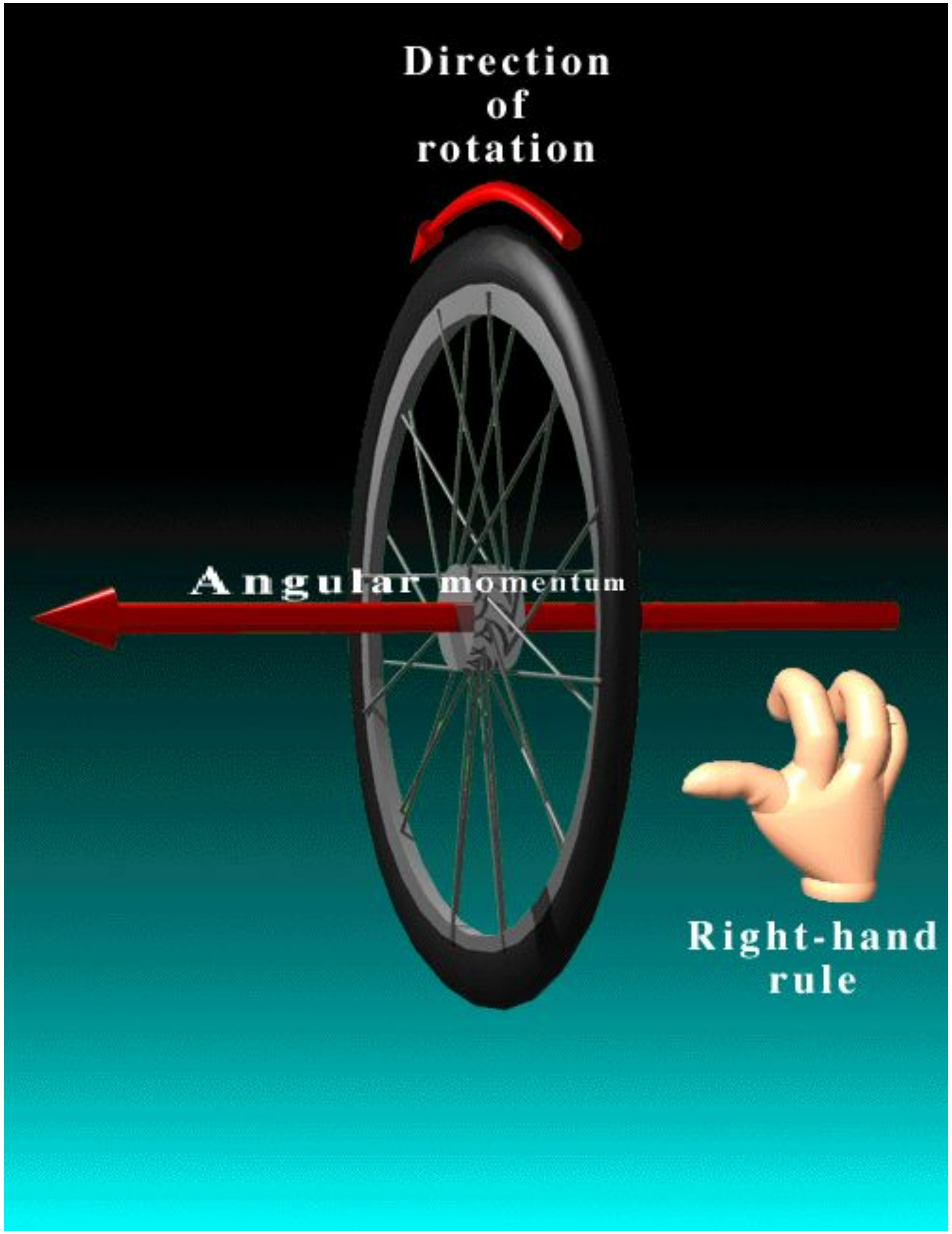
Note: We can also define work done by a constant torque in moving through an angle θ as: $W = \tau\theta$

