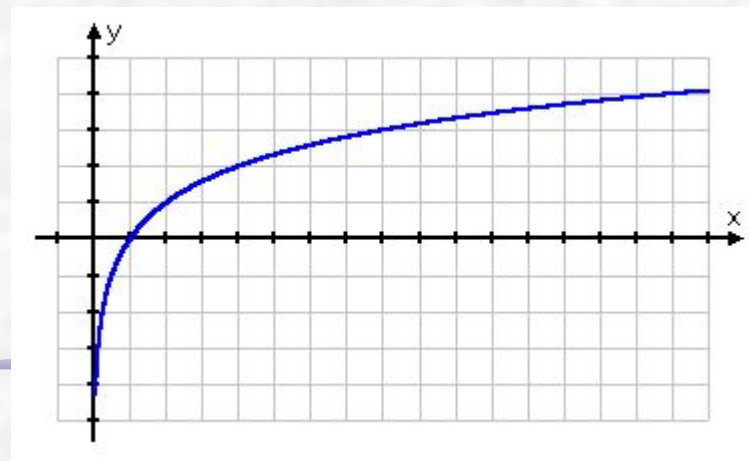
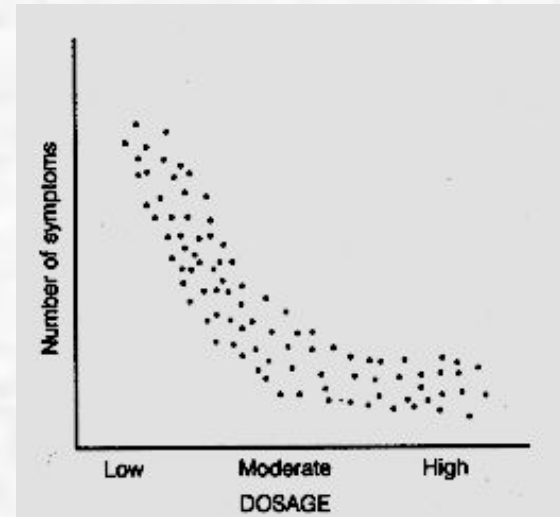
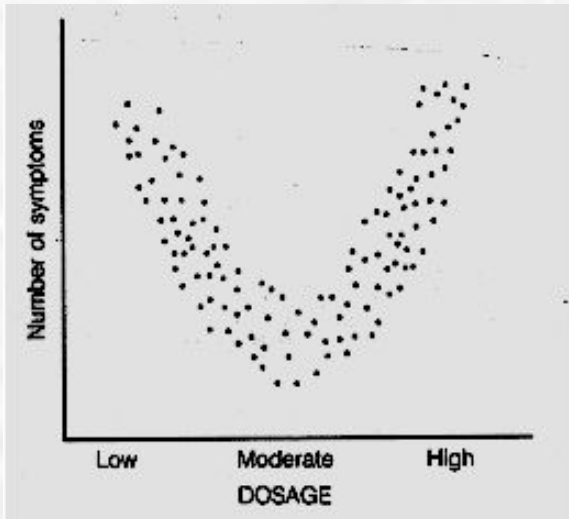


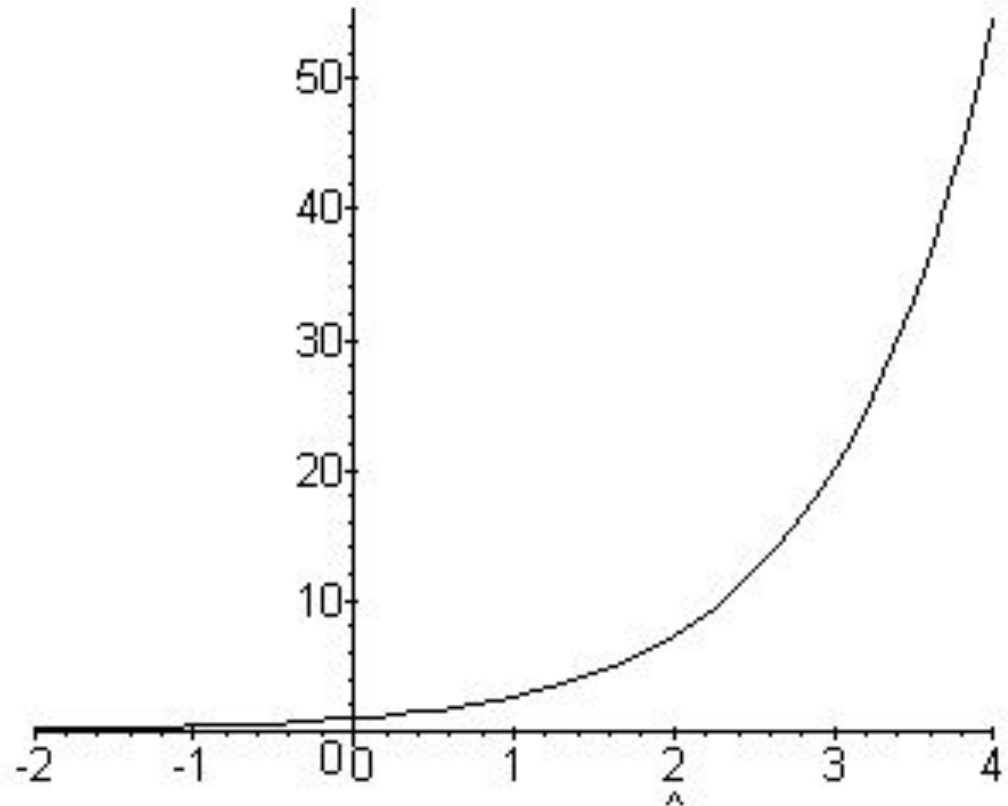
Choice of the functional form

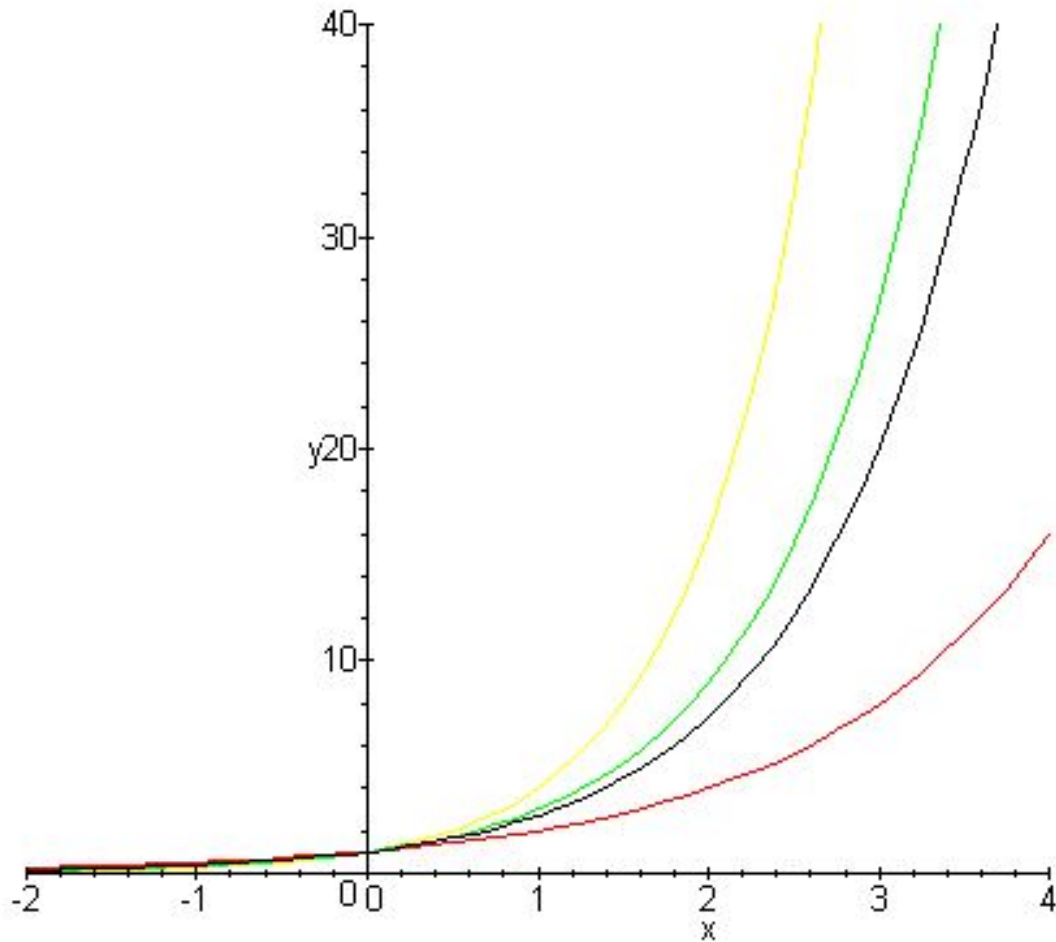
What if...



Exponential function

Exponential functions are functions which can be represented by graphs similar to the graph on the right





Yellow = 4^x

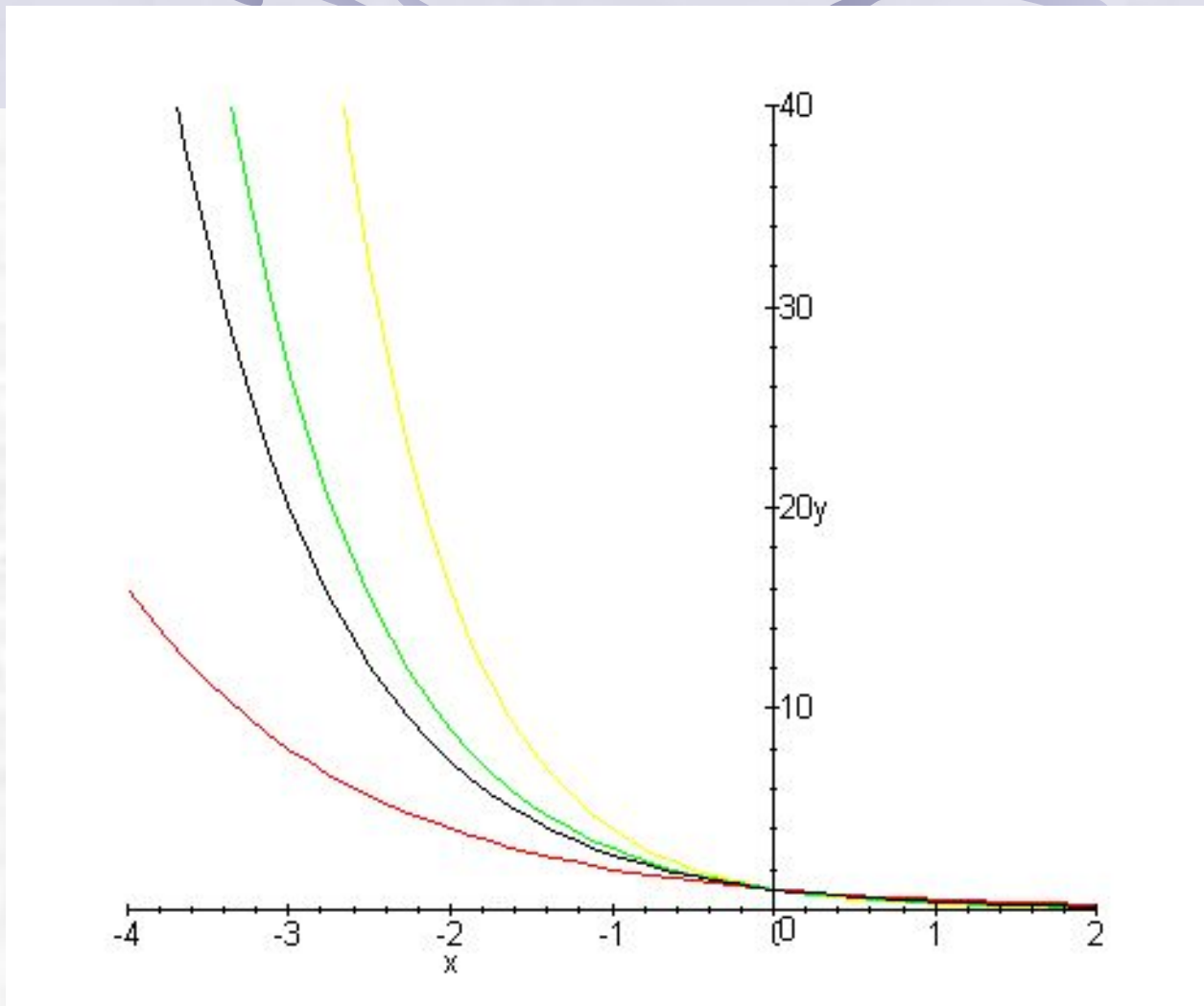
Green = e^x

Black = 3^x

Red = 2^x

As you could see in the graph, the larger the base, the faster the function increased

If we place a negative sign in front of the x , the graphs will be reflected(flipped) across the y -axis



Yellow = 4^{-x}

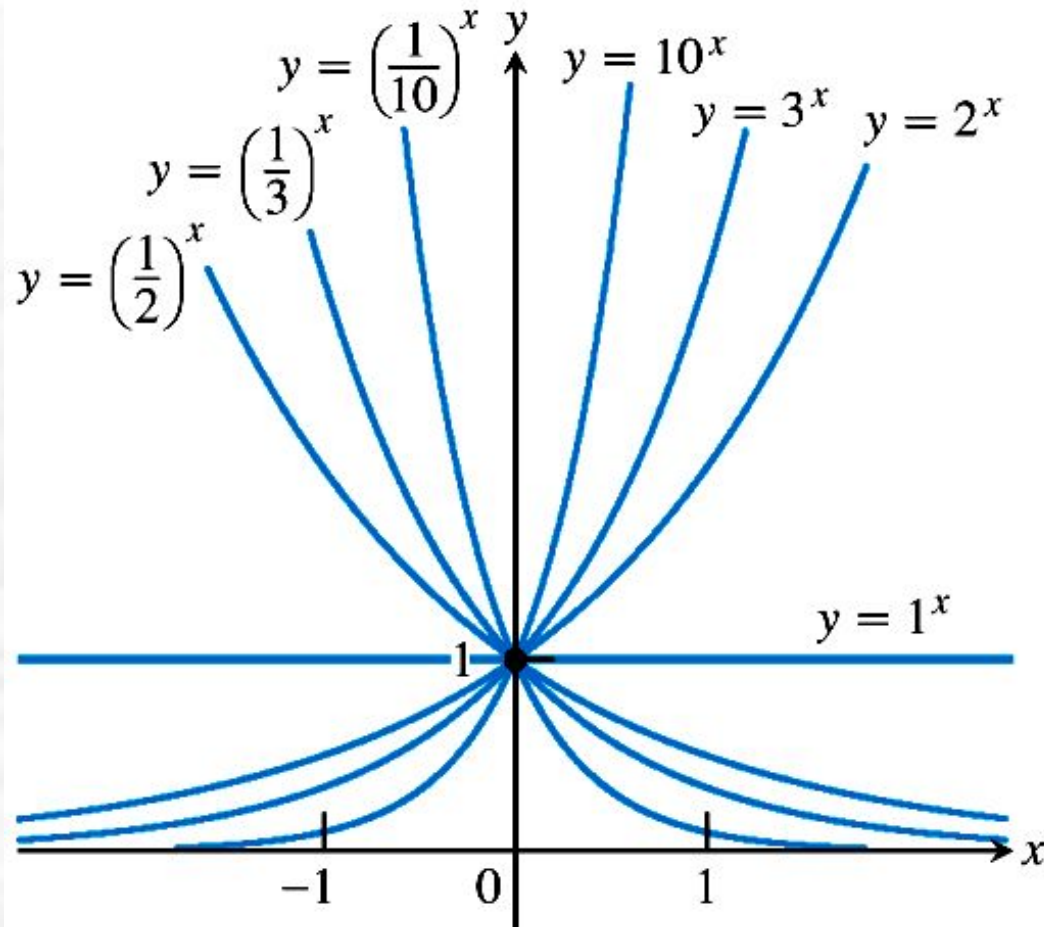
Green = e^{-x}

Black = 3^{-x}

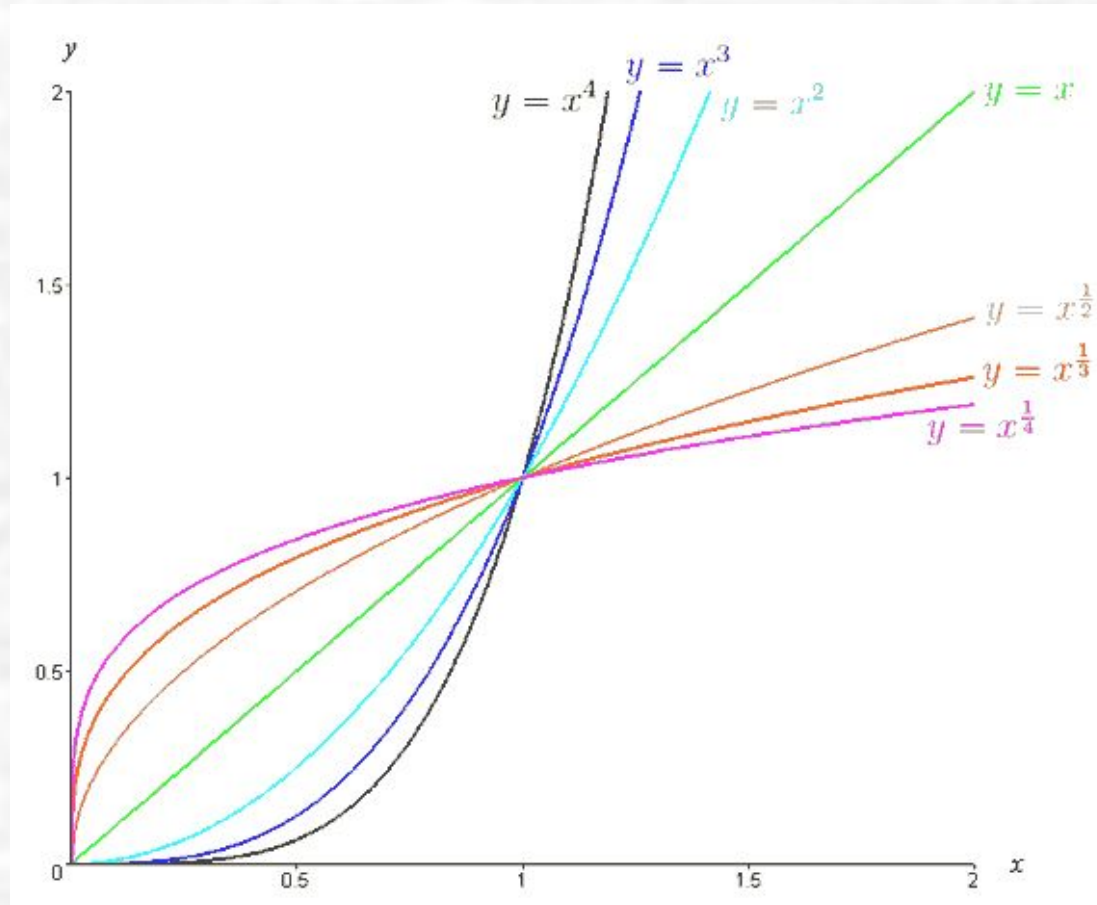
Red = 2^{-x}

Exponential function

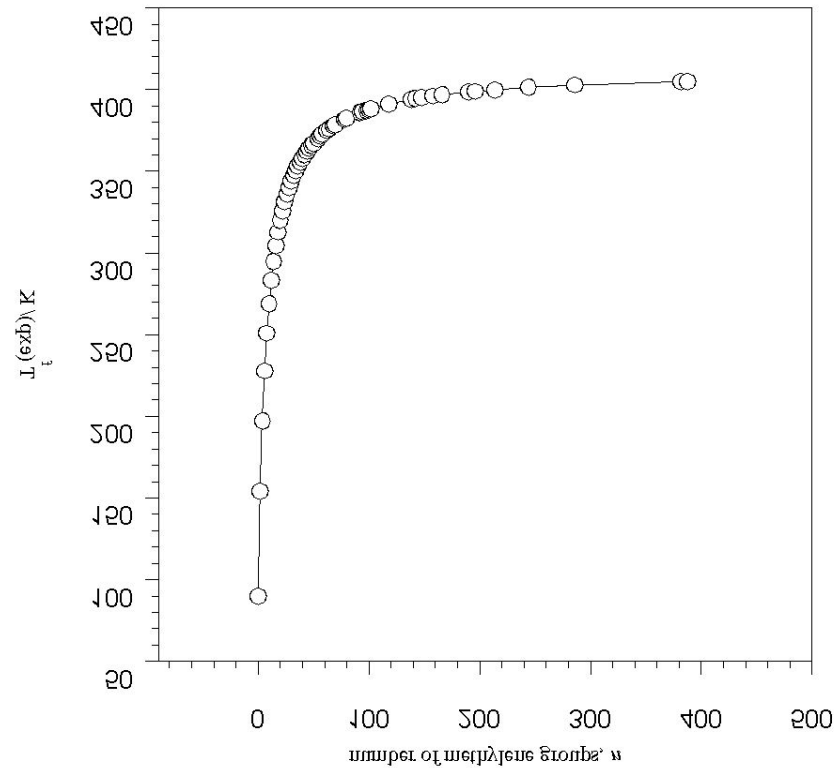
- Exponential functions decrease if $0 < b_1 < 1$ and increase if $b_1 > 1$



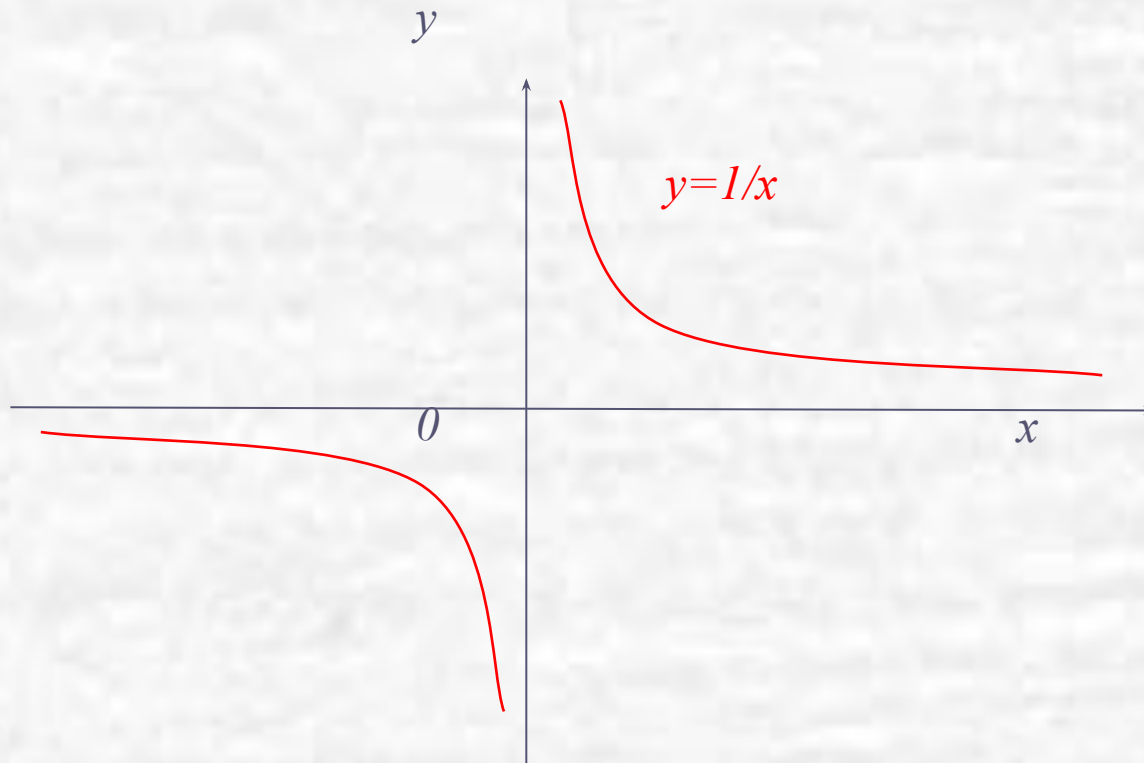
Power function



Logarithmic function



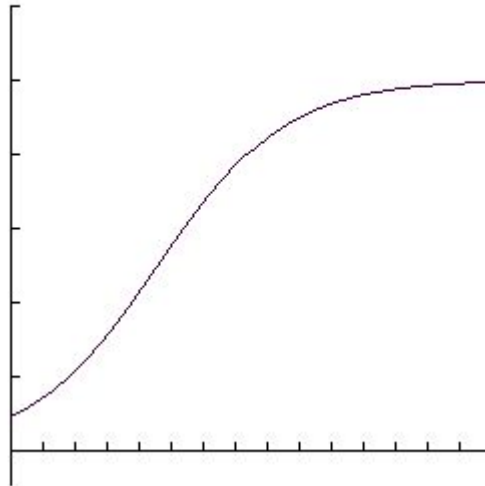
Hyperbolic function



Quadratic function

Logistic function

Logistic Growth Function



If growth begins slowly, then increases rapidly and eventually levels off, the data often can be model by an “S-curve”, or a logistic function.

General information

- NONLINEAR MODELS OFTEN ARE USED FOR SITUATION IN WHICH THE RATE OF INCREASE OR DECREASE IN THE DEPENDENT VARIABLE (WHEN PLOTTED AGAINST A PARTICULAR INDEPENDENT VARIABLE) IS NOT CONSTANT.

General information

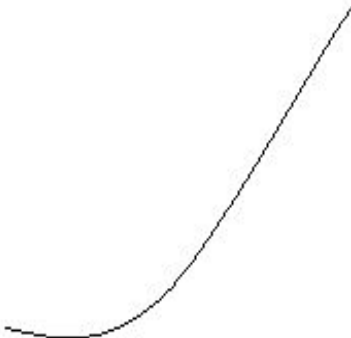
- SOME OF THESE MODELS REQUIRED A TRANSFORMATION TO THE INDEPENDENT VARIABLE.

Transformation

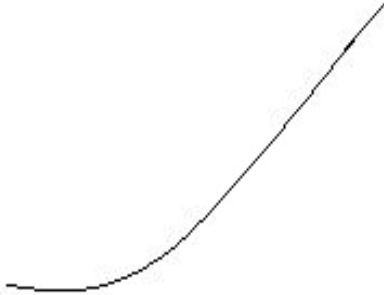
- Logarithms
- Substitution

Data transformations can be used to convert an equation into a linear form

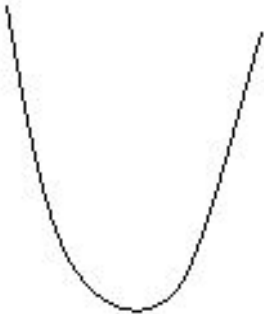
Exponential function

Model	Scatter plot	Transformation	Model after transformation
$\hat{y} = b_0 b_1^x$ exponential function		Logarithms	$y' = b_0' + b_1' x'$ where $y' = \log y$ $b_0' = \log b_0$ $b_1' = \log b_1$


Power function

Model	Scatter plot	Transformation	Model after transformation
$\hat{y} = b_0 x^{b_1}$ power function		Logarithms	$y' = b_0' + b_1 x'$ where $y' = \log y$ $x' = \log x$ $b_0' = \log b_0$


Quadratic function

Model	Scatter plot	Transformation	Model after transformation
$\hat{y} = b_0 + b_1x + b_2x^2$ quadratic function		Substitution	$y = b_0 + b_1x_1 + b_2x_2$ where $x_1 = x$ $x_2 = x^2$

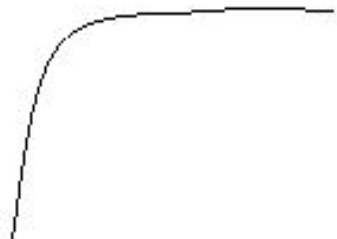
Polynomial function

Model	Scatter plot	Transformation	Model after transformation
$\hat{y} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots + b_kx^k$ polynomial function		Substitution	$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \dots + b_kx_k$ where $x_1 = x$ $x_2 = x^2$ $x_3 = x^3$ $x_k = x^k$

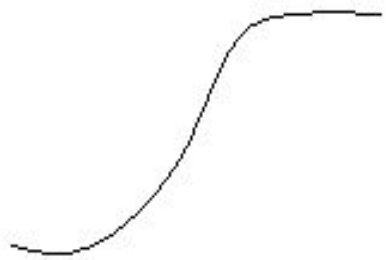
Hyperbolic function

Model	Scatter plot	Transformation	Model after transformation
$\hat{y} = b_0 + b_1 \frac{1}{x}$ hyperbolic function		Substitution	$y = b_0 + b_1 x'$ where $x' = 1/x$

Logarithmic function

Model	Scatter plot	Transformation	Model after transformation
$\hat{y} = b_0 + b_1 \ln x$ logarithmic function		Substitution	$y = b_0 + b_1 x'$ where $x' = \ln x$

Logistic function

Model	Scatter plot	Transformation	Model after transformation
$\hat{y} = \frac{b_0}{1 + b_1 e^{-x}}$ logistic function		Substitution	$y' = b_0' + b_1' x'$ where $y' = 1/y$ $x' = e^{-x}$ $b_0' = 1/b_0$ $b_1' = b_1/b_0$

Linear function

X	Y	ΔX	ΔY	$\Delta Y / \Delta X$
1,2	3,08	-	-	-
1,4	3,58	0,2	0,5	2,50
1,6	4,08	0,2	0,5	2,50
1,8	4,58	0,2	0,5	2,50

$$b_1 = 2,50$$

Exponential function

X	Y	ΔX	ΔY	$\Delta Y / \Delta X$	log y	log y/x
1,2	4,963	-	-	-	0,6957	0,580
1,4	6,482	0,2	1,519	7,595	0,8117	0,580
1,8	11,086	0,4	4,604	11,51	1,0448	0,580
3	54,872	1,2	43,786	36,4883	1,7394	0,580

$$b_1 = 10^{0,580} = 3,8$$

Power function

X	Y	ΔX	ΔY	$\Delta Y / \Delta X$	$\log y$	$\log y/x$	$\log x$	$\log y / \log x$
1,4	2,027	-	-	-	0,3069	0,219	0,146	2,10
2	4,28	0,6	2,253	3,755	0,6314	0,316	0,301	2,10
5	29,37	3	25,085	8,36167	1,4678	0,294	0,699	2,10
9	100,9	4	71,535	17,88375	2,0039	0,223	0,954	2,10

$$b_1 = 2,10$$

Comparison

EXPONENTIAL	POWER
Independent variable is a power exponent	Independent variable is a power base
<u>Form of model:</u>	
$\hat{y} = b_0 \cdot b_1^x$	$\hat{y} = b_0 \cdot x^{b_1}$
<u>Interpretation of the coefficients</u>	
<p>b_0- is the value of Y if independent variable is equal to zero.</p>	<p>b_0- is the value of Y if independent variable is equal to one</p>
<p>b_1- is the growth rate Y. If the independent variable increases 1 unit, the dependent variable will change (increase, if $b_1 > 1$, or decrease, if $b_1 < 1$) b_1 times, on average {or $(b_1 - 1) \times 100[\%]$, on average}.</p>	<p>b_1 - is the elasticity Y. If the independent variable increases 1 %, the dependent variable will change (increase, if $b_1 > 0$, or decrease, if $b_1 < 0$) $b_1\%$, on average.</p>

Comparison

EXPONENTIAL

POWER

Linear transformation - logarithms

$$\log \hat{y} = \log b_0 + x \cdot \log b_1$$

Linear form

$$\log \hat{y} = \log b_0 + b_1 \cdot \log x$$

Parameters estimation – OLS:

$$\begin{bmatrix} \log b_0 \\ \log b_1 \end{bmatrix} = (X^T X)^{-1} X^T \log Y$$

$$\begin{bmatrix} \log b_0 \\ b_1 \end{bmatrix} = (\log X^T \log X)^{-1} \log X^T \log Y$$

Matrix and vector:

$$X^T X = \begin{bmatrix} n & \sum x \\ \sum x & \sum x^2 \end{bmatrix}$$

$$\log X^T \log X = \begin{bmatrix} n & \sum \log x \\ \sum \log x & \sum (\log x)^2 \end{bmatrix}$$

$$X^T \log Y = \begin{bmatrix} \sum \log y \\ \sum x \cdot \log y \end{bmatrix}$$

$$\log X^T \log Y = \begin{bmatrix} \sum \log y \\ \sum \log x \cdot \log y \end{bmatrix}$$

Comparison

EXPONENTIAL

POWER

After $\log b_0$ and $\log b_1$ are estimated we should check goodness of fit (standard error of the estimate, indetermination coefficient, determination coefficient, test parameters individually and check residuals' characteristics – at least linearity) for the linear form.

$$\log \hat{y} = \log b_0 + x \cdot \log b_1$$

$$S_e = \sqrt{\frac{\sum (\log y_i - \log \hat{y}_i)^2}{n - k - 1}}$$

$$\log \hat{y} = \log b_0 + b_1 \cdot \log x$$

$$S_e = \sqrt{\frac{\sum (\log y_i - \log \hat{y}_i)^2}{n - k - 1}}$$

To interpret the results, antilog b_0 and b_1 should be calculated

To interpret the results, antilog b_0 should be calculated