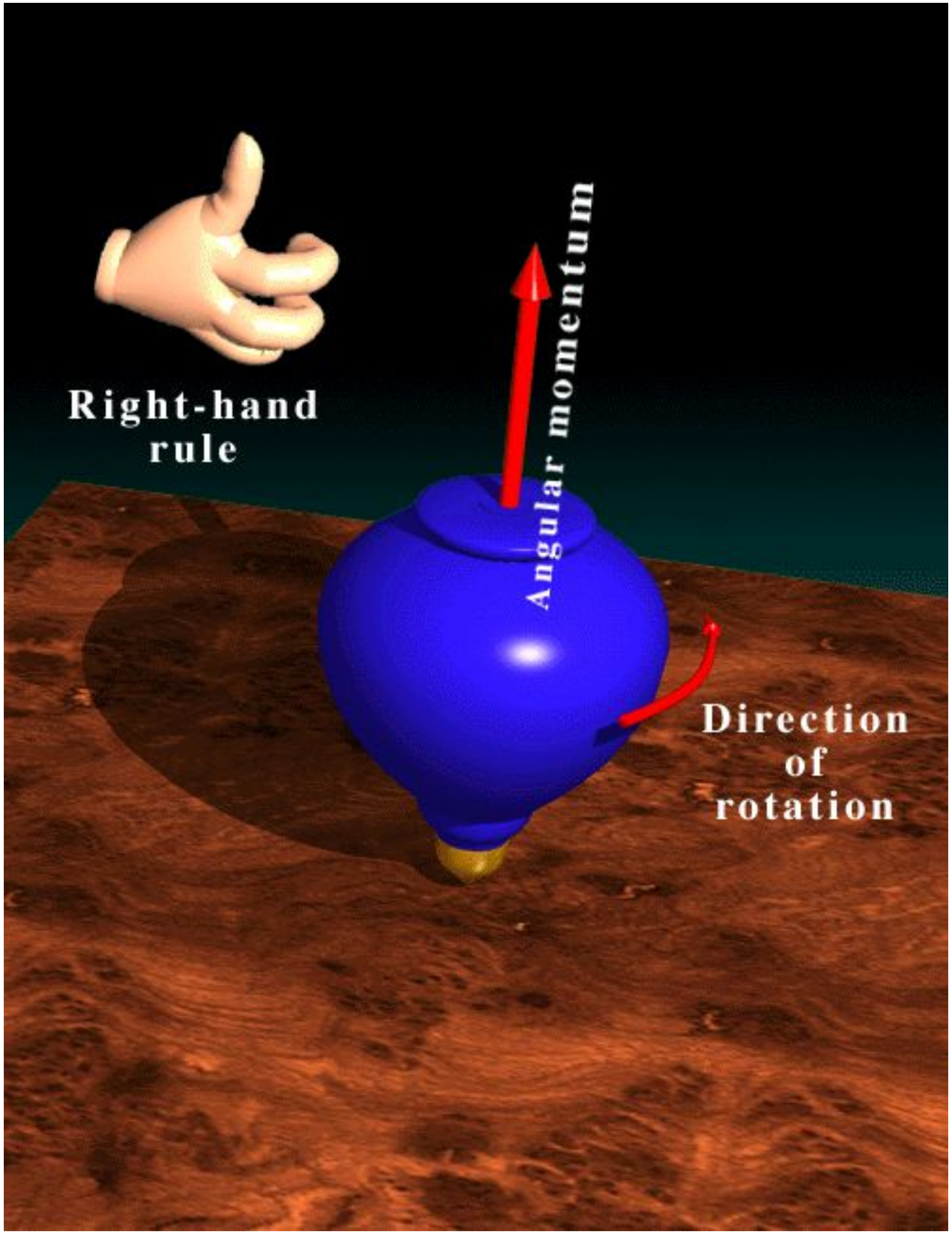
The power $P = \frac{dW}{dt}$

Example 2

An electric motor supplies a power of 500 W to drive a flywheel of I = 2.0 kgm^2 at a speed of 600 rev/min. How long will it take the flywheel to come to rest after the power is switched off, assuming the retarding torque, due to friction, is constant?




Direction of rotation
Angular momentum
Right-hand rule


Right-hand
rule
Angular momentum
Direction
of
rotation

CHECKLIST

READING Adams and Allday: 3.30, 3.31

At the end of this lecture you should

